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Evidential Reasoning for the Treatment of Incoherent Terminologies

ABSTRACT

Many reasoning algorithms and techniques require consistent terminologies to be able to operate correctly and efficiently. However, many ontologies become inconsistent during their evolution and lifecycle. Many methods have been proposed to handle inconsistent terminologies including those that tolerate or repair inconsistencies. Most of these approaches focus on the syntactic properties of ontology terminologies and attempt to address inconsistency from that perspective and satisfy postulates such as the principle of minimal change. In this paper, we will employ evidential reasoning to take into account assertional statements of an ontology as observations and probable indications for the correctness and validity of one axiom over other competing axioms. We will show how ontology assertions are beneficial in ranking axioms to be used in Reiter's hitting set algorithm.

Categories and Subject Descriptors

I.2.4 [Knowledge Representation Formalisms and Methods]: Representation languages, Representations; H.4.m [Information Systems Applications]: Miscellaneous; H.4 [Information Systems Applications]: Miscellaneous

General Terms

Theory

Keywords

Evidential Reasoning, Inconsistency Resolution, Debugging Incoherent Terminologies, Subjective Logic, Description Logic

1. INTRODUCTION

The growth and effectiveness of core semantic technologies has paved the way for rich semantic-driven applications that require rich structured knowledge in the form of ontologies. For this reason, various efficient ontology development and maintenance methodologies have been developed to support experts in their pursuit of the creation of complete and consistent domain specific ontologies [9, 22]. However, the problem with the development of such com-

plete ontologies is as the sage persian minister, Bozorgmehr¹, puts it *everyone knows everything, and everyone has not yet been born*. In other words and for our context, the required knowledge about a specific domain is usually dispersed between different groups of experts and often captured in fragmented pieces of formulated ontologies. So, a reasonable way to approach domain specific ontology development would be the employment of ontology modularization and distributed ontology development techniques where various teams contribute to the advancement of the knowledge modeling process [10]. Another viable solution is to use the World Wide Web as a very rich source of mostly unstructured or semi-structured information and try to learn structured information (ontologies) from its content [7]. Such approaches guarantee that a diverse range of information are gathered and the formed ontologies are rather comprehensive. However, the integration of information from multiple sources can entail inconsistencies in the structure of knowledge. For instance, one group of experts may perceive and hence define a concept different from another group giving way to inconsistencies [2, 1].

There have been two fundamental approaches for dealing with inconsistency or incoherency in description logic ontologies [4]. Some approaches tend to tolerate inconsistencies and allow for logical contradictions to exist in the knowledge structure and employ non-standard reasoning methods, e.g. based on multi-valued semantics [20], to infer meaningful relevant information for a given query. The basic idea here is to find the most relevant consistent sub-theory of the description logic knowledge base to the query at hand based on some measure of relevance that could be employed in the reasoning procedure. The other strand of work, often referred to as inconsistency intolerant approaches, enforce the resolution of logical inconsistencies as soon as they are encountered. Such approaches attempt to debug inconsistent ontologies to find the most precise justification as a diagnosis for the cause of the problem and provide techniques for repairing and resolving the issue. Most of the techniques in both approaches consider syntactical ontology information to resolve inconsistency. For instance, some techniques focus on respecting the *principle of minimality of change* or some on a revised subset of the AGM postulates in belief revision [11, 12]. Such principles can be regarded as the most rational in the absence of higher level semantic information. The restriction of the approaches based on these principles is that although they preserve interesting syntactical properties, they fail to find a semantic relevance for the inconsistency resolution strategies. One of the reasons for this may be that the resolution techniques concentrate only on the terminological component of the ontologies. However, it can be interesting to look at the ontology assertion component as

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well and use evidential reasoning to resolve inconsistencies². Resolution strategies in such an approach may be closer to the semantic level.

Evidential reasoning has close resemblance to the issue of inconsistency resolution. Within its context, reasoners often have to make decisions based on incomplete or conflicting information; therefore, a typical reasoning process involves the formulation of several possible hypotheses, which is accompanied by a search for facts that corroborate or dissonate with them [29]. For the reasoning process to make sense to human users, the process for finding relevant facts to the hypotheses would need to identify with the strategies that human beings adopt in decision making in face of incomplete and competing goals and values. Piatelli-Palmerini [23] argues that in this regard three major strategies exist: 1) finding the minimum number of evidence to support the first good enough alternative; 2) looking at all possible evidence to find the hypothesis that benefits from the greatest support; 3) choosing the hypothesis that is most likely to prevent some known undesired entailments. So as we can see real-life evidence-based reasoning is rarely about establishing *the ultimate truth* about some state, rather it is about collecting evidence to reach an acceptable conclusion [8].

As it may turn out to be in real-life scenarios or critical decision making situations, as the information becomes more complete, previous evidences may lose their credibility or get proven to be incorrect due to various reasons such as the inaccuracy of the measurement device, etc, but nevertheless the factual support based on the current evidence is the best tool that one can employ to base his/her judgments on. Formally stated, in evidential reasoning, hypotheses are evaluated based on the current hypothesis space that constitutes a set of finite observations and a likelihood function over the observations. The reasoning process is itself a cognitive process that acts over the hypothesis space to evaluate the possible hypotheses [29].

In this paper, we intend to build on top of the currently available syntactical approaches for resolving incoherency and move towards a more naturally intuitive way to resolve incoherencies in the terminological component of the ontologies by using an evidence-based reasoning process that views the ontology assertion component as the finite set of observations of the hypothesis space. The contributions of this paper can be summarized as follows:

- We provide a formalization of the resolution of incoherencies in the ontology terminology component based on an evidence-based reasoning process supported by assertions from the corresponding ontology assertion component;
- We show how Subjective logic, an extension to the Dempster-Shafer theory of evidence, can be used to compute the degree of *evidential harmony* or *evidential contradiction* for terminological axioms;
- A modified version of Reiter's Hitting Set algorithm is presented that supports for finding the most suitable resolution strategy based on evidential information.

The rest of the paper is organized as follows. In the next section, we give some preliminaries on Description Logics and Subjective

² As will be defined formally in the preliminaries, this paper focuses on incoherencies. The introduction section uses inconsistency and incoherency interchangeably.

Logic. Section 3 provides a formalization of incoherency resolution as an evidential reasoning process. In Section 4, suitable definitions for evidential harmony and evidential contradiction are provided, followed by a brief introduction to the concept of minimal unsatisfiability-preserving sub-TBoxes. The modified version of Reiter's algorithm is given in Section 6. Finally, in Section 7 we discuss some related work and concluded the paper in Section 8.

2. PRELIMINARIES

2.1 Description Logics

The knowledge base of a description logic based ontology is defined as $\Psi = \langle \mathcal{T}, \mathcal{A} \rangle$, where $\mathcal{T}$ denotes TBox and comprises of a set of general inclusion axioms and $\mathcal{A}$ stands for ABox and comprise of a set of instance assertions. The signature of an ontology is defined as a set of all concept names (C_N), role names (R_N) and individuals (I_N) which are included its knowledge base. The semantic of an ontology is defined by an interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ where $\Delta^{\mathcal{I}}$ is a non-empty set of individuals and $\cdot^{\mathcal{I}}$ is a function which maps each $C \in C_N$ to $C^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$, each $R \in R_N$ to $R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ and each $a \in I_N$ to an $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$. An interpretation $\mathcal{I}$ satisfies a TBox axiom $C \sqsubseteq D$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$, satisfies an ABox assertion $C(a)$ iff $a^{\mathcal{I}} \in C^{\mathcal{I}}$ and an ABox assertion $R(x, y)$ iff $\langle x^{\mathcal{I}}, y^{\mathcal{I}} \rangle \in R^{\mathcal{I}}$.

An interpretation $\mathcal{I}$ is a model of a TBox $\mathcal{T}$ iff it satisfies all of its axioms. An interpretation $\mathcal{I}$ is a model of a knowledge base Ψ if it satisfies every TBox axiom and ABox assertion of Ψ . A concept C is *satisfiable* with regards to a TBox $\mathcal{T}$ iff there exists a model $\mathcal{I}$ for $\mathcal{T}$ such that $C^{\mathcal{I}} \neq \emptyset$.

In this paper, we distinguish between the notions of incoherency and inconsistency in DL-based ontologies. A DL knowledge base Ψ is inconsistent iff there is no model for it such that satisfies all of its axioms and assertions. A DL knowledge base $\Psi = \langle \mathcal{T}, \mathcal{A} \rangle$ is incoherence iff there is a concept name in its signature that is not satisfiable with regards to $\mathcal{T}$. In other words, the notion of coherency is related to the TBox axioms and the satisfiability of concept names and is independent from the ABox assertions. Observably, a knowledge base can be coherent even if it is not consistent.

Let $\mathcal{T}$ be an incoherent TBox and $\mathcal{H}$ be an incoherent preserving subset of $\mathcal{T}$. Furthermore let $\mathcal{H}$ comprises of a set of axioms $\{\sigma_1, \dots, \sigma_n\}$. We say $\{\sigma_i\}$ are contradicting axioms if $\mathcal{H}$ be coherent after taking out one or a subset of them from $\mathcal{H}$.

2.2 Subjective Logic

Dempster-Shafer (DS) theory is basically an extension to probability theory where probabilities are assigned to sets as opposed to singleton elements [30]. The employment of the DS theory requires the definition of the set of all possible states in a given setting, referred to as the frame of discernment represented by θ . The powerset of θ , denoted 2^θ , incorporates all possible unions of the sets in θ that can receive belief mass. The truthful subsets of the powerset can receive a degree of belief mass; therefore, the belief mass assigned to an atomic set such as $\psi \in 2^\theta$ is taken as the belief that the given set is true. The belief in ψ is interpreted as the absolute faith in the truthfulness of ψ , which not only relies on the belief mass assigned to ψ , but also to belief masses assigned to subsets of ψ . DS theory provides two important functions: Belief (Bel) and Plausibility (Pl) functions. The belief and plausibility functions are interrelated, i.e., $Pl(\psi) = 1 - Bel(\psi')$.

Subjective logic [17] adds to DS theory by formally defining two extra functions: Disbelief and Uncertainty. A belief expression in Subjective logic is therefore defined as a triple $\chi_\psi^A = (b_\psi^A, d_\psi^A, u_\psi^A)$ also known as the opinion of expert A about hypothesis ψ .

DEFINITION 2.1. Let $\chi_\psi = (b_\psi, d_\psi, u_\psi)$ be a subjective opinion. The elements of χ_ψ can be calculated from a DS basic belief mass assignment by allowing:

$$b_\psi = \text{Bel}(\psi). \quad (1)$$

$$d_\psi = \sum_{\varphi \cap \psi = \emptyset} m_\theta(\varphi), \quad \varphi, \psi \in 2^\theta. \quad (2)$$

$$u_\psi = \sum_{\varphi \cap \psi \neq \emptyset, \varphi \not\subseteq \psi} m_\theta(\varphi), \quad \varphi, \psi \in 2^\theta. \quad (3)$$

It can be shown with this definition that belief, disbelief, and uncertainty elements of an opinion should satisfy $b_\psi^A + d_\psi^A + u_\psi^A = 1$. There are various operators such as combination, discount, deduction, and others in Subjective logic that allow for reasoning over Subjective opinions.

3. PROBLEM FORMALIZATION

Incoherencies in knowledge structure and terminologies can be viewed as discrepant or dissimilar abstractions of observations made over a phenomenon. For this reason, the trace of observations can be considered as evidences to reason about Incoherencies. Here, we consider the set of assertions in the ontology ABox to serve as the set of observations made over the terminologies; therefore, we can summarily view terminologies as hypotheses that need to be evaluated in an evidence-based reasoning process. It is natural to assume that assertions are real-world evidences of the occurrence of the terminology axioms. An important point to mention is that Description Logic ontologies make the open world assumption that states there may exist some additional information that have not been captured in the ontology yet, and hence reasoning over incomplete information to resolve incoherencies may not be very reliable. This is clearly the case for evidence-based reasoning as well. The assumption in such reasoning is that although the current set of observations may not be complete, there are no other sources of information to base reasoning decisions upon and therefore, *an acceptable conclusion* is one based on the current knowledge.

We define the evidence space for an incoherencies terminology as follows:

DEFINITION 3.1. Let $\mathcal{H}$ be a finite subset of $\mathcal{T}$, where $\mathcal{H}$ is incoherent, i.e. it includes a concept name which is not satisfiable w.r.t. $\mathcal{T}$, and $\mathcal{O}$ be a finite set of assertions relevant to the axioms in $\mathcal{H}$. We also define all members of $\mathcal{H}$ as competing or incoherent terminological axioms, then the evidence space for the competing (incoherent) terminological axioms is defined as a pair $\mathcal{E} = (\mathcal{H}, \mathcal{O})$.

An evidence-based reasoning process will need to consider all of the facts and counter-facts expressed in the form of ontology assertions into account to evaluate the incoherent terminology ($\mathcal{H}$). The appropriateness and suitability of an axiom in an incoherent set is relative to the other axioms present in $\mathcal{H}$; therefore, the reasoning process will need to create a likelihood distribution to represent the chances of each axiom being correct.

DEFINITION 3.2. Let $\forall \sigma_i \in \mathcal{H}$, where σ_i are contradicting axioms, and $\mathcal{O}$ be the observation space of $\mathcal{E}$, which provides support and context for σ_i . Now, let $\tau_1, \dots, \tau_n \in \mathcal{O}$ be assertions in $\mathcal{A}$ that are indicators or counter-indicators of σ_1 , i.e., items of information in $\mathcal{O}$ that can be taken into account for justifying the validity of σ_1 , then $\Phi(\sigma_1 | \tau_1, \tau_2, \dots, \tau_n, \sigma_2, \sigma_3, \dots)$ is the conditional likelihood of the correctness of σ_1 in the light of assertions (observations) $\tau_1, \tau_2, \dots, \tau_n$ and the other axioms $\sigma_2, \sigma_3, \dots$ in contrast with σ_1 .

It is now possible to form an evidence-based reasoning process over the evidence space.

DEFINITION 3.3. Let $\Phi(\mathcal{H})$ be the conditional likelihood vector of the correctness of all axioms in $\mathcal{H}$, and $\mathcal{O}$ be the observation space of $\mathcal{E}$ then the evidence-based reasoning process for evaluating the appropriateness of an axiom such as $\sigma \in \mathcal{H}$ is defined as:

$$\mathcal{O} \models_{\Phi(\mathcal{H})} \sigma$$

which informally represents the possibility of entailment of σ from the set of observations seen in $\mathcal{O}$ with a likelihood represented as $\Phi(\mathcal{H})$.

The evidence-based reasoning process over the set of contradicting axioms in an incoherent terminology operates similar to the *inclusion function*, which selects formulas to be discarded from among the inconsistent formulas [24, 14]. Here, the reasoning process considers the likelihood of the correctness of all the alternative axioms and can hence also be viewed as an ordering function. In the following, we will show how evidential support can be derived for an incoherent terminology from the ontology assertion component.

4. EVIDENTIAL SUPPORT

Regardless of the coherency or incoherency of the ontology TBox, the axioms in the TBox can themselves be supported or contradicted by the assertions expressed in the ABox. For instance, consider the well-known example of penguins and flying. Suppose that the following axioms exist in the TBox:

$$\text{Bird} \sqsubseteq \text{Flying}, \text{NonFlying} \equiv \neg \text{Flying}, \text{Penguin} \sqsubseteq \text{Bird}. \quad (4)$$

Furthermore, in the ABox assertions there are:

$$\begin{aligned} &\text{Bird}(\text{TUX}), \text{Penguin}(\text{TUX}), \text{NonFlying}(\text{TUX}), \\ &\text{Bird}(\text{TWEETY}), \text{Flying}(\text{TWEETY}). \end{aligned} \quad (5)$$

So, as it can be seen the terminological component is not incoherent by itself. If we consider the individual TWEETY, it is possible to see that this individual is a supportive observation for the inclusion axiom that states *birds can fly*. However, if TUX is considered, it would serve as an invalidating observation for that statement, but is a supportive statement for the axiom suggesting that *penguins are birds*. Let us now define the notions of evidential harmony and contradiction.

DEFINITION 4.1. Let $C \sqsubseteq D$ be a general inclusion axiom in $\mathcal{T}$, where C, D are concepts. An individual such as b is called an evidential harmony for $C \sqsubseteq D$, denoted $\triangle^{C \sqsubseteq D}$, if both $C(b) \in \mathcal{A}$ and $D(b) \in \mathcal{A}$.

In the above example, TUX is an evidential harmony for $\text{Penguin} \sqsubseteq \text{Bird}$ to its ABox. It is easy to see that the TBox is incoherent; therefore, and TWEETY is an evidential harmony for $\text{Bird} \sqsubseteq \text{Flying}$.

DEFINITION 4.2. Let $C \sqsubseteq D$ be a general inclusion axiom in $\mathcal{T}$, where C, D are concepts. An individual such as b is called an evidential contradiction for $C \sqsubseteq D$, denoted $\nabla^{C \sqsubseteq D}$, if $C(b) \in \mathcal{A}$ and $D(b) \notin \mathcal{A}$.

So, TUX is an evidential contradiction for $\text{Bird} \sqsubseteq \text{Flying}$. It is important to note that evidential harmony and contradiction are only derived from the extensional information in the ABox and not the intensional inferrable knowledge. This is because the use of intensional information would create a link between the information in the ABox and the TBox and can lead to the creation of assertions that have not been directly observed, which cannot be used to base an evidential reasoning process on.

Evidential harmony and contradictions of an axiom such as σ are the required indicators or counter-indicators in Definition 3.2.

It is now possible to develop the conditional likelihood distribution based on evidential harmony and contradiction. For this purpose, we employ the correspondence between Beta Probability Density Functions (PDF) and Subjective opinions. As is known, the probability density over binary event spaces can be expressed as Beta PDFs denoted by $\text{Beta}(\alpha, \beta)$. Let $|\Delta^\sigma|$ and $|\nabla^\sigma|$ express the number of past observations in terms of evidential harmony and contradictions for axiom σ and let a_σ express the a priori or base rate, then α and β can be determined as:

$$\alpha = |\Delta^\sigma| + 2|\nabla^\sigma|, \quad \beta = |\nabla^\sigma| + 2(1 - a_\sigma).$$

Consequently, based on the restriction $b_\sigma + d_\sigma + u_\sigma = 1$, Jøsang [16] establishes the following mapping between Subjective opinion dimensions and the Beta PDF parameters:

$$b_\sigma = \frac{|\Delta^\sigma|}{|\Delta^\sigma| + |\nabla^\sigma| + 1} \quad (6)$$

$$d_\sigma = \frac{|\nabla^\sigma|}{|\Delta^\sigma| + |\nabla^\sigma| + 1} \quad (7)$$

$$u_\sigma = \frac{1}{|\Delta^\sigma| + |\nabla^\sigma| + 1} \quad (8)$$

With this correspondence, we can define the evidential support for a terminological axiom as follows:

DEFINITION 4.3. The evidential support for $\sigma_1 \in \mathcal{H}$ is a Subjective opinion computed by Equations 6-8 based on the number of past observations in terms of evidential harmony and contradictions for σ_1 ; therefore, we have:

$$\Phi(\sigma_1 | \tau_1, \tau_2, \dots, \tau_n, \sigma_2, \sigma_3, \dots) = (b_{\sigma_1}, d_{\sigma_1}, u_{\sigma_1}).$$

Moreover, the conditional likelihood vector of the correctness of all axioms in $\mathcal{H}$, $\Phi(\mathcal{H})$, is a vector of all evidential support values for $\forall \sigma_i \in \mathcal{H}$.

EXAMPLE 4.1. Let us add $\text{Bird} \sqsubseteq \text{NonFlying}$ to the TBox of the earlier example, and also $\text{Flying}(\text{JECKLE})$, $\text{Bird}(\text{JECKLE})$

$\mathcal{H} = \{\text{Bird} \sqsubseteq \text{NonFlying}, \text{Bird} \sqsubseteq \text{Flying}, \text{NonFlying} \equiv \neg \text{Flying}\}$, which means that the two members of $\mathcal{H}$ are competing axioms. We can now compute the evidential harmony and contradiction of each axiom and develop their evidential support: $\Delta^{\text{Bird} \sqsubseteq \text{NonFlying}} = \{\text{TUX}\}$, $\nabla^{\text{Bird} \sqsubseteq \text{NonFlying}} = \{\text{TWEETY}, \text{JECKLE}\}$, $\Delta^{\text{Bird} \sqsubseteq \text{Flying}} = \{\text{TWEETY}, \text{JECKLE}\}$, $\nabla^{\text{Bird} \sqsubseteq \text{Flying}} = \{\text{TUX}\}$; hence, $|\Delta^{\text{Bird} \sqsubseteq \text{NonFlying}}| = 1$, $|\nabla^{\text{Bird} \sqsubseteq \text{NonFlying}}| = 2$, $|\Delta^{\text{Bird} \sqsubseteq \text{Flying}}| = 2$, $|\nabla^{\text{Bird} \sqsubseteq \text{Flying}}| = 1$. Consequently, the evidential support for $\text{Bird} \sqsubseteq \text{NonFlying}$ is $(0.25, 0.5, 0.25)$, which has a higher degree of disbelief assigned to it than belief that shows a higher likelihood that this axiom is not correct, and also the evidential support for $\text{Bird} \sqsubseteq \text{Flying}$ is $(0.5, 0.25, 0.25)$, which has a higher belief mass assigned to it.

The high degree of belief mass assigned to uncertainty in these examples is due to the very limited number of observations (individuals). As the number of observations increase, the degree of dogmatism of the beliefs increases as well. Technically, uncertainty approaches zero when infinite observations have been made. This is the advantage of this formalization with Subjective logic over pure probabilistic measures, since immature conclusions cannot be made about the hypotheses with a small amount of observations.

With the above definitions, it is now possible to perform the evidential reasoning process over contradicting axioms of the terminological component, i.e., $\sigma : \mathcal{O} \models_{\Phi(\mathcal{H})} \sigma$. In the following, we will first introduce the notions of MUPS and MIPS from [28], which are used to identify incoherencies in the terminology component of an ontology.

5. PINPOINTING INCOHERENCIES

Schlobach et al have employed Reiter's generic method for calculating diagnoses, which is based on conflict sets and their minimal hitting sets [28]. A *conflict set* is a set of elements of a system description that can lead to incoherencies. Furthermore, a *hitting set* is a set that contains at least one element from the collection of conflict sets, and is minimal iff no subset of it is a hitting set. Accordingly, Minimal Unsatisfiability-Preserving Sub-TBoxes (MUPS) denote minimal conflict sets, i.e., MUPS of a TBox for an unsatisfiable concept such as A are the subsets of the TBox where A is satisfiable.

DEFINITION 5.1. Let A be an unsatisfiable concept in a TBox $\mathcal{T}$. A sub-TBox $\mathcal{T}' \subseteq \mathcal{T}$ is a minimal unsatisfiability-preserving sub-TBox for A in $\mathcal{T}$ iff A is unsatisfiable in $\mathcal{T}'$, but is satisfiable in every $\mathcal{T}'' \subset \mathcal{T}'$, and is denoted by $\text{mups}(\mathcal{T}, A)$.

MUPS are useful for explaining the underlying reason for the unsatisfiability of a specific concept through a set of TBox axioms. In addition, they can be used to compute the Minimal Incoherence-Preserving Sub-TBox (MIPS), which explains the incoherence of a TBox in general.

DEFINITION 5.2. A sub-TBox $\mathcal{T}' \subseteq \mathcal{T}$ is a minimal incoherence-preserving sub-TBox of $\mathcal{T}$ iff $\mathcal{T}'$ is incoherent, but every $\mathcal{T}'' \subset \mathcal{T}'$ is coherent, and is denoted by $\text{mips}(\mathcal{T})$.

Each element in $\text{mips}(\mathcal{T})$ is a minimal conflict set with regards to the incoherence of $\mathcal{T}$.

EXAMPLE 5.1. Consider the following TBox:

$$\begin{aligned} ax1 : A_1 &\sqsubseteq \neg A \sqcap A_2 \sqcap A_3; ax2 : A_2 \sqsubseteq A \sqcap A_4; \\ ax4 : A_4 &\sqsubseteq \forall s. B \sqcap C; ax5 : A_5 \sqsubseteq \exists s. \neg B; \\ ax6 : A_6 &\sqsubseteq A_1 \sqcup \exists r. (A_3 \sqcap \neg C \sqcap A_4) \end{aligned}$$

The unsatisfiable concepts are A_1, A_3, A_6, A_7 , e.g., $mups(A_3, T) = \{ax_3, ax_4, ax_5\}$. Also the $mips(T) = \{\{ax_1, ax_2\}, \{ax_3, ax_4, ax_5\}, \{ax_4, ax_7\}\}$.

Since MIPS is a minimal conflict set, the minimal hitting set needs to be developed based on the selection of at least one element from each of its subsets. The selection of the members of the hitting set are often based on syntactic measures such as the *scoring functions on axioms of MIPS* or *MIPS core arities*. In the following, we propose to annotate the axioms in MIPS with evidential support information for finding the minimal hitting set.

6. SELECTING REPAIR STRATEGIES

In order to generate repair strategies based on the evidential information gathered from $\mathcal{A}$, we propose to annotate the axioms within MIPS with evidential support information. Evidential support for an axioms such as σ is in the form of a Subjective opinion $(b_\sigma, d_\sigma, u_\sigma)$, which depicts the amount of corroborative and/or contradictory observations for hypothesis σ . Evidential support values allow us to reason about the comparative degree of validity of an axiom in the context of MIPS.

Kalyanpour et al. have previously proposed a modification to Reiter's algorithm [27] in which axiom ranking information are employed to minimize *path rank* instead of *path length* in the developed hitting set tree of this algorithm [19]. They argue that in the standard Reiter's algorithm there is no basis for selecting one axiom over the others while building the edges of the hitting set tree, whereas axiom ranking information can be used to select the next appropriate axiom, i.e., in each step the axiom with the lowest rank is selected, which means that the smallest harm will be made by the removal of this axiom compared with the others.

In our approach, we employ the evidential support for an axiom as its ranking and employ the modified version of Reiter's algorithm. The intuition behind this approach is that axioms with fewer number of evidential contradictions and more evidential harmony need to have higher ranking, which is reflected in their evidential support value; therefore, while selecting the axioms to be included in the minimal hitting set, such axioms will have a lower probability and weaker chance of being selected. Instead other competing axioms with weaker evidential support will be chosen for the minimal hitting set and will eventually be removed.

So given the minimal incoherence-preserving sub-TBox of $\mathcal{T}$, the modified Reiter's algorithm employs the hitting set tree data structure, which is the smallest edge-labeled and node-labeled tree such that its nodes are labeled by any set in $mips(\mathcal{T})$. For each node n , we allow $H(n)$ to be the set of edge labels from the root to node n . The label selected for n is any set $m \in MIPS$ that respects $m \cap H(n) = \emptyset$. If such a set does not exist the node is labeled by a thumbs up. Further, for a node labeled by m , $\forall \sigma \in m$, a follow-up node connected to m through label σ is created. For the nodes with a thumbs up, their path edge labels from the root are a hitting set for $mips(\mathcal{T})$. The optimality of a branch is computed as the sum of the axiom rankings on the edge labels; therefore for optimization purposes, if the path edge label ranking sum exceeds the current

Figure 1: The behavior of the potency metric for different numbers of evidential harmony and contradiction.

optimal hitting set ranking sum value, the search is terminated for that branch (thumbs down).

Now, the difference between the axiom rankings used in the method proposed by Kalyanpour and that of ours is that in their method they employ a value in $[0, 1]$; while in our approach the axiom ranking information is in the form of evidential support represented in Subjective logic opinions. For this reason, computing the path edge label ranking sums needs to be performed over Subjective opinions, which can be done using the *Subjective Addition operator*, denoted $\sigma_1 + \sigma_2$ ³. The addition operator returns a new Subjective opinion as a result. Besides computing the sum of the path edge label rankings, we need to be able to compare two ranking sums in the form of Subjective opinions. This is done rather easily when ranking information are numerical values, but for comparing Subjective opinions to see which one is weaker, we need to employ a corresponding comparable form of the opinion for our purpose; therefore, it is possible to interpret evidential support information in the form of Subjective opinions as the potential that a hypothesis has towards revealing the correct terminological axiom. For this purpose, the *potency metric* [1] can be used.

DEFINITION 6.1. Let σ be a terminological axiom, and $\phi = (b_\sigma, d_\sigma, u_\sigma)$ be its corresponding evidential support. The potency metric for σ , denoted as $\Upsilon_\phi(\sigma)$, is defined as:

$$\Upsilon_\phi(\sigma) = \begin{cases} 0 & \text{if } u_\sigma = 1 \\ \frac{b_\sigma - d_\sigma}{1 - u_\sigma} & \text{else} \end{cases}$$

As it can be seen in Figure 1, the potency metric defines that axioms with higher number of evidential harmony (higher b_σ) and more number of evidential observations (lower u_σ) have a higher likelihood of being the correct hypothesis; hence, this metric can be used to compare evidential support information in the form of Subjective opinions. Now, with the employment of the Subjective addition operator and the potency metric, we can benefit from the modified Reiter's algorithm to find the minimal hitting set with the use of the evidential support information of the competing terminological axioms.

EXAMPLE 6.1. Lets assume that the evidential support for axioms in the MIPS of Example 5.1 are represented by $\phi_i = (b_i, d_i, u_i)$, where i denotes the index of the axiom. For instance, the evidential support for ax_1 is ϕ_1 . Further assume that the following relationship exists between the potency metric of the evidential support

³For a formal definition of the addition operator, see [21]

Figure 2: Reiter's hitting set tree for MIPS in Example 6.1.

information: $\Upsilon_{\phi_6}(\sigma_6) < \Upsilon_{\phi_3}(\sigma_3) < \Upsilon_{\phi_1}(\sigma_1) < \Upsilon_{\phi_4}(\sigma_4) < \Upsilon_{\phi_5}(\sigma_5) < \Upsilon_{\phi_2}(\sigma_2) < \Upsilon_{\phi_7}(\sigma_7)$.

As it can be seen, the hitting sets developed in the first three branches of the hitting set tree namely (ax_1, ax_3, ax_4) , (ax_1, ax_3, ax_7) , and (ax_1, ax_4) are all labeled with thumbs up which means that they held the local minimum path edge label ranking sum at the time of their formation. However, the rest of the edges all possessed higher rank sums based on the general order shown between their potency metric values, and since they exceeded an already lower ranking sum, they were discarded from further processing. From the three viable hitting sets and based on the potency metric rankings, it can be seen that (ax_1, ax_4) is the better hitting set to choose, since $\Upsilon_{\phi_1+\phi_4}(\sigma_1 + \sigma_4)$ is smaller than the potency of the other hitting set axioms, and is therefore selected as the repair strategy.

The benefit of using the modified Reiter's algorithm along with evidential observation information is that axioms with the least number of corroborative evidence and most contradictory indications will form the hitting set and will be removed as a part of the repair solution. To put this in context, let's consider the principle of minimality of change. Many revision and inconsistency resolution techniques focus on the satisfaction of this principle, which entails the removal of the least number of axioms; however, in our approach we take a more semantic approach where we rely on the assumption that real-world observations reveal the correct and valid knowledge structure. Furthermore, it seems more natural for logics such as DL, which have an assertion component, as opposed to other logics such as propositional logic, to take such information into account. Based on this, we define the *minimality of semantic change* as follows.

DEFINITION 6.2. Let $O = \langle T, \mathcal{A} \rangle$ be an ontology and $T', T'' \subset T$. T'' is a **greater deviation** from T than T' iff $\forall \sigma_i \in T', \forall \gamma_j \in T''$

1. $\sum_i |\Delta^{\sigma_i}| > \sum_j |\Delta^{\gamma_j}|$,
2. $\sum_i |\nabla^{\sigma_i}| < \sum_j |\nabla^{\gamma_j}|$.

This definition shows that a sub-terminology is further away from the initial terminological component of the ontology compared with another sub-terminology if in comparison it has more differences with the assertions of the ontology than the other sub-terminology.

DEFINITION 6.3. Let T be an incoherent TBox of an ontology, and $T', T'' \subset T$. T' is a **minimal semantic change** for T , iff

1. T' is coherent,
2. there is no other coherent sub-terminology of T such as T'' where T' is a greater deviation from T than T'' .

Definition 6.3 characterizes the principle of minimal semantic change. It is easy to show, as we have explained above, that the proposed evidential reasoning process for terminology incoherency resolution respects this principle.

PROPOSITION 6.1. Let $O = \langle T, \mathcal{A} \rangle$ be an ontology with an incoherent terminological component, and T' be the outcome of the evidential reasoning process for terminology incoherency resolution, then T' is a minimal semantic change for T .

It is important to note that the evidential incoherency resolution process does not necessarily respect the principle of minimal change unless the minimal change axioms are also minimal semantic changes.

7. RELATED WORK

There have been some work in the area of resolving inconsistency in ontology terminologies. Schlobach et al. have proposed a technique that identifies the position of a contradiction within the axioms of a terminology by finding minimal conflicting concepts, and introduce the concept of MIPS and MUPS [28]. Further they propose the employment of Reiter's hitting set algorithm to compute repair solutions. Their technique is called *concept pinpointing*. Kalyanpour et al. focus on a more expressive description logic, *SHOIN*, and propose axiom ranking based on various criteria such as minimal impact changes, authored important test cases, provenance information, and others [19]. With the use of the ranking information, they are able to modify Reiter's algorithm to perform more efficiently. In addition, they introduce strategies to rewrite erroneous axioms instead of completely removing them from the terminology.

In another attempt, the authors in [18] propose a black-box technique to find the justifications of an entailment, which can be used for debugging unsatisfiable concepts. They define justification as the minimal subset of the ontology that is sufficient for an entailment to hold. Built on this concept, precise and laconic justifications were later introduced, which are more fine-grained justifications that facilitate the generation of semantically minimal repairs [15]. A complete review of techniques for handling inconsistency in DL-based ontologies is given in [4].

Other work in the field of belief revision can also be considered relevant where an incoming axiom or piece of information causes inconsistency in the knowledge base. Most revision operators attempt to satisfy all or a subset of the eight AGM postulates [13]. However, operators that consider AGM postulates may not be directly applicable to DL ontologies. Flouris et al. have argued that description logics *SHIF* and *SHOIN* are not AGM compliant and therefore replacing postulates have been proposed [12, 11]. Other techniques such as model-based [6, 26] and iterative knowledge base revision [3, 5] have also been proposed in the literature. Qi and Yang have reviewed some of the important belief revision approaches in the context of description logics [25].

8. CONCLUDING REMARKS

In this paper, we have opened up a new perspective towards the resolution of incoherencies in description logic terminologies, where we view assertional statements in ontologies as a useful source of information, which allow for higher level semantic decision making. The proposed approach is based on concepts from evidence-based reasoning and views ontology assertions as real-world observations and indications of the correctness or validity of a terminological axiom in contrast with the other competing axioms. In other words, the more a knowledge structure is viewed in reality, the higher its probability of correctness would be. Thus, in an incoherent subset of a terminology, we will be able to reason over the competing terminologies in light of the assertional observations. We introduce two forms of observations namely, evidential harmony and evidential contradiction that depict the corroborative and contradictory evidences for an axiom, respectively.

Along the same paths as the modified Reiter's algorithm, we have employed path rank instead of path length on the hitting set tree. The path ranks are computed using the evidential harmony and evidential contradiction information for the axioms in the conflict sets. Since evidences obtained to support the competing axioms are in Subjective logic form in correspondence with the probability density over binary event spaces expressed as Beta PDFs, we employ and introduce operators to adapt Reiter's hitting set algorithm for Subjective path rank information.

It has been discussed that many of humans daily decisions are based on the incomplete facts or information that is at their disposal; therefore, their decisions is based on the semantic interpretation of their observations. We believe that a rather up-to-date assertional component of an ontology can serve as observations to back its terminological component. For this reason, our approach does not necessarily conform to the widely used principle of minimal change because this principle only focuses on syntactical aspects of the knowledge base and may not always entail semantically correct results, instead we introduce a postulate for minimal semantic change that is respected by our proposed approach. Currently, we are in the process of developing a tool for supporting the approach proposed in this paper in Protege 4.0. The tool would allow us to formally analyze the behavior of the evidence-based incoherency resolution technique.

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