NRC Publications Archive Archives des publications du CNRC

Water distribution network renewal planning

Kleiner, Y.; Adams, B. J.; Rogers, J. S.

This publication could be one of several versions: author's original, accepted manuscript or the publisher's version. / La version de cette publication peut être l'une des suivantes : la version prépublication de l'auteur, la version acceptée du manuscrit ou la version de l'éditeur.

For the publisher's version, please access the DOI link below. / Pour consulter la version de l'éditeur, utilisez le lien DOI ci-dessous.

Publisher's version / Version de l'éditeur:

[https://doi.org/10.1061/\(ASCE\)0887-3801\(2001\)15:1\(15\)](https://doi.org/10.1061/(ASCE)0887-3801(2001)15:1(15))

Journal of Computing in Civil Engineering, 15, January 1, pp. 15-26, 2001-01-01

NRC Publications Record / Notice d'Archives des publications de CNRC:

<https://nrc-publications.canada.ca/eng/view/object/?id=ef0230cc-55d9-45f3-b096-a286eed0abf>

<https://publications-cnrc.canada.ca/fra/voir/objet/?id=ef0230cc-55d9-45f3-b096-a286eed0abf>

Access and use of this website and the material on it are subject to the Terms and Conditions set forth at

<https://nrc-publications.canada.ca/eng/copyright>

READ THESE TERMS AND CONDITIONS CAREFULLY BEFORE USING THIS WEBSITE.

L'accès à ce site Web et l'utilisation de son contenu sont assujettis aux conditions présentées dans le site

<https://publications-cnrc.canada.ca/fra/droits>

LISEZ CES CONDITIONS ATTENTIVEMENT AVANT D'UTILISER CE SITE WEB.

Questions? Contact the NRC Publications Archive team at

PublicationsArchive-ArchivesPublications@nrc-cnrc.gc.ca. If you wish to email the authors directly, please see the first page of the publication for their contact information.

Vous avez des questions? Nous pouvons vous aider. Pour communiquer directement avec un auteur, consultez la première page de la revue dans laquelle son article a été publié afin de trouver ses coordonnées. Si vous n'arrivez pas à les repérer, communiquez avec nous à PublicationsArchive-ArchivesPublications@nrc-cnrc.gc.ca.

National Research
Council Canada

Conseil national
de recherches Canada

NRC - CNRC

Water distribution network renewal planning

Kleiner, Y. ; Adams, B. J. ; Rogers, J. S.

NRCC-43369

A version of this paper is published in / Une version de ce document se trouve dans :
Journal of Computing in Civil Engineering, v. 15, no. 1, January 2001, pp. 15-26

www.nrc.ca/irc/ircpubs

Water Distribution Network Renewal Planning

By Y. Kleiner, Member, ASCE¹ B.J. Adams, Member, ASCE² and J.S. Rogers³

Published in ASCE Journal of Computing in Civil Engineering, Vol. 15, No. 1, pp. 15-26, 2001

Abstract

This paper provides an overview of the authors' previous work in formulating a comprehensive approach to the important problem of water distribution network renewal planning, with a particular emphasis on the computing aspects involved. As pipes in a water distribution network age in service, they are characterized by increased frequency of breakage and decreased hydraulic capacity. The resulting service failures incur utility costs for the repair or rehabilitation of the pipe systems and consumer costs for degraded system performance. The challenge to the decision maker is to determine the most cost-effective plan in terms of what pipes in the network to rehabilitate, by which rehabilitation alternative and at what time in the planning horizon, subject to the constraints of service requirements (system reliability, service pressure, etc.) A dynamic programming approach, combined with partial and implicit enumeration schemes, was used to search the vast combinatorial solution space that this problem presents. A computer program was written to implement these concepts. A hydraulic network solver is used by the program to assure the network conformance to hydraulic constraints during the search for a solution. The outcome is a strategy that identifies, for each pipe in the network, the optimal rehabilitation/renewal alternative and its optimal time of implementation. The significance of this method is in its ability to identify an optimal rehabilitation strategy while considering the deterioration of both structural integrity and hydraulic capacity of the entire network. The best current heuristic method is limited in practical studies to a network of up to 15-20 pipe links. A more efficient heuristic method is required for implementing these principles in a larger-scale water distribution system and is the subject of current research.

Key words: water distribution system, water main rehabilitation, water main renewal, long-term planning, numerical optimization, large scale systems analysis

¹ Research Officer, Institute for Research in Construction, National Research Council of Canada, Ottawa, Canada, K1A 0R6

² Professor & Chair, Department of Civil Engineering University of Toronto, Toronto, Ontario, Canada M5S 1A4

³ Professor, Dept of Mechanical and Industrial Engineering, University of Toronto, Toronto, Ontario, Canada, M5S 3G8

Introduction

Water supply systems are the lifeline of urban and rural communities. The water distribution network, which is typically the most expensive component of a water supply system, is continuously subject to environmental and operational stresses which lead to its deterioration. Increased operation and maintenance costs, water losses, reduction in the quality of service and reduction in the quality of water are typical outcomes of this deterioration. A recent USEPA survey estimated the cost requirements for upgrading water distribution and transmission systems in the United States at US \$77 billion over the next 20 years (Davies et al., 1997). In Canada, CWWA (Canadian Water and Wastewater Association) estimates that CAN \$11.5 billion will be required for watermain upgrading over the next 15 years (CWWA, 1997).

There is little doubt that costs of such magnitude warrant careful planning and great diligence in considering all the alternative courses of action, while addressing the issues of safety, reliability, quality and efficiency in water supply. The state of practice in the long-term planning of water main renewal is still evolving. Although an all-encompassing, quantitative decision model is yet to be developed, the approach described in this paper considers the structural and hydraulic state of the network, while providing a framework for the future inclusion of other considerations (reliability, water quality) as well.

Pipe Deterioration and failure. The deterioration of pipes may be classified into structural deterioration which diminishes their structural resiliency and their ability to withstand various types of stress, and internal deterioration resulting in diminishing hydraulic capacity, degradation of water quality and even diminishing structural resiliency in cases of severe internal corrosion. Structural deterioration mechanisms are affected by many factors, including the type of pipe, its surrounding environment and its operational conditions. The deterioration processes as well as pipe structural failure modes are therefore very complex and difficult to model.

Although significant work has been done in modeling the physical process of pipe deterioration and failure (e.g., Doleac et al., 1980; Bahmanyar and Edil, 1983; Kumar et al., 1984; Ahamed and Melchers, 1994; Rajani et al., 1996; and others), the complex processes, lack of pertinent data and

highly variable environmental conditions posed severe challenges to these research efforts and a comprehensive model is yet to be developed. Consequently, much effort has been dedicated to using statistical methods for explaining and predicting pipe breakage rates. Some of these effects are discussed in the following.

Statistical methods to predict water main breaks. Shamir and Howard (1979), Walski and Pelliccia (1982) and Clark et al. (1982) observed exponential increases in watermain breakage rates over time (the latter also observed a time lag between main installation and first breakage). Kettler and Goulter (1985) observed a linear relationship between pipe breaks and age. Andreou et al. (1987) proposed a “proportional hazards” model to predict failure probabilities of pipes in the early stages of deterioration and a Poisson-type model for the later stages of pipe deterioration. Goulter and Kazemi (1988) and Goulter et al. (1993) observed temporal and spatial clustering of water main failures, which they modeled by a non-homogeneous Poisson distribution. Constantine and Darroch (1993) and Constantine et al. (1996) proposed a time-dependent Poisson distribution, in which the cumulative number of breaks in the pipe is a power function of the pipe age. Herz (1996) developed a new probability distribution function to be applied in a cohort survival model to an entire stock of pipes in a distribution system. Deb et al. (1998) applied the Herz (1996) model to several distribution systems.

Water Distribution Network Rehabilitation. Since the performance of the distribution system depends on the performance of every single pipe, the decision on pipe rehabilitation or renewal should consider the individual pipe in the context of the network performance. Several models have been developed for this purpose. Woodburn et al. (1987) combined a nonlinear programming procedure with a hydraulic simulation program in a model designed to determine which pipes should be replaced, rehabilitated or left alone in order to minimize cost. Su and Mays (1988) introduced some probabilistic considerations to the Woodburn et al. model. Male et al. (1990) addressed the structural deterioration of watermains over time but did not consider the hydraulic capacity of the distribution system. Kim and Mays (1994) proposed a branch and bound scheme to improve the Su and Mays (1988) model. Arulraj and Suresh (1995) introduced the concept of significance index (SI) which is an optimality criterion that can be applied heuristically to prioritize pipe rehabilitation. Schneider et al. (1996) introduced the capacity reliability concept, which is the probability that the carrying capacity of a network meets the flow demand.

The time dimension is a crucial aspect of an effective pipe rehabilitation strategy because of ongoing (and often independent) deterioration mechanisms that affect pipe structural integrity and hydraulic capacity. For instance, hydraulic analysis of a water distribution system may show that the hydraulic capacity of a pipe should be improved, and relining the pipe may be found to be the least cost solution. However, if deterioration in the structural integrity of the pipe is considered, it may be economical to replace this pipe in say, five or ten years due to the costs associated with high breakage frequency. Thus, it may be more economical to replace the pipe at the appropriate time in the future while in the meantime the hydraulic capacity of the network is improved by rehabilitating another pipe. This scenario may be further complicated if a future increase in demand is expected, in which case it may be more economical to replace the pipe with a larger diameter pipe in the future.

Kleiner et al. (1998a, 1998b) proposed an approach in which the network economics and hydraulic capacity is analyzed simultaneously over a pre-defined analysis period, while explicitly considering the deterioration over time of both the structural integrity and the hydraulic capacity of every pipe in the system. A brief description of this approach is provided here with a special emphasis on the computer program created for the application of this approach, including description of the data structure, program flow chart and speed of execution. Comments are provided about the application of the approach to large systems.

Life-cycle of a Single Pipe

Pipe replacement. In a water distribution network with p pipes (links) and n nodes, every pipe in the network may be rehabilitated by one of R rehabilitation alternatives which may be implemented any number of times, at any year from the present to planning horizon H . Pipe relining is considered as a hydraulic capacity improvement measure, while pipe replacement (with the same or a larger diameter pipe) improves both the structural integrity and hydraulic capacity of the link. If an exponential increase in pipe breakage rate is assumed, after Shamir and Howard (1979), and the pipe is replaced at year T , with replacement alternative j , then the present value of all the costs associated with the pipe from the present to year T is denoted by

$$C(T_{ij}) = L_i C_{r_{ij}} \cdot e^{-rT_{ij}} + \int_0^{T_{ij}} L_i \cdot C_{b_i} \cdot N(t_0)_i \cdot e^{A_i(t+g_i)} \cdot e^{-rt} dt \quad (1)$$

where C_{bi} = cost of a single breakage repair in existing pipe i (\$)
 C_{rij} = the cost to replace pipe i with replacement alternative j (\$/km)
 T_{ij} = the year at which pipe i is replaced with replacement alternative j
 L_i = length of pipe i (km)
 $N(t)_i$ = number of breaks per unit length per year in existing pipe i ($\text{km}^{-1} \text{ year}^{-1}$)
 $N(t_0)_i$ = $N(t)_i$ at the year of installation of pipe i (i.e., when the pipe is new)
 A_i = coefficient of breakage rate growth in existing pipe i (year^{-1})
 g_i = age (years) of existing pipe i at present
 r = equivalent continuous discount rate

After the pipe is initially replaced at time T^f , (see Figure 1), the new pipe will have a low breakage rate, which will grow as this replacement pipe ages until it too is replaced, and so on in perpetuity. Assuming that all future replacement cycles are identical with duration T^c , as is illustrated in Figure 1, it can be shown that the total cost of all replacement cycles of the pipe to infinity,

Figure 1. Stream of costs (steady state) associated with a single pipe

discounted to the time of first replacement, is given by

$$C_{inf}(T_{ij}^c) = \frac{L_i C_{rij} \cdot e^{-rT_{ij}^c} + \int_0^{T_{ij}^c} L_i \cdot C_{bij} \cdot N(t_0)_{ij} \cdot e^{A_{ij}(t-r)} dt}{(1 - e^{-rT_{ij}^c})} \quad (2)$$

where C_{bij} = cost of a single breakage repair in replacement pipe ij (\$)
 T_j^c = the duration of a replacement cycle of pipe i with alternative j (years)
 $N(t)_{ij}$ = number of breaks per unit length per year in replacement pipe ij ($\text{km}^{-1} \text{ year}^{-1}$)
 $N(t_0)_i$ = $N(t)_i$ at the year of installation of pipe ij (i.e., when the pipe is new)
 A_{ij} = coefficient of breakage rate growth in replacement pipe ij (year^{-1})
 r = equivalent continuous discount rate

Equating to zero the derivative of (2) with respect to T_j^c , and solving for T_j^c , one can find numerically the cycle duration T_j^{**} that minimizes the total cost of pipe replacement cycles to infinity. This requires that $r \neq 0$ and $r \neq A_{ij}$. The present value of the total cost associated with a pipe from the present to infinity is thus

$$C_{ij}^{tot}(T_{ij}^f) = C_{r_{ij}} \cdot e^{-rT_{ij}^f} + \int_0^{T_{ij}^f} C_{b_{ij}} \cdot N(t_0)_i e^{A_i(t+g_i)} e^{-rt} dt + C_{inf}(T_{ij}^{**}) e^{-rT_{ij}^f} \quad (3)$$

Equating to zero the derivative of (3) with respect to T_j^f , and solving for T_j^f , one can find the time of first replacement T_j^* that minimizes the present value of the total cost of pipe i replaced with alternative j from the present to infinity.

$$T_{ij}^* = -g_i + \frac{1}{A_i} \ln \left[r \frac{C_{r_{ij}} + C_{inf}(T_{ij}^{**})}{C_{b_{ij}} \cdot N(t_0)_i} \right] \equiv T_{ij}^* \quad (4)$$

The value T_j^* is also referred to as minimum cost replacement timing (MCRT) of pipe i and replacement alternative j . Graphically, the total cost function of a single pipe vs. year of first replacement may be represented as in Figure 2.

Figure 2. Total cost vs. replacement time.

Pipe Relining. As previously

mentioned, pipe relining assumes no structural improvement in the pipe, thus it may be economically feasible to reline a pipe whose hydraulic capacity deteriorates much faster than its structural integrity. As the maintenance costs of a relined pipe are assumed to continue rising with age, it is further assumed that the relined pipe will have to be replaced at some future time. Consequently, if

the hydraulic capacity of a pipe needs to be improved well before this time in the future, relining should be considered as a feasible alternative. The schematic in Figure 3 is used to illustrate this concept.

As a first approximation, it is assumed that if a pipe is relined it will subsequently be replaced by the same diameter pipe at time T^* and after that every T^{**} in perpetuity to infinity. Thus, if the hydraulic

Figure 3. The feasibility of relining

capacity of a pipe displays early deterioration and that the hydraulic capacity of the network must be improved no later than time T_l , the relining alternative is compared with an early replacement with same diameter pipe alternative, which hydraulically is roughly equivalent to relining. An early replacement would incur an additional cost $\Delta cost_{replace}$ above the cost of replacement at T^* , whereas relining at time T_l would incur an additional cost equal to the capital investment for relining CAP_{reline} . Subsequently, if $\Delta cost_{replace} > CAP_{reline}$, then the reline alternative is chosen and vice versa. Consequently, the total cost of the reline alternative is

$$C_{rel}^{tot}(T_{ij}, T_{ik}) = CAP_{reline} e^{-rT_{ij}} + C_{ik}^{tot}(T_{ik}^*) \quad (5)$$

where j is the relining alternative ($j \in R$) and k denotes the alternative of replacement with the same diameter pipe ($k \in R$).

Deterioration of Pipe Hydraulic Capacity

The head loss h (m) in any pipe i is calculated (following the Hazen-Williams equation) by

$$h_i = 10.653 \left(\frac{Q_i}{C_i^{HW}} \right)^{1.852} D_i^{-4.87} L_i \quad (6)$$

where Q_i = flow rate in pipe i (m^3/s)
 C_i^{HW} = Hazen Williams hydraulic conductivity coefficient of pipe i
 D_i = diameter of pipe i (m)
 L_i = length of pipe i (m)

The conductivity coefficient deteriorates with the age of the system. The rate of deterioration will vary according to the type of pipe, the quality of supply water and operation and maintenance practices. Sharp and Walski (1988) proposed a C^{HW} deterioration model, depicted in equation (7), for an existing pipe i before rehabilitation (replacement or relining). When applying the model to a rehabilitated pipe, index i is substituted with index ij to reflect the appropriate rehabilitation alternative, and t should be greater than T_{ij} .

$$C_i^{HW}(t) = 18.0 - 37.2 \log \left(\frac{e_{0i} + a_i(t + g_i)}{D_i} \right) \quad (7)$$

where e_{0i} = initial roughness in pipe i at the time of installation when it was new (m)
 $C_i^{HW}(t)$ = Hazen Williams hydraulic conductivity coefficient in pipe i
 t = time elapsed from present time to future periods ($t < T_{ij}$) (years)
 a_i = roughness growth rate in pipe i (m/yr)
 g_i = age of pipe i at the present time (time of analysis) (years)
 D_i = diameter of pipe i (m)

The method described here is not restricted to any one hydraulic deterioration model and, should a more suitable equation for predicting C^{HW} be developed, it could easily be accommodated.

Complete Problem Statement

Equation (8) provides the formal mathematical statement of the problem described to this point. The objective function is to minimize the total discounted costs associated with rehabilitation (initial relining and subsequent replacement cycles to infinity or initial replacement and subsequent replacement cycles to infinity) and breakage repair for all the pipes in the distribution system. The decision variables are the respective rehabilitation times of all the pipes with the various rehabilitation alternatives. The constraints are (1) conservation of mass (continuity equations) in the distribution network, (2) conservation of energy (where the friction headlosses in the pipes increase in time and decrease upon pipe rehabilitation) and, (3) minimum residual nodal pressure at the critical nodes.

$$\text{minimize : } \sum_{\substack{i=1 \\ j \in R}}^P L_i \cdot C_{r_{ij}} \cdot e^{-rT_{ij}} + C^M(T_{ij}) + C_{inf}(T_{ij}^{**})e^{-rT_{ij}} \quad (8)$$

where if j is a replacement alternative

$$C^M(T_{ij}) = \int_0^{T_{ij}} L_i \cdot C_{b_i} \cdot N(t_o)_i e^{A_i(t+g_i)-rt} dt$$

$$\text{and } C_{inf}(T_{ij}^{**}) = \sum_{m=0}^{\infty} \left[e^{-mrT_{ij}^{**}} \cdot L_i \left(C_{r_{ij}} \cdot e^{-rT_{ij}^{**}} + \int_0^{T_{ij}^{**}} C_{b_{ij}} \cdot N(t_o)_{ij} e^{(A_{ij}-r)t} dt \right) \right]$$

or if j is a relining alternative

$$C^M(T_{ij}) = \int_0^{T_{ik}^*} L_i \cdot C_{b_i} \cdot N(t_o)_i e^{A_i(t+g_i)-rt} dt \quad (\text{where } k \text{ denotes the alternative of replacement})$$

$$\text{and } C_{inf}(T_{ij}^{**})e^{-rT_{ij}} = L_i C_{r_{ik}} e^{T_{ik}^*} + C_{inf}(T_{ik}^{**})e^{-rT_{ik}^*} \quad \text{with the same diameter pipe}$$

subject to

$$(a) \quad Q_{in_y} = Q_{out_y} \quad y \in \{1, 2, \dots, n\} \quad \text{mass conservation in every node } y$$

$$(b) \quad h_1 = \text{constant} \quad \text{energy conservation, in every path } l$$

$$\text{where } h_1 = 10.653 \left(\frac{Q_{ij}}{C_{ij}^{HW}} \right)^{1.852} D_{ij}^{-4.87} L_i \quad i \in p, \quad j \in R \quad \text{friction headloss}$$

$$\text{and } C_{ij}^{HW}(t) = 18.0 - 37.2 \log \left(\frac{e_{o_{ij}} + a_{ij}(t - T_{ij})}{D_{ij}} \right) \quad \text{for a pipe } i \text{ after rehabilitation } j$$

$$\text{or } C_i^{HW}(t) = 18.0 - 37.2 \log \left(\frac{e_{o_i} + a_i(t + g_i)}{D_i} \right) \quad \text{for a pipe } i \text{ before rehabilitation}$$

$$\text{for } t \in \{1, 2, \dots, H\}$$

$$(c) \quad P_{yt} \geq P_{min_y} \quad y \in \{1, 2, \dots, n\}, \quad t \in \{1, 2, \dots, H\} \quad \text{minimum supply pressure in every node } y, \text{ during planning horizon } H$$

<u>Notation</u>	<u>Comments</u>
T_{ij}	- Decision variables - timing of rehabilitation of pipe i by alternative j .
$CAP(T_{ij})$	- Capital cost of implementing rehabilitation of pipe i by alternative j at time T_{ij} .
$C^M(T_{ij})$	- Cost of maintenance associated with pipe i and alternative j from the present to time T_{ij} .
$C_{inf}(T_{ij}^{**})$	- Total cost stream (capital + maintenance) of replacement cycles (with duration T^{**}) from T_{ij} to infinity.

Simultaneous Consideration of Economics and Hydraulics

The deterioration in the hydraulic capacity of pipes results in a reduction of the supply pressure, and the deterioration of the structural integrity causes increased breakage rates which result in increased pipe maintenance costs. The principles of jointly considering the network hydraulics and the individual pipe economics to determine the timing and sequencing of rehabilitation projects are best

Figure 4. A simple water distribution system

explained with the aid of a simple example.

Figure 4 illustrates a two-pipe water distribution system with one source node and one demand node. Assuming constant demand and supply pressure at the source node, the residual pressure at the demand node depends only on the hydraulic capacity of the “distribution network”. Since the

hydraulic capacity of the pipes diminishes over time, so does the residual pressure at the demand node. At some point in time, the diminished hydraulic capacity of the system will have to be increased to avoid the residual pressure at the demand node dropping below P_{min} , which is the minimum residual pressure required at the demand node to maintain adequate service. This can be achieved by (1) relining one or both pipes, (2) replacing one or both pipes with a new pipe of the same diameter, or (3) replace one or both pipes with larger diameter pipes. If the goal is to maintain adequate pressure at the demand node during a planning period of, say, 30 years, it may be necessary to implement rehabilitation projects for both pipes (possibly even more than once), depending on the deterioration rates of both the existing pipes and the rehabilitated pipes. This process is graphically illustrated in Figure 5. It should be emphasized that the hydraulic behavior of the network is determined simultaneously by all pipes and the interactions among them, because any change in the hydraulic properties of any pipe in the network causes a redistribution of flows in all pipes in the network.

Within the hydraulic framework described above, the actual timing of the rehabilitation project is determined using economic considerations as well. The total cost of a pipe comprises (a) the maintenance costs of the existing pipe until it is initially rehabilitated (including possible pipe relining), (b) the cost of the first replacement, and (c) the costs of maintenance and replacement in

subsequent replacement cycles to infinity, as is depicted in equation (3). It can be shown that under certain conditions this total cost function is convex with respect to the timing of the first

Figure 5. The hydraulic behavior of a water distribution network

replacement, and has a minimum cost point depicting a time at which, if the first replacement of a pipe is indeed implemented, the total cost of this pipe is minimized.

This Minimum Cost Replacement Timing (MCRT - denoted by T^* in equation 4) depends on the structural properties and the costs associated with the existing individual pipe and the replacement pipe; hence, every pipe in the network has a unique MCRT for every replacement alternative. For example, suppose that for pipe 1, two replacement alternatives are considered (e.g., same diameter and a larger diameter pipe), then each alternative has its own MCRT. Pipe relining is handled somewhat differently (when the pipe is initially relined, the relining cost is added to the total cost) as described later in this section. Consequently, in determining the timing of a replacement project there are two “motivations”: (a) implement it as late as possible (without violating hydraulic integrity) in order to delay the TMP (time of minimum pressure) as much as possible, and (b) implement the replacement as close as possible to the pertinent MCRT in order to reduce cost. This is demonstrated in Figure 6 for the two pipe system. In this numerical example, the MCRT of replacement alternative 2 (year 20) happens to be later than the previous TMP (year 15). This is not always the case as other situations may exist (e.g., if the MCRT is in year 14 or earlier). Detailed rules on how to determine replacement timing, when the MCRT of the next replacement alternative happens to be before the previous TMP, are provided in Kleiner et al. (1998a). The relining of a pipe is assumed to be hydraulically equivalent to replacing it with the same diameter pipe, although this is only an approximation. From a structural integrity viewpoint however, the relining alternative is different than replacement because it provides no reduction in breakage rate. It is consequently assumed that if a pipe is relined, it will eventually be replaced, and this time of (eventual) replacement is assumed to be the MCRT of the replacement with the same diameter alternative. From this point on, a series of replacement cycles to infinity is assumed. The details are discussed earlier and the total cost for this alternative is given in equation (5).

The approach described above was termed Procedure for Rehabilitation Analysis of Water Distribution Systems (PRAWDS). The use of an infinite series approach provides two distinct advantages. Firstly, each pipe in the network has a single, a priori determined cost curve for every rehabilitation alternative. This cost curve is independent of the planning horizon and thus does not require that the residual value of each pipe at the end of the planning horizon be quantified. Secondly, this approach automatically considers the possibility of more than one replacement for each pipe when the planning horizon is long and pipe deterioration is rapid, which would otherwise dramatically increase the dimensionality of the problem.

Figure 6. Example determination of replacement timing

Considering an Entire Water Distribution Network

The solution space to the cost minimization problem that is depicted by equation (8) is essentially a combinatorial space, which consists of all the combinations of pipes, rehabilitation alternatives and the sequencing thereof. In a network of p pipes, R rehabilitation alternatives and a planning horizon of H timesteps, the number of combinations is $(R+1)^{pH}$, which is vast even for a very small system. Kleiner et al. (1998b) proposed a methodology termed multistage PRAW (M-PRAW) to find a global minimum in this combinatorial solution space. M-PRAW is based on a dynamic programming approach combined with partial and (sometimes) implicit enumeration schemes. The principles of jointly considering the network hydraulics and the individual pipe economics to determine the timing and sequencing of rehabilitation projects were described in detail and can be summarized as follows:

- (a) The network hydraulics determines the latest time to implement the next rehabilitation project. It is the time at which, due to the hydraulic deterioration of the pipes, the hydraulic capacity of the network becomes inadequate, as illustrated in Figure 5.
- (b) The individual pipe economics determines the actual timing of the next rehabilitation project within the upper boundary set by the network hydraulics, as illustrated in Figure 6.

The heuristic process of systematically considering all possible sequences of rehabilitation alternatives is briefly described, using the two-pipe network in Figure 4 and the illustration in Figure 6. The following terms are defined:

Stage - The number of pipes that have been rehabilitated (e.g., stage 0 is the initial stage in which no pipe has yet been rehabilitated; stage 1 is the stage at which one pipe has been rehabilitated; etc.).

State - A unique sequence of rehabilitated pipes at a given stage (e.g., in the example above, possible states in stage 1 could be pipe 1 rehabilitated with alternative 1 at year 15; pipe 2 rehabilitated with alternative 2 at year 5; pipe 1 rehabilitated with alternative 2 at year 10; etc.). A state has three main attributes:

- (a) the unique sequence of pipes it comprises,

- (b) the state cost, consisting of the costs (maintenance and rehabilitation) of all individual pipes in the sequence
- (c) the time of minimum pressure (TMP) resulting from the state, e.g., in the example, the state (in stage 1) comprising pipe 1 with rehabilitation alternative 2 at year 15, has a TMP equal to year 25, as is illustrated in Figure 6.

Transition - adding a pipe to the states in one stage to generate states in the next stage.

Inferior state - if in a given stage there are two states that comprise the same pipes (yet with different rehabilitation schedules) in their sequences, and if one of these states has an earlier TMP yet a higher cost than the other, it is inferior to the other and can be discarded.

The flow chart in Figure 7 illustrates the conceptual procedure of systematically considering all the possible sequences of rehabilitation alternatives. After the minimum cost replacement timings

Figure 7. A conceptual description of the optimization procedure

(MCRTs, also denoted by T_{ij}^* for pipe i rehabilitated by alternative j) for all combinations of pipes and rehabilitation alternatives in the network are calculated, the procedure is initialized by determining the TMP at stage zero (which is the latest time allowed for implementing the next

rehabilitation project). The transition to stage 1 involves adding to the (empty at this stage) sequence all possible combinations of pipes and rehabilitation alternatives with timings that are determined using the principles of network hydraulics and individual pipe economics discussed earlier. This operation establishes the states in the first stage.

These states are then examined, respective total costs and TMPs are compared, and the inferior states are identified and discarded. The remaining states are now a starting point for generating new states in the transition to the next stage. Each stage represents an addition of one pipe. The transition to a new stage involves adding each of the remaining pipes with all possible combinations of rehabilitation alternatives, thus generating the states in the next stage.

The transition process is repeated until all pipes are considered in the sequences. The last stage comprises states whose respective sequences consist of all pipes in the network. Among all the states whose TMP extends at least until the pre-defined time horizon, the one with the least cost is selected as the optimal sequence of rehabilitated pipes.

Computer program

A computer program was developed for implementing M-PRAWDS using the process shown in the flow-chart in Figure 7. The network simulator used for calculating nodal residual pressures was EPANET by the US Environmental Protection Agency, (Rossman, 1993), which was modified to run as a subroutine of the main program.

Since the number of states varies from stage to stage in a manner that cannot be predicted prior to run-time, the data were structure as ‘doubly-linked lists’, which can efficiently store data that dynamically expand and shrink during run-time. Thus, at every stage a list is created to store the states at this stage. Every link (state) in this list contains the necessary information that is relevant to the state, and information (pointers) about the location in memory of the previous link (state) and the next link, as is illustrated in Figure 8. The relevant information of every state must also include the subset of pipes that have already been rehabilitated in the previous stages. This subset is stored as a (singly) linked list that branches from every link (state) in the main list. It expands as the stage advances, until it contains a full set in the last stage.

Figure 8. Graphical representation of the data structure

Procedure Validation

Given that an alternative for solving this type of problem does not exist, the most credible validation would be to compare the results obtained by M-PRAWDS to results obtained by exhaustively enumerating all possible combinations of rehabilitation strategies. However, the dimensionality of even a trivial network would render this course of action computationally prohibitive. Consequently, a three-pipe (one source node and two demand nodes) network with three rehabilitation alternatives (reline, replace with same diameter, replace with one larger diameter) was used for validation. It should be noted that even for this small network the solution space is in the order of magnitude of

10^{72} combinations for a 40-year (single year timestep) time horizon. Exhaustively enumerating this problem would take many centuries for even the fastest computer available today. Thus, instead of exhaustive enumeration, two schemes of restricted enumeration were applied to this network of three pipes, as follows:

- In the first restricted enumeration scheme, pipe parameters were assigned so that the replacement cycles were long enough to ensure no more than one rehabilitation per pipe per analysis period. This scheme reduces the number of combinations to $(RH+1)^p$ or about 800,000. A total of 144 scenarios were enumerated in this manner consisting of random combinations of pipe parameters with analysis time horizons of 30 or 40 years. Subsequently, M-PRAWDS was applied to the same 144 scenarios. Comparison of the results of the two processes revealed that (a) of the 144 scenarios, 122 least-cost solutions identified by M-PRAWDS were identical to those identified by this enumeration scheme; and (b) in the remaining 22 scenarios, the same pipes were identified for optimal rehabilitation, with slight differences in scheduling (up to one year) in one or more of the pipes, due to the precision and rounding-off of the hydraulic calculations. It was thus confirmed that under the stated restrictions of the first enumeration scheme (i.e., one rehabilitation per pipe per analysis period and all the assumptions in the cost calculations) M-PRAWDS appeared to be capable of identifying the least cost rehabilitation strategy.
- The second restricted enumeration scheme allowed for the consideration of more than one rehabilitation measure per pipe per the analysis period; however, it did not allow more than one rehabilitation measure per timestep. In this scheme, the total number of combinations is $(Rp+1)^H$. The 3-pipe system was enumerated for the same three rehabilitation alternatives, and for an analysis period of 32 years divided into 8 timesteps of 4 years each. This brought the number of combinations to 10^8 . Several scenarios were enumerated in this manner and the results compared to those obtained by M-PRAWDS. The comparison demonstrated that under these restrictions M-PRAWDS appears to be capable in identifying the least cost combination. In addition, some of the replacement pipes' parameters were deliberately selected to represent rapidly deteriorating pipes. This selection imposed two pipe replacements (in some of the scenarios) during the analysis period. When the second replacement occurred, it was consistently T^{**} years after the first replacement. This appeared to validate the concept of the optimal replacement cycles (where the least cost timing of the second replacement of a pipe is T^{**} years after the first replacement).

Furthermore, in some scenarios, where pipe relining was part of the optimal solution, it was subsequently replaced by the same diameter pipe at the respective MCRT. This appeared to validate the concept that pipe replacement (at year T^*) will follow the initial relining of the pipe under certain conditions.

In addition, M-PRAWDS was also compared to existing analysis practices in the following manner. A hypothetical distribution system was prepared (12 pipes, 8 demand nodes and one source node) including network layout – see Figure 9, pipe characteristics, pipe history, costs, etc.

* These values were recorded in a survey conducted in 1987.

** All pipes are cast iron and assumed to have a friction coefficient of $C=130$ upon installation.

Figure 9. Distribution network for comparison to existing analysis practices.

The data for the existing pipes was based upon typical data obtained from the distribution system in Pressure District 5S of the City of Scarborough, in Ontario. This sample was sent to six water utility managers in Southern Ontario willing to participate in the experiment. The participants were required to apply their existing analysis and decision making methods to devise an optimal rehabilitation strategy (ORS) while conforming to the stated hydraulic constraints. The respective results were then compared to the results obtained by applying M-PRAWDS to the sample system.

A direct comparison on an equal basis of all ORSs was not possible due to the fact that most participants made additional assumptions necessary for their respective analysis processes, and some deviated from the guidelines and constraints. The following was noted:

- None of the participants considered repair costs after pipe replacement, even when recommending an immediate replacement.
- Four out of six participants failed to consider or under-estimated the deterioration in the hydraulic capacity of the existing pipes. The other two participants neglected the hydraulic deterioration of the replacement or relined pipes.
- Four out of six did not explicitly consider the economics of replace versus continue to repair. The other two participants made assumptions that resulted in projected breakage rates far below those predicted by the exponential model.
- One participant used a point scoring system to prioritise pipes for replacement, but the replacement timing was not determined.

The survey results clearly demonstrated that the state of the practice in planning the renewal of water mains lacks rigour and that M-PRAWDS offers significant advantages in (a) its ability to explicitly consider the deterioration over time of both the structural integrity and the hydraulic capacity of the pipes in the water distribution network, (b) its ability (by considering rehabilitation cycles to infinity) to compare projected cost streams that are independent of the selected analysis period, (c) its ability to consider the economics and performance of the entire network while regarding each pipe as a separate entity with its own characteristics and parameters, and (d) its ability to determine each rehabilitation action, not only by the current state of the system but by future rehabilitation actions as well, by continually considering the entire analysis period.

Computer Program Performance

M-PRAWDS reduces the dimensionality of the problem significantly. The following are the results of some test runs performed on the 12-pipe network (Figure 9) using a 500 MHz Pentium III PC. The total enumeration of this network, with 3 rehabilitation alternatives and 1 year timestep (30 year planning horizon) is $\sim 5.5 \cdot 10^{216}$. For comparison, if we could examine one trillion alternatives per second, it would take one century to examine “only” $\sim 3 \cdot 10^{21}$ alternatives. M-PRAWDS solved this network in 32 minutes. The dimensionality and run-time of the problem can be reduced by using longer timesteps. Figure 10 illustrates the run-times for various timesteps. The total cost of the optimal solution is seen to decline as the timestep increases in length. This may, at first glance, seem counter-intuitive, because longer timesteps would be expected to result in a less accurate (thus a higher cost) solution. However, M-PRAWDS checks for hydraulic integrity only on transition from one stage to the next, and if TMP is reached in a stage, the network is upgraded in the next stage. Thus, when using long timesteps, network upgrade can be delayed, resulting in lower costs but at the same time the hydraulic integrity of the network may be breached towards the end of the stage. It is possible to reduce total run-time by using a long timestep to filter out improbable states and then running the system with a short timestep and a reduced set of alternatives for the final results.

M-PRAWDS run-time is also affected by the network’s minimum allowed pressure, as illustrated in Figure 11. The lower the minimum pressure allowed, the later TMP and the more states have to be evaluated at each stage. This has a dramatic effect on the run-time. Of course, things could even be more complex if a booster pump was to be considered as an alternative, but this is beyond the scope of M-PRAWDS in its current form.

Figure 10. Runtime (squares) and cost (circles) vary with length of timestep (min. pressure is 35m).

Figure 11. Runtime (squares) and cost (circles) vary with min. pressure (timestep is 1 year).

Discussion

PRAWDS assumes an exponential increase in pipe breakage rate over time. The parameters for the exponential function are derived by regression analysis on an individual pipe basis, which may present a problem in pipes that had few or no breaks. Some bundling or grouping of pipes may then be necessary.

The assumption of identical replacement cycles to infinity presents a limitation in the methodology. In urban centers experiencing a period of expansion, the design maximum flow in a given pipe may increase substantially over a period equal to T^{**} of the pipe. Consequently, it is reasonable to assume that in many instances, while the initial pipe replacement would be implemented with a certain diameter pipe, subsequent replacements may be with larger diameter pipes. Furthermore, advancements in pipe material and operation and management techniques could allow for longer replacement cycles in the future. These uncertainties in the future performance of replacement pipes seem, however, to have relatively little effect on total cost values due to the typically long periods of time which result in deep discounting of future costs (Kleiner, 1996). Further, it is possible to re-derive equations (1) and (2) with parameters $N(t_0)$ and A that are time-dependent, to reflect improvements in pipe materials, or even events like the application of cathodic protection to the pipe. A method to consider time-dependent variables in predicting water main breaks was recently

introduced by Kleiner and Rajani (2000), however, this method has not yet been incorporated into M-PRAWDS.

An alternative to address the uncertainties in the future conditions is the manner in which M-PRAWDS is implemented in practice. The optimal rehabilitation strategy (ORS) obtained by M-PRAWDS for the entire analysis period may be divided into a short-term segment (comprising say, three to six years) and the long-term segment comprising the rest of the analysis period. The rehabilitation measures depicted in the short-term segment would then be considered for actual implementation. Towards the end of the short-term segment, M-PRAWDS is reapplied to the water distribution network, utilizing up-to-date data pertaining to pipes as well as demand flows. A new and revised ORS is then obtained, which again is divided into a short-term and a long-term segment, and so on. Factors such as economies of scale and pipe replacement scheduling in conjunction with other public works (e.g. road renewal) could also be considered in this way.

M-PRAWDS can be applied with various size timesteps. In general, the shorter the timestep, the more likely the results are to be closer to the true minimum cost solution, but the longer it would take to arrive at this solution. Consequently, running the system with longer timesteps could be useful to screen out inferior alternatives, thus reducing runtimes of shorter timesteps.

Although M-PRAWDS considers infinite replacement cycles, the system is not guaranteed to be in a steady state hydraulically (the hydraulic capacity is assured only for the analysis period). Consequently, the issue of residual hydraulic capacity should be considered. The residual capacity of a system can be determined by applying M-PRAWDS with increasingly longer time horizons (H). Experience shows, however, that the impact of residual capacity considerations is typically not great because it is discounted by H years (typical values of H are 30 to 60 years) and because it is diminished by the infinite stream of replacement cycles.

The sensitivity of M-PRAWDS to the discount rate was examined as well. Generally, the lower the discount rate, the sooner the MCRT and the shorter the replacement cycle of each pipe. Consequently, implementation times arrive sooner. Nevertheless, it appears that the infinite replacement cycles approach has a buffering effect that reduces the sensitivity of M-PRAWDS results with respect to the discount rate.

Energy costs are not explicitly considered in the proposed methodology (PRAWDS) because evaluating the cost of energy in a system over an analysis period which is typically 30 to 60 years,

involves numerous extended time simulations that would make the model computationally prohibitive. This omission of energy costs is discussed at length in Kleiner et al. (1998a) where it was explained why in most practical cases this might not be a significant limitation.

It appears that M-PRAWDS is not fast enough to solve a network with, say, more than 20 links, as the dimensionality of the problem increases very fast with the number of pipes. Consequently, a more efficient procedure for combinatorial space search would be required in order to apply PRAWDS to a larger system. Random search heuristics such as genetic algorithms or tabu-search may be useful although they do not guarantee global minima.

Summary and Conclusions

The long term planning of the rehabilitation and upgrading of a water distribution system involves the selection of an appropriate rehabilitation measure for each pipe in the network and the implementation timing thereof, all the while maintaining adequate supply pressures in the system. Finding an optimal strategy requires identification of a minimum cost solution in a vast combinatorial (and not ‘well behaved’) solution space. This paper outlines research performed to address this need. The network economics and hydraulic capacity are analyzed simultaneously over a pre-defined analysis period, while explicitly considering the deterioration over time of both the structural integrity and the hydraulic capacity of every pipe in the system.

A pipe cost function was developed that considers an infinite time stream of costs that is associated with infinite rehabilitation cycles for every pipe. A multistage procedure (M-PRAWDS) was developed based on a dynamic programming approach combined with partial and (sometimes) implicit enumeration schemes. This multistage procedure can identify an optimal rehabilitation strategy for the water distribution network.

A computer program was written for the application of the method. The dynamic nature of the data during interim processing required a unique data structure comprising link lists that are branching from doubly linked lists.

The advantages of M-PRAWDS are (a) its ability to explicitly consider the deterioration over time of both the structural integrity and the hydraulic capacity of the pipes in the water distribution network, (b) its ability (by considering rehabilitation cycles to infinity) to compare projected cost streams that are independent of the selected analysis period, (c) its ability to consider the economics and

performance of the entire network while regarding each pipe as a separate entity with its own characteristics and parameters, and (d) each rehabilitation action is determined not only by the current state of the system but by future rehabilitation actions as well. It is, however, suitable for relatively small distribution systems with present computational techniques and equipment.

References

- Ahamad, M., & Melchers, R.E., **Reliability of Underground Pipelines Subject to Corrosion**, Journal of Transportation Engineering, ASCE, 120:6:989, (1994).
- Andreou, S. A., D. H. Marks, and R. M. Clark, **A New Methodology for Modelling Break Failure Patterns in Deteriorating Water Distribution Systems: Theory**, *Advance in Water Resources*, Vol. 10, pp. 2-10, Mar., 1987.
- Arulraj, G. P., and H.R. Suresh **Concept of Significance Index for Maintenance and Design of Pipe Networks**, *Journal of Hydraulic Engineering*, ASCE, Vol. 121 No. 11, pp. 833-837, Nov., 1995.
- Bahmanyar, H.G. & Edil, T.B., **Cold Weather Effects on Underground Pipeline Failures**. *Proceedings of the 1983 ASCE Conference on Pipelines in Adverse Environments Vol.2*, pp. 579-593 New York, 1983.
- Canadian Water and Wastewater Association, **Municipal Water and Wastewater Infrastructure: Estimated Investment Needs 1997 – 2012**, *A report partially sponsored by the Canadian Mortgage and Housing Corporation*, Apr., 1997.
- Clark, R. M., C. L. Stafford, and J. A. Goodrich, **Water Distribution Systems: A Spatial and Cost Evaluation**, *Journal of Water Resources Planning and Management Division*, ASCE, Vol 108, No. WR3, pp. 243-256, Oct. 1982.
- Colebrook, C. F., and C. M. White, **The Reduction of Carrying Capacity of Pipes With Age**, *Journal of the Inst. Civil Engrs.*, London, 10:99 pp. 97-118, 1937.
- Constantine, A. G., and J. N. Darroch, **Pipeline Reliability: Stochastic Models in Engineering Technology and Management**, edited by S. Osaki, D.N.P. Murthy, *World Scientific Publishing Co.*, 1993.
- Constantine, A. G., J. N. Darroch, and R. Miller, **Predicting Underground Pipe Failure**, *Australian Water Works Association*, Mar./Apr., 1996.
- Davies, C., D. L. Fraser, P. C. Hertzler, and R. T. Jones **USEPA's Infrastructure Needs Survey**, *Journal AWWA*, Vol. 89, No. 12, pp. 30-38, Dec., 1997.
- Deb, A. K.; Y. J. Hasit, F. M. Grablutz, and R. K. Herz., **Quantifying Future Rehabilitation and Replacement Needs of Water Mains**, *AWWA Research Foundation*, Denver, 1998.
- Doleac, M. L., S. L. Lackey, and G. Bratton, **Prediction of Time-to-Failure for Buried Cast Iron Pipes**, *AWWA Annual Conference Proceedings*, Atlanta, pp. 31-38, GA., June, (1980).
- Goulter, I. C., and A. Kazemi, **Spatial and Temporal Groupings of Water Main Pipe Breakage in Winnipeg**, *Canadian Journal of Civil Eng.*, 15(1), pp. 91-97, 1988.

- Goulter, I. C., J. Davidson J. and P. Jacobs, **Predicting Water-Main Breakage Rate**, *Journal of Water Resources Planning and Management*, ASCE, Vol. 119, No. 4, pp. 419-436, Jul./Aug., 1993.
- Herz, R. K. **Ageing Process and Rehabilitation Needs of Drinking Water Distribution Networks**, *Journal Water SRT – Aqua*, 45:5:221, 1996.
- Kettler, A. J., and I. C. Goulter **An Analysis of Pipe Breakage in Urban Water Distribution Networks**, *Can. Journal of Civil Eng.*, 12, 286-293, 1985.
- Kim, J. H., and L. W. Mays, **Optimal Rehabilitation Model for Water-Distribution Systems**, *Journal of Water Resources Planning and Management*, ASCE, Vol. 120 No. 5, pp. 674-692, Sept./Oct. 1994.
- Kleiner, Y. **Water Distribution Network Rehabilitation: Selection and Scheduling of Pipe Rehabilitation Alternatives**, *Ph.D. Thesis*, Department of Civil Engineering, University of Toronto, Toronto, 1996.
- Kleiner Y., B. J Adams, and J. S Rogers, **Long-term Planning Methodology for Water Distribution System Rehabilitation**, *Water Resources Research*, 34(8), pp. 2039-2051, Aug. 1998.
- Kleiner, Y., B. J. Adams, and J. S. Rogers, **Selection and Scheduling of Rehabilitation Alternatives for Water Distribution Systems**, *Water Resources Research*, 34(8), pp 2053-2061, Aug. 1998.
- Kleiner, Y., and B. Rajani, **Considering Time-dependent Factors in the Statistical Prediction of Water Main Breaks**, *AWWA Infrastructure Conference Proceedings*, Baltimore, Mar., 2000.
- Kumar, A., E. Meronkly, and E. Segan, **Development of Concepts for Corrosion Assessment and Evaluation of Underground Pipelines**, *US Army Corps of Engineers, Construction Engineering Research Laboratory*, Champaign Illinois, (1984).
- Male, J. W., T. M. Walski, and A. H. Slutski **Analyzing Watermain Replacement Policies**, *Journal of Water Resources Planning and Management*, ASCE, Vol. 116, No. 3, pp. 363-374, May/June, 1990.
- Rajani, B., C. Zhan, and S. Kuraoka **Pipe-soil Interaction Analysis of Jointed Watermains**, *Canadian Geotechnical Jour.*, 33:(3), pp. 393-404, (1996).
- Rossman, L. A., **EPANET User Manual**, Drinking Water Research Division, Risk Reduction Engineering Laboratory, Office of Research and Development, U.S. Environmental Protection Agency, Cincinnati, OH., July, 1993.
- Schneider, C. R., Y. Y. Haims, D. Li, and J. H. Lambert **Capacity Reliability of Water Distribution Networks and Optimum Rehabilitation Decision Making**, *Water Resources Research*, Vol. 32, No. 7, pp. 2271-2278, Jul., 1996.
- Shamir U., and C.D.D. Howard, **An Analytic Approach to Scheduling Pipe Replacement**, *Journal of AWWA*, Vol. 71, No. 5, pp. 248-258, May, 1979.
- Sharp, W. W., and T. M. Walski, **Predicting Internal Roughness in Water Mains**, *Journal of AWWA*, Vol. 80, No. 11, pp. 34-40, Nov., 1988.

Su Y.C., and L. W. Mays, **New Methodology for Determining the Optimal Rehabilitation and Replacement of Water Distribution System Components**, *Hydraulic Engineering - Proceedings of the 1988 ASCE National Conference*, Edited by S. R. Abt & J. Gessler, pp. 1149-1154, ASCE, New York, N.Y., 1988.

Walski, T. M., and A. Pelliccia, **Economic Analysis of Water Main Breaks**, *Journal of the AWWA*, Vol 74, No. 3, pp. 140-147, Mar., 1982

Woodburn, J., K. E. Lansey, and L. W. Mays, **Model for the Optimal Rehabilitation and Replacement of Water Distribution System Components**, *Proceedings, ASCE 1987 National Conference on Hydraulic Engineering*, Williamsburg, Virginia, 1987.

LIST OF NOTATION

p	number of pipes in the network
n	number of nodes in the network
R	number of rehabilitation alternatives to consider
H	analysis time horizon (years)
T_{ij}	time from present that pipe i is rehabilitated with alternative j (years)
C_{rij}	cost of rehabilitation measure j in pipe i (\$ /km)
L_i	length of pipe i (km)
r	discount rate
t	time elapsed (years)
$N(t)_i$	number of breaks per unit length per year in existing pipe i (km ⁻¹ year ⁻¹)
$N(t_0)_i$	$N(t)_i$ at the year of installation of existing pipe i (i.e., when the pipe was new)
g_i	age of existing pipe i at the present time (years)
A_i	coefficient of breakage rate growth in existing pipe i (year ⁻¹)
$N(t)_{ij}$	same as $N(t)_i$ except in pipe i , rehabilitated with alternative j (km ⁻¹ year ⁻¹)
$N(t_0)_{ij}$	$N(t)_i$ at the year of installation of pipe i alternative j (i.e., upon replacement)
g_{ij}	age of replacement pipe i replaced by alternative j
A_{ij}	coefficient of breakage rate growth in existing pipe i (year ⁻¹)
$C^M(T_{ij})$	the present value of breakage repairs in pipe i for the years elapsed from the present to the year of rehabilitation with alternative j
C_{bi}	cost of a single breakage repair in existing pipe i (\$)

T^f	time of first replacement of a pipe (year)
T^c	duration of a replacement cycle (years)
$C_{inf}(T_{ij}^c)$	PV of total cost associated with an infinite series of replacement cycles
T_{ij}^{**}	the value of T^c that minimizes C_{inf} for pipe i replaced by rehabilitation alt. j
T_{ij}^f	the time of first replacement of pipe i with replacement alternative j
T_{ij}^*	the value of T_{ij}^f that minimizes the total cost associated with the pipe from the present to infinity, also referred to as MCRT (minimum cost replacement timing)
$C_{ij}^{tot}(T_{ij}^f)$	the PV of the total cost (present to infinity) associated with pipe i rehabilitated with alternative j at time T_{ij}
Q_i	flow rate in pipe i (m ³ /s)
D_i	diameter of pipe i (m) or (ft)
$C_i^{HW}(t)$	Hazen-Williams friction coefficient pipe i at year t
E_{0i}	initial roughness (ft) in existing pipe i when it was installed new
a_i	roughness growth rate (ft/yr) in pipe i
$C_{ij}^{HW}(t)$	Hazen-Williams friction coefficient in pipe i , alternative j , at year t
E_{0ij}	initial roughness (ft) in pipe i alternative j
a_{ij}	roughness growth rate (ft/yr) in pipe i with rehabilitation alternative j
D_{ij}	diameter (m) or (ft) of pipe i rehabilitated with rehabilitation alternative j
$Q_{in,y}$	flow rate into node y
$Q_{out,y}$	flow rate out of node y
h_l	head changes in a pipe or a component that belongs to path l

P_{yt}	residual supply pressure at node y in year t
P_{miny}	minimum residual pressure allowed at node y in the system
$CAP_{reline}(T_i)$	capital cost of relining pipe i at time T_i
TMP	time of minimum pressure (defined in Figure 5)
MCRT	minimum cost replacement timing (see also T_{ij}^*)