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Confined Compression Tests: Outlining the Failure Envelope of Columnar Sea Ice

by G.W. Timco and R.M.W. Frederking

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RÉSUMÉ

Une série d'essais de compression en confinement a été réalisée sur de la glace marine prismatique dans un éventail de vitesses nominales de déformation de 10^{-5} à 10^{-3} s^{-1} à une température de -2°C . On a mesuré la charge appliquée et la charge sous confinement latéral. Ces essais servent à déterminer l'enveloppe globale de rupture de la glace dans les trois dimensions. Les résultats sont étudiés au point de vue effet de la vitesse de chargement, température, volume d'eau salée et porosité de la glace. On établit un indice de résistance qui met en corrélation les résultats actuels sur la base de ces paramètres. Cela conduit à une relation empirique qui indique la dépendance fonctionnelle de la résistance en compression uniaxiale en termes de vitesse de déformation, de direction de chargement, de salinité, de température et de densité de la glace. Une description mathématique de l'enveloppe de rupture est faite à l'aide d'une fonction limite élastique modifiée de type n.

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CONFINED COMPRESSION TESTS: OUTLINING THE FAILURE ENVELOPE OF COLUMNAR SEA ICE

G.W. Timco and R.M.W. Frederking

National Research Council of Canada, Ottawa, Ontario K1A 0R6 (Canada)

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ABSTRACT

A series of confined compression tests has been performed on columnar sea ice over a range of nominal strain rates of 10^{-5} to 10^{-3} s^{-1} at a temperature of -2°C . Both the applied load and side confining load were measured. These are used to outline the full 3-dimensional failure envelope of the ice. The results are discussed in terms of loading rate effects, temperature, brine volume and air porosity. A strength index is introduced which correlates the present results in terms of these parameters. This leads to an empirical relationship which gives the functional dependence of the uni-axial compressive strength in terms of strain rate, loading direction, salinity, temperature and ice density. A mathematical description of the failure envelope is presented using a modified n-type yield function.

INTRODUCTION

When an icebreaker travels through a floating ice sheet, or when a moving ice sheet interacts with a stationary structure, the breaking of the ice is an important part of the interaction process. The stress levels at which the ice fails can influence both the global loads and local pressures on the structure. This failure stress is controlled by the mechanical properties of the ice. Ice, however, can exhibit a variety of behaviour such as elasticity, brittle fracture, viscoelasticity, plasticity, etc. Moreover, for sea ice, the mechanical properties are influenced by the loading strain rate, loading stress rate, ice salinity, ice temperature, grain structure, grain size, grain orientation, brine porosity and air content. Thus, in order to be

able to understand the interaction process, it is necessary to study the mechanical properties of ice for a wide range of conditions. Because of its importance, there have been several investigations of the mechanical properties of ice. These are usually performed by taking a sample of ice and, under a well-defined loading arrangement, subjecting it to an increasing load until it fails. For simplicity in testing and analysis, the stress field is usually applied only in one direction, for example, in uni-axial testing of the compressive strength of ice (Peyton, 1966; Schwarz, 1970, 1983; Wang, 1979; Sinha, 1983a, 1983b, 1984; Frederking and Timco, 1980, 1983, 1984a).

In nature, however, the stress field in the ice can be very complex and the response of the ice cannot, therefore, be described uniquely by the uni-axial stress properties of the ice. Because of this, information is required on the behaviour of ice under complex stress states. This behaviour is usually described in terms of the *failure envelope* for the material (Paul, 1968). This is a description of the stress levels at which the ice yields for any combination of compressive or tensile stress states. One technique for investigating the failure envelope of a material is through the use of confined compression tests. In these tests, the material is loaded in one direction while confined in a second direction. This allows deformation of the material in only one direction. By measuring both the applied and confining loads, and knowing the orientation of the ice in the confinement apparatus, the failure stresses for the ice can be determined for various combinations of stress states. These define the failure envelope for the ice in the 3-dimensional compression-compression-compression octant. A knowledge of the failure envelope for ice is important since it indicates the maximum stress which the ice can

sustain before failure. In addition, if it is described in a convenient mathematical form in terms of the principal and shear stresses, it can be used to predict analytically the ice loads for various ice-structure interaction scenarios. The failure envelope for *columnar* sea ice is particularly important since columnar-structured ice is predominant in many areas of the Arctic.

To date, there has been only a handful of confined compression tests on sea ice. These include tests by Timco and Frederking (1983) and Blanchet and Hamza (1983) who measured the confined compressive strength of ice with horizontal loading at one temperature and over a very limited range of strain rates. A more thorough investigation was carried out by Timco and Frederking (1984a) who measured the full failure envelope using both vertical and horizontal loading arrangements for granular/discontinuous columnar sea ice at one temperature over a range of loading rates. Their tests were analysed using plasticity theory and fitted successfully to an n -type yield function. The present tests are concerned with the failure envelope of *columnar* sea ice. The tests were performed in the field on warm ($T = -2^{\circ}\text{C}$) columnar sea ice from Northern Labrador along the east coast of Canada. In this paper, the techniques used for measuring the strength are described and the results for five different confinement conditions are given in relation to loading rate at constant temperature. In addition, a series of uni-axial tests were performed at different temperatures, and the results are presented in terms of temperature, loading rate, brine volume and total porosity of the ice. The results from all of the tests are used to outline the 3-dimensional failure envelope for warm columnar sea ice, and the corresponding envelope at -10°C is inferred. Finally, the results are analysed to fit a modified n -type yield function for the ice at both temperatures.

EXPERIMENTAL

The tests described here were carried out in late May, 1984 in northern Labrador, Canada during the icebreaker trials of the German icebreaking research vessel *Polarstern*. Since the tests were carried out late in the spring, the air temperature was relatively high (-5 to $+5^{\circ}\text{C}$) and the ice had undergone some deterior-

ation. To get specimens of ice, a large block $1.2\text{ m} \times 1.2\text{ m} \times 1.0\text{ m}$ (ice thickness) was cut from the ice cover with a chain saw, and brought on board using the crane of the vessel. There, the bottom half was immediately cut into smaller pieces, $30\text{ cm} \times 50\text{ cm} \times 30\text{ cm}$, labelled and put in an onboard cold chamber at a temperature of -15°C . These blocks were then cut into $19\text{ cm} \times 8\text{ cm} \times 5\text{ cm}$ samples using a band saw. The large faces of each sample were smoothed using a power planer. Once cut to sample dimensions, the ice was transferred to a second cold room and allowed to sit overnight at the ambient temperature of the test. All samples tested consisted of strictly columnar S2 ice with random orientation of the c -axis in the horizontal plane (see Fig. 1). For this test series, the majority of tests were performed at -2°C , although a small number of samples were tested in uni-axial compression at -4°C and -12°C .

Fig. 1. Thin section of ice through crossed polaroids showing the columnar structure of the ice. The width of the sample is approximately 10 cm.

The temperature of -2°C was chosen to correspond to the average temperature of the ice in the Labrador Sea at that time. For all ice specimens, the salinity and density were determined. The average salinities were $1.6 \pm 0.4\text{‰}$ and $1.7 \pm 0.4\text{‰}$, and the average densities were $0.847 \pm 0.036 \text{ g cm}^{-3}$ and $0.829 \pm 0.031 \text{ g cm}^{-3}$ at -12°C and -2°C respectively. Salinity and density histograms are shown for both temperatures in Figs. 2 and 3 respectively. There were several brine drainage channels and large voids evident throughout the ice. The relatively low values for both salinity and density, as well as the general appearance of the ice, confirm that it had undergone considerable deterioration. All prepared samples were tested, even if they contained large drainage channels, voids, etc. In total 93 samples were tested.

The tests were performed using a 0.05 MN capacity Soiltest CT-405 compression tester which has a screw-driven actuator powered by a 1/4 H.P. motor. With this motor drive, actuator speeds ($\dot{\chi}$) of 3×10^{-3} to $7 \times 10^{-2} \text{ mms}^{-1}$ are obtainable. During this test series, intermittent problems with the motor necessitated conversion of the press to a hand-driven machine. This modification worked very well and it allowed higher cross-head speeds up to 1.9×10^{-1}

Fig. 2. Salinity histogram for all ice samples at $T = -12^{\circ}\text{C}$ and $T = -2^{\circ}\text{C}$.

Fig. 3. Density histogram for all ice samples at $T = -12^{\circ}\text{C}$ and $T = -2^{\circ}\text{C}$.

mms^{-1} . With samples of length $L = 19 \text{ cm}$, this gives a range of nominal strain rates ($\dot{\epsilon}_n = \dot{\chi}/L$) of 1×10^{-5} to $1 \times 10^{-3} \text{ s}^{-1}$. The testing frame stiffness and loading system stiffness were measured using the approach described by Frederking and Timco (1983) to be 200 and 100 MNm^{-1} , respectively. In order to create the confined conditions for the ice, a specially built aluminum sub-press was used (see Fig. 4). In confinement, the ice specimen was put between the two confining walls of the sub-press which were firmly retained with an initial pressure of 0.1 MPa using four large threaded rods with nuts. This restraining system is stiffer than the system of C-clamps used by the authors in earlier tests (Timco and Frederking, 1983, 1984a). Both the applied load and side confining load were measured using two different load cells. The output from the load cells was fed into a strip-chart recorder. From the load-time curves, the stresses and strengths were determined as the load divided by the initial cross-sectional area of the sample. The average stress-rate ($\dot{\sigma}_a$) which is defined as the stress at yield divided by the time to failure was also determined from the curves.

For this columnar-structured ice, there is a high degree of anisotropy. In the plane of the ice cover,

Fig. 4. Photograph of an ice piece sandwiched between the confining plates in the sub-press. Note the two load cells which measure the applied and confining loads. The cloth at the bottom was used to absorb the large amount of liquid brine squeezed from the sample in the A-type tests.

the ice can be considered to be isotropic since there was random orientation of the c -axis of the ice grains in this plane. Because of the vertical columnar structure, there is a unique direction which is in the long direction of the grains (i.e. in the direction of growth of the ice cover). With this ice structure, there are five different test configurations that can be used. These are shown in Fig. 5 as:

Type A: Both loading and confinement in the plane of the ice cover. This measures the confined strength in the x -direction and side-loading in the y -direction, or, since the ice is isotropic in this plane, the strength in the y -direction and the side-loading in the x -direction.

Type B: Loading in the plane of the ice cover with confinement in the vertical direction. This measures the strength in either the x or y direction, and the side loading in the z -direction.

Type C: Loading in the plane of the ice cover with no confinement. This test measures the uni-axial compressive strength for horizontal loading (i.e. perpendicular to the long direction of the grains).

Type D: Loading parallel to the growth of the ice cover with confinement in the plane of the ice cover. This measures the confined strength in the z -direction and the side loading in either the x or y direction.

Type E: Loading in the direction of growth of the ice with no confinement. This measures the uni-axial compressive strength for vertical loading (i.e. parallel to the long direction of the grains).

Using these five different loading arrangements,

Fig. 5. Geometry for confined compression tests showing the five confinement arrangements.

the size and general shape of the failure envelope can be determined for columnar ice for the x - y , x - z and y - z planes in the compression-compression quadrants. When combined, they outline the failure envelope of the ice in 3-dimensional stress space.

RESULTS

Effects of confinement

Figure 6 shows typical stress-time curves for each of the five types of confinement at a nominal strain rate of $2 \times 10^{-4} \text{ s}^{-1}$ and temperature of -2°C . For confined conditions, both the applied stress and side-confining stress are shown. A comparison of their values at yield for all investigated loading rates is shown in Fig. 7 for A, B and D type confinement. From these figures, there are several things to note. For C-type confinement, which is the conventional uni-axial loading in the horizontal direction, the stress-time curve illustrates typical ductile failure at this strain rate and temperature. For B-type confinement, the loading characteristics are similar with only very small ($<0.2 \text{ MPa}$) side confining stress, even at yield. A-type confinement, on the other hand, produces strain-hardening and appreciably higher stresses at yield for both the applied and side-loading directions. In addition, the time to failure is considerably longer than for either the B-type or C-type confinement. For this columnar ice, A-type confinement produces yield stresses which are 4–5 times higher than the stress necessary for ice failure in unconfined conditions. In addition, the confinement stress is typically 50% of the applied stress.

S2 columnar ice is characterized by a preferred basal-plane orientation and grain boundary direction such that the "easy-fail" direction is in the plane of the ice cover. In A-type confinement, the confining plates and loading platens are oriented in such a way that they prevent deformation in this plane. As such, this deformation is restricted, and the ice must deform in the long direction of the grains. This is observed after a test by examining the ice surface in the unconfined direction. In many cases, the ice deformed to give a "mottled" appearance in which individual grains were clearly evident. Because of the orientation of the grain boundaries and basal planes for S2

columnar ice, this type of deformation by elongation of the columns requires some type of non basal-plane glide. The stress for this is much greater than that associated with basal-plane glide (Higashi, 1967).

Fig. 6. Stress-time curves for each of the five confinement conditions showing both the applied and confining stress in each case at $\dot{\epsilon}_n = 2 \times 10^{-4} \text{ s}^{-1}$ and $T = -2^\circ\text{C}$.

Fig. 7. Applied stress versus side-confining stress at yield for A, B and D-type confinement.

Therefore, the stresses necessary for yield of the columnar ice with A-type confinement are significantly higher than for unconfined or B-type confinement where deformation with basal glide is allowed. This also explains the similarity in strength between B and C-type confinement for this columnar ice. For the same reasons, loading perpendicular to this direction, with either D or E-type confinement results in failure stresses which are considerably higher than for horizontal C-type loading, but very similar to strengths for A-type loading. With regard to the confining stress, it is relatively small for D-type loading.

Effects of loading rate

Figure 8 shows the change in yield stress as a function of nominal strain rate ($\dot{\epsilon}_n$) for a constant temperature of -2°C for each of the five types of confinement. For both B and C-type confinement, there is a definite increase in strength with increasing loading rate with a functional form

$$\sigma = 55(\dot{\epsilon}_n)^{0.46} \quad \text{for B-type} \quad (1)$$

and

$$\sigma = 8.9(\dot{\epsilon}_n)^{0.26} \quad \text{for C-type} \quad (2)$$

where σ is the MPa and $\dot{\epsilon}_n$ is in s^{-1} . For the other three types of confinement, however, there is little

Fig. 8. Strength versus nominal strain rate for each of the five confinement conditions at $T = -2^\circ\text{C}$.

evidence of any loading rate effects for this ice. The strength values for A, D or E-type confinement are very similar for all strain rates. Figure 8 shows that within the range of loading rates investigated, the difference between strength values decreases with either the type of confinement or loading direction as the loading rate is increased. A similar trend has been noted by Frederking (1977) in his laboratory tests studying the confined compressive strength of fresh-water ice. Note that at the higher strain rates in this test series, the E-type test produce brittle failure of the ice.

The results have also been interpreted in terms of average stress rate to failure, i.e. $\dot{\sigma}_a = \sigma/t_f$ where σ is the stress at yield and t_f is the time to failure. This was done since a comparison on a stress rate basis largely eliminates the influence of the test machine stiffness thereby allowing a ready comparison with previous tests reported in the literature (Sinha and Frederking, 1979). Figure 9 shows the yield stress as a function of average stress rate for all five types of confinement for $T = -2^\circ\text{C}$. Note the relatively low stress-rates for the A-type loading. This is a reflection of the very long times-to-failure for this type of confinement. In general, the results interpreted in terms of stress rate show the same tendencies exhibited in Figure 8 in terms of strain rate; viz, dependence of strength on stress rate for both B and C type confine-

ment with functional forms

$$\sigma = 3.3(\dot{\sigma}_a)^{0.29} \quad \text{for B-type} \quad (3)$$

and

$$\sigma = 1.9(\dot{\sigma}_a)^{0.22} \quad \text{for C-type} \quad (4)$$

where σ is in MPa and $\dot{\sigma}_a$ is in MPa s^{-1} ; no stress rate effect for any of the other three types of confinement; similar strength values for A, D and E-type confinement over the whole range of stress rates and a gradual decrease in the difference of the strength with type of confinement or loading direction with increasing stress rate.

Fig. 9. Strength versus average stress rate for each of the five confinement conditions at $T = -2^\circ\text{C}$.

Effects of temperature

In addition to the full set of tests for the five different confinement conditions for the warm columnar ice at -2°C , a separate series was performed at -12°C for the unconfined (C and E-type) conditions. This was done primarily for correlation with the existing information on columnar sea ice at that temperature. The results are presented in terms of average stress rate in Figs. 10 and 11 for C-type and E-type respectively. For both loading directions, the strength of the ice is higher for the colder temperatures over the whole range of stress rates, as expected. At the colder temperatures, relatively few tests were done, so curve fitting of the test data is not considered to be reliable. These data are compared to previously published results at a similar temperature. For the C-type tests, previous tests in which the strength is

presented in terms of average stress rate include those of Sinha (1983a, 1984) and Frederking and Timco (1983, 1984a), given in Table 1, and plotted in Fig. 10. The present tests at -12°C are in reasonable agreement with these curves. Note that for both temperatures, all C-type failures were ductile in nature. For the E-type test, the results can be compared to previous tests of the strength of vertically loaded columnar sea ice by Sinha (1983b) and Frederking and Timco (1984a). The present tests are also in reasonable agreement with these results. Note that for the E-type loading, the failure of the ice was ductile for $\dot{\sigma}_a \leq 0.1 \text{ MPa s}^{-1}$ and brittle for stress rates above this value regardless of test temperature.

Fig. 10. Strength versus average stress rate for two different temperatures for C-type loading.

Fig. 11. Strength versus average stress rate for two different temperatures for E-type loading.

TABLE 1

Summary of previous tests

Temperature (°C)	Ice salinity (‰)	Average density (Mg m ⁻³)	Test type	Number of tests	Results*	Investigators
-26	3-5	(0.90)	C	7	$\bar{\sigma} = 3.9$ at $\dot{\sigma}_a = 0.074$	Frederking and Timco (1980)
-10	6-9	0.91	C	30	$\sigma = 4.5$ $(\dot{\sigma}_a)^{0.28}$	Sinha (1983a)
-11	4.5 ± 0.5	0.90	C	10	$\sigma = 5.2$ $(\dot{\sigma}_a)^{0.36}$	Frederking and Timco (1983)
-11	3.2-4.5	(0.85)	C	18	$\sigma = 4.1$ $(\dot{\sigma}_a)^{0.20}$	Frederking and Timco (1984a)
-10	4-7	0.91	C	10	$\sigma = 4.6$ $(\dot{\sigma}_a)^{0.29}$	Sinha (1984)
-26	3-5	(0.90)	E	16	$\bar{\sigma} = 13$ at $\dot{\sigma}_a = 0.27$	Frederking and Timco (1980)
-11	3.2-4.5	(0.85)	E	13	$\sigma = 14$ $(\dot{\sigma}_a)^{0.22}$	Frederking and Timco (1984a)
-10	3.0-4.5	0.90	E	20	$\sigma = 25$ $(\dot{\sigma}_a)^{0.34}$	Sinha (1983b)

* σ = MPa, $\dot{\sigma}_a$ = MPas⁻¹

Effects of brine volume and porosity

Results of strength tests on sea ice are frequently presented in terms of the brine volume (or square root of the brine volume fraction) of the ice. This is done since sea ice consists of a complex mixture of ice, air, salts and liquid brine such that the relative amount of each of these constituents is determined by the temperature, salinity, density and the corresponding phase diagrams of the various salts in the ice (see e.g. Weeks and Ackley, 1982). With increasing temperature, the liquid brine inclusions within the ice increase in size. Since this increases the total porosity of the ice, the strength of the ice should exhibit a functional dependence on brine volume such that it decreases with increasing brine volume. When interpreting the test results in terms of brine volume, however, it is not always possible to do this directly since the strength of the ice is also a function of the rate of loading. As a first approximation to eliminate this effect, the present test results were normalized with respect to average stress rate. This produces a "strength number (σ_n)" which is independent of loading rate and defined as

$$\sigma_n = \sigma(\dot{\sigma}_{a,r}/\dot{\sigma}_a)^b \quad (5)$$

where σ is the measured compressive strength, $\dot{\sigma}_a$ the corresponding average stress rate, $\dot{\sigma}_{a,r}$ is a reference average stress rate, and b is some exponent. Based on the tests for the warm sea ice, and considering past tests on sea ice (see Figs. 10 and 11), a value of $b = 0.22$ for the exponent seems reasonable. As a reference stress rate, $\dot{\sigma}_{a,r} = 1 \text{ MPas}^{-1}$ is chosen for simplicity in comparison, even though it is beyond the range of actual measurements. Using this approach, the effects of loading rate are largely eliminated, thereby allowing an evaluation of the influence of brine volume and total porosity on the strength of the ice. It should be noted that Peyton (1966) has introduced a similar approach to eliminate stress rate effects (as discussed by Weeks and Assur (1967), Schwarz and Weeks (1977)). His results have been normalized using a multiple linear regression analysis and they are therefore not compatible with the present approach. The reader is cautioned not to confuse the present strength number (σ_n) with Peyton's strength index.

Figure 12 shows the strength number (σ_n) versus the brine volume in the ice for the present tests. Included in these results are a few tests which were per-

formed at a temperature of -4°C . In general, there is a decrease in strength with increasing brine volume, with large scatter in the data. For this ice, which had undergone some deterioration, it was evident that there was a considerable volume of air within the ice. As such, the total porosity of the ice would be much higher than that predicted solely on the basis of brine volume alone (i.e. liquid inclusions). Recently, Cox and Weeks (1983) have derived simple expressions which can be used to determine the amount of both brine and air within the ice, if the temperature, salinity and density are known. Using these equations and these measured quantities, the air and brine content were determined for each sample tested in the present test series. The results of the C-type compression tests are shown in Fig. 12 as a function of the total porosity (air plus brine) in the ice. From this figure, there are two things to note. Firstly, there is a much better correlation of the strength data with total porosity than with the brine porosity. Secondly, the amount of air porosity for this ice was higher by a factor of 3 to 4 than the brine porosity. This reflects the relatively low salinities and densities measured (Fig. 2 and 3). The results of Fig. 12 emphasize the need to know the total porosity of the ice in order to be able to interpret properly the strength measurement. Clearly, the density of the ice should be measured for any strength test on sea ice.

Since there was an apparent strong correlation between the uni-axial compressive strength and the total

Fig. 12. Strength number (σ_n) versus brine volume (asterisks — lower axis) and total porosity (circles — upper axis) for the present tests.

porosity, the strength number was plotted versus the square root of the total porosity for both C- and E-type loading. These are shown in Figs. 13 and 14 respectively. For both test types, there appears to

Fig. 13. Strength number versus the square root of the total porosity for the present tests for C-type loading. Included on this figure are the envelopes of data from previously published results expressed in terms of the strength number. The number beside each box represents the number of data points for each of these studies.

Fig. 14. Strength number versus the square root of the total porosity for the present tests for E-type loading. Included on this figure are the envelopes of data from previously published results expressed in terms of the strength number. The number beside each box represents the number of data points for each of these studies.

be a good correlation for all the test data with an apparent linear dependence between σ_n and $\sqrt{\nu_T}$. The present approach, therefore, seems to provide a method of comparison of compressive behaviour of sea ice in terms of brine volume, air porosity and loading rate. It is interesting to include in these figures the results of previous tests which have been made on columnar sea ice with different salinities and densities. For the C-type loading, this includes the work of Sinha (1983a, 1984), and Frederking and Timco (1980, 1983, 1984a) as summarized in Table 1. For these, if the functional dependence between the strength and stress rate was given, the strength number was taken as the value at 1 MPas^{-1} using the regression equation. If this relationship was not given, the strength values were normalized to this stress rate value using eqn. (5) and $b = 0.22$. The resulting compression strength number is plotted versus the square root of the total porosity based on the average salinity and densities for the ice. It should be noted that for the test series of Frederking and Timco (1980, 1984a), the density of the ice was not measured. Therefore, for these test series, the density was estimated based on the time of the year and the physical description of the ice. The test results for vertically-loaded (E-type) compressive strength are shown in Fig. 14 with the additional test results from Sinha (1983b) and Frederking and Timco (1980, 1984a). Examination of these figures indicates that there is a good correlation for all of the test series using this approach. A linear regression analysis of the data, with appropriate weighing according to the number of data points from each test series yields, for the C-type

$$\sigma_n = 8.4 - 14.9 \sqrt{\nu_T} \quad (6)$$

and, for the E-type

$$\sigma_n = 32.6 - 61.8 \sqrt{\nu_T} \quad (7)$$

These equations can be re-arranged as

$$\sigma_n = 8.4(1 - \sqrt{\nu_T/320}) \text{ for C-type} \quad (8)$$

$$\sigma_n = 32.6(1 - \sqrt{\nu_T/280}) \text{ for E-type} \quad (9)$$

where ν_T is in parts per thousand in eqns. (8) and (9).

Use can be made of eq. (5) to re-express these strength numbers (eqns. (8) and (9)) in terms of the uni-axial compressive strength of columnar sea ice

for the two loading directions. That is, since $\sigma_n = \sigma(\dot{\sigma}_{ax}/\dot{\sigma}_a)^b$ and $\dot{\sigma}_{ax} = 1 \text{ MPas}^{-1}$, these equations become

$$\sigma = 8.4(\dot{\sigma}_a)^{0.22}(1 - \sqrt{\nu_T/320}) \quad (10)$$

for C-type, and

$$\sigma = 32.6(\dot{\sigma}_a)^{0.22}(1 - \sqrt{\nu_T/280}) \quad (11)$$

for E-type, where σ is MPa, $\dot{\sigma}_a$ is in MPas^{-1} such that $10^{-3} \leq \dot{\sigma}_a \leq 10^0$ and ν_T is in parts per thousand. These expressions relate the uni-axial compressive strength of columnar sea ice for both horizontal and vertical loading in terms of the total porosity of the ice and the average stress-rate for loading. These equations indicate that the ratio of the uni-axial compressive strength for vertical and horizontal loading is 3.9 independent of both the porosity of the ice and loading rate. Moreover, extrapolation of both equations to high porosities indicates that the strength of the ice approaches zero for porosities above $\approx 300\%$. Extrapolation of the equations to low porosities suggests that for $\nu_{\text{tot}} = 0$, $\sigma = 8.4(\dot{\sigma}_a)^{0.22}$ and $\sigma = 32.6(\dot{\sigma}_a)^{0.22}$ for C-type and E-type loading respectively. The former is in reasonable agreement with but less than the strength measured on columnar *fresh-water* ice loaded horizontally at $T = -10^\circ\text{C}$ where $\sigma_c = 11.7(\dot{\sigma}_a)^{0.30}$ (Sinha, 1981).

In many instances, it is more useful to express the uni-axial compressive strength in terms of strain rate rather than stress rate. This can be done for both loading directions. For C-type loading, a comparison of the rate-dependence for both nominal strain rate (eqn. (2)) and average stress-rate (eqn. (4)) gives $\sigma = 8.9(\dot{\epsilon}_n)^{0.26} = 1.9(\dot{\sigma}_a)^{0.22}$ which, when substituted into eqn. (10) yields

$$\sigma = 39(\dot{\epsilon}_n)^{0.26}(1 - \sqrt{\nu_T/320}) \quad (12)$$

where σ is in MPa, $\dot{\epsilon}_n$ is in s^{-1} such that $10^{-5} \leq \dot{\epsilon}_n \leq 10^{-3}$ and ν_T is in parts per thousand. For the E-type loading, there was no rate dependence evident. In comparing Figs. 8 and 9, it would seem that for each individual test with E-type loading, a typical ratio of the loading stress-rate to strain-rate is of the order of $1 \times 10^3 \text{ MPa}$. Thus for E-type loading, eqn. (11) becomes

$$\sigma = 150(\dot{\epsilon}_n)^{0.22}(1 - \sqrt{\nu_T/280}) \quad (13)$$

where σ is in MPa, $\dot{\epsilon}_n$ is in s^{-1} such that $10^{-5} < \dot{\epsilon}_n < 10^{-3}$ and ν_T is in parts per thousand. These equations express the strength of the ice in terms of the nominal strain rate and total porosity of the ice.

It is known that because conventional test machines are not significantly stiffer than the ice sample under test, the applied strain rate is lower than the nominal strain rate (Sinha and Frederking, 1979). Recently, Timco and Frederking (1984b) have proposed a simple model to account for this effect. They found that the results of tests using conventional machines were in good agreement with similar tests performed on closed-loop machines at truly constant strain rates if the results were interpreted in terms of a sample strain rate ($\dot{\epsilon}_s$). This is a function of the nominal strain rate, physical size and strain modulus of the ice, and stiffness of the test machine. For the uni-axial tests in this series, $\dot{\epsilon}_s \approx 0.5 \dot{\epsilon}_n$ and $\dot{\epsilon}_s \approx 0.6 \dot{\epsilon}_n$ for C and E-type confinement respectively. This formulation can be used to convert these tests in terms of sample strain rate. Thus eqns. (12) and (13) become

$$\sigma = 47(\dot{\epsilon}_s)^{0.26} (1 - \sqrt{\nu_T/320}) \quad (14)$$

for C-type, and

$$\sigma = 168(\dot{\epsilon}_s)^{0.22} (1 - \sqrt{\nu_T/280}) \quad (15)$$

for E-type, where σ is in MPa, $\dot{\epsilon}_s$ is in s^{-1} such that $5 \times 10^{-6} \leq \dot{\epsilon}_s \leq 5 \times 10^{-4}$, and ν_T is in parts per thousand. These two equations ((14) and (15)) provide a wealth of information. They relate the uni-axial compressive strength of columnar sea ice explicitly in terms of loading direction, loading strain rate and total porosity in the ice, and implicitly in terms of ice salinity, temperature and density. This approach, therefore, ties together a large number of tests performed by different investigations over a range of temperatures at several locations in the Canadian Arctic.

THE FAILURE ENVELOPE OF THE ICE

Having established the yield stress and confining stresses for the ice with different confinement conditions for various loading rates, temperatures and ice porosities, it is possible to delineate the failure envelope of the ice. Both loads for any loading con-

figuration may be plotted as yield points in a two-dimensional plane (the third principal stress is zero). These then define the failure envelope of the material. Using the reference system of Fig. 5, the points on the failure envelope are shown in Fig. 15a for the $\sigma_x - \sigma_y$ plane (i.e. the plane of the ice cover) and in Fig. 15b for the $\sigma_x - \sigma_z$ or $\sigma_y - \sigma_z$ plane (i.e. perpendicular to the plane of the ice cover) for a nominal strain rate of $2 \times 10^{-4} s^{-1}$ and a temperature of $-2^\circ C$. Included on the figure are stress paths which were followed by hand-tightening the confining plates after yield for one test each of B and D-type confinement. These stress paths follow the failure envelope for these cases and illustrate the general shape of the surface in the $\sigma_x - \sigma_z$ or $\sigma_y - \sigma_z$ plane. This figure also includes the results of some shear tests performed on the same ice at the same time using the experimental approach recently described by Frederking and Timco (1984b). These give points on the failure envelope in each of the compression-tension quadrants at the point where the two are equal. For this ice, the average value for the shear strength was 500 kPa at $-2^\circ C$ (Frederking and Timco, to be published). A general outline of the failure envelope based on all of the available test data is indicated by the heavy dashed lines for each of the $\sigma_x - \sigma_y$, $\sigma_x - \sigma_z$ and $\sigma_y - \sigma_z$ planes in Fig. 15. The differences in the size and shape for each plane emphasizes the structural and mechanical anisotropy of the ice. When combined, the parts of this figure indicate the general shape and extent of the three-dimensional failure envelope of columnar sea ice at a nominal strain rate of $2 \times 10^{-4} s^{-1}$ and a temperature of $-2^\circ C$. The shape is complex, but similar to an inverted, truncated cone which is offset from the origin.

The results of the tests at different strain rates and temperatures can be used to predict the change in size and shape of the envelope with these parameters. For example, Fig. 8 which shows the strain rate effect, indicates that with a decrease in strain rate (at least for the range investigated in the present test series) the position of the A-type strengths does not change, but the C-type decreases and thus moves in along the σ_x and σ_y axes. This results in a more "pointed" shape of the envelope in the $\sigma_x - \sigma_y$ plane. In a similar manner, decreasing strain rate will result in a more slender envelope in the $\sigma_x - \sigma_z$ and $\sigma_y - \sigma_z$ planes. With regard to temperature effects, the present tests

Fig. 15. Test results showing points on the failure envelope for columnar sea ice in the (a) x - y plane and (b) x - z or y - z plane at $\dot{\epsilon}_n = 2 \times 10^{-4} \text{ s}^{-1}$ and $T = -2^\circ\text{C}$. The solid light line represents the fit of the data to an n -type yield function. The dotted line represents a fit of a von Mises criterion to the A-type data in 15(a). The heavy dashed line is a modified n -type function which gives a good representation of the total failure envelope.

and some previous work on the confined compressive strength of columnar sea ice can be used to predict the failure envelope at lower temperatures. For example, at -10°C , Timco and Frederking (1983) found that the A-type confined strength is ~ 4 times higher than the unconfined (C-type) strength at the same nominal strain rate. This is in good agreement with the present data at the higher ice temperature. Also, the C and E-type strengths have been measured and these provide points on the failure envelope. For a nominal strain rate of $2 \times 10^{-4} \text{ s}^{-1}$ and a test temperature of -10°C , the strength of the ice is $\sim 9.5 \text{ MPa}$, 2.5 MPa and 10.5 MPa for A, C and E-type respectively. The results of shear tests on columnar sea ice at this temperature give values of 625 kPa for shear in the σ_x - σ_z or σ_y - σ_z plane, and 750 kPa for shear in the σ_x - σ_y plane (Frederking and Timco, to be published). Assuming that the ratio of the applied to confining loads for A-type tests is independent of temperature, the failure envelope for ice at $T = -10^\circ\text{C}$ and $\dot{\epsilon}_n = 2 \times 10^{-4} \text{ s}^{-1}$ can be predicted as shown in Fig. 16. Included on this figure is the outline of the failure envelope for granular/discontinuous-columnar ice at the same temperature and loading rate (Timco and Frederking, 1984a). Comparing Figs. 15 and 16, it is clear that although the overall shape of the failure envelope is the same for columnar ice, it is

substantially larger at the lower temperature. Moreover, a comparison of the failure envelope for columnar ice to that for granular/discontinuous columnar sea ice illustrates that significantly higher stresses can be sustained by columnar ice before it will fail. This has important implications with regard to the loads which an ice sheet can exert on an icebreaker or an Arctic structure. It clearly emphasizes the need to take into account both the uni-axial compressive strength and the grain structure of the ice in evaluating the results of icebreaker trials, or in estimating the loads that an ice sheet can exert on a structure.

APPLICATION OF RESULTS

The results of these confined compression tests provide information on the mechanical behaviour of sea ice which can be used in various analytical approaches to predict ice loads on structures. For their use, appropriate representations of the mechanical properties are necessary. These are usually presented in terms of either a mathematical description of the failure envelope or the constitutive relationship of the ice. The present tests can be used to evaluate the types of yield functions which are most suitable for describing ice strengths and to determine the appro-

Fig. 16. Outline of the failure envelope based on the present and previous tests on columnar sea ice at $\dot{\epsilon}_n = 2 \times 10^{-4} \text{ s}^{-1}$ and $T = -10^\circ\text{C}$. The solid line represent the extent of the failure envelope for granular/discontinuous-columnar sea ice at the same temperature and strain rate.

appropriate values of the yield function coefficients. In addition, the tests can be used to provide information on the constitutive relationships for ice. The general procedure for using the present test results for the former is briefly outlined below. For the latter application, the reader is referred to the review article by Sanderson (1984).

Several different formulations have been used to mathematically describe the failure envelope for ice including the von Mises (Michel and Toussaint, 1977), Tresca (Croasdale et al., 1977), n -type (Reinicke and Ralston, 1977; Ralston, 1978) and modified Drucker-Prager (Karr and Das, 1983a, 1983b). Of all of these methods, only the n -type yield function can represent the anisotropic nature of the ice. An accurate description of the failure envelope is important since several problems in ice mechanics can be treated analytically using plasticity theory if the failure envelope for the ice is known. Typical examples include the determination of the ice forces on a flat indenter (Ralston, 1978; Reinicke, 1980) or on a conical structure (Ralston, 1980).

In brief, yield functions for elastic-perfectly plastic materials are usually presented in the form

$$f(\sigma_{ij}) = 0 \quad (16)$$

where $f(\sigma_{ij})$ is some algebraic combination of stress components σ_{ij} . For $f < 0$, the stress state is elastic

whereas for $f = 0$, the material is at yield (i.e. in the plastic state). For columnar ice which has transverse isotropy (i.e. isotropic in the plane of the ice cover), Ralston (1978) has shown that a function of the following form can be used to describe the failure envelope in terms of the normal stresses (σ) and shear stresses (τ)

$$f(\sigma_{ij}) = a_1[(\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2] + a_3(\sigma_x - \sigma_y)^2 + a_4(\tau_{yz}^2 + \tau_{zx}^2) + a_6\tau_{xy}^2 + a_7(\sigma_x + \sigma_y) + a_9\sigma_z - 1 \quad (17)$$

where $a_6 = 2(a_1 + 2a_3)$. The values of the coefficients a_1 , a_3 , a_4 , a_7 and a_9 can be determined from the present test results. To represent the failure envelope in the 2-dimensional σ_x - σ_y plane, $\sigma_z = 0$ and eqn. (17) is simplified to

$$f(\sigma) = a_1[\sigma_y^2 + \sigma_x^2] + a_3(\sigma_x - \sigma_y)^2 + a_6\tau_{xy}^2 + a_7(\sigma_x + \sigma_y) - 1 \quad (18)$$

where it is assumed that $\tau_{xz} = \tau_{yz} = 0$. In a similar manner, for either the σ_x - σ_z or σ_y - σ_z plane, eqn. (17) reduces to

$$f(\sigma) = a_1[\sigma_z^2 + (\sigma_z - \sigma_x)^2] + a_3\sigma_x^2 + a_4\tau_{xz}^2 + a_7\sigma_x + a_9\sigma_z - 1 \quad (19)$$

and

$$f(\sigma) = a_1[\sigma_z^2 + (\sigma_y - \sigma_z)^2] + a_3\sigma_y^2 + a_4\tau_{yz}^2 + a_7\sigma_y + a_9\sigma_z - 1 \quad (20)$$

respectively. Using the approach outlined by Ralston (1978) and Timco and Frederking (1984a), the values of the coefficients can be determined. For the $\sigma_x - \sigma_y$ plane, the test results for the A-type, C-type and shear tests are substituted into eqn. (18). Similarly for the $\sigma_x - \sigma_z$ and $\sigma_y - \sigma_z$ planes, the results of the E-type and shear tests are substituted into either eqn. (19) or (20). This gives five equations for the five unknown coefficients. Solving the equations to fit elliptical functions yields $a_1 = 0.1 \text{ MPa}^{-2}$, $a_3 = 1.0 \text{ MPa}^{-2}$, $a_4 = 4.0 \text{ MPa}^{-2}$, $a_7 = 0.9 \text{ MPa}^{-1}$ and $a_9 = 1.2 \text{ MPa}^{-1}$. Substituting these values into eqns. (17) gives the description of the failure envelope in terms of an n -type yield function. With these coefficients, eqns. (18)–(20) become

$$f(\sigma) = 0.1(\sigma_x^2 + \sigma_y^2) + (\sigma_x - \sigma_y)^2 + 0.9(\sigma_x + \sigma_y) - 1 = 0 \quad (21)$$

$$f(\sigma) = 0.1 \sigma_z^2 + 0.1(\sigma_z - \sigma_x)^2 + \sigma_x^2 + 0.9 \sigma_x + 1.2 \sigma_z - 1 = 0 \quad (22)$$

$$f(\sigma) = 0.1 \sigma_z^2 + 0.1(\sigma_z - \sigma_y)^2 + \sigma_y^2 + 0.9 \sigma_y + 1.2 \sigma_z - 1 = 0 \quad (23)$$

for the $\sigma_x - \sigma_y$, $\sigma_x - \sigma_z$ and $\sigma_y - \sigma_z$ planes respectively. These curves are shown assuming $\tau = 0$ as the light solid lines in Fig. 15 along with the test data used to determine the coefficients. In examining the shape of the failure envelope for the columnar sea ice as derived using this approach, it would seem that although the fit is reasonable for the $\sigma_x - \sigma_z$ and $\sigma_y - \sigma_z$ planes, the derived function does not adequately describe the shape of the failure envelope in the $\sigma_x - \sigma_y$ plane. This occurs because the general shape of the envelope is not elliptical in this plane. Because of this, using this curve fitting procedure to fit the test data to an elliptical shape produces a poor fit. Clearly, the ellipse described by eqn. (21) extends much too far into the compression–compression quadrant. As such, it appears that although an n -type yield function is a suitable description of the failure envelope for granular/discontinuous columnar ice (Timco and Frederking, 1984a), it does not adequately represent the failure envelope for columnar ice.

Recently, Karr and Das (1983a, 1983b) have suggested that a more accurate description of the failure envelope for ice may be obtained by combining two different yield functions. This can be done in the

present case. From Fig. 15, it is evident that the n -type function gives a good representation of the envelope in the region where the stresses are equal to or less than the A-type strength of the ice. Thus, the failure criterion could be expressed as a *modified n*-type function by combining the n -type failure criterion with another type of failure criterion which would limit the envelope in the compression–compression quadrant. To do this, for example, a von Mises criterion can be used. The generalized von Mises criterion can be expressed as

$$(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 = 6K^2 \quad (24)$$

where K is a strength parameter constant (Paul, 1968). This approach implies that the yield behaviour can be approximated by assuming that the ice failure is influenced by both the normal and shear stresses under low hydrostatic pressure, but only by the shear stresses above a critical level of hydrostatic stress. In order to obtain a continuous function for the failure surface, it is necessary that an appropriate value of K be chosen. This can be done for the plane of the ice cover where $\sigma_z = 0$ and eqn. (24) becomes

$$(\sigma_x - \sigma_y)^2 + \sigma_x^2 + \sigma_y^2 = 6K^2 \quad (25)$$

K can be determined by substituting into this equation the appropriate values for the applied and confining stresses for the A-type failure for the ice. From Fig. 15, these would be -5.5 MPa and -3.0 MPa respectively. Thus, from eqn. (25), $K = 2.8 \text{ MPa}$ and eqn. (25) becomes

$$(\sigma_x - \sigma_y)^2 + \sigma_x^2 + \sigma_y^2 - 45.6 = 0 \quad (26)$$

This curve is shown in fig. 15 as the thin dotted line. The failure envelope in the $\sigma_x - \sigma_y$ plane for warm ($T = -2^\circ\text{C}$) columnar sea ice at $\dot{\epsilon}_n = 2 \times 10^{-4} \text{ s}^{-1}$ can be represented as

$$\begin{aligned} f_1 &= 0.1(\sigma_x^2 + \sigma_y^2) + (\sigma_x - \sigma_y)^2 + 0.9(\sigma_x + \sigma_y) - 1 \\ &\quad \text{for } \sigma_x + \sigma_y \geq \gamma \\ f_2 &= (\sigma_x - \sigma_y)^2 + \sigma_x^2 + \sigma_y^2 - 45.6 \\ &\quad \text{for } \sigma_x + \sigma_y < \gamma \end{aligned} \quad (27)$$

where γ is the sum of the applied and confining load for A-type compression ($= -8.5 \text{ MPa}$ in this case). This expression which represents the case of plane stress is shown in Fig. 15a as the heavy dashed line. This modified n -type function appears to be a reason-

able representation of the failure envelope for plane stress conditions. In a similar manner, the full failure envelope for colder ($T = -10^\circ\text{C}$) columnar sea ice can be determined as

$$f_1 = 0.04[(\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2] + 0.43(\sigma_x - \sigma_y)^2 + 2.52(\tau_{yz}^2 + \tau_{xz}^2) + 1.78\tau_{xy}^2 + 0.78(\sigma_x + \sigma_y) + 0.74\sigma_z - 1 \quad \text{for } \sigma_x + \sigma_y \geq \gamma$$

$$f_2 = (\sigma_x - \sigma_y)^2 + \sigma_x^2 + \sigma_y^2 - 136 \quad \text{for } \sigma_x + \sigma_y < \gamma \quad (28)$$

where $\gamma = -14.7$ MPa in this case. This is shown in Fig. 16.

CONCLUSIONS

The results of the present tests indicate the general behaviour and strength of columnar sea ice under confined conditions. The test results indicate:

(1) Confinement conditions do not appreciably affect the strength of the ice if the confinement is of either B or D-type. For A-type confinement, however, the stress level in the ice can be over four times higher than the strength of the ice with no confinement.

(2) A-type confinement on columnar sea ice produces strain-hardening with correspondingly longer times to failure.

(3) The ratio of the confinement stress to applied stress at yield for the test apparatus used is of the order of 58%, 10% and 4% for A-type, B-type and D-type confinement, respectively.

(4) Comparison of unconfined tests indicates that the strength of columnar ice loaded vertically is approximately four times higher than the strength of the ice loaded horizontally.

(5) A "strength number" has been introduced which ties together the results of the present tests and a number of previous tests for uni-axial loading. This leads to an empirical relationship which gives the functional dependence of the uni-axial compressive strength in terms of loading direction, loading rate, temperature, salinity, density and total porosity.

(6) For columnar ice, over the range of loading rates of 10^{-5} to 10^{-3} s^{-1} , there is a power law dependence of the strength of the ice for B and C-type loading only. For the other three types of confinement, there is no apparent loading rate effect.

(7) The three-dimensional yield surface for columnar ice changes both size and shape with changes in loading rate.

(8) With changes in temperature, the failure envelope remains similar in shape, but becomes appreciably larger as the temperature decreases.

(9) The results of confined compression tests can be used to evaluate the general applicability of various formulations describing the failure envelope of the ice. Moreover, the results can be used to evaluate the coefficients which give the best description of the yield surface.

(10) The results of the present tests have been analyzed and fit to a modified n -type yield function for temperatures of -2 and -10°C at $\dot{\epsilon}_n = 2 \times 10^{-4} \text{ s}^{-1}$. These expressions, together with an appropriate representation of the rheological behaviour of the ice should provide a solid basis for the use of plasticity theory to analyze various problems in ice mechanics and Arctic engineering.

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