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Dynamic Modeling of Hydraulic Shovel Excavators for Geomaterials

Samuel Frimpong¹; Yafei Hu²; and Hilary Inyang³

Abstract: The hydraulic shovel excavator has found significant applications in surface mining, construction, and geotechnical operations due to its flexibility and mobility. The key to high availability and utilization of this shovel is adequate understanding of machine dynamics and machine-formation interactions among other technical, operating, safety, and economic factors. These shovels are capital intensive, complex in design and operation within severely constrained environments. Detailed dynamic modeling and analysis are required to understand their effective utilization for achieving efficient operating performance and economic useful lives. Previous attempts at solving these problems are limited because they do not provide knowledge on the resistive forces and moments for efficient excavation. In this paper, the Newton-Euler techniques are used to develop hydraulic shovel dynamic models with numerical examples. Detailed analysis of the results shows that: (1) the kinematics of the stick-bucket joint (joint 3) is the most critical and effective control of this joint and is important input into efficient excavation design and execution; and (2) the highest resistive moments occur between the duration of 1.5 and 2.0 s after the start of formation excavation and the highest magnitudes are 1,500 Nm (for stick), 900 Nm (for bucket), and 600 Nm (for boom). Based on these results, the path trajectories, dynamic velocity and acceleration profiles, and dimensioned parameters for optimum feed force, torques, and momentum of shovel boom-bucket assembly can be modeled and used for efficient excavation. The optimum digging forces and resistances for the hydraulic shovel excavator can also be modeled and used to predict optimum excavation performance.

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Introduction

Hydraulic shovel excavators are widely used as primary production equipment in surface mining for removing overburden and ore materials and for construction and geotechnical applications. Fig. 1 illustrates the nomenclature of a hydraulic shovel. This shovel consists of three major assemblies: The lower frame, upper frame, and the attachment. The lower frame provides a stable base for the machine and includes the propel drive and the crawler system. The upper frame provides an attachment point for the power plant, pump drive, hoist and swing machinery, boom attachment, control valves, fuel and oil reservoirs, operator's cab, and supporting equipment. The attachment consists of the boom, stick, bucket, and associated cylinders and valves. Hydraulic shovel excavation is flexible, efficient, and easily relocated to different operating environments. However, the shovel-truck mining method can be rendered inefficient from operating, environmental, topography, and operator constraints.

Any naturally occurring formation is characterized by the defining properties of the constituent soils and rocks. These properties are shaped by the pre- and postformation chemical and mechanical processes to yield the relative ease of digging or excavating the formation. Thus, an excavator's cutting force is a function of the formation properties, machine-formation interactions, and the operating parameters of an excavator. The formation parameters include cohesion, internal friction angle, density, water saturation, formation hardness and compaction, abrasiveness, the angle of formation failure wedge, and shear plane angle. The machine-formation interaction parameters include adhesion and external friction angle. The operating parameters also include blade travel velocity, cutting angle, tool working depth beneath surface, and surcharge pressure acting vertically on formation surface. These defining characteristics must be controlled through optimization to yield efficient excavation.

Efficient shovel utilization and performance require detailed understanding of machine kinematics and dynamics given the underlying field constraints. These dynamic models describe the evolution of an excavator's trajectory for efficient excavation. It also provides an indication of the active and reactive forces, and the moments and resistive moments on machine components and joints for efficient equipment performance and longevity (Frimpong and Hu 2004; Frimpong et al. 2003). The hydraulic shovel excavator is designed to operate within certain designed specifications, which may include optimum loading of its components, angular displacements, velocities and accelerations, resistive forces, moments, and torque. Dynamic modeling provides comparative analysis to avoid excessive equipment utilization and burdening to achieve optimum performance and the expected economic useful lives. Dynamic model results may also be used for

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Fig. 1. Hydraulic shovel nomenclature

planning the 3D space geometry requirements for optimum shovel performance. The results may also be used to develop virtual prototype simulators for training operators within safety and cost constraints. Frimpong and Hu (2004) and Frimpong et al. (2003) have articulated these issues for hydraulic shovel performance modeling. Hendricks et al. (1993) developed a dynamic model for cable shovels using Lagrange formulations without the resistive forces from the in situ materials. Daneshmend et al. (1993) dealt with the same problem using an iterative Newton-Euler formulation. The latter did not include the stick, which is very important for the complete description of the dynamic behavior of a hydraulic shovel. These models contain no predictions. Araya et al. (1988) gave a simplified dynamic model for hydraulic shovel excavators using Lagrange's equations of motion.

Machine optimization provides the optimum feed force required to excavate a unit volume of material within an optimized trajectory for a given geological formation, with its geomechanical characteristics. This optimum feed force, within an optimized trajectory, is governed by a set of optimized machine parameters, which results in the least breakout force or energy during excavation. A hydraulic shovel uses diesel engines or electric motors to drive hydraulic pumps, motors, and cylinders that in turn actuate the motions required for machine propel, stick extension and retraction, hoisting, swinging, and excavation. Inefficient shovel excavation results in faster machine wear and tear and fatigue failure with corresponding higher maintenance and energy costs. The optimized parameters include crowd, hoist and swing speeds, and the configuration of the optimized trajectory. Awuah-Offei (2005) showed that shovel optimization could result in significant reductions in shovel digging time, digging energy per cycle, and energy costs, with a corresponding increase in shovel production and productivity.

In this paper, the dynamic models of a hydraulic shovel excavator are derived using the Newton-Euler formulation. These models are based on the main functional components (the boom, the stick, and the bucket) of the hydraulic shovel as free bodies. Responses are predicted for given digging trajectories. These models form the basis for developing comprehensive simulator models for efficient shovel operations in constrained surface mining and construction and geotechnical environments. The primary objectives of the study include (1) detailed dynamic modeling of a hydraulic shovel; (2) validation with real-world data; (3) analysis of dynamic performance based on results; and (4) recommendations on optimum equipment utilization. The paper focuses on the formulation of the generalized and detailed dynamic models

Fig. 2. Hydraulic shovel front end assembly for dynamic modeling

of a shovel excavator. Comprehensive dynamic models of the hydraulic shovel excavator are contained in the Appendix. The Appendix also provides numerical examples of the components of hydraulic shovel dynamics. The paper is concluded with appropriate references and notation.

Dynamic Modeling of Shovel Excavators

Hydraulic shovel excavators are primary production equipment in most surface mining operations and construction and geotechnical applications. Hydraulic shovel excavators are useful for specific applications in surface mining operations, where mobility, flexibility, and selectivity are important in design and production. The dynamic models of this shovel will center on the front-end assembly illustrated in Fig. 2. The front-end assembly, consisting of the boom, stick, and bucket with the appropriate cylinders, are controlled to effect dynamic motion for excavation. Fig. 3 also illustrates the reference coordinate systems for various movements of the boom, stick, and bucket assembly. The coordinate systems include the boom-machinery joint (X_1, Y_1, Z_1), boom-stick joint (X_2, Y_2, Z_2), stick-bucket joint (X_3, Y_3, Z_3), the cutting edge (X_4, Y_4, Z_4), and the bottom-door hinge of the bucket (X_5, Y_5, Z_5). Fig. 4 illustrates the motion dynamics of the front-end assembly of the hydraulic shovel during excavation. Hydraulic cylinders control the boom, stick, and bucket components to cause outward and backward motions during the excavation process. The overall dynamic model for an excavator (cable shovel, hydraulic shovel, and the backhoe) is given as Eq. (1)

$$D(\Theta)\ddot{\Theta} + C(\Theta, \dot{\Theta})\dot{\Theta} + G(\Theta) = F - F_{\text{load}}(F_r, F_n) \quad (1)$$

F is related to the hydraulic force effects; and $F_{\text{load}}(F_r, F_n)$ = resistive force due to soil-bucket interaction. It is the effective force engaged in the process of excavation, as illustrated in Fig. 5. The dynamic equations can be obtained by applying either the Lagrange or the iterative Newton-Euler methods. The iterative Newton-Euler method is used because the formulation lends itself to efficient software implementation, which is of importance for real-time simulation and parameter estimation (Frimpong and Hu 2004; Daneshmend et al. 1993; Koivo 1994; Koivo et al. 1996; Vaha and Shibniewski 1990).

Dynamic Models of the Hydraulic Shovel

The Newton-Euler iteration method is utilized to analyze the kinematics and dynamic behavior of the hydraulic shovel. The ad-

Fig. 3. Elements of hydraulic dynamic model

Fig. 4. Motion dynamics of boom-stick-bucket assembly

Fig. 5. Interaction between bucket and environment ($\theta_2 + \theta_3 + \theta_4 = \theta_{dg} - \theta_b$)

vantage of this method is that it follows an iterative procedure and it is easy to obtain solutions using numerical and stochastic simulation-modeling techniques. It also provides detailed information on all linkages and joints, which will be useful in later finite element analysis of the stress and/or strength problems during excavation (Craig 1986).

Hydraulic Shovel Kinematics

The shovel kinematics models are obtained for the front-end assembly, which comprises the boom, stick, and bucket components as illustrated in Figs. 1 and 2. The centers of rotational and linear motions are used as the reference coordinate systems for the shovel kinematics. These centers include the boom-machinery attachment, boom-stick joint, stick-bucket joint, the bucket cutting edge, and the bottom door latch of the shovel bucket. The angular and linear displacements, velocities, and acceleration are established for the shovel base, boom, stick, bucket, and the cutting edge of the bucket, as illustrated in the Appendix. The Newton-Euler formulations are obtained for the outward and inner iterations, as illustrated in the Appendix. The equations of the outward iterations govern the motion of the forward assembly when it is being extended to begin the excavation process. An empty bucket, higher angular and linear displacements and velocities, and smaller moments about the centers of rotation characterize this motion. The equations of the inward iterations govern the motion of the forward assembly when it is being retracted after the excavation process. A loaded bucket, relatively slower angular and linear displacements and velocities, and higher moments about the centers of rotation characterize this motion (Craig 1986).

Hydraulic Shovel Dynamics

The shovel dynamic models are obtained for the front-end assembly in Figs. 1 and 2. The dynamic models use the results of the kinematics as input to develop the models for active and resistive forces, moments, and resistive moments. The active forces are

Fig. 6. Angular displacement (θ) variations with time

used to generate the breakout forces required for the excavation process. The breakout force must exceed the resistive force for the formation to be removed from in situ. The resistive force is a resultant of the tangential and normal force incident on the plane of excavation, as illustrated in Fig. 5 (Zelenin et al. 1985). The equations governing the active and reactive forces are contained in the Appendix. The moments, generated as a result of the active forces and the centers of rotation about various joints in the frame, and the resistive moments, are important in the overall performance of the shovel. The equations governing the moments and reactive moments are also contained in the Appendix. The dynamic models are also developed for outward or backward and inner or inward iterations to examine the dynamics of the excavation process in the extension, excavation, and retraction phases.

Numerical Examples of Hydraulic Shovel Dynamics

The models are validated using real-world data from Frimpong and Hu (2004). A number of experiments are run to generate various results for comprehensive analysis. The digging trajectory shown in Fig. 5 is used here to demonstrate the dynamic models in this study. Figs. 6–8 show the change of θ , $\dot{\theta}$, and $\ddot{\theta}$, respectively with time. Fig. 6 illustrates the angular displacement of Joints 1, 2, and 3 with time in the vertical plane. For optimum performance, the variation of θ_1 with time is marginal (below $5^\circ/\text{s}$), which indicates that the relative increase is very small. θ_2 decreases to a small value and begins to rise at a marginal gradient in time (below $20^\circ/\text{s}$). θ_3 increases very fast in a relatively short period of time (around $50^\circ/\text{s}$). Thus, for the excavation

Fig. 7. Angular velocity ($\dot{\theta}$) variations with time

Fig. 8. Angular acceleration ($\ddot{\theta}$) variations with time

process, the major action point is the stick-bucket joint, and thus, effective control of this joint is significant for optimum operations. Figs. 7 and 8 show the respective variations of angular velocities and accelerations for Joints 1 and 2 with time. The variations in the angular velocities of joints 1 and 2 are marginal, over the duration of excavation, within 0 to $20^\circ/\text{s}^2$ for Joint 1 and 0 to 30 for Joint 2. However, Joint 3 has a wide variation in velocity with time within 50 and $60^\circ/\text{s}^2$. The velocity profiles also show the importance of joint 3 (stick-bucket joint or wrist) in the excavation process. The angular acceleration curves for the three joints show fairly wide variations with joint 3 recording the largest variations, indicating joint 3 as very critical for optimum shovel performance. When θ , $\dot{\theta}$, and $\ddot{\theta}$ are determined at each time, the left-hand side of Eq. (1) can be calculated and its change with time is shown in Fig. 9.

The inertia moments experienced by Joints 1, 2, and 3 are illustrated in Fig. 9. The moments about the joints are very significant because in order for the shovel to achieve the expected economic useful life, these moments must not exceed the recommended design and tested specifications by the manufacturer. The moments about Joint 1 (machinery boom) are the highest in terms of magnitude and variation over time. This joint is heavily supported to avoid crack initiation and propagation and wear and tear due to large variable moments. The moments about Joint 2 (boom stick) and Joint 3 (stick bucket) are relatively smaller with very small variations in magnitude over time. The resistive moment according to the current trajectory is calculated and shown in Fig. 10. This resistive moment is caused by the resistive forces due to the material being excavated from its natural environment. As the

Fig. 9. Inertial part moment variations with time

Fig. 10. Resistive moment variations with time

bucket engages the formation, the magnitudes of the resistive moments increase significantly in the stick and then the boom, followed by the bucket in order of decreasing resistance. The highest magnitudes 1,500 Nm (for stick), 900 Nm (for bucket), and 600 Nm (for boom) occur within the duration from 1.5 sec and 2.0 sec after starting the excavation process, during which the bucket is engaged in formation excavation.

Finally, F may be obtained from Eq. (1) since all other terms are already determined through the above analysis. The relationship between the stick force and hoist torque from stick and hoist hydraulic cylinders and the resistance force F_r may be established as illustrated in Fig. 5. Fig. 5 shows the normal and tangential force, whose resultant force is the resistive force F_r . The incident angle, θ_{dg} , F_n , and F_t depend on the diggability of the materials being excavated at any time. This dynamic force is important for effective excavation process and optimum equipment utilization. By measuring F_n and F_t , the current resistance force F_r can be estimated as the resultant of the two forces. These results are very important for the effective planning, implementation, and assessment of the hydraulic shovel utilization.

Optimum Shovel Performance

The results of the machinery-formation interactions define the envelope for optimum digging performance. As illustrated by equations (50)–(52), the formation-resistant force or moments depends on several variables that govern the geometry of machine-formation interactions. For efficient excavation, the results show that the resistive moments experienced by the shovel boom, stick, and bucket, as a result of the digging process, must be governed by Fig. 10. The maximum resistive moments of the boom, stick, and bucket lie between 1.25 and 2.25 s of the 3-s digging time. Figs. 6–9 show the respective periodic angular displacement, velocity and acceleration, and the inertia moment profiles for the hydraulic Joints 1, 2, and 3 for optimum excavation.

Conclusions

Dynamic models of the hydraulic shovel excavator have been derived for its boom, stick, and bucket, and are analyzed as free body diagrams using the iterative Newton-Euler method. The responses from a given digging trajectory are predicted as an illustration. Detailed analysis of the results shows that: (1) the kinematics of the stick-bucket joint (Joint 3) is the most critical, and effective control of this joint is important input into efficient excavation design and execution; and (2) the highest resistive mo-

ments occur between the duration of 1.5 and 2.0 sec after the start of formation excavation, and the highest magnitudes are 1,500 Nm (for stick), 900 Nm (for bucket), and 600 Nm (for boom). These resistive moments are appropriate based on manufacturers' design and tested specifications, and it can be concluded that the machine is utilized under acceptable operating performance conditions. The next step in this research is the characterizing, modeling, and simulation of the interaction between the shovel and the mined environments. This will eventually enable complete simulation of the shovel operation, as well as assessment of ore bench digging conditions based on online measurements of the information from the main functional components of the shovel excavators.

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Appendix. Hydraulic Shovel Dynamic Equations

The following equations describe the hydraulic shovel kinematics and dynamic models in a typical operating environment. The base of the shovel is fixed, and hence does not rotate, during operation, which results in Eq. (2)

$$\omega_1 = 0 \quad \text{and} \quad \dot{\omega}_1 = 0 \quad (2)$$

The iterative Newton-Euler dynamics algorithm for the shovel is obtained using Fig. 4. The outward Newton-Euler iterations for the boom can be modeled as Equations (3)–(8), which define the angular and linear motions

$${}^2\omega_2 = {}^2R^1\omega_1 + \dot{\theta}_2 {}^2\hat{Z}_2 = \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{pmatrix} \quad (3)$$

$${}^2\dot{\omega}_2 = {}^2R^1\dot{\omega}_1 + \dot{\theta}_2 {}^2\hat{Z}_2 + {}^2R^1\dot{\omega}_1 + \ddot{\theta}_2 {}^2\hat{Z}_2 = \begin{pmatrix} 0 \\ 0 \\ \ddot{\theta}_2 \end{pmatrix} \quad (4)$$

$$\begin{aligned} {}^2\dot{v}_2 &= {}^2R[{}^1\dot{\omega}_1 \times {}^1P_2 + {}^1\omega_1 \times ({}^1\omega_1 \times {}^1P_2) + {}^1\dot{v}_1] \\ &= \begin{pmatrix} c_{\theta 2} & s_{\theta 2} & 0 \\ -s_{\theta 2} & c_{\theta 2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ g \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} gs_{\theta 2} \\ gc_{\theta 2} \\ 0 \end{pmatrix} \end{aligned} \quad (5)$$

$$\begin{aligned} {}^2\dot{v}_{c_2} &= {}^2\dot{\omega}_2 \times {}^2P_{c_2} + {}^2\omega_2 \times ({}^2\omega_2 \times {}^2P_{c_2}) + {}^2\dot{v}_2 \\ &= \begin{pmatrix} -d_2\ddot{\theta}_2s_{\alpha 2} - d_2\dot{\theta}_2^2c_{\alpha 2} + gs_{\theta 2} \\ d_2\ddot{\theta}_2c_{\alpha 2} - d_2\dot{\theta}_2^2s_{\alpha 2} + gc_{\theta 2} \\ 0 \end{pmatrix} \end{aligned} \quad (6)$$

$$\begin{aligned}
{}^2N_2 &= {}^2I_2 {}^2\dot{\omega}_2 + {}^2\omega_2 \times {}^2I_2 {}^2\omega_2 \\
&= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I_{zz2} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \ddot{\theta}_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{pmatrix} \times \left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I_{zz2} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{pmatrix} \right] \\
&= \begin{pmatrix} 0 \\ 0 \\ I_{zz2} \ddot{\theta}_2 \end{pmatrix}
\end{aligned} \quad (7)$$

$${}^2F_2 = m_2 {}^2\dot{v}_{c_2} = \begin{pmatrix} -m_2 d_2 \ddot{\theta}_2 s_{\alpha 2} - m_2 d_2 \dot{\theta}_2^2 c_{\alpha 2} + m_2 g s_{\theta 2} \\ m_2 d_2 \ddot{\theta}_2 c_{\alpha 2} - m_2 d_2 \dot{\theta}_2^2 s_{\alpha 2} + m_2 g c_{\theta 2} \\ 0 \end{pmatrix} \quad (8)$$

The outward Newton-Euler iterations for the stick can be obtained as equations (9)–(14). These equations describe the angular and linear motions in the outward direction

$${}^3\omega_3 = {}^3R^2 \omega_2 + \dot{\theta}_3 {}^3\hat{Z}_3 = \begin{pmatrix} c_{\theta 3} & s_{\theta 3} & 0 \\ -s_{\theta 3} & c_{\theta 3} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_{23} \end{pmatrix} \quad (9)$$

$$\begin{aligned}
{}^3\dot{\omega}_3 &= {}^3R^2 \dot{\omega}_2 + \dot{\theta}_3 {}^3\hat{Z}_3 + {}^3R^2 \dot{\omega}_2 + \ddot{\theta}_3 {}^3\hat{Z}_3 \\
&= \begin{pmatrix} c_{\theta 3} & s_{\theta 3} & 0 \\ -s_{\theta 3} & c_{\theta 3} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_3 \end{pmatrix} \\
&\quad + \begin{pmatrix} c_{\theta 3} & s_{\theta 3} & 0 \\ -s_{\theta 3} & c_{\theta 3} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \ddot{\theta}_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \ddot{\theta}_3 \end{pmatrix} \\
&= \begin{pmatrix} 0 \\ 0 \\ \ddot{\theta}_{23} \end{pmatrix}
\end{aligned} \quad (10)$$

$$\begin{aligned}
{}^3\dot{v}_3 &= {}^3R[{}^2\dot{\omega}_2 \times {}^2P_3 + {}^2\omega_2 \times ({}^2\omega_2 \times {}^2P_3) + {}^2\dot{v}_2] \\
&= \begin{pmatrix} -r_2 \ddot{\theta}_2^2 c_{\theta 3} + r_2 \ddot{\theta}_2 s_{\theta 3} + g s_{\theta 2 \theta 3} \\ r_2 \ddot{\theta}_2^2 s_{\theta 3} + r_2 \ddot{\theta}_2 c_{\theta 3} + g c_{\theta 2 \theta 3} \\ 0 \end{pmatrix}
\end{aligned} \quad (11)$$

$$\begin{aligned}
{}^3\dot{v}_{c_3} &= {}^3\dot{\omega}_3 \times {}^3P_{c_3} + {}^3\omega_3 \times ({}^3\omega_3 \times {}^3P_{c_3}) + {}^3\dot{v}_3 \\
&= \begin{pmatrix} -d_3 \ddot{\theta}_{23} s_{\alpha 3} - d_3 \dot{\theta}_{23}^2 c_{\alpha 3} - r_2 \ddot{\theta}_2^2 c_{\theta 3} + r_2 \ddot{\theta}_2 s_{\theta 3} + g s_{\theta 2 \theta 3} \\ d_3 \ddot{\theta}_{23} c_{\alpha 3} - d_3 \dot{\theta}_{23}^2 s_{\alpha 3} + r_2 \ddot{\theta}_2^2 s_{\theta 3} + r_2 \ddot{\theta}_2 c_{\theta 3} + g c_{\theta 2 \theta 3} \\ 0 \end{pmatrix}
\end{aligned} \quad (12)$$

$${}^3N_3 = {}^3I_3 {}^3\dot{\omega}_3 + {}^3\omega_3 \times {}^3I_3 {}^3\omega_3 = \begin{pmatrix} 0 \\ 0 \\ I_{zz3} \ddot{\theta}_{23} \end{pmatrix} \quad (13)$$

$$\begin{aligned}
{}^3F_4 &= m_3 {}^3\dot{v}_{c_3} \\
&= \begin{pmatrix} -m_3 d_3 \ddot{\theta}_{23} s_{\alpha 3} - m_3 d_3 \dot{\theta}_{23}^2 c_{\alpha 3} - m_3 r_2 \ddot{\theta}_2^2 c_{\theta 3} + m_3 r_2 \ddot{\theta}_2 s_{\theta 3} + m_3 g s_{\theta 2 \theta 3} \\ m_3 d_3 \ddot{\theta}_{23} c_{\alpha 3} - m_3 d_3 \dot{\theta}_{23}^2 s_{\alpha 3} + m_3 r_2 \ddot{\theta}_2^2 s_{\theta 3} + m_3 r_2 \ddot{\theta}_2 c_{\theta 3} + m_3 g c_{\theta 2 \theta 3} \\ 0 \end{pmatrix}
\end{aligned} \quad (14)$$

The outward Newton-Euler iterations for the bucket can be obtained using equations (15)–(20)

$${}^4\omega_4 = {}^4R^3 \omega_3 + \dot{\theta}_4 {}^4\hat{Z}_4 = \begin{pmatrix} c_{\theta 4} & s_{\theta 4} & 0 \\ -s_{\theta 4} & c_{\theta 4} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_{23} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_{234} \end{pmatrix} \quad (15)$$

$$\begin{aligned}
{}^4\dot{\omega}_4 &= {}^4R^3 \dot{\omega}_3 + \dot{\theta}_4 {}^4\hat{Z}_4 + {}^4R^3 \dot{\omega}_3 + \ddot{\theta}_4 {}^4\hat{Z}_4 = \begin{pmatrix} c_{\theta 4} & s_{\theta 4} & 0 \\ -s_{\theta 4} & c_{\theta 4} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_{23} \end{pmatrix} \\
&\quad \times \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_4 \end{pmatrix} + \begin{pmatrix} c_{\theta 4} & s_{\theta 4} & 0 \\ -s_{\theta 4} & c_{\theta 4} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \ddot{\theta}_{23} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \ddot{\theta}_4 \end{pmatrix}
\end{aligned} \quad (16)$$

$$\begin{aligned}
{}^4\dot{v}_4 &= {}^4R[{}^3\dot{\omega}_3 \times {}^3P_4 + {}^3\omega_3 \times ({}^3\omega_3 \times {}^3P_4) + {}^3\dot{v}_3] \\
&= \begin{pmatrix} -r_2 \ddot{\theta}_2^2 c_{\theta 3 \theta 4} + r_2 \ddot{\theta}_2 s_{\theta 3 \theta 4} - r_3 \ddot{\theta}_{23}^2 c_{\theta 4} + r_3 \ddot{\theta}_{23} s_{\theta 4} + g s_{\theta 2 \theta 3 \theta 4} \\ r_2 \ddot{\theta}_2^2 s_{\theta 3 \theta 4} + r_2 \ddot{\theta}_2 c_{\theta 3 \theta 4} + r_3 \ddot{\theta}_{23}^2 s_{\theta 4} + r_3 \ddot{\theta}_{23} c_{\theta 4} + g c_{\theta 2 \theta 3 \theta 4} \\ 0 \end{pmatrix}
\end{aligned} \quad (17)$$

$${}^4\dot{v}_{c_4} = {}^4\dot{\omega}_4 \times {}^4P_{c_4} + {}^4\omega_4 \times ({}^4\omega_4 \times {}^4P_{c_4}) + {}^4\dot{v}_4 = \begin{pmatrix} -d_4 \ddot{\theta}_{234} s_{\alpha 4} - d_4 \dot{\theta}_{234}^2 c_{\alpha 4} - r_2 \ddot{\theta}_2^2 c_{\theta 3 \theta 4} + r_2 \ddot{\theta}_2 s_{\theta 3 \theta 4} - r_3 \ddot{\theta}_{23}^2 c_{\theta 4} + r_3 \ddot{\theta}_{23} s_{\theta 4} + g s_{\theta 2 \theta 3 \theta 4} \\ d_4 \ddot{\theta}_{234} c_{\alpha 4} - d_4 \dot{\theta}_{234}^2 s_{\alpha 4} + r_2 \ddot{\theta}_2^2 s_{\theta 3 \theta 4} + r_2 \ddot{\theta}_2 c_{\theta 3 \theta 4} + r_3 \ddot{\theta}_{23}^2 s_{\theta 4} + r_3 \ddot{\theta}_{23} c_{\theta 4} + g c_{\theta 2 \theta 3 \theta 4} \\ 0 \end{pmatrix} \quad (18)$$

$${}^4N_4 = {}^4I_4 {}^4\dot{\omega}_4 + {}^4\omega_4 \times {}^4I_4 {}^4\omega_4 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I_{zz4} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \ddot{\theta}_{234} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_{234} \end{pmatrix} \times \left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I_{zz4} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \dot{\theta}_{234} \end{pmatrix} \right] \quad (19)$$

$${}^4F_4 = m_4 {}^4\ddot{v}_{c_4} = m_4 \begin{pmatrix} -d_4\ddot{\theta}_{234}s_{\alpha 4} - d_4\dot{\theta}_{234}^2c_{\alpha 4} - r_2\ddot{\theta}_{23}c_{0304} + r_2\dot{\theta}_{23}s_{0304} - r_3\ddot{\theta}_{23}c_{04} + r_3\dot{\theta}_{23}s_{04} + gs_{020304} \\ d_4\ddot{\theta}_{234}c_{\alpha 4} - d_4\dot{\theta}_{234}^2s_{\alpha 4} + r_2\ddot{\theta}_{23}s_{0304} + r_2\dot{\theta}_{23}c_{0304} + r_3\ddot{\theta}_{23}s_{04} + r_3\dot{\theta}_{23}c_{04} + gc_{020304} \\ 0 \end{pmatrix} \quad (20)$$

For the backward iteration, the inertial part of the dynamic model is first analyzed. By the analysis at the bucket, the effects of the inertial forces and inertial moments are summarized in Eq. (21)

$$f_5 = \begin{Bmatrix} f_{51} \\ f_{52} \\ f_{53} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad \text{and} \quad n_5 = \begin{Bmatrix} n_{51} \\ n_{52} \\ n_{53} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad (21)$$

The inward part of the Newton-Euler method can be used to obtain the iterative Newton-Euler dynamics algorithm for the hydraulic shovel. The inward Newton-Euler iterations for the bucket can be obtained using Eqs. (22)–(24)

$${}^4f_4 = {}^4R^5f_5 + {}^4F_4 = \begin{pmatrix} c_{05} & -s_{05} & 0 \\ s_{05} & c_{05} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} F_{41} \\ F_{42} \\ 0 \end{pmatrix} = \begin{pmatrix} F_{41} \\ F_{42} \\ 0 \end{pmatrix} \quad (22)$$

$${}^4n_4 = {}^4N_4 + {}^4R^5n_5 + {}^4P_{c_4} \times {}^4F_4 + {}^4P_5 \times {}^4R^5f_5 = \begin{pmatrix} 0 \\ 0 \\ I_{zz4}\ddot{\theta}_{234} + F_{42}d_4c_{\alpha 4} - F_{41}d_4s_{\alpha 4} \end{pmatrix} \quad (23)$$

$$\tau_4 = {}^4n_4 {}^4\hat{Z}_4 = (I_{zz4} + m_4d_4^2)\ddot{\theta}_{234} + m_4r_2d_4(\dot{\theta}_{23}^2s_{0304\alpha 4} + \ddot{\theta}_{23}c_{0304\alpha 4}) + m_4r_3d_4(\dot{\theta}_{23}^2s_{04\alpha 4} + \ddot{\theta}_{23}c_{04\alpha 4}) + m_4gd_4c_{020304\alpha 4} \quad (24)$$

The inward Newton-Euler iterations for the stick can be obtained as equations (25)–(27)

$${}^3f_3 = {}^4R^4f_4 + {}^3F_3 = \begin{pmatrix} c_{04} & -s_{04} & 0 \\ s_{04} & c_{04} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} F_{41} \\ F_{42} \\ 0 \end{pmatrix} + \begin{pmatrix} F_{31} \\ F_{32} \\ 0 \end{pmatrix} = \begin{pmatrix} F_{41}c_{04} - F_{42}s_{04} + F_{31} \\ F_{41}s_{04} + F_{42}c_{04} + F_{32} \\ 0 \end{pmatrix} \quad (25)$$

$${}^3n_3 = {}^3N_3 + {}^3R^4n_4 + {}^3P_{c_3} \times {}^3F_3 + {}^3P_4 \times {}^3R^4f_4 = \begin{pmatrix} 0 \\ 0 \\ \tau_4 + I_{zz3}\ddot{\theta}_{23} + F_{32}d_3c_{\alpha 3} - F_{31}d_3s_{\alpha 3} + F_{41}r_3s_{04} + F_{42}r_3c_{04} \end{pmatrix} \quad (26)$$

$$\begin{aligned} \tau_3 &= {}^3n_3 {}^3\hat{Z}_3 \\ &= \tau_4 + I_{zz3}\ddot{\theta}_{23} + F_{32}d_3c_{\alpha 3} - F_{31}d_3s_{\alpha 3} + F_{41}r_3s_{04} + F_{42}r_3c_{04} \\ &= \tau_4 + (I_{zz3} + m_3d_3^2 + m_4r_3^2)\ddot{\theta}_{23} + m_3r_2d_3(\dot{\theta}_{23}^2s_{03\alpha 3} + \ddot{\theta}_{23}c_{03\alpha 3}) \\ &\quad + m_4r_3d_4(\ddot{\theta}_{234}c_{04\alpha 4} - \dot{\theta}_{234}^2s_{04\alpha 4}) + m_4r_2r_3(\dot{\theta}_{23}^2s_{03} + \ddot{\theta}_{23}c_{03}) \\ &\quad + m_3gd_3c_{0203\alpha 3} + m_4gr_3c_{0203} \end{aligned} \quad (27)$$

The inward Newton-Euler iterations for the boom can be obtained as Eqs. (28)–(30)

$${}^2f_2 = {}^2R^3f_3 + {}^2F_2 = \begin{pmatrix} F_{41}c_{0304} - F_{42}s_{0304} + F_{31}c_{03} - F_{32}s_{03} + F_{21} \\ F_{41}s_{0304} + F_{42}c_{0304} + F_{31}s_{03} + F_{32}c_{03} + F_{22} \\ 0 \end{pmatrix} \quad (28)$$

$$\begin{aligned} {}^2n_2 &= {}^2N_2 + {}^2R^3n_3 + {}^2P_{c_2} \times {}^2F_2 + {}^2P_3 \times {}^2R^3f_3 \\ &= \begin{pmatrix} 0 \\ 0 \\ \tau_3 + I_{zz2}\ddot{\theta}_2 + F_{22}d_2c_{\alpha 2} - F_{21}d_2s_{\alpha 2} \end{pmatrix} \\ &\quad + \begin{pmatrix} 0 \\ 0 \\ F_{41}r_2s_{0304} + F_{42}r_2c_{0304} + F_{31}r_2s_{03} + F_{32}r_2c_{03} \end{pmatrix} \end{aligned} \quad (29)$$

$$\begin{aligned} \tau_2 &= {}^2n_2 {}^2\hat{Z}_2 \\ &= \tau_3 + [I_{zz2} + m_2d_2^2 + (m_3 + m_4)r_2^2]\ddot{\theta}_2 \\ &\quad + m_3r_2d_3(\ddot{\theta}_{23}c_{03\alpha 3} - \dot{\theta}_{23}^2s_{03\alpha 3}) \\ &\quad + m_4r_2d_4(\ddot{\theta}_{234}c_{0304\alpha 4} - \dot{\theta}_{234}^2s_{0304\alpha 4}) + m_4r_2r_3(\ddot{\theta}_{23}c_{03} - \dot{\theta}_{23}^2s_{03}) \\ &\quad + m_2gd_2c_{02\alpha 2} + (m_3 + m_4)gr_2c_{02} \end{aligned} \quad (30)$$

Analysis of the hydraulic forces and interaction forces follow the same procedure as the dynamic model analysis. The initial condition of the forces and moments for the bucket are given by Eq. (31)

$$f_5 = \begin{Bmatrix} f_{51} \\ f_{52} \\ f_{53} \end{Bmatrix} = \begin{Bmatrix} f_t \\ f_n \\ 0 \end{Bmatrix} \quad \text{and} \quad n_5 = \begin{Bmatrix} n_{51} \\ n_{52} \\ n_{53} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad (31)$$

The inward Newton-Euler iterations for the bucket can be obtained as Eqs. (32)–(34)

$${}^4f_4 = {}^4R^5f_5 + {}^4F_4 = \begin{pmatrix} F_r \cos(\theta_b + 0.1) + F_{JK} \cos(\theta_5 + \sigma_{25}) \\ F_r \sin(\theta_b + 0.1) + F_{JK} \sin(\theta_5 + \sigma_{25}) \\ 0 \end{pmatrix} \quad (32)$$

$${}^4n_4 = {}^4P_5 \times {}^4R^5f_5 + {}^4P_6 \times {}^4R^6f_6$$

$$= \begin{pmatrix} 0 \\ 0 \\ F_r r_4 \sin(\theta_b + 0.1) + F_{JK} r_5 \sin(\sigma_{25}) \end{pmatrix} \quad (33)$$

$$\tau_4 = {}^4n_4^T \hat{Z}_4 = F_r r_4 \sin(\theta_b + 0.1) + F_{JK} r_5 \sin(\sigma_{25}) \quad (34)$$

The inward Newton-Euler iterations for the stick can be obtained as Eqs. (35)–(37)

$${}^3f_3 = {}^3R^4f_4 + {}^3R^6f_6 + {}^3R^7f_7$$

$$= \begin{pmatrix} F_r \cos(\theta_4 + \theta_b + 0.1) + F_{JK} \cos(\theta_4 + \theta_5 + \sigma_{25}) \\ F_r \sin(\theta_4 + \theta_b + 0.1) + F_{JK} \sin(\theta_4 + \theta_5 + \sigma_{25}) \\ 0 \end{pmatrix}$$

$$+ \begin{pmatrix} F_{HI} \cos(\theta_6 + \sigma_{26}) \\ F_{HI} \sin(\theta_6 + \sigma_{26}) \\ 0 \end{pmatrix} + \begin{pmatrix} F_{FG} \cos(\theta_7 + \sigma_{27}) \\ F_{FG} \sin(\theta_7 + \sigma_{27}) \\ 0 \end{pmatrix} \quad (35)$$

$${}^3n_3 = {}^3R^4n_4 + {}^3P_4 \times {}^3R^4f_4 + {}^3P_6 \times {}^3R^6f_6 + {}^3P_7 \times {}^3R^7f_7$$

$$= \begin{pmatrix} 0 \\ 0 \\ F_r(r_3 \sin(\theta_4 + \theta_b + 0.1) + r_4 \sin(\theta_b + 0.1)) \end{pmatrix}$$

$$+ \begin{pmatrix} 0 \\ 0 \\ F_{JK}(r_3 \sin(\theta_4 + \theta_5 + \sigma_{25}) + r_5 \sin(\sigma_{25})) \end{pmatrix}$$

$$+ \begin{pmatrix} 0 \\ 0 \\ F_{HI} r_6 \sin(\sigma_{26}) + F_{FG} r_7 \sin(\sigma_{27}) \end{pmatrix} \quad (36)$$

$${}^2n_2 = {}^2R^3n_3 + {}^2P_3 \times {}^2R^3f_3 + {}^2P_8 \times {}^2R^8f_8 + {}^2P_9 \times {}^2R^9f_9 + {}^2P_{10} \times {}^2R^{10}f_{10} + {}^2P_{11} \times {}^2R^{11}f_{11}$$

$$= \begin{pmatrix} 0 \\ 0 \\ F_r(r_2 \sin(\theta_3 + \theta_4 + \theta_b + 0.1) + r_3 \sin(\theta_4 + \theta_b + 0.1) + r_4 \sin(\theta_b + 0.1)) \end{pmatrix}$$

$$+ \begin{pmatrix} 0 \\ 0 \\ F_{JK}[r_2 \sin(\theta_3 + \theta_4 + \theta_5 + \sigma_{25}) + r_3 \sin(\theta_4 + \theta_5 + \sigma_{25}) + r_5 \sin(\sigma_{25}) + r_8 \sin(\sigma_{28})] \end{pmatrix}$$

$$+ \begin{pmatrix} 0 \\ 0 \\ F_{HI}[r_2 \sin(\theta_3 + \theta_6 + \sigma_{26}) + r_6 \sin(\sigma_{26}) + r_{11} \sin(\sigma_{211})] \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ F_{FG}[r_2 \sin(\theta_3 + \theta_7 + \sigma_{27}) + F_{FG} r_7 \sin(\sigma_{27}) + r_{10} \sin(\sigma_{210})] \end{pmatrix}$$

$$+ \begin{pmatrix} 0 \\ 0 \\ F_{DE} r_9 \sin(\sigma_{29}) \end{pmatrix} \quad (39)$$

$$\tau_2 = {}^2n_2^T \hat{Z}_2 = F_r[r_2 \sin(\theta_3 + \theta_4 + \theta_b + 0.1) + r_3 \sin(\theta_4 + \theta_b + 0.1)$$

$$+ r_4 \sin(\theta_b + 0.1)] + F_{JK}[r_2 \sin(\theta_3 + \theta_4 + \theta_5 + \sigma_{25}) + r_3 \sin(\theta_4$$

$$+ \theta_5 + \sigma_{25}) + r_5 \sin(\sigma_{25}) + r_8 \sin(\sigma_{28})] + F_{HI}[r_2 \sin(\theta_3 + \theta_6$$

$$+ \sigma_{26}) + r_6 \sin(\sigma_{26}) + r_{11} \sin(\sigma_{211})] + F_{FG}[r_2 \sin(\theta_3 + \theta_7 + \sigma_{27})$$

$$+ F_{FG} r_7 \sin(\sigma_{27}) + r_{10} \sin(\sigma_{210})] + F_{DE} r_9 \sin(\sigma_{29}) \quad (40)$$

$$\tau_3 = {}^3n_3^T \hat{Z}_3$$

$$= F_r[r_3 \sin(\theta_4 + \theta_b + 0.1) + r_4 \sin(\theta_b + 0.1)]$$

$$+ F_{JK}[r_3 \sin(\theta_4 + \theta_5 + \sigma_{25}) + r_5 \sin(\sigma_{25})] + F_{HI} r_6 \sin(\sigma_{26})$$

$$+ F_{FG} r_7 \sin(\sigma_{27}) \quad (37)$$

The inward Newton-Euler iterations for the boom can be obtained as Eqs. (38)–(40)

$${}^2f_2 = {}^2R^3f_3 + {}^2R^8f_8 + {}^2R^9f_9 + {}^2R^{10}f_{10} + {}^2R^{11}f_{11}$$

$$= \begin{pmatrix} F_r \cos(\theta_3 + \theta_4 + \theta_b + 0.1) \\ F_r \sin(\theta_3 + \theta_4 + \theta_b + 0.1) \\ 0 \end{pmatrix}$$

$$+ \begin{pmatrix} F_{JK}[\cos(\theta_3 + \theta_4 + \theta_5 + \sigma_{25}) + \cos(\theta_8 + \sigma_{28})] \\ F_{JK}[\sin(\theta_3 + \theta_4 + \theta_5 + \sigma_{25}) + \sin(\theta_8 + \sigma_{28})] \\ 0 \end{pmatrix}$$

$$+ \begin{pmatrix} F_{HI}[\cos(\theta_3 + \theta_6 + \sigma_{26}) + \cos(\theta_{11} + \sigma_{211})] \\ F_{HI}[\sin(\theta_3 + \theta_6 + \sigma_{26}) + \sin(\theta_{11} + \sigma_{211})] \\ 0 \end{pmatrix}$$

$$+ \begin{pmatrix} F_{FG}[\cos(\theta_3 + \theta_7 + \sigma_{27}) + \cos(\theta_{10} + \sigma_{210})] \\ F_{FG}[\sin(\theta_3 + \theta_7 + \sigma_{27}) + \sin(\theta_{10} + \sigma_{210})] \\ 0 \end{pmatrix}$$

$$+ \begin{pmatrix} F_{DE} \cos(\theta_9 + \sigma_{29}) \\ F_{DE} \sin(\theta_9 + \sigma_{29}) \\ 0 \end{pmatrix} \quad (38)$$

As a summary of Eqs. (34), (37), and (40), F_{BE} , F_{FI} , F_{JK} , F_{LK} , and F_{PK} can be obtained as Eqs. (41)–(43)

$$\tau_4 = {}^4n_4^T \hat{Z}_4 = F_r r_4 \sin(\theta_b + 0.1) + F_{JK} r_5 \sin(\sigma_{25}) \quad (41)$$

$$\tau_3 = F_r[r_3 \sin(\theta_4 + \theta_b + 0.1) + r_4 \sin(\theta_b + 0.1)] + F_{JK}[r_3 \sin(\theta_4 + \theta_5$$

$$+ \sigma_{25}) + r_5 \sin(\sigma_{25})] + F_{HI} r_6 \sin(\sigma_{26}) + F_{FG} r_7 \sin(\sigma_{27}) \quad (42)$$

$$\begin{aligned}\tau_2 = & F_r[r_2 \sin(\theta_3 + \theta_4 + \theta_b + 0.1) + r_3 \sin(\theta_4 + \theta_b + 0.1) + r_4 \sin(\theta_b \\ & + 0.1)] + F_{JK}[r_2 \sin(\theta_3 + \theta_4 + \theta_5 + \sigma_{25}) + r_3 \sin(\theta_4 + \theta_5 + \sigma_{25}) \\ & + r_5 \sin(\sigma_{25}) + r_8 \sin(\sigma_{28})] + F_{HI}[r_2 \sin(\theta_3 + \theta_6 + \sigma_{26}) \\ & + r_6 \sin(\sigma_{26}) + r_{11} \sin(\sigma_{211})] + F_{FG}[r_2 \sin(\theta_3 + \theta_7 + \sigma_{27}) \\ & + r_7 \sin(\sigma_{27}) + r_{10} \sin(\sigma_{210})] + F_{DE}r_9 \sin(\sigma_{29})\end{aligned}\quad (43)$$

Their corresponding matrix form is given by equation (44)

$$\begin{Bmatrix} \tau_4^2 \\ \tau_3^2 \\ \tau_2^2 \end{Bmatrix} = \begin{Bmatrix} \Gamma_{11} & 0 & 0 & 0 \\ \Gamma_{21} & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} \\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33} & 0 \end{Bmatrix} \begin{Bmatrix} F_{JK} \\ F_{HI} \\ F_{FG} \\ F_{DE} \end{Bmatrix} - F_{load}(F_t, F_n) \quad (44)$$

$$\Gamma_{33} = r_2 \sin(\theta_3 + \theta_7 + \sigma_{27}) + r_7 \sin(\sigma_{27}) + r_{10} \sin(\sigma_{210}) \quad (45)$$

$$\Gamma_{32} = r_2 \sin(\theta_3 + \theta_6 + \sigma_{26}) + r_6 \sin(\sigma_{26}) + r_{11} \sin(\sigma_{211}) \quad (46)$$

$$\Gamma_{31} = r_2 \sin(\theta_3 + \theta_4 + \theta_5 + \sigma_{25}) + r_3 \sin(\theta_4 + \theta_5 + \sigma_{25}) + r_5 \sin(\sigma_{25}) \quad (47)$$

$$\Gamma_{24} = r_9 \sin(\sigma_{29}) \quad \Gamma_{23} = r_7 \sin(\sigma_{27}) \quad \Gamma_{22} = r_6 \sin(\sigma_{26}) \quad (48)$$

$$\Gamma_{21} = r_3 \sin(\theta_4 + \theta_5 + \sigma_{25}) + r_5 \sin(\sigma_{25}) \quad \Gamma_{11} = r_5 \sin(\theta_{25}) \quad (49)$$

and

$$F_{load}(F_t, F_n) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{Bmatrix} -F_r r_4 [\sin \theta_b \cos(0.1) + \cos \theta_b \sin(0.1)] \\ -F_r r_3 [\sin(\theta_4 + \theta_b) \cos(0.1) + \cos(\theta_4 + \theta_b) \sin(0.1)] \\ -F_r r_2 [\sin(\theta_3 + \theta_4 + \theta_b) \cos(0.1) + \cos(\theta_3 + \theta_4 + \theta_b) \sin(0.1)] \end{Bmatrix} \quad (50)$$

Assuming that $F_t = F_r \cos(0.1)$, and $F_n = F_r \sin(0.1)$, then

$$\begin{aligned}F_{load}(F_t, F_n) = & \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \\ & \times \begin{Bmatrix} r_4(-F_t \sin \theta_b - F_n \cos \theta_b) \\ r_3[-F_t \sin(\theta_4 + \theta_b) - F_n \cos(\theta_4 + \theta_b)] \\ r_2[-F_t \sin(\theta_3 + \theta_4 + \theta_b) - F_n \cos(\theta_3 + \theta_4 + \theta_b)] \end{Bmatrix}\end{aligned}\quad (51)$$

The bucket-environment geometrical relationships are illustrated in Fig. 5. Therefore, Eq. (51) is formulated as Eq. (52)

$$\begin{aligned}F_{load}(F_t, F_n) = & \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \\ & \times \begin{Bmatrix} r_4(-F_t \sin \theta_b - F_n \cos \theta_b) \\ r_3[F_t \sin(\theta_2 + \theta_3 - \theta_{dg}) - F_n \cos(\theta_2 + \theta_3 - \theta_{dg})] \\ r_2[F_t \sin(\theta_2 - \theta_{dg}) - F_n \cos(\theta_2 - \theta_{dg})] \end{Bmatrix}\end{aligned}\quad (52)$$

Notation

The following symbols are used in this paper:

- $C(\Theta, \dot{\Theta})$ = generalized Coriolis and centripetal torque;
- C_{ij} = (i, j) element of Coriolis and centripetal torque $C(\Theta, \dot{\Theta})$;
- c_1, s_1 = $\cos \theta_1, \sin \theta_1$, respectively;
- $c_{\theta_i}, s_{\theta_i}$ = $\cos \theta_i, \sin \theta_i$ (i, j), respectively;
- $c_{\theta_i \theta_j}, s_{\theta_i \theta_j}$ = $\cos(\theta_i + \theta_j), \sin(\theta_i + \theta_j)$ (i, j), respectively;
- $D(\Theta)$ = generalized inertia matrix;
- D_{ij} = (i, j) element of inertia matrix $D(\Theta)$;
- $d_1, \dot{d}_1, \ddot{d}_1$ = length, velocity, and acceleration of the stick;
- d_i = offset distance of the gravity center in link i ;

- F = function of the hydraulic force effects;
- F_{BE} = piston force of hydraulic activator connected between B and E, and B, E signify any two points on excavator (Fig. 3);
- $F_{load}(F_t, F_n)$ = resistive force due to soil-bucket interaction;
- F_n = normal reaction force;
- F_r = loading force acting on bucket from the soil;
- F_t = tangential reaction force;
- $G(\Theta)$ = generalized gravity torque;
- G_i = i th component of gravity torque $G(\Theta)$;
- I_i = moment of inertia of link I about the centroid axis parallel to z_i -axis;
- I_{zz1} = moment of inertia of the stick about centroidal axis parallel to z_1 -axis;
- I_{zz2} = moment of inertia of bucket about centroidal axis parallel to z_2 -axis;
- $\bar{k}_{zi}$ = unit vector on the z_i -axis;
- $\bar{k}_{zi}$ = unit vector on the z_i -axis;
- L_1 = length of the stick from pivotal point to the connection point between arm and bucket ($L_1 = l_1$);
- L_2 = length between bucket tip and connect point of arm and bucket ($L_2 = l_2$);
- m_i = mass of link i ;
- m_1 = mass of the stick;
- m_2 = mass of bucket;
- r_i = the length between the hydraulic force action point and the rotation point of a link ($i=5, 6, 7, 8, 9, 10, 11$);
- Γ_{ij} = (i, j) element of control matrix $\Gamma(\Theta)$;
- $\theta_1, \dot{\theta}_1, \ddot{\theta}_1$ = angular displacement, velocity, and acceleration of the stick, respectively;
- θ_b = angle between bucket bottom and X_4 -axis;
- θ_{dg} = angle between bucket edge and horizontal line (digging angle);
- $\theta_i, \dot{\theta}_i, \ddot{\theta}_i$ = angular displacement, velocity, and acceleration of joint i ($i=1, 2, 3$), respectively.

Θ = vector of generalized variables; and
 σ_i = the angle between r_i and the direction of a hydraulic force ($i=25,26,27,28,29,210,211$).

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