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Effective Bloch equations for strongly driven modulation-doped quantum wells

Alexandra Olaya-Castro,¹ Marek Korkusinski,¹ Pawel Hawrylak,¹ and Misha Yu. Ivanov²

¹*Institute for Microstructural Sciences, National Research Council of Canada, Ottawa, Canada K1A 0R6*

²*Steacie Institute for Molecular Sciences, National Research Council of Canada, Ottawa, Canada K1A 0R6*

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We study the effects of macroscopic carrier density on intersubband transitions in a modulation-doped quantum well with two electronic subbands coupled by an intense ac electric field. We propose an ansatz wave function consistent with the Hartree-Fock approximation, and we use it to derive and analyze the effective nonlinear Bloch equations.

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I. INTRODUCTION

There is currently an interest in exploiting intersubband transitions in semiconductor quantum wells for detectors, emitters, quantum cascade lasers, and lasers without inversion in the far-infrared and terahertz region.¹ Many of the physical concepts considered in this context originate from atomic physics, where one manipulates well-defined quantum levels.² In quantum wells, however, a macroscopic number of carriers participates in intersubband transitions³ and analogies with atomic physics are not obvious. We are particularly motivated by recent experiments on quantum fountain laser in coupled quantum wells by Liu *et al.*⁴ In these experiments, two subbands of a modulation-doped quantum well are optically driven by a CO₂ laser. In related experiments, intersubband transitions in intense laser fields have been studied experimentally^{5–8} and theoretically.^{9–14} In theoretical works, the effects of electron-electron interactions have been included mainly in the Hartree approximation by, e.g., Zaluzny¹⁰ and Birnir and Galdrikian.¹³ The effect of the exchange and correlations were studied using semiconductor Bloch equations (SBE's) (exchange only)^{11,12,14,15} and the time-dependent density functional approach.¹⁶ The effects of electron-electron correlations in quantum dots in intense laser field have also been studied using exact diagonalization techniques.^{17,18} The SBE approach typically relies on the equation of motion approach which is not transparent to experimentalists, as it resorts early on to numerical analysis, and often neglects exchange and correlations.

In this work we formulate the analogy of intersubband transitions in intense laser fields with two-level atoms starting from the ansatz many-body wave function. We postulate a correlated wave function, show how to reduce it to a Hartree-Fock (HF) approach, derive SBE directly from the HF wave function, and reduce SBE to an effective Bloch equation for a two-level system familiar from the atomic physics.¹⁹ The effect of a macroscopic number of carriers translates into a nonlinear Bloch equation. The nonlinearity is carefully traced to depolarization effects, i.e., screening of the applied electric field, and to self-energy and vertex corrections. The self-energy and vertex corrections are natural and compensating components of any response from the Hartree-Fock ground state. Here we clarify how they contribute to the response to intense ac fields.

II. THE MODEL

In order to make a clear analogy with two-level systems we focus on a symmetric double quantum well (DQW) for which only two subbands are relevant, as illustrated in Fig. 1(a). The single-particle states of the DQW are $|kn\sigma\rangle$ with energies given by $\epsilon(kn\sigma) = E_n + k^2/2m^*$. Here $n=0,1$ is the subband index, E_n is the quantized subband energy, $\vec{k}$ is the wave vector parallel to the plane, σ is the z th component of the spin, and m^* is the electronic effective mass. The lowest subband $n=0$ has even parity, and the first excited subband $n=1$ has odd parity. The two lowest levels are well separated from other subbands, with spacing tunable by the barrier width and height. The lowest level is filled with electrons, with Fermi energy less than the energy of the $n=1$ subband. The two levels $n=0$ and $n=1$ are coupled through a time-dependent field $\mathcal{E}(t)$ applied along the growth direction (z axis). We assume the radiation field to induce wave vector and spin-conserving transitions between states with different parity, as illustrated in Fig. 1(b).

The Hamiltonian for the system of electrons in a double quantum well has the form

FIG. 1. (a) Potential profile, energy levels, and wave functions of the lowest (solid line) and first excited (dotted line) subbands in a double symmetric quantum well. (b) Schematic energy dispersion $E(k)$ for the first two subbands. In the lowest order, the electric field induces spin and wave vector-conserving transitions and each pair of occupied $\vec{k}$ levels forms a driven two-level system. (c) Pair excitations with net wave vector $\vec{k}=0$ from the “light dressed” occupied levels to bare undressed unoccupied levels.

$$\begin{aligned}
2H = & \sum_n \sum_{\vec{k}\sigma} \epsilon(kn\sigma) c_{\vec{k}n\sigma}^+ c_{\vec{k}n\sigma} \\
& + \sum_{nn'} \sum_{\vec{k}\vec{k}'\sigma} \langle \vec{k}n | ez\mathcal{E}(t) | \vec{k}'n' \rangle c_{\vec{k}n\sigma}^+ c_{\vec{k}'n'\sigma} \\
& + \frac{1}{2} \sum_{n_1 n_2 n_3 n_4} \sum_{\vec{k}_1 \vec{k}_2 \vec{k}_3 \vec{k}_4 \sigma \sigma'} \langle \vec{k}_1 n_1, \vec{k}_2 n_2 | V | \vec{k}_3 n_3, \vec{k}_4 n_4 \rangle \\
& \times c_{\vec{k}_1 n_1 \sigma}^+ c_{\vec{k}_2 n_2 \sigma'}^+ c_{\vec{k}_3 n_3 \sigma} c_{\vec{k}_4 n_4 \sigma'}. \quad (1)
\end{aligned}$$

The operator $c_{\vec{k}n\sigma}^+$ ($c_{\vec{k}n\sigma}$) creates (annihilates) a particle in a state $|\vec{k}n\sigma\rangle$. We can introduce two new creation operators identified with the two subbands: for $n=0$, $c_{\vec{k}n\sigma}^+ = a_{\vec{k}\sigma}^+$, and for $n=1$, $c_{\vec{k}n\sigma}^+ = b_{\vec{k}\sigma}^+$. The matrix elements between different states coupled by the ac electric field are defined as $\langle \sigma' n' \vec{k}' | ez\mathcal{E}(t) | \vec{k}n\sigma \rangle = \delta_{\vec{k},\vec{k}'} \delta_{\sigma,\sigma'} \delta_{n',n\pm 1} \mu \mathcal{E}(t)$, and the Coulomb matrix elements are written as $\langle \vec{k}_1 n_1, \vec{k}_2 n_2 | V | \vec{k}_3 n_3, \vec{k}_4 n_4 \rangle = v(q) F_{n_1, n_2, n_3, n_4}(q)$, with $\vec{k}_1 = \vec{k} - \vec{q}$, $\vec{k}_2 = \vec{k}' + \vec{q}$, $\vec{k}_3 = \vec{k}'$, and $\vec{k}_4 = \vec{k}$. The form factor $F_{n_1 n_2 n_3 n_4}(q) = \int \int dz dz' \xi_{n_1}(z) \xi_{n_2}(z') e^{-q|z-z'|} \xi_{n_3}(z') \xi_{n_4}(z)$ and $v(q) = 2\pi e^2 / \epsilon_0 q$, where ϵ_0 is the dielectric constant and e is the electronic charge. The form factor is nonzero for the combination of indices conserving parity in scattering: $F_{0000}, F_{1001} = F_{0110}, F_{0101} = F_{1010}, F_{1100} = F_{0011}, F_{1111}$.

III. THE WAVE FUNCTION AND BLOCH EQUATIONS

We now proceed to an investigation of the time dependence of the system. Before the external field is turned on the ground state of the system has a definite parity, i.e., all electrons are in the $n=0$ subband, filling a Fermi disk $\vec{k} \leq \vec{k}_F$. This state is $|0\rangle = \prod_{\vec{k} \leq \vec{k}_F, \sigma} a_{\vec{k}\sigma}^+ |v\rangle$, where $|v\rangle$ is the vacuum. Since the radiation induces wave vector and spin-conserving intersubband transitions between different parity states, we are only interested in states with zero total wave vector but undetermined parity. The simplest ansatz for the Hartree-Fock wave function which satisfies these requirements can be written as

$$|\Psi(t)\rangle_{\text{HF}} = \prod_{\vec{k} < \vec{k}_F, \sigma} [\alpha_{\vec{k}\sigma}(t) a_{\vec{k}\sigma}^+ + \beta_{\vec{k}\sigma}(t) b_{\vec{k}\sigma}^+] |v\rangle. \quad (2)$$

This ansatz is a HF combination of two-level states without definite parity but preserving the Fermi disk, with states occupied with probability 1 up to $k=k_F$ and 0 outside. Only occupied states are dressed by the light and renormalized. Electronic correlations change this sharp distinction between the occupied and empty states by smearing the occupation probability due to the admixture of multipair excitations. We can include these correlations by expanding the wave function in multiples of pair excitations chosen to preserve the total wave vector. This procedure is schematically illustrated in Fig. 1(c). Note that the light-dressed states are occupied states which are renormalized differently from the unoccupied undressed states, which is illustrated in Fig. 1(c) as a discontinuity of the quasiparticle dispersion at the Fermi level. If we extend the definition of HF operators into the

entire lowest “light-dressed band” as $A_{\vec{k}\sigma}^+ = \alpha_{\vec{k}\sigma}(t) a_{\vec{k}\sigma}^+ + \beta_{\vec{k}\sigma}(t) b_{\vec{k}\sigma}^+$ corresponding to each state $\vec{k}$ at a given time, the HF wave function can now be written as $|\Psi(t)\rangle_{\text{HF}} = \prod_{\vec{k} < \vec{k}_F, \sigma} [A_{\vec{k}\sigma}^+ |v\rangle]$. In a similar way we can define operators B^+ of the second light-dressed band. To account for correlations, the wave function may be expanded in pair excitations from the light-dressed Fermi disk with no net wave vector

$$\begin{aligned}
|\Psi(t)\rangle = & |\Psi(t)\rangle_{\text{HF}} + \sum_{\vec{q}\vec{k}\vec{k}'\sigma\sigma'} B(\vec{k}, \vec{k}', \vec{q}, t) A_{(\vec{k}+\vec{q})\sigma}^+ A_{(\vec{k}-\vec{q})\sigma'}^+ |\Psi(t)\rangle_{\text{HF}} + \dots \quad (3) \\
& \times A_{(\vec{k}-\vec{q})\sigma'}^+ A_{\vec{k}'\sigma'} A_{\vec{k}\sigma} |\Psi(t)\rangle_{\text{HF}} + \dots
\end{aligned}$$

The interband excitations can be added in a similar way. It is clear that one way to include pair excitations to all orders but without higher order correlations is to sum up the pair excitations in a way analogous to the linked cluster expansion

$$\begin{aligned}
|\Psi(t)\rangle = & \exp \left[\sum_{\vec{q}\vec{k}\vec{k}'\sigma\sigma'} B(\vec{k}, \vec{k}', \vec{q}, t) A_{(\vec{k}+\vec{q})\sigma}^+ A_{(\vec{k}-\vec{q})\sigma'}^+ A_{\vec{k}'\sigma'} A_{\vec{k}\sigma} \right] \\
& \times |\Psi(t)\rangle_{\text{HF}}. \quad (4)
\end{aligned}$$

The time evolution of this wave function is described by the time evolution of each two level system given by the time dependence of HF coefficients $\alpha_{\vec{k}\sigma}(t)$, $\beta_{\vec{k}\sigma}(t)$ and by correlations among excitations expressed by the coefficient $B(\vec{k}, \vec{k}', \vec{q}, t)$. Even though this is an approximate ansatz, the determination of all the coefficients is still a very complicated problem which will be tackled elsewhere. Our goal in this work is the opposite, i.e., to provide the simplest possible description of the problem with full understanding of the limitations of such a description. The first step in this direction is the reduction of the problem to as few parameters as possible, bringing out the analogy with atomic two-level systems. Hence, we neglect correlations given by coefficients $B(\vec{k}, \vec{k}', \vec{q}, t)$ and consider only parameters of each two-level system (the HF approximation). These parameters are related to the expectation values of both particle number operators $\langle \Psi(t) | a_{\vec{k}\sigma}^+ a_{\vec{k}\sigma} | \Psi(t) \rangle$, and the off-diagonal matrix elements, e.g., $\langle \Psi(t) | b_{\vec{k}\sigma}^+ a_{\vec{k}\sigma} | \Psi(t) \rangle$. To see that the off-diagonal matrix elements of this form survive, let us evaluate $\langle \Psi(t) | b_{\vec{l}}^+ a_{\vec{l}} | \Psi(t) \rangle$:

$$\begin{aligned}
\langle \Psi(t) | b_{\vec{l}}^+ a_{\vec{l}} | \Psi(t) \rangle = & \prod_{\vec{k}, \vec{k}'} \langle v | (\alpha_{\vec{k}}^* a_{\vec{k}} + \beta_{\vec{k}}^* b_{\vec{k}}) b_{\vec{l}}^+ a_{\vec{l}} (\alpha_{\vec{k}} a_{\vec{k}}^+ + \beta_{\vec{k}} b_{\vec{k}}^+) | v \rangle \\
= & \prod_{\vec{k}, \vec{k}'} [\alpha_{\vec{k}}^* \alpha_{\vec{k}} \langle v | a_{\vec{k}} b_{\vec{l}}^+ a_{\vec{l}} a_{\vec{k}}^+ | v \rangle \\
& + \alpha_{\vec{k}}^* \beta_{\vec{k}} \langle v | a_{\vec{k}} b_{\vec{l}}^+ a_{\vec{l}} b_{\vec{k}}^+ | v \rangle + \beta_{\vec{k}}^* \alpha_{\vec{k}} \langle v | b_{\vec{k}} b_{\vec{l}}^+ a_{\vec{l}} a_{\vec{k}}^+ | v \rangle \\
& + \beta_{\vec{k}}^* \beta_{\vec{k}} \langle v | b_{\vec{k}} b_{\vec{l}}^+ | v \rangle] \\
= & \beta_{\vec{l}}^* \alpha_{\vec{l}} \langle v | b_{\vec{l}} b_{\vec{l}}^+ a_{\vec{l}} a_{\vec{l}}^+ | v \rangle = \beta_{\vec{l}}^* \alpha_{\vec{l}}. \quad (5)
\end{aligned}$$

The expectation values of operators $n_0(k) = \langle a_{\vec{k},\sigma}^+ a_{\vec{k},\sigma} \rangle$, $n_1(k) = \langle b_{\vec{k},\sigma}^+ b_{\vec{k},\sigma} \rangle$, $p(k) = \langle b_{\vec{k},\sigma}^+ a_{\vec{k},\sigma} \rangle$, $p^+(k) = \langle a_{\vec{k},\sigma}^+ b_{\vec{k},\sigma} \rangle$ are simply products of corresponding coefficients α and β . Their physical meaning is related to the number of particles, e.g., in the second subband $N_1(t) = \sum_{\vec{k},\sigma} \langle b_{\vec{k},\sigma}^+ b_{\vec{k},\sigma} \rangle$, and to the complex intersubband polarization $P(t) = \sum_{\vec{k},\sigma} \langle b_{\vec{k},\sigma}^+ a_{\vec{k},\sigma} \rangle = \sum_{\vec{k},\sigma} \beta_{\vec{k},\sigma}^* \alpha_{\vec{k},\sigma}$. Their time evolution is derived from the equations of motion

$$i \frac{\partial}{\partial t} \langle \Psi(t) | b_{\vec{k},\sigma}^+ a_{\vec{k},\sigma} | \Psi(t) \rangle = \langle \Psi(t) | [b_{\vec{k},\sigma}^+ a_{\vec{k},\sigma}, H] | \Psi(t) \rangle. \quad (6)$$

Through direct evaluation of the commutators and expectation values the evolution of the density matrix is found to be given by

$$\begin{aligned} i p^+(\vec{k}, \sigma) &= -\Omega_{10}(\vec{k}, \sigma) p^+(\vec{k}, \sigma) + \Delta_{10}(\vec{k}, \sigma) [n_1(\vec{k}, \sigma) - n_0(\vec{k}, \sigma)], \\ i p(\vec{k}, \sigma) &= \Omega_{10}(\vec{k}, \sigma) p(\vec{k}, \sigma) - \Delta_{01}(\vec{k}, \sigma) [n_1(\vec{k}, \sigma) - n_0(\vec{k}, \sigma)], \\ i n_0(\vec{k}, \sigma) &= -p^+(\vec{k}, \sigma) \Delta_{01}(\vec{k}, \sigma) + p(\vec{k}, \sigma) \Delta_{10}(\vec{k}, \sigma), \\ i n_1(\vec{k}, \sigma) &= +p^+(\vec{k}, \sigma) \Delta_{01}(\vec{k}, \sigma) - p(\vec{k}, \sigma) \Delta_{10}(\vec{k}, \sigma). \end{aligned} \quad (7)$$

Here the effective energies and effective dipole moments are given by $\Omega_{10}(\vec{k}, \sigma) = E_1(\vec{k}, \sigma) + \Sigma_{11}(\vec{k}, \sigma) - E_0(\vec{k}, \sigma) - \Sigma_{00}(\vec{k}, \sigma)$, $\Delta_{01}(\vec{k}, \sigma) = \mu \mathcal{E} + V_{01}(\vec{k}, \sigma)$, and $\Delta_{10}(\vec{k}, \sigma) = \mu E(t) + V_{10}(\vec{k}, \sigma)$. The self-energies of the two levels are defined as

$$\begin{aligned} \Sigma_{00}(k, \sigma) &= \sum_{k', \sigma'} (\langle k' 0, k' 0 | V | k' 0, k' 0 \rangle \\ &\quad - \langle k' 0, k' 0 | V | k' 0, k' 0 \rangle \delta_{\sigma \sigma'} n_0(k', \sigma') \\ &\quad + (\langle k' 1, k' 0 | V | k' 0, k' 1 \rangle \\ &\quad - \langle k' 1, k' 0 | V | k' 1, k' 0 \rangle \delta_{\sigma \sigma'}) n_1(k', \sigma'), \\ \Sigma_{11}(k, \sigma) &= \sum_{k', \sigma'} (\langle k' 1, k' 1 | V | k' 1, k' 1 \rangle \\ &\quad - \langle k' 1, k' 1 | V | k' 1, k' 1 \rangle \delta_{\sigma \sigma'} n_1(k', \sigma') \\ &\quad + (\langle k' 0, k' 1 | V | k' 1, k' 0 \rangle \\ &\quad - \langle k' 0, k' 1 | V | k' 0, k' 1 \rangle \delta_{\sigma \sigma'}) n_0(k', \sigma'). \end{aligned} \quad (8)$$

The off-diagonal elements of the Coulomb interaction renormalizing the applied field are defined as

$$\begin{aligned} V_{01}(k, \sigma) &= \sum_{k', \sigma'} (\langle k' 1, k' 0 | V | k' 1, k' 0 \rangle \\ &\quad - \langle k' 1, k' 0 | V | k' 0, k' 1 \rangle \delta_{\sigma \sigma'}) p(k', \sigma') \\ &\quad + (\langle k' 1, k' 1 | V | k' 0, k' 0 \rangle \\ &\quad - \langle k' 1, k' 1 | V | k' 0, k' 0 \rangle \delta_{\sigma \sigma'}) p^+(k', \sigma'), \end{aligned}$$

$$\begin{aligned} V_{10}(k, \sigma) &= \sum_{k', \sigma'} (\langle k' 1, k' 0 | V | k' 1, k' 0 \rangle \\ &\quad - \langle k' 1, k' 0 | V | k' 0, k' 1 \rangle \delta_{\sigma \sigma'}) p^+(k', \sigma') \\ &\quad + (\langle k' 0, k' 0 | V | k' 1, k' 1 \rangle \\ &\quad - \langle k' 0, k' 0 | V | k' 1, k' 1 \rangle \delta_{\sigma \sigma'}) p(k', \sigma'). \end{aligned} \quad (9)$$

These Bloch equations are a direct consequence of the form of the HF wave function. They were derived by a different method in Ref. 11. These equations carry the information about time evolution of each occupied $\vec{k}$ state while neglecting correlations among them. This detailed microscopic information is often not needed; rather information about the macroscopic quantities such as the population inversion and the polarization of the system is important. We therefore proceed to reducing the SBE equations to a comprehensive form of effective two-level Bloch equations for total population inversion and total polarization of the two subbands.

IV. EFFECTIVE NONLINEAR BLOCH EQUATIONS

We define the three-component Bloch vector $\mathbf{S}(t)$ in terms of mean real and imaginary parts of polarization and mean population inversion per electron as

$$S_1 = [\sum_{\vec{k}, \sigma} p^+(\vec{k}, \sigma) + p(\vec{k}, \sigma)] / \sum_{\vec{k}, \sigma} n(\vec{k}, \sigma),$$

$$S_2 = \sum_{\vec{k}, \sigma} i [p^+(\vec{k}, \sigma) - p(\vec{k}, \sigma)] / \sum_{\vec{k}, \sigma} n(\vec{k}, \sigma),$$

and

$$S_3(k) = [\sum_{\vec{k}, \sigma} n_1(\vec{k}, \sigma) - n_0(\vec{k}, \sigma)] / \sum_{\vec{k}, \sigma} n(\vec{k}, \sigma).$$

Due to the conservation of the number of particles the Bloch vector satisfies $S_1^2 + S_2^2 + S_3^2 = 1$. To obtain equations for $\mathbf{S}(t)$ we replace the local behavior in Eq. (7) by corresponding mean field values and close equations. For example, when we evaluate $\sum_{\vec{k}} \Omega_{10}(\vec{k}) p_{\vec{k}}$ we use the mean value theorem to write $\sum_{\vec{k}} \Omega_{10}(\vec{k}) p_{\vec{k}} = \langle \Omega \rangle \sum_{\vec{k}} p_{\vec{k}}$, and replace the unknown value of $\langle \Omega \rangle$ with the average transition energy $\langle \Omega \rangle \approx \sum_{\vec{k}} \Omega_{10}(\vec{k}) n(\vec{k}) / \sum_{\vec{k}} n(\vec{k})$. The resulting equations lead to effective nonlinear Bloch equations for the driven double quantum well system

$$\dot{S}_1 = (\omega_{10} - \gamma S_3) S_2,$$

$$\dot{S}_2 = -(\omega_{10} - \gamma S_3) S_1 + [2\mu \mathcal{E}(t) - 2\beta S_1] S_3,$$

$$\dot{S}_3 = -[2\mu \mathcal{E}(t) - 2\beta S_1] S_2. \quad (10)$$

Here ω_{10} is the renormalized but time-independent transition energy, β is the coefficient of the nonlinear term that renormalizes the applied field due to induced polarization, and γ is a result of the interplay between vertex and self-energy corrections to the transition energy. The nonlinear coefficients can be evaluated as a function of the total number of electrons and parameters of a double quantum well. We find it sufficient to expand the form factors $F(q)$ to the lowest

order in q and define the relevant length elements $L_{n_1 n_2 n_3 n_4} = \iint dz dz' \xi_{n_1}(z) \xi_{n_2}(z') |z - z'| \xi_{n_3}(z') \xi_{n_4}(z)$ to obtain

$$\begin{aligned}\omega_{10} &= (E_1 - E_0) + \frac{1}{2} \frac{2\pi e^2}{\epsilon_0} N \left(\frac{L_{1111} - L_{0000}}{2} \right), \\ \gamma &= \frac{1}{2} \frac{2\pi e^2}{\epsilon_0} N \left(L_{1001} - \frac{L_{1111} + L_{0000}}{2} \right), \\ \beta &= \frac{1}{2} \frac{2\pi e^2}{\epsilon_0} N L_{1100}.\end{aligned}\quad (11)$$

We see that the population inversion S_3 renormalizes the transition energy ω_{10} and that the induced polarization S_1 screens the applied electric field. The renormalization of the transition energy (coefficient γ) consists of two compensating terms: the self-energy and the vertex correction. A similar compensating process takes place in the screening of applied field. The parameter β , accounting for screening, turns out to be exactly half the Hartree contribution $(2\pi e^2/\epsilon_0)NL_{1100}$, where N is the electron density. The reduction in the screening is due to the exchange correction. The exchange corrections appear as terms linear in the number of electrons N . This is rather surprising as the exchange self-energy depends on N as $N^{1/2}$. All exchange terms nonlinear in N cancelled out exactly due to the interplay of self-energy and vertex corrections. We are left only with linear terms which are the same order as Hartree contribution.

The structure of the set of equations (10) is $\vec{S} = \vec{\Omega}(\vec{S}) \times \vec{S}$, i.e., these are truly nonlinear Bloch equations.¹⁵ The driving field has the form $\vec{\Omega}(\vec{S}) = -(2\mu\mathcal{E}(t) - 2\beta) \hat{x} - \omega_{10}(S_3) \hat{z}$, where $\omega_{10}(S_3) = \omega_{10} - \gamma S_3$ is the renormalized and time-dependent transition energy. We emphasize that this renormalization is different from just a difference in self-energies and includes vertex corrections.

Finally, we model the relaxation phenomena in terms of dephasing (T_2) and population decay (T_1) times, which leads to a set of effective Bloch equations

$$\begin{aligned}\dot{S}_1 &= \omega_{10}(S_3) S_2 - \frac{S_1}{T_2}, \\ \dot{S}_2 &= -\omega_{10}(S_3) S_1 + [2\mu\mathcal{E}(t) - 2\beta S_1] S_3 - \frac{S_2}{T_2}, \\ \dot{S}_3 &= -[2\mu\mathcal{E}(t) - 2\beta S_1] S_2 - \frac{S_3 + 1}{T_1}.\end{aligned}\quad (12)$$

This treatment of relaxation draws on analogies to two-level systems but the issue of relaxation is a difficult one and is closely related to the extension of the proposed wave function to include correlations and the interaction with phonons.

V. ROTATING WAVE APPROXIMATION

The Bloch equations can be analyzed in the rotating frame of the driving field, which has been assumed to be $\mathcal{E}(t) = \mathcal{E}_0 \sin(\omega t)$. The Bloch vector can be rewritten in terms of

in-phase and out of phase components with the driving field, in the following way:

$$\begin{aligned}S_1 &= U \sin(\omega t) - V \cos(\omega t), \\ S_2 &= V \sin(\omega t) + U \cos(\omega t), \\ S_3 &= Z.\end{aligned}\quad (13)$$

In this rotating frame, the Bloch equations for U , V , and Z can be written as a combination of slowly and fast oscillating terms:

$$\begin{aligned}\dot{U} &= \widetilde{\Delta\omega_{10}}(Z) V - \frac{U}{T_2} - [\mu\mathcal{E}_0 - \beta U] Z \cos(2\omega t) \\ &\quad + \beta Z V \sin(2\omega t), \\ \dot{V} &= -\widetilde{\Delta\omega_{10}}(Z) U + \mu\mathcal{E}_0 Z - \frac{V}{T_2} + [\mu\mathcal{E}_0 - \beta U] Z \sin(2\omega t) \\ &\quad + \beta Z V \cos(2\omega t), \\ \dot{Z} &= -\frac{Z+1}{T_1} - \mu\mathcal{E}_0 V + [\mu\mathcal{E}_0 - \beta U] V \cos(2\omega t) \\ &\quad - [\mu\mathcal{E}_0 U - \beta(U^2 - V^2)] \sin(2\omega t),\end{aligned}\quad (14)$$

where $\widetilde{\Delta\omega_{10}}(Z) = \omega_{10} - \omega - \gamma Z + \beta Z$ is the renormalized transition energy. This frequency shift accounts for screening as well as vertex and self-energy effects. Neglecting fast oscillating terms in Eq. (14) leads to the rotating wave approximation (RWA). The RWA set of equations has the form $\vec{R} = \vec{\Omega}(\vec{R}) \times \vec{R}$, with $\vec{\Omega}(\vec{R}) = -\mu\mathcal{E}_0 \hat{x} - \widetilde{\Delta\omega_{10}}(Z) \hat{z}$.

VI. PUMP AND PROBE ANALYSIS

In this section we explore the response of the driven DQW system to a weak probe field $\delta\vec{S}$. In the presence of the probe field the total Bloch vector satisfies $\vec{S}_{\text{total}} = \vec{S}_{\text{pump}} + \delta\vec{S}$. Neglecting second order terms, the time evolution of the probe field is

$$\delta\dot{\vec{S}} = \vec{\Omega}(\vec{S}_{\text{pump}}) \times \delta\vec{S} + \delta\vec{\Omega}(\vec{S}) \times \vec{S}_{\text{pump}}, \quad (15)$$

where $\delta\vec{\Omega}(\vec{S}) = \delta\vec{\Omega}_0 + \delta\vec{M}_p(\vec{S})$, with $\delta\vec{\Omega}_0 = -2\mu\mathcal{E}_{\text{probe}}(t) \hat{x}$ and $\delta\vec{M}_p(\vec{S}) = 2\beta\delta S_1 \hat{x} + \gamma\delta S_3 \hat{z}$. Therefore the equations for the perturbation become

$$\begin{aligned}\delta\dot{S}_1 &= (\omega_{10} - \gamma S_3^p) \delta S_2 - \gamma S_2^p \delta S_3, \\ \delta\dot{S}_2 &= -[(\omega_{10} - \gamma S_3^p) + 2\beta S_1^p] \delta S_1 \\ &\quad + [(2\mu\mathcal{E}(t) - 2\beta S_1^p) + \gamma S_1^p] \delta S_3 + 2\mu S_3^p \mathcal{E}_{\text{probe}}(t), \\ \delta\dot{S}_3 &= -\{[2\mu\mathcal{E}(t) - 2\beta S_1^p]\} \delta S_2 + 2\beta S_2^p \delta S_1 - 2\mu S_2^p \mathcal{E}_{\text{probe}}(t).\end{aligned}\quad (16)$$

We see that the test field does not just probe the renormalized frequency $(\omega_{10} - \gamma S_3^p)$, nor the renormalized field

FIG. 2. Length element L_{1100} (a) and dipole moment z_{10} (b) as a function of the barrier width relative to the well width for a symmetric DQW.

$[2\mu\mathcal{E}(t) - 2\beta S_1^p]$ of the electronic system. It probes the combined electronic and pump photon system which leads to cancellation effects in frequency $[(\omega_{10} - \gamma S_3^p) + 2\beta S_3^p]$ due to the screening of applied field (parameter β), and in applied field $\{[2\mu\mathcal{E}(t) - 2\beta S_1^p] + \gamma S_1^p\}$ due to the renormalization of electronic transition energy (parameter γ).

VII. DISCUSSION OF RESULTS

We now turn to the numerical analysis of our nonlinear Bloch equations. We rescale the time in terms of the bare transition energy, i.e., $\tau_{10} = 1/E_{10}$ ($\hbar = 1$), measure energies in units of the effective Rydberg $\mathcal{R} = m^* e^4 / 2\epsilon_0^2 \hbar^2$, and lengths in units of the effective Bohr radius $a_B = \epsilon_0 \hbar^2 / m^* e^2$. In these units explicit expressions for coefficients are

$$\begin{aligned} \omega_{10}(S_3) &= 1 + (E_F/E_{10})(L_{1001} - L_{0000}) - (E_F/E_{10})[L_{1001} \\ &\quad - (L_{1111} + L_{0000})/2](1 + S_3), \\ \beta &= (E_F/E_{10})L_{1100}, \end{aligned}$$

here E_F is the Fermi energy, and $\mu\mathcal{E}(t) = (z_{10}/E_{10})\mathcal{E}_0 \sin(\omega t) = \mu\mathcal{E}_0 \sin(\omega t)$. As an example we compute these coefficients for a double quantum well shown in Fig. 1. We find $z_{10} = -0.51a_B$, $E_{10} = 1.12\mathcal{R}$, $\beta = -0.41(E_F/E_{10})$, $\omega_{10}(S_3) = 1 - 0.0124(E_F/E_{10})S_3$ (since $L_{1111} \approx L_{0000}$). For a density of $\approx 1.2 \times 10^{11} \text{ cm}^{-2}$, i.e., $E_F/E_{10} = 0.5$, the nonlinear coefficient $\beta = -0.2$ and $\omega_{10}(t$

FIG. 3. Real part of mean polarization S_1 as a function of time for different values of parameter β . In this calculation $\omega/\omega_{10} = 0.1$ and $\mu\mathcal{E}_0 = 0.1$.

$= 0) = 1.0124$. It is clear that for a symmetric double quantum well with these parameters the energy renormalization is very small. However, if the barrier separating the two wells is removed, we find $\beta = -0.15(E_F/E_{10})$ and $\omega_{10}(S_3) = 1 - 0.11(E_F/E_{10})S_3$, i.e., the nonlinearity in screening of the field is reduced, while the energy renormalization is increased.

In Fig. 2 we show the dipole moment z_{10} and the parameter L_{1100} as a function of the barrier width relative to the well width, for a double symmetric quantum well. As can be seen, the screening of the applied field, $\beta = (E_F/E_{10})L_{1100}$, varies linearly with the barrier width [see Fig. 2(a)], as does the dipole moment z_{10} [see Fig. 2(b)].

We now turn to explore the coherent long-time behavior of the system. In the RWA the $\dot{S} = 0$ and the adiabatic condition $V = 0$ gives the remaining U and Z components of the Bloch vector. Since $U^2 + Z^2 = 1$, we can write $U = \sin(\theta)$, $Z = \cos(\theta)$, where θ is the Euler angle. This reduces the Bloch equations to one transcendental equation

$$\tan(\theta) = \frac{\mu\mathcal{E}_0 - \beta \sin(\theta)}{\Delta\omega_{10} - \gamma \cos(\theta)}. \quad (17)$$

Its solutions, if they exist, give the stationary state of the system. Without nonlinearity the solution takes the form $\tan(\theta) = \mu\mathcal{E}_0/\Delta\omega_{10}$, i.e., $\theta = 0, \pi$, or $Z = \pm 1$ in the absence of the external field. The system is thus either in its ground state ($Z = -1$) or is inverted ($Z = +1$). When the external

FIG. 4. Imaginary part of mean polarization S_2 as a function of time, for different values of parameter β . In this calculation $\omega/\omega_{10}=0.1$ and $\mu\mathcal{E}_0=0.1$.

FIG. 5. Population inversion S_3 as a function of time, for different values of parameter β . In this calculation $\omega/\omega_{10}=0.1$ and $\mu\mathcal{E}_0=0.1$.

FIG. 6. In-phase (a) and out of phase (b) components of the Bloch vector as a function of time, for different values of parameters β , in the RWA. Time is measured in units of the effective frequency ω_{eff} . In this calculation $\omega/\omega_{10}=0.1$ and $\mu\mathcal{E}_0=0.1$.

field is very small but $\beta \neq 0$, two new solutions appear: $Z = -E_{10}/\beta$, $U = \pm \sqrt{1 - (E_{10}/\beta)^2}$. These solutions are characterized by opposite sign of polarization. They exist only when $|E_{10}|/|\beta| < 1$.

In Figs. 3, 4, and 5 we show the coherent time evolution of the Bloch vector components S_1 , S_2 , and S_3 , respectively, for slow ($\omega/\omega_{10}=0.1$) and weak ($\mu\mathcal{E}_0=0.1$) driving field, and for different values of $\beta=0, -0.2, -2.0$. Figure 4 indicates that the absorption of the radiation is very small, as is expected for this frequency. Figure 3 shows that the induced polarization follows and reduces the applied field. Such behavior is superimposed with rapid oscillations, whose frequency increases with increasing nonlinearity. These rapid oscillations can be better understood in the RWA. Since the frequency and strength of the field are small, the population inversion remains almost constant (see Fig. 5) and the renormalized transition energy becomes $\widetilde{\Delta}\omega_{10} \approx \omega_{10} - \omega + \gamma - \beta$. Therefore, the RWA equations become linear and their solutions are of the form $V = \exp(i\omega_{\text{eff}}t)$, with $\omega_{\text{eff}} = \sqrt{(\mu\mathcal{E}_0)^2 + \widetilde{\Delta}\omega_{10}^2}$. This is illustrated in Fig. 6 which shows U and V as a function of time for different values of the parameters β and γ .

We now turn to the analysis of the long-time behavior of induced polarization S_1 as a function of the strength of the applied field. In Fig. 7 we show the time evolution of S_1 for different field strengths obtained by integrating the Bloch equations without RWA. The long-time behavior of S_1 is

FIG. 7. Real part of mean polarization S_1 as a function of time for $\beta = -0.2$ and relaxation times $T_1 = T_2 = 30\tau_{10}$, and different strengths of the applied field.

extracted for times much longer than the relaxation times: first, the system is allowed to evolve 200 cycles, and then the values of S_1 are stored at fixed times equal to half a cycle of the driving field, i.e., at times when the driving field attains its maximal value and is directed either parallel or antiparallel to the z axis. The polarization as a function of field strength is shown in Fig. 8. As can be seen, if the field strength exceeds a critical value, S_1 oscillates between two opposite values, corresponding to the same population inversion. Such critical behavior depends on the value of the nonlinearity β as a function of the applied field. The values of the steady-state solution can be obtained by solving the equation $T^{-1}\vec{S} = \vec{\Omega}(\vec{S}) \times \vec{S}$ with constant driving field. The results shown here agree qualitatively with those of Ref. 13, obtained in Hartree approximation and for finite temperatures. The main difference is the lack of chaotic behavior observed in Ref. 13.

VIII. CONCLUSIONS

In conclusion, we derived effective nonlinear Bloch equations describing the effect of macroscopic carrier density on intersubband transitions in modulation doped quantum well

FIG. 8. Real part of mean polarization S_1 as a function of the strength of the applied field for $\beta = -0.2$ (a) and $\beta = -2.0$ (b). In this calculation the relaxation times $T_1 = T_2 = 10\tau_{10}$ and $\omega/\omega_{10} = 0.1$.

with two electronic subbands coupled by an intense ac electric field. We proposed the correlated wave function and systematically reduced it to an effective two-level nonlinear Bloch equation. Further, we analyzed the properties of this equation and calculated the response of the system to the test field. Our future work will focus on the effect of correlations and coupling to optical and acoustical phonons.

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¹For review and references see *Intersubband Transitions in Quantum Wells: Physics and Device Applications*, Vol. 62 of *Semiconductors and Semimetals*, edited by H.C. Liu and F. Capasso.

²M. Gavrilu and J.Z. Kaminski, Phys. Rev. Lett. **52**, 613 (1984); M. Pont, N.R. Walet, M. Gavrilu, and C.W. McCurdy, *ibid.* **61**, 939 (1988); J. Purvis, M. Dörr, M. Terao-Dunseath, C.J. Joachain, P.G. Burke, and C.J. Noble, *ibid.* **71**, 3943 (1993); P.B. Corkum, *ibid.* **71**, 1994 (1993).

³For a review of excitations in quantum wells see, e.g., J.J. Quinn, G. Eliasson, and P. Hawrylak, in *Electromagnetic Waves: Recent Developments in Research*, edited by P. Halevi (North Holland,

Amsterdam, 1992), Vol. 1.

⁴H.C. Liu, I.V. Cheung, A.J. SpringThorpe, C. Dharma-wardana, Z.R. Wasilewski, D.J. Lockwood, and G.C. Aers, Appl. Phys. Lett. **78**, 3580 (2001).

⁵D. Fröhlich, R. Wille, W. Schlapp, and G. Weimann, Phys. Rev. Lett. **59**, 1748 (1987).

⁶K. Unterrainer, B.J. Keay, M.C. Wanke, S.J. Allen, D. Leonard, G. Medeiros-Ribeiro, U. Bhattacharya, and M.J.W. Rodwell, Phys. Rev. Lett. **76**, 2973 (1996).

⁷E. Dupont, P.B. Corkum, H.C. Liu, M. Buchanan, and Z.R. Wasilewski, Phys. Rev. Lett. **74**, 3596 (1995).

- ⁸J.N. Heyman, K. Craig, B. Galdrikian, M.S. Sherwin, K. Campman, P.F. Hopkins, S. Fafard, and A.C. Gossard, Phys. Rev. Lett. **72**, 2183 (1994); K. Craig, B. Galdrikian, J.N. Heyman, A.G. Markelz, J.B. Williams, M.S. Sherwin, K. Campman, P.F. Hopkins, and A.C. Gossard, *ibid.* **76**, 2382 (1996).
- ⁹M. Holthaus and D. Hone, Phys. Rev. B **47**, 6499 (1993); R. Bavli and H. Metiu, Phys. Rev. Lett. **69**, 1986 (1992); D.D. Coon and H.C. Liu, J. Appl. Phys. **58**, 2230 (1985).
- ¹⁰M. Zaluzny, Phys. Rev. B **47**, 3995 (1993).
- ¹¹S.L. Chuang, M.S.C. Luo, S. Schmitt-Rink, and A. Pinczuk, Phys. Rev. B **46**, 1897 (1992).
- ¹²D.E. Nikonov, A. Imamoglu, L.V. Butov, and H. Schmidt, Phys. Rev. Lett. **79**, 4633 (1997).
- ¹³B. Galdrikian and B. Birnir, Phys. Rev. Lett. **76**, 3308 (1996).
- ¹⁴A. Batista, B. Birnir, and P. Tamborenea, cond-mat/0102263 (unpublished).
- ¹⁵P. Hawrylak, in *Light Scattering in Semiconductor Structures and Superlattices*, edited by D.J. Lockwood and J.F. Young (Plenum, New York, 1991), p. 479.
- ¹⁶G. Vignale and W. Kohn, Phys. Rev. Lett. **77**, 2037 (1996); G. Vignale, C.A. Ullrich, and S. Conti, *ibid.* **79**, 4878 (1997); C.A. Ullrich and G. Vignale *ibid.* **87**, 037402 (2001).
- ¹⁷P.I. Tamborenea and H. Metiu, Phys. Rev. Lett. **83**, 3912 (1999).
- ¹⁸P. Hawrylak, P.A. Schultz, and J.J. Palacios, Solid State Commun. **93**, 909 (1995); P. Hawrylak and L. Rego, Physica E (Amsterdam) **3**, 198 (1998).
- ¹⁹L. Allen and J.H. Eberly, *Optical Resonance and Two-level Atoms* (Dover, New York, 1987).