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COMMENTS ON THE COHERENT AND INCOHERENT
NATURE OF A REVERBERANT SOUND FIELD

by W. T. Chu

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SOMMAIRE

L'auteur de cette note propose de mieux caractériser les champs de réverbération en les classifiant selon leur nature grâce aux expressions "champ cohérent" et "champ incohérent". La nature des champs de réverbération est vérifiée pour divers types de signaux sonores et diverses salles acoustiques pour déterminer les conditions dans lesquelles les résultats de la théorie statistique s'appliquent.

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Comments on the coherent and incoherent nature of a reverberant sound field

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It is proposed that reverberant sound fields should be classified with better-defined characteristics in terms of coherence and incoherence. Their exact nature created under different types of signal excitation and room condition are explored with the object of clarifying the conditions for which various results from the statistical theory apply.

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INTRODUCTION

There is little doubt that statistical room acoustics has made great advances since its inception in the mid-fifties, following early work by Schroeder.¹ It has yielded results for the cross-correlation function,² the interference patterns at the boundaries of the room,³ and the sampling distribution of the reverberant field.^{1,4,5} It appears, however, that the general association of the theory of statistical room acoustics with random reverberant sound fields, together with the different possible interpretations of the definition of a diffuse sound field,⁶ has created some misunderstanding in the practical application of the statistical theory. For example, in the current standards^{7,8} for sound power measurements in reverberation rooms the requirement for microphone spacing and the boundary interference corrections based on the results of Refs. 2 and 3 have been adopted indiscriminately for measurement of random noise sources as well as discrete frequency sources. The reverberant sound field created by the two types of sources are, however, rather different.

This paper attempts to classify the reverberant sound fields with a better defined characteristics in terms of coherence and incoherence and to clarify the conditions for which various results from the statistical theory apply.

I. DEFINITIONS

Morse and Bolt⁹ initially considered the direct field as "coherent" and the reverberant field as "incoherent." This definition, however, is too general for present use. Instead, the original definition from the classical wave theory will be adopted in this paper.

Consider two harmonic waves of the same frequency

$$\begin{aligned} p_1(x, t) &= \text{Re}\{A(x)e^{-i\omega t}\}, \\ p_2(x, t) &= \text{Re}\{B(x)e^{-i\omega t}\}, \end{aligned} \quad (1)$$

where $A(x)$ and $B(x)$ are complex quantities and $\text{Re}\{\}$ denotes "real part of." The total excitation at a point x is given by

$$p(x, t) = p_1 + p_2 = \text{Re}\{A(x) + B(x)\}e^{-i\omega t}.$$

It follows for the mean-square pressure that

$$\bar{p}_1^2(x) \sim |A(x)|^2,$$

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$$\bar{p}_2^2(x) \sim |B(x)|^2,$$

and

$$\bar{p}^2(x) \sim |A(x) + B(x)|^2 = [A(x) + B(x)][A^*(x) + B^*(x)],$$

where $*$ denotes complex conjugate and overbar defines time average. Therefore

$$\bar{p}^2(x) = \bar{p}_1^2(x) + \bar{p}_2^2(x) + 2[\bar{p}_1^2(x)\bar{p}_2^2(x)]^{1/2} \cos \delta. \quad (2)$$

δ represents the phase difference of the arguments of A and B . If this phase difference has a fixed value, the nonadditivity of the mean-square pressure is evident. The waves are said to be coherently related. If the phases of the original waves are random functions of time such that $\cos \delta$ averages to zero, they are then referred to as incoherently related.

Extending the above formulation to the study of spatial cross correlation at two points,

$$\begin{aligned} \overline{p(x_1)p(x_2)} &= \overline{p_1(x_1)p_1(x_2)} + \overline{p_2(x_1)p_2(x_2)} \\ &+ [\bar{p}_1^2(x_1)\bar{p}_2^2(x_2)]^{1/2} + [\bar{p}_1^2(x_2)\bar{p}_2^2(x_1)]^{1/2} \cos \delta. \end{aligned} \quad (3)$$

If the waves are incoherent, the last term will average to zero. In this case each wave contributes independently to the total cross correlation.

The above definitions of coherence and incoherence have to be modified slightly for the discussion of reverberant sound fields in an enclosure. In addition, there is need to discuss the behavior of the wave components that make up the individual modes. If contributions to the mean-square pressure or the spatial cross correlation from the different modes cannot be considered as independent, the field is labeled "coherent"; otherwise, it is "incoherent." If, in addition, the contributions from the wave components within each mode can be shown to be independent, the field is considered to be "incoherent in the strict sense."

The cross-product terms between modes or those in Eqs. (2) and (3) can sometimes be eliminated by some kind of spatial averaging or ensemble averaging. These special cases will be termed "equivalent incoherent."

II. EIGENMODE ANALYSIS OF STEADY-STATE SOUND

To illustrate the coherent or incoherent nature of the sound field in a room, the space-time cross-correla-

tion function will be used. As the sound field is not necessarily homogeneous in space, the proper normalization of the cross-correlation function by the local root-mean-square values is required. For simplicity, a rectangular room of sides L_x, L_y, L_z and volume $V = L_x L_y L_z$ was chosen. The sound field inside such a room can readily be analyzed by means of the eigenmode theory. The sound field at the point $\mathbf{r} = (x, y, z)$ due to a simple source $Qe^{-i\omega t}$ at $\mathbf{r}_0 = (x_0, y_0, z_0)$ has been written as¹⁰

$$p = \frac{i\rho\omega Q}{V} e^{-i\omega t} \sum_N \frac{\psi_N(\mathbf{r})\psi_N(\mathbf{r}_0)}{\Lambda_N(k^2 - k_N^2 + 2ik\delta_N)}. \quad (4)$$

Where the origin of coordinates is at one corner of the chamber, N stands for the trio of numbers l, m, n , and

$$\Lambda_N = 1/\epsilon_l \epsilon_m \epsilon_n, \quad \epsilon_0 = 1, \quad \text{and } \epsilon_i = 2, \quad \text{for } i \neq 0,$$

$$\psi_N(x, y, z) = \cos k_x x \cos k_y y \cos k_z z,$$

$$k_x = \frac{l\pi}{L_x}, \quad k_y = \frac{m\pi}{L_y}, \quad k_z = \frac{n\pi}{L_z}, \quad k_N^2 = k_x^2 + k_y^2 + k_z^2,$$

$$\delta_N = \beta(\epsilon_l/L_x + \epsilon_m/L_y + \epsilon_n/L_z),$$

$k = \omega/c$ is the wavenumber, ρ is the density of the medium, and β is the specific normal admittance of the chamber surface assumed to be real and small. Equation (4) can be rewritten as

$$p(\mathbf{r}) = \frac{\rho\omega Q}{V} \sum_N \frac{e^{-i(\omega t - \tau/2 + \phi_N)} \psi_N(\mathbf{r}) \psi_N(\mathbf{r}_0)}{\Lambda_N[(k^2 - k_N^2)^2 + (2k\delta_N)^2]^{1/2}}, \quad (5)$$

where

$$\phi_N = \tan^{-1}[2k\delta_N/(k^2 - k_N^2)].$$

The space-time cross-correlation function of the acoustic pressure at two points $\mathbf{r}_1$ and $\mathbf{r}_2$ is given by

$$\overline{p(\mathbf{r}_1)p(\mathbf{r}_2, \tau)} = \frac{1}{2} \text{Re}[p(\mathbf{r}_1)p^*(\mathbf{r}_2, \tau)].$$

Thus, using Eq. (5),

$$\overline{p(\mathbf{r}_1)p(\mathbf{r}_2, \tau)} = \frac{1}{2} \left(\frac{\rho\omega Q}{V} \right)^2 \sum_N \sum_M \frac{\psi_N(\mathbf{r}_1)\psi_M(\mathbf{r}_2)\psi_N(\mathbf{r}_0)\psi_M(\mathbf{r}_0)\cos(\phi_M - \phi_N + \omega\tau)}{\Lambda_N\Lambda_M[(k^2 - k_N^2)^2 + (2k\delta_N)^2]^{1/2}[(k^2 - k_M^2)^2 + (2k\delta_M)^2]^{1/2}}. \quad (6)$$

Define

$$B_N = \left(\frac{\rho\omega Q}{V} \right) \frac{\psi_N(\mathbf{r}_1)\psi_N(\mathbf{r}_0)}{\Lambda_N[(k^2 - k_N^2)^2 + (2k\delta_N)^2]^{1/2}}, \quad C_M = \left(\frac{\rho\omega Q}{V} \right) \frac{\psi_M(\mathbf{r}_2)\psi_M(\mathbf{r}_0)}{\Lambda_M[(k^2 - k_M^2)^2 + (2k\delta_M)^2]^{1/2}}. \quad (7)$$

Equation (6) can be written, with the cosine term expanded, as

$$\begin{aligned} \overline{p(\mathbf{r}_1)p(\mathbf{r}_2, \tau)} = & \frac{1}{2} \left(\sum_N B_N \cos \phi_N \sum_M C_M \cos \phi_M + \sum_N B_N \sin \phi_N \sum_M C_M \sin \phi_M \right) \cos \omega\tau \\ & - \frac{1}{2} \left(\sum_N B_N \cos \phi_N \sum_M C_M \sin \phi_M - \sum_N B_N \sin \phi_N \sum_M C_M \cos \phi_M \right) \sin \omega\tau. \end{aligned} \quad (8)$$

In addition, if

$$\begin{aligned} \tan \eta &= \frac{\sum_N B_N \sin \phi_N}{\sum_N B_N \cos \phi_N}, \\ \tan \alpha &= \frac{\sum_M C_M \sin \phi_M}{\sum_M C_M \cos \phi_M}, \\ D &= \left[\left(\sum_N B_N \sin \phi_N \right)^2 + \left(\sum_N B_N \cos \phi_N \right)^2 \right]^{1/2}, \\ E &= \left[\left(\sum_M C_M \sin \phi_M \right)^2 + \left(\sum_M C_M \cos \phi_M \right)^2 \right]^{1/2}, \end{aligned} \quad (9)$$

Eq. (8) can be written as

$$\begin{aligned} \overline{p(\mathbf{r}_1)p(\mathbf{r}_2, \tau)} &= \frac{1}{2} DE [(\cos \eta \cos \alpha + \sin \eta \sin \alpha) \cos \omega\tau \\ &\quad - (\cos \eta \sin \alpha - \sin \eta \cos \alpha) \sin \omega\tau], \\ &= \frac{1}{2} DE \cos(\eta - \alpha - \omega\tau). \end{aligned}$$

As

$$\begin{aligned} \bar{p}^2(\mathbf{r}_1) &= \frac{1}{2} D^2, \\ \bar{p}^2(\mathbf{r}_2) &= \frac{1}{2} E^2, \end{aligned}$$

the normalized cross-correlation coefficient is

$$\begin{aligned} R &= \overline{p(\mathbf{r}_1)p(\mathbf{r}_2, \tau)} / [\bar{p}^2(\mathbf{r}_1)\bar{p}^2(\mathbf{r}_2)]^{1/2}, \\ &= \cos(\eta - \alpha - \omega\tau). \end{aligned} \quad (10)$$

Equation (10) shows that R varies between ± 1 , corresponding to a coherent field. This is in agreement with the definition set out in Sec. I, since contributions from different modes have not been considered to be independent here. Thus the sound field created under pure-tone excitation is definitely coherent.

On the other hand, if the field were incoherent, the cross-correlation coefficient would have a different form. To illustrate this point, an equivalent incoherent field obtained by averaging the previous result over many source positions will be used. The cross-product terms between modes were eliminated by the orthogonality of the characteristic functions associated with the source.¹⁰ Thus

$$[\overline{p(\mathbf{r}_1)p(\mathbf{r}_2)}]_s = \frac{1}{2} \left(\frac{\rho\omega Q}{V} \right)^2 \sum_N \frac{\psi_N(\mathbf{r}_1)\psi_N(\mathbf{r}_2)\cos \omega\tau}{\Lambda_N[(k^2 - k_N^2)^2 + (2k\delta_N)^2]}, \quad (11)$$

where $[\ ]_s$ signifies averaging over all possible source positions in the room. Chien and Soroka¹¹ have shown

that for high frequencies, by replacing the summation by integration, Eq. (11) can be written as

$$[\rho(r_1)\rho(r_2)]_s$$

$$= \frac{1}{2} \left(\frac{\rho\omega Q}{V} \right)^2 \left(\frac{V^2}{8\pi\beta S} \right) \left(\frac{\sin kr}{kr} + \text{other terms} \right) \cos\omega\tau, \quad (12)$$

where S is the surface area of the chamber. Chien and Soroka heuristically assumed the "other terms" to be negligible. A better argument for dropping them, together with observations concerning these terms, will be given in Appendix A. As indicated there, the "other terms" can be ignored in the central region of the room where the equivalent incoherent field is nearly homogeneous. Thus the cross-correlation coefficient becomes

$$[R]_s = (\sin kr / kr) \cos\omega\tau. \quad (13)$$

In contrast with Eq. (10), $[R]_s$ varies between $\pm \sin kr / kr$, a magnitude that depends on k and r . Chien and Soroka¹¹ derived a result similar to Eq. (10) for $\tau = 0$ by carrying out the actual summation over the modes. Their analysis was for the nearfield of the source. The present experimental results, however, indicate that it is also true in the reverberant field.

It is worth noting that although the analysis is for a rectangular room, the result should be applicable to nonrectangular rooms or rooms with fixed diffusers. For these cases the only difference is that D , E , η , and α of Eq. (9) become unknown functions of the room characteristics.

III. EXPERIMENTAL RESULTS

The experiment was carried out in one of the reverberation chambers at DBR/NRC. It is equipped with fixed diffusers and a rotating vane. For this particular experiment the rotating vane was not in operation and the room was excited with pure tone using a single loudspeaker at the corner. Space-time correlation coefficients were obtained for six pairs of microphones scattered throughout the middle of the room. Their space separations ranged from 1.65 to 3.95 m. The nearest microphone was 3 m from the source, a distance greater than the nearfield criteria set out in Ref. 11. Figure 1 shows that the correlation coefficients are in agreement with Eq. (10) rather than with Eq. (13). The reverberant field is definitely coherent. At 1.6-kHz excitation frequency the modal overlap for the room is about 90. Similar results are obtained for 1 kHz.

To provide further experimental evidence about the nature of the reverberant field, spatial cross-correlation coefficients between a fixed pair of microphones have been obtained under various conditions. In these experiments the two $\frac{1}{2}$ -in. B & K microphones were fixed 10 cm apart, located near the center of the room. The separation vector between them was not parallel to any boundary of the room and different frequencies were used to generate various kr values. Three room and signal conditions were investigated.

(1) The room was excited with a pure tone using a

FIG. 1. Space-time correlation coefficients of six pairs of microphones in a reverberation chamber equipped with fixed diffusers. Single frequency excitation at 1.6 kHz.

single loudspeaker at the corner and the rotating vane not in operation.

(2) The test was repeated for 12 fixed orientations of the rotating vane and the results from the 12 tests were averaged to simulate the effect of averaging over source positions. The assumption is that modifying the room geometry and thus the arrangement of the image sources is equivalent to changing the source position.

(3) Third-octave band random noise was used to excite the room under the first condition. Some preliminary results are presented in Figs. 2 and 3, together with the theoretical prediction ($\sin kr / kr$) for an incoherent field.

Although these experimental results appear to be rather limited, they are sufficient to indicate the different nature of the reverberant sound field. Results represented by the open circles in Fig. 2 support the conclusion drawn from Fig. 1 that the sound field created by the pure-tone excitation is coherent. An equivalent incoherent field can be approached, however, if averaging over source positions is taken as indicated by the results depicted as solid triangles in Fig. 2. When the exciting signal is band-limited random noise, the sound field is definitely incoherent, as shown by the results presented in Fig. 3. Discrepancy at large values of kr is probably caused by diffraction effects (as explained by Cook *et al.*³) that have not been corrected in the present case. Better agreement with theory was obtained by Balachandran¹² in the DBR facility some twenty years ago.

FIG. 2. Spatial correlation coefficient between a pair of fixed microphones under different conditions. —○— pure-tone excitation, rotating vane not in operation, ▼ pure-tone excitation, averaged result over twelve fixed rotating vane orientations.

FIG. 3. Spatial correlation coefficient between a pair of fixed microphones under $\frac{1}{3}$ -octave-band noise excitation, rotating vane not in operation.

IV. DISCUSSION

It has been established that the reverberant field created under pure-tone excitation is coherent. Recent studies, experimental¹³ and computer simulation,¹⁴ have confirmed the sampling distribution derived by Schroeder¹ and Waterhouse⁴ for such a coherent field in rooms with a certain finite boundary absorption. Their assumptions that the modes or wave components were statistically independent should not, however, be taken to imply an incoherent field. It is interesting that the mean-square pressure can be written as

$$\bar{p}^2(\mathbf{r}) = \frac{1}{2} D^2 = \frac{1}{2} \left[\left(\sum_N B_N \sin \phi_N \right)^2 + \left(\sum_N B_N \cos \phi_N \right)^2 \right],$$

from the present eigenmode analysis of Sec. II. This formulation is similar to those used by both Schroeder and Waterhouse. The reconciliation of the two models for the sampling distributions will be the subject of another study.

When the room is excited by band-limited random noise, the reverberant field will become incoherent because a large number of modes will be excited and resonant at their own characteristic frequencies. As harmonic waves at different frequencies are uncorrelated, the modes become noninterfering. Their contribution to both mean-square pressure and spatial correlation can be considered independent. This argument is supported by the more quantitative analysis given in Ref. 10. In addition, it is shown in Appendix A that in the central region of the room most of the coherent contributions from the wave components of the individual independent modes are also very small and can be neglected. This region of the sound field can be called "incoherent in the strict sense"; only here will Cook's theory of spatial cross correlation² be applicable. In spaces close to the boundaries these coherent contributions from the wave components cannot be ignored. In fact, they are responsible for the interference pattern and build up of the mean-square pressure near the boundaries described by Waterhouse.³

To explain this situation using the free-wave model, consider the free waves coming from different directions as being generated by the images of the real source in the room. When the source emits band-limited random noise, it will have a finite coherence time (correlation time length) that is inversely proportional to the bandwidth. Thus the free-wave packets coming from different image sources will be incoherent when the time delays between them, in reaching a certain region inside the room, are longer than the coherence time of the real source. This will be true for the majority of the wave packets crisscrossing the central region of the room, but not for the space close to boundaries. Here, because of the symmetrical disposition of the images with respect to these surfaces, many of the differences in time delay become small or zero. On the other hand, if the source emits only a single frequency, all the waves coming from the image sources will be coherent in the sense that they are only shifted in phase and will still interfere with each other. It is therefore not possible to neglect cross terms be-

tween any two of them in forming spatial cross-correlations or interference patterns near the boundaries. If the bandwidth of the random noise used is very narrow, a partially coherent situation will result. This particular case has not been discussed.

As indicated here, however, and in Ref. 10, an equivalent incoherent condition for even a coherent field can be approached by applying some kind of averaging technique over frequency or space. Averaging over source positions has been chosen for illustration because of its practical application, but another spatial averaging technique has been used in Ref. 10.

So far, the word "random" has been avoided in describing the sound field because of its ambiguity. For example, the term "random sound field" used in Ref. 2 is to be interpreted as a "field containing many uncorrelated plane waves," whereas the term "random phase" used in Ref. 4 in describing the plane waves should be interpreted as "the phase relations of the plane waves constitute a *fixed* set of random variables and they are not uncorrelated." One must be very specific when using the word "random" to describe the reverberant sound field.

$$\begin{aligned} \psi_N(\mathbf{r}_1)\psi_N(\mathbf{r}_2) = & \frac{1}{8} [C_x(D_x)C_y(D_y)C_z(D_z) + C_x(S_x)C_y(D_y)C_z(D_z) + C_x(D_x)C_y(S_y)C_z(D_z) + C_x(D_x)C_y(D_y)C_z(S_z) \\ & + C_x(S_x)C_y(S_y)C_z(D_z) + C_x(S_x)C_y(D_y)C_z(S_z) + C_x(D_x)C_y(S_y)C_z(S_z) + C_x(S_x)C_y(S_y)C_z(S_z)]. \end{aligned} \quad (\text{A2})$$

Chien and Soroka¹¹ assumed that the first term is the dominant term, following an argument presented originally by Morrow.¹⁵ A more quantitative assessment of the importance of these terms will be given here. A better understanding of the sound field can be obtained by carefully studying the other terms. Rewrite the cross correlation as a function of the separation distance, r , between the two points and the position vector, $\mathbf{r}_c$, of the midpoint between them. Referring to Fig. A1,

$$\begin{aligned} x_1 &= x_c + (r/2)\sin\beta\cos\gamma, & x_2 &= x_c - (r/2)\sin\beta\cos\gamma, \\ y_1 &= y_c - (r/2)\sin\beta\sin\gamma, & y_2 &= y_c + (r/2)\sin\beta\sin\gamma, \quad (\text{A3}) \\ z_1 &= z_c - (r/2)\cos\beta, & z_2 &= z_c + (r/2)\cos\beta. \end{aligned}$$

Changing the wavenumber space from rectangular to spherical-coordinate system it follows

$$\begin{aligned} k_x &= k_N \sin\theta \cos\phi, \\ k_y &= k_N \sin\theta \sin\phi, \\ k_z &= k_N \cos\theta. \end{aligned}$$

Thus the first term of Eq. (A2) becomes

$$\begin{aligned} C_x(D_x)C_y(D_y)C_z(D_z) &= \cos(k_N r \sin\theta \cos\phi \sin\beta \cos\gamma) \\ &\times \cos(k_N r \sin\theta \sin\phi \sin\beta \sin\gamma) \\ &\times \cos(k_N r \cos\theta \cos\beta), \end{aligned} \quad (\text{A4})$$

which is the same as that used by Chien and Soroka in obtaining the $\sin kr/kr$ term in Eq. (12) of this paper. Similarly, expanding the second term of Eq. (A2), gives

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APPENDIX A:

Let the two position vectors be $\mathbf{r}_1 = (x_1, y_1, z_1)$ and $\mathbf{r}_2 = (x_2, y_2, z_2)$. According to definition

$$\begin{aligned} \psi_N(\mathbf{r}_1)\psi_N(\mathbf{r}_2) &= (\cos k_x x_1 \cos k_y y_1 \cos k_z z_1) \\ &\times (\cos k_x x_2 \cos k_y y_2 \cos k_z z_2). \end{aligned} \quad (\text{A1})$$

Using the trigonometric relation

$$\cos k_x x_1 \cos k_x x_2 = \frac{1}{2} \cos[k_x(x_1 - x_2)] + \frac{1}{2} \cos[k_x(x_1 + x_2)],$$

and the definitions

$$\begin{aligned} C_x(D_x) &= \cos k_x(x_1 - x_2), \\ C_x(S_x) &= \cos k_x(x_1 + x_2), \\ C_y(D_y) &= \cos k_y(y_1 - y_2), \text{ etc.}, \end{aligned}$$

Eq. (A1) can be rewritten as

$$\begin{aligned} C_x(S_x)C_y(D_y)C_z(D_z) &= \cos(2k_N x_c \sin\theta \sin\phi) \\ &\times \cos(k_N r \sin\theta \sin\phi \sin\beta \sin\gamma) \\ &\times \cos(k_N r \cos\theta \cos\beta). \end{aligned} \quad (\text{A5})$$

When the summation over all possible modes is carried out by means of integration over the wavenumber space for high frequencies, as for the procedure used in Ref.

FIG. A1. Coordinate system.

10, the following expression is obtained

$$j_0\{k[4x_c^2 + (r \sin\beta \sin\gamma)^2 + (r \cos\beta)^2]^{1/2}\}, \quad (\text{A6})$$

where $j_0(A) = \sin A/A$ for the contribution of the second

term to the total cross-correlation function. A similar derivation can be performed for the other terms. Thus the seven expressions that constitute the "other terms" in Eq. (12) are

$$\begin{aligned} j_0\{k[4x_c^2 + (r \sin\beta \sin\gamma)^2 + (r \cos\beta)^2]^{1/2}\} + j_0\{k[(r \sin\beta \cos\gamma)^2 + 4y_c^2 + (r \cos\beta)^2]^{1/2}\} \\ + j_0\{k[(r \sin\beta \cos\gamma)^2 + (r \sin\beta \sin\gamma)^2 + 4z_c^2]^{1/2}\} + j_0\{k[4x_c^2 + 4y_c^2 + (r \cos\beta)^2]^{1/2}\} \\ + j_0\{k[4x_c^2 + 4z_c^2 + (r \sin\beta \sin\gamma)^2]^{1/2}\} + j_0\{k[4y_c^2 + 4z_c^2 + (r \sin\beta \cos\gamma)^2]^{1/2}\} + j_0[2k(x_c^2 + y_c^2 + z_c^2)^{1/2}]. \end{aligned} \quad (\text{A7})$$

One may observe that the "other terms" become negligible when the cross correlation is considered in the central portion of the room, but not close to the boundaries. In fact, these terms are responsible for the interference pattern near the boundaries, as derived in Ref. 10. The same result can be obtained by putting $r = 0$ in Eq. (A7) and in the $\sin kr/kr$ term.

Another observation may be made by decomposing the modes into their wave components. As was demonstrated by Lubman,¹⁶ an oblique mode can be written as the summation of eight traveling waves. That is

$$\begin{aligned} \psi_N(\mathbf{r})e^{i\omega t} &= \cos k_x x \cos k_y y \cos k_z z e^{i\omega t} \\ &= \frac{1}{8} \{ \exp[i(k_x x + k_y y + k_z z)] + \exp[-i(k_x x + k_y y + k_z z)] + \exp[i(k_x x - k_y y - k_z z)] + \exp[-i(k_x x - k_y y - k_z z)] \\ &\quad + \exp[i(k_x x + k_y y - k_z z)] + \exp[-i(k_x x + k_y y - k_z z)] + \exp[i(k_x x - k_y y + k_z z)] + \exp[-i(k_x x - k_y y + k_z z)] \} e^{i\omega t}. \end{aligned} \quad (\text{A8})$$

The spatial cross correlation due to the first wave component can be written as

$$\overline{p_1(\mathbf{r}_1)p_1(\mathbf{r}_2)} = \{ \cos[k_x(x_1 - x_2) + k_y(y_1 - y_2) + k_z(z_1 - z_2)] \} / 128.$$

If the eight wave components are considered as incoherent, so that their contribution to the cross correlation can be summed independently,

$$\begin{aligned} \overline{p(\mathbf{r}_1)p(\mathbf{r}_2)} &= \{ 2 \cos[k_x(x_1 - x_2) + k_y(y_1 - y_2) + k_z(z_1 - z_2)] + 2 \cos[k_x(x_1 - x_2) - k_y(y_1 - y_2) - k_z(z_1 - z_2)] \\ &\quad + 2 \cos[k_x(x_1 - x_2) + k_y(y_1 - y_2) - k_z(z_1 - z_2)] + 2 \cos[k_x(x_1 - x_2) - k_y(y_1 - y_2) + k_z(z_1 - z_2)] \} / 128, \\ &= [\cos k_x(x_1 - x_2) \cos k_y(y_1 - y_2) \cos k_z(z_1 - z_2)] / 16. \end{aligned}$$

This is the first term of Eq. (A2) that gives rise to $\sin kr/kr$ except for a factor of $\frac{1}{2}$ which comes from the additional time averaging applied to Eq. (A8). The "other terms" then must be due to the coherent interference of the wave components within each individual mode. The interesting fact is that in an incoherent reverberant sound field there is a region where the wave components of each individual mode can also be treated as independent or incoherent. This region is "incoherent in the strict sense."

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