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Design Sensitivity Analysis for the Optimization of the Injection Molding Process

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Abstract

This paper presents an application of the Continuous Sensitivity Equation Method (CSEM) for the optimization of the injection molding process and its three-dimensional (3D) simulation by the finite element method. Finding the proper combination of process parameters such as injection speed, and melt and mold temperatures is critical to achieving a part that minimizes warpage and has the desired mechanical properties. Very often a successful design in injection molding comes at the end of a long trial and error process. Design Sensitivity Analysis (DSA) can help manufacturers improve their designs and can produce substantial savings in terms of both time and money. This work explores the ability of sensitivity analysis to predict the effects of design parameters on the performance of an injection molding process. The paper presents results of a 3D finite element solution of the filling stage of the injection molding process. Sensitivities of the solution with respect to different process parameters are computed using the continuous sensitivity equation method. Solutions are shown for the non-isothermal filling of a rectangular plate with a polymer melt behaving as a non-Newtonian fluid. The paper presents the equations for the sensitivity of the velocity, pressure and temperature and their solution by a finite element method. Sensitivities of the solution with respect to the injection speed, the melt and mold temperatures and to the heat transfer coefficient at the cavity/mold interface are shown.

1 Introduction

Injection molding of polymeric materials is an important manufacturing process used in a large number of industries. Molded parts must satisfy stringent quality requirements in terms of their mechanical behavior and dimensional characteristics. Computer simulations are now performed at continuously lower cost and with smaller turnaround time, making them an increasingly valuable tool for the design and evaluation of manufacturing processes. Moreover, numerical investigations now provide practical quantitative information about the physical model that would otherwise be difficult to obtain. Computations are sufficiently sophisticated to provide insight into the effects of parameter changes on the characteristics of the final part. Hence, the time is ripe for using simulation in an automated optimal design loop to achieve improved products or processes.

A faithful model and reliable numerical predictions of the injection molding process is extremely important to predict the characteristics of the final product with confidence. For instance, while the filling stage determines whether or not the part is completely filled, where the weld lines are located and what the temperature distribution is at the end of filling, the packing and cooling stages affect the shrinkage, the presence of residual stresses and warpage. Much work has been done on the simulation of the injection molding process with most studies making use of the Hele-Shaw approximation [1, 2, 3]. Such an approach cannot predict important effects such as fountain flow and transverse pressure gradients; it also involves simplifications of the model near corners, bifurcations and changes in the part thickness. This paper uses a full 3D numerical approach to solve the injection molding process. While the methodology is ideally suited for three-dimensional parts for which the simplifying lubrication hypothesis is inappropriate, it is also useful for thin parts in which three-dimensional effects are important.

Even using the latest advances in 3D modeling techniques to simulate injection molding, the number of simulations required to optimize the design of a process is too large for routine applications. DSA can be of great help in the design of the process because it provides quantitative information at a reasonable cost on how modifications to parameters will affect the overall behavior of the process. For example, DSA can provide information on how an increase of 1% in the temperature of the injected melt will affect the temperature distribution in the part at the end of filling. This type of information is referred to as the sensitivity of the part temperature with respect to the melt injection temperature. One can use this information to make decisions about how to change the process parameters to achieve a particular objective. Furthermore, once these sensitivities are known, they can be used in formal gradient based optimization algorithms to minimize a given cost function.

There are several means of computing sensitivities: finite difference of solutions [4], the complex step method [5], and sensitivity equation methods [6, 7, 8]. The first approach is costly because the injection simulation must be carried out for 2 or more values of each parameter of interest. The complex step method is code invasive, as it requires a complete rewriting of the simulation software. Automatic differentiation is powerful but requires human intervention to ensure code efficiency. Problems are encountered when applying automatic differentiation to commercial software, since source code is not available.

Approaches also differ depending on the order of the operation of approximation and differentiation. In the Discrete Sensitivity approach the total derivative of the problem equation with respect to a parameter is approximated [9] whereas in the Continuous Sensitivity Equation Method (CSEM) one differentiates the continuum equations to yield differential equations for the continuous sensitivities. See reference [10] for a discussion of the two approaches.

Initial work on sensitivity analysis for material processing can be found in references [11, 12, 13]. McDavid and Dantzig [11] present the solution of sensitivities for free surface flows with application to casting processes. The numerical sensitivities are coupled with a commercial optimizer and applications are shown for the optimization of a runner and gating system. Smith et al. [12, 13] used design sensitivity analysis for optimizing the design of polymer sheeting dies. Both adjoint and direct sensitivities are used with the Hele-Shaw equations to minimize the pressure drop and velocity variation across the die exit. Applications of the CSEM to flows with variable fluid properties are shown in references [14, 15] for adaptive meshes. In recent work Ilinca and Héту [16] presented results for the CSEM applied to gas-assisted injection problems. The procedure provided the position of the polymer/air and gas/polymer interfaces and its sensitivity to gas pressure and melt temperature. Sensitivities obtained by the CSEM were compared with those obtained using finite differences. Sensitivity solutions compare well for the initial phase of the process but errors increase with time through the end of the simulation.

For this work, a finite element solution algorithm is used to solve three-dimensional flow and heat transfer problems. The Stokes and energy equations are solved in a segregated manner. For the momentum and continuity equations a Galerkin Least-Squares (GLS) method [17] is used. Such stabilization methods allow the use of linear velocity and pressure interpolants that do not satisfy the Babuška-Brezzi condition. These elements are both computationally effective and easy to implement, especially for three-dimensional cases. The heat transfer problem is decomposed using a SUPG/GGLS approach [18]. During the filling stage, the position of the flow front in the mold cavity is computed using a pseudo-concentration technique [19]. Equations are integrated in time using a first order implicit scheme. The paper also presents the sensitivity equations of the mathematical filling model. Computations are carried out for the

filling of a rectangular plate through a center gate.

2 Mathematical Modeling

The material behavior during injection molding is modeled by the momentum-continuity and energy equations which provide velocity, pressure and temperature fields. During the filling stage, a free surface problem must also be considered to identify the location of the polymer/air front. Front tracking is performed by solving a pseudo-concentration function.

2.1 Model equation for the mold filling problem

The governing equations for the flow of an incompressible polymer melt are

$$0 = -\nabla p + \nabla \cdot (2\eta \dot{\gamma}(\mathbf{u})), \quad (1)$$

$$-\nabla \cdot \mathbf{u} = 0, \quad (2)$$

where the strain rate tensor is given by $\dot{\gamma}(\mathbf{u}) = (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)/2$. Heat transfer effects are modeled by the energy equation:

$$\rho c_p \left(\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) = \nabla \cdot (k \nabla T) + \phi, \quad (3)$$

where $\phi = 2\eta \dot{\gamma} : \dot{\gamma}$ is the viscous dissipation function. In the above equations, t , $\mathbf{u}$, p , T , ρ , η , c_p and k denote time, velocity, pressure, temperature, density, viscosity, specific heat and conductivity respectively.

The heat transfer between the cavity and the mold is modeled by imposing a convection heat flux at the cavity/mold interface

$$q_m = k \frac{\partial T}{\partial n} = h_c (T - T_m) \quad \text{on } \Gamma_{\text{mold}} \quad (4)$$

where h_c is the surface heat transfer coefficient, T is the melt temperature at the interface part/mold and T_m is the mold temperature. The momentum, continuity and energy equations are solved in a segregated manner [20].

The position of the polymer/air interface is modeled using a pseudo-concentration method. The pseudo-concentration function is convected using [20]:

$$\frac{\partial F}{\partial t} + \mathbf{u} \cdot \nabla F = 0. \quad (5)$$

where $\mathbf{u}$ is the velocity field provided by the solution of the momentum-continuity equations. Regions of the mold cavity where the tracking function F is larger than a preset critical value F_c are identified as being filled with polymer, while the rest of the cavity is considered empty.

2.2 Continuous sensitivity equations

The continuous sensitivity equations are derived formally by implicit differentiation of the equations (1), (2), (3) and (5) with respect to parameter a . We treat dependent variables as functions of space, time and a . This dependence is denoted by

$$\mathbf{u} = \mathbf{u}(x, t; a), \quad p = p(x, t; a), \quad T = T(x, t; a), \quad F = F(x, t; a). \quad (6)$$

Defining the system sensitivities as

$$\mathbf{s}_u = \frac{\partial \mathbf{u}}{\partial a}, \quad s_p = \frac{\partial p}{\partial a}, \quad s_T = \frac{\partial T}{\partial a}, \quad s_F = \frac{\partial F}{\partial a} \quad (7)$$

and denoting the total derivatives of fluid properties and other flow parameter by a prime ('), the system of sensitivity equations is given by [16]:

$$0 = -\nabla s_p + \nabla \cdot (2\eta \dot{\gamma}(\mathbf{s}_u)) + \nabla \cdot (2\eta' \dot{\gamma}(\mathbf{u})), \quad (8)$$

$$-\nabla \cdot \mathbf{s}_u = 0, \quad (9)$$

$$\rho c_p \left(\frac{\partial s_T}{\partial t} + \mathbf{u} \cdot \nabla s_T + \mathbf{s}_u \cdot \nabla T \right) = \nabla \cdot (k \nabla s_T) + \phi', \quad (10)$$

$$\frac{\partial s_F}{\partial t} + \mathbf{u} \cdot \nabla s_F + \mathbf{s}_u \cdot \nabla F = 0. \quad (11)$$

The term ϕ' , in the equation for the temperature sensitivity, denotes the sensitivity of the viscous dissipation function:

$$\phi' = \frac{\partial \phi}{\partial a} = \frac{\partial \phi}{\partial \eta} s_\eta + \frac{\partial \phi}{\partial |\dot{\gamma}|} \frac{\partial |\dot{\gamma}|}{\partial \mathbf{u}} \mathbf{s}_u. \quad (12)$$

Note that the sensitivity equations are linear but coupled. The melt is a PC200 grade polymer behaving as a generalized Newtonian fluid for which the viscosity is a function of the shear rate and temperature. For this work the rheological behavior of the melt is described by the Cross-WLF model:

$$\eta = \frac{\eta_0}{1 + (\eta_0 |\dot{\gamma}| / \tau^*)^{-n}}, \quad (13)$$

$$\eta_0 = D_1 \exp \left\{ \frac{-A_1 (T - T^*)}{A_2 + (T - T^*)} \right\}, \quad (14)$$

where $T^* = D_2 + D_3 p$, and $A_2 = \tilde{A}_2 + D_3 p$. The rate of deformation $|\dot{\gamma}|$ is computed from the strain rate tensor as $|\dot{\gamma}| = \sqrt{2\dot{\gamma} : \dot{\gamma}}$. Model constants used for the numerical simulation are summarized in Table 1. The sensitivity of the viscosity, η' , depends on the shear rate and temperature but also on the sensitivity of the shear rate and temperature:

$$\eta = \eta(\eta_0(T(a)), |\dot{\gamma}|(\mathbf{u}(a)), a)$$

$$\eta' = \frac{d\eta}{da} = \frac{\partial \eta}{\partial \eta_0} \frac{\partial \eta_0}{\partial T} s_T + \frac{\partial \eta}{\partial |\dot{\gamma}|} \frac{\partial |\dot{\gamma}|}{\partial \mathbf{u}} \mathbf{s}_u + \frac{\partial \eta}{\partial a} \quad (15)$$

Density, specific heat and thermal conductivity were considered constant, with values of 1162 kg/m^3 , $2000 \text{ J/(kg}\cdot^\circ\text{C)}$, and $0.25 \text{ W/(m}\cdot^\circ\text{C)}$ respectively. Since they do not depend on the parameter α , their sensitivities are zero.

A special treatment is required when the filling time depends on the design parameter as is the case when the injection speed or the filling time are considered as design parameter. In this case the time dependent equations (3) and (5) are rewritten explicitly in terms of the injection time.

This is done through the change of variable $\bar{t} = t / t_{fill}$, where t_{fill} is the filling time, so that $\bar{t}$ varies from 0 to 1 when t evolves from 0 to t_{fill} . In this manner $\bar{t}$ does not depend on the design parameter. Equations (3) and (5) then become

$$\rho c_p \left(\frac{\partial T}{\partial \bar{t}} \frac{1}{t_{fill}} + \mathbf{u} \cdot \nabla T \right) = \nabla \cdot (k \nabla T) + \phi, \quad (16)$$

$$\frac{\partial F}{\partial \bar{t}} \frac{1}{t_{fill}} + \mathbf{u} \cdot \nabla F = 0, \quad (17)$$

The corresponding sensitivity equations are

$$\rho c_p \left(\frac{\partial s_T}{\partial \bar{t}} \frac{1}{t_{fill}} - \frac{\partial T}{\partial \bar{t}} \frac{1}{t_{fill}^2} \frac{\partial t_{fill}}{\partial \alpha} + \mathbf{u} \cdot \nabla s_T + \mathbf{s}_u \cdot \nabla T \right) = \nabla \cdot (k \nabla s_T) + \phi', \quad (18)$$

$$\frac{\partial s_F}{\partial \bar{t}} \frac{1}{t_{fill}} - \frac{\partial F}{\partial \bar{t}} \frac{1}{t_{fill}^2} \frac{\partial t_{fill}}{\partial \alpha} + \mathbf{u} \cdot \nabla s_F + \mathbf{s}_u \cdot \nabla F = 0. \quad (19)$$

Rewriting equations (18) and (19) in terms of the real time t we obtain:

$$\rho c_p \left(\frac{\partial s_T}{\partial t} - \frac{\partial T}{\partial t} \frac{1}{t_{fill}} \frac{\partial t_{fill}}{\partial \alpha} + \mathbf{u} \cdot \nabla s_T + \mathbf{s}_u \cdot \nabla T \right) = \nabla \cdot (k \nabla s_T) + \phi', \quad (20)$$

$$\frac{\partial s_F}{\partial t} - \frac{\partial F}{\partial t} \frac{1}{t_{fill}} \frac{\partial t_{fill}}{\partial \alpha} + \mathbf{u} \cdot \nabla s_F + \mathbf{s}_u \cdot \nabla F = 0. \quad (21)$$

The sensitivity of the filling time $\partial t_{fill} / \partial a$ is equal to unity if the design parameter is the filling time ($a = t_{fill}$) and is given by $\partial t_{fill} / \partial a = -t_{fill} / V_{inj}$ when the design parameter is the injection speed ($a = V_{inj}$). When the design parameter does not affect the filling time, as is the case when considering for the melt and mold temperatures as parameters, the sensitivity of the filling time is zero and equations (20) and (21) reduce to equations (10) and (11), respectively.

3 Application

The mould for this application consists of a centrally gated rectangular cavity with dimensions of 76x165x7 mm as shown in Fig. 1. The sprue is made of a divergent 40 mm long channel with a rectangular cross-section. The height of the channel was kept constant at 5.56 mm, but the width varied from 5.56 mm at the entrance of the channel to 8.28 mm at the gate. The flat walls of the sprue channel were required so that two pressure transducers can be mounted flush, 25 mm apart. Fig. 1 shows a schematic of the cavity and sprue with the location of the Kistler pressure transducers labeled K1 and K2. The material used is an injection grade Polycarbonate (PC Calibre 200-14) from Dow Chemical (see Ref. [21]). The pressure at K1 and K2 locations was measured on moldings made under the following conditions: 300°C melt temperature, 50°C mould temperature and injection speeds of 5, 10, 20, 35, 50, 65, and 80mm/s.

4 Results and discussion

Simulations were carried out for a range of molding conditions and solution sensitivities were computed with respect to the injection speed V_{inj} , melt injection temperature T_{inj} , mold temperature T_{mold} , and to the heat transfer coefficient h_c at the cavity/mold interface. Two configurations were considered as shown in Fig. 1. The first one solves only inside the sprue and

the plate (*without nozzle*), while the second one considers also the runner and the nozzle (*with nozzle*). The reference conditions were injection speed of 50 mm/s, injection temperature of 300°C, mold temperature of 90°C and cavity/mold heat transfer coefficient of 1000 W/m²·°C. The mold temperature was taken higher than the cooling temperature to account for the mold heating during repeated injection molding cycles. The time variation of the pressure drop between locations K1 and K2 is compared with the experimental measurements in Fig. 2. The simulation without nozzle overestimates the pressure drop, while the pressure predicted when solving with nozzle is in better agreement with the experiment especially at latter times ($t > 0.4$ s). The differences are due to the important thermal heating occurring upstream of the sprue that is neglected when the model does not include the nozzle. To illustrate this effect, the temperature distribution inside the sprue is shown in Fig. 3 for the configuration without nozzle and in Fig. 4 for the case with nozzle. When solving without the nozzle, the temperature in the boundary layer increases by 15°C at the K1 location and by 40°C at the K2 location, while the temperature in the core remains equal to the injection temperature. The behavior is changed drastically when the CFD model accounts for the flow in the nozzle. The maximum temperature is reached at the center of the sprue cross-section because of the viscous heating on the inner wall of the annular region upstream of the nozzle. In this region the temperature increases by as much as 100°C. Large temperatures are also observed near the cavity walls where shear heating is important.

4.1 Sensitivity with respect to the injection speed

In this case the fill time depends on the design parameter, namely the injection speed. Hence sensitivity equations are solved in the form given by equations (8), (9), (20) and (21). The evolution in time of the sensitivity of the pressure drop with respect to the injection speed is

shown in Fig. 5. The sensitivity values reflect the change in the pressure drop (in percentage) for a 1% increase in the injection speed. Here $\delta\Delta p$ is the change in pressure drop and Δp is the pressure drop at the nominal value of the injection speed. The relative change in the pressure drop is computed as follows

$$\frac{\delta\Delta p}{\Delta p} = \frac{1}{\Delta p} \frac{\partial \Delta p}{\partial V_{inj}} \delta V_{inj} = \frac{1}{\Delta p} \Delta \left(\frac{\partial p}{\partial V_{inj}} \right) \delta V_{inj} = \frac{\Delta s_p}{\Delta p} \delta V_{inj}, \quad (22)$$

where $\delta V_{inj} = 0.01 V_{inj}$ and Δs_p is the drop in the sensitivity of the pressure between the two locations. At the end of the filling, the changes in the pressure drop are similar for the two configurations indicating a 0.17% increase in the pressure drop for a 1% increase in the injection speed. In Fig. 6 the pressure drop in the sprue at the end of the filling is compared with the experimental data. For the numerical solution we also show the slope at each point obtained by solving the sensitivity equations. The sensitivity indicates how the pressure drop reacts to changes in the injection speed. As can be seen the slope from the sensitivity of the solution correlates well with the variation of the pressure drop for all injection speeds. For the configuration with nozzle the sensitivity analysis indicates that the smallest pressure drop in the sprue is obtained for an injection speed of about 13mm/s. Decreasing the injection speed below this value results in an increase in the pressure drop. A similar behavior is observed for the solution without nozzle. Note that the behavior of the two solutions is different at the lower limit of the injection speeds range. The solution with the nozzle indicates that the pressure drop increases when the injection speed is decreased, while the solution without nozzle results in a decrease of the pressure drop when the injection speed decreases.

4.2 Sensitivity with respect to the melt temperature

The sensitivity of the solution with respect to the melt temperature is also computed. In this case the filling time does not depend on the design parameter. Hence, sensitivity equations (8)-(11) are used. Fig. 7 shows the time variation of the sensitivity with respect to the melt temperature for a melt temperature of 300°C and an injection speed of 50 mm/s. Again the values plotted represent the pressure drop change (in percentage) for a 1% change in the injection temperature.

The relative change in the pressure drop is computed as:

$$\frac{\delta \Delta p}{\Delta p} = \frac{1}{\Delta p} \frac{\partial \Delta p}{\partial T_{inj}} \delta T_{inj} = \frac{1}{\Delta p} \Delta \left(\frac{\partial p}{\partial T_{inj}} \right) \delta T_{inj} = \frac{\Delta s_p}{\Delta p} \delta T_{inj}, \quad (23)$$

where $\delta T_{inj} = 0.01 T_{inj}$. The sensitivity solution indicates that the pressure drop is more pronounced for the solution with a nozzle than for the solution without a nozzle. In both cases, negative values are obtained for the sensitivity indicating that the pressure drop decreases with an increase in the melt temperature, as one would intuitively expect. Fig. 8 illustrates the pressure drop in the sprue at the end of the filling for injection temperatures of 260°C, 280°C, and 300°C. The solution of the sensitivity equations yields good estimates of the slope of the pressure drop with respect to the injection temperature at each operating point. Excellent quantitative trends are observed for both configurations.

4.3 Sensitivity with respect to the mold temperature

Sensitivities of the pressure drop in the sprue with respect to the mold temperature are shown in Figs. 9 and 10. Sensitivity equations (8)-(11) are solved, and the relative change in the pressure drop is computed as:

$$\frac{\delta \Delta p}{\Delta p} = \frac{1}{\Delta p} \frac{\partial \Delta p}{\partial T_{mold}} \delta T_{mold} = \frac{1}{\Delta p} \Delta \left(\frac{\partial p}{\partial T_{mold}} \right) \delta T_{mold} = \frac{\Delta s_p}{\Delta p} \delta T_{mold}, \quad (24)$$

where $\delta T_{mold} = 0.01 T_{mold}$. Fig. 9 shows the relative variation of the pressure drop during the filling time for a mold temperature of 90°C and an injection speed of 50 mm/s. Again negative sensitivities indicate that the pressure drop decreases when the mold temperature is increased, as one would anticipate. In Fig. 10 the pressure drop in the sprue at the end of the filling stage is given for mold temperatures of 50°C, 70°C, and 90°C. For both configurations, good agreement is observed between the computed sensitivity and the observed variation of the solution.

4.4 Sensitivity with respect to the cavity/mold heat transfer coefficient

The sensitivity with respect to the cavity/mold heat transfer coefficient is shown in Fig. 11. Again sensitivity equations are solved in the form given by equations (8)-(11). The relative change in the pressure drop is computed as:

$$\frac{\delta \Delta p}{\Delta p} = \frac{1}{\Delta p} \frac{\partial \Delta p}{\partial h_c} \delta h_c = \frac{1}{\Delta p} \Delta \left(\frac{\partial p}{\partial h_c} \right) \delta h_c = \frac{\Delta s_p}{\Delta p} \delta h_c, \quad (25)$$

where $\delta h_c = 0.01 h_c$. An increase in the cavity/mold heat transfer coefficient results in increased polymer cooling at the wall, thus lower temperature of the melt and hence, higher pressure. Sensitivity analysis indicates that, for the configuration with a nozzle, the pressure drop in the sprue increases by 0.28% for a 1% increase in the heat transfer coefficient. The sensitivity of the configuration without nozzle behaves in the same way as the one that solves the flow prior to the sprue. The solutions for various cavity/mold heat transfer coefficients are shown in Fig. 12. Here again the computed sensitivity is an accurate representation of the slope of the curve Δp vs h_c . Observe that the sensitivity of the pressure drop with respect to the heat transfer coefficient is

larger at smaller values of the heat transfer coefficient (large slopes of Δp vs h_c) and becomes almost constant for heat transfer coefficients larger than 2000 W/m²·°C.

5 Conclusion

This work presents an application of the Continuous Sensitivity Equation Method to the injection molding process. The solution of the CSEM provides the sensitivity of the solution with respect to design variables. More specifically, the sensitivity of the pressure drop with respect to the injection speed $\partial\Delta p / \partial V_{inj}$, melt temperature $\partial\Delta p / \partial T_{inj}$, mold temperature $\partial\Delta p / \partial T_{mold}$ and the cavity/mold heat transfer coefficient $\partial\Delta p / \partial h_c$ has been presented. The sensitivities provide accurate estimates of the solution response when the design parameters are varied. In future work, gradient information obtained by the CSEM will serve as input to gradient based optimizers to achieve optimal processing conditions.

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Model constant	Value
$n; \tau^* (Pa)$	$0.12; 8.0 \cdot 10^5$
$D_1 (Pa \cdot s); D_2 (^{\circ}C); D_3 (^{\circ}C/Pa)$	$3.4 \cdot 10^9; 175.0; 0.0$
$A_1; \tilde{A}_2 (^{\circ}C)$	$21.0; 40.0$

Table 1. Cross-WLF model constants for PC200

Fig. 1. Computational domain without nozzle (left) and with nozzle (right)

Fig. 2. Pressure drop in the sprue during filling

Fig. 3. Temperature distribution inside the sprue and plate for the solution without nozzle

Fig. 4. Effect of the nozzle on the temperature distribution inside the sprue and plate

Fig. 5. Relative variation of the pressure drop for a 1% change in the injection speed

Fig. 6. Pressure drop at the end of the filling for various injection speeds

Fig. 7. Relative variation of the pressure drop for a 1% change in the injection temperature

Fig. 8. Pressure drop at the end of the filling for various injection temperatures

Fig. 9. Relative variation of the pressure drop for a 1% change in the mold temperature

Fig. 10. Pressure drop at the end of the filling for various mold temperatures

Fig. 11. Relative variation of the pressure drop for a 1% change in the cavity/mold heat transfer coefficient

Fig. 12. Pressure drop at the end of the filling for various cavity/mold heat transfer coefficients