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ANALYSIS OF ANNUAL VARIATIONS IN GROUND
TEMPERATURE AT AN OTTAWA SITE

BY

L. W. GOLD

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SOMMAIRE

Des observations effectuées sur une période de cinq ans en un emplacement situé à Ottawa ont montré que la température du sol avait des composantes Fourier importantes dont les périodes étaient $\frac{1}{2}$ et de 2 ans. La température moyenne annuelle du sol et les amplitudes des composantes Fourier dont les périodes étaient de 1 an et de 6 mois ont subi des fluctuations non périodiques de presque 1°C à une profondeur de 10 cm. L'amplitude de cette fluctuation diminuait avec la profondeur et son maximum s'est produit avec un décalage de temps. On a eu la preuve d'une augmentation graduelle de la température moyenne annuelle du sol correspondant à environ 0,2°C sur la période de 5 ans à une profondeur de 610 cm. La signification de si petits changements de température, dans des zones où la température du sol est d'environ 0°C, est indiquée.

ANALYZED

**ANALYSIS OF ANNUAL VARIATIONS IN GROUND
TEMPERATURE AT AN OTTAWA SITE**

L. W. GOLD

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ANALYSIS OF ANNUAL VARIATIONS IN GROUND TEMPERATURE AT AN OTTAWA SITE¹

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ABSTRACT

Observations over a 5-year period at a site at Ottawa showed that the ground temperature had significant Fourier components with period $\frac{1}{2}$ and 2 years. The average annual ground temperature and amplitudes of the Fourier components of period 1 year and $\frac{1}{2}$ year underwent non-periodic fluctuations of almost 1 °C degree at a depth of 10 cm. The amplitude of this fluctuation decreased with depth, and its maximum occurred later in time. There was evidence of a gradual increase in average annual ground temperature amounting to about 0.2 °C degree over the 5-year period at the 610-cm depth. The significance of such small temperature changes in areas where the ground temperature is close to 0 °C is pointed out.

INTRODUCTION

The temperature of the ground near the surface undergoes fluctuations in response to changes in the heat transfer processes at the surface. The major fluctuation in ground temperature has a period of 1 year, corresponding to the annual variation in solar radiation. There is also a significant diurnal component corresponding to the daily variation in radiation. Superimposed on periodic fluctuations are non-periodic fluctuations whose duration covers the time range of seconds to years. The non-periodic changes can have a variety of causes such as sporadic cloud cover, and changes in weather and climate.

During the 5-year period, 1 Nov. 1957 to 31 Oct. 1962, observations were made on ground temperature at a site in Ottawa, Canada, to a depth of 610 cm. These observations were analyzed to determine the average annual temperature and the Fourier components of period 1, $\frac{1}{2}$, and 2 years, and the non-periodic fluctuations that occurred in these quantities. The results of this analysis are presented in this paper. The significance of the non-periodic fluctuations with respect to the ground thermal regime and observations of it are discussed briefly.

INSTALLATION

All measurements were made with No. 20 gauge copper-constantan thermocouples located in a sensitive marine clay at least 10 m deep overlying limestone formations. The surface cover in summer was natural grass that was allowed to grow to a height of between 15 and 20 cm before cutting. Temperatures were recorded once every 20 min on a 16-point Leeds and Northrup recorder located in an instrument tunnel about 46 m (150 ft) from the ground temperature installation.

Temperatures were measured at two adjacent installations. At one, installed in 1956, the thermocouples were placed in duraluminum tubes and inserted into the vertical face of a pit at various levels down to 90 cm below the surface.

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A description of this installation, together with some results from observations taken during the first year, is given by Pearce and Gold (1959). Temperatures measured at depths of 10, 40, and 90 cm were analyzed.

For the second installation, the thermocouples were placed in a groove 1 in sq. cut into a 3- by 3-in pine pole about 20 ft long. The thermocouples were brought to the surface of the pole at appropriate locations, the groove filled with a plastic and capped with a piece of 1- by 3-in pine. At each thermocouple location a groove was cut circumferentially about the pole and the thermocouple wound into it and sealed in place. The pole was painted with a preservative and installed in a hole drilled about 6 months earlier. Temperatures measured 213 cm (7 ft), 366 cm (12 ft), and 610 cm (20 ft) below the surface were analyzed.

ANALYSIS OF OBSERVATIONS

The technique of Fourier analyses is usually used in studies of ground temperatures. The dependence of temperature on depth and time is assumed to satisfy simple diffusion theory and to be given by a series of the form

$$T(z, t) = T_0 + \sum_n T_n \exp [-\sqrt{(nwz^2/2\lambda)}] \sin [nwt + \delta_n - \sqrt{(nwz^2/2\lambda)}]$$

where

$T(z, t)$ is the temperature at depth z and time t ,

T_0 is the average temperature at depth z , which for this study was the average annual temperature,

T_n is the amplitude at the soil surface of the Fourier component having n complete cycles in a given period,

w is the angular frequency,

λ is the thermal diffusivity of the soil,

t is the time,

δ_n is the phase angle for the n th Fourier component.

A program was available for the Bendix G-15D computer with which the average annual temperature and Fourier components could be calculated from average daily temperatures (Stephenson 1960). The program required that the observations be grouped into blocks containing 50 average daily temperatures each in chronological order. As 15 days of observations distributed uniformly throughout the year were rejected, there were seven blocks of observations for each 1-year period. The average daily temperature 10 cm below the surface was assumed to be equal to the mean of the daily maximum and minimum. At the other depths the temperature changed so slowly that the daily average could be read directly from the recorder charts.

The computer was programmed to determine the average annual temperature, and amplitude and phase of the specified Fourier components for consecutive periods of 1 year (2 years for the Fourier component with period of 2 years). Each 1-year period overlapped the previously analyzed period by 6/7 year and included 1/7 year of new observations. The analysis for the 5-year period, therefore, gave 29 consecutive values of average annual temperature and Fourier components with period of $\frac{1}{2}$ and 1 year and 22 values for the component of period 2 years.

One disturbing fact arose during the plotting of profiles of average annual ground temperature: a discontinuity was present between the profiles obtained from the shallow installation and those from the deep one. Temperatures were also measured at the 76- and 107-cm depths at the deep installation, and there the average annual temperatures were continuous with the deeper values. When profiles were plotted of average daily temperature it was found that for the months of July to December, inclusive, the profiles obtained from the deep installation were consistent with those obtained from the shallow installation. For the months of January to June, inclusive, the temperatures measured at the 76- and 107-cm depths were usually 0.2 to 0.8 C degrees warmer than those measured at the same depth at the shallow installation.

There was no obvious reason why temperatures measured at the same depth in winter at the two installations should be different. The distance between them was only about 4 m and the snow cover at the site was quite uniform. It is possible that the rise in the groundwater table that occurs in winter could have an effect, particularly if there was poor contact between the deep hole probe and the walls of the hole. Conduction of heat up or down the probe appears to be ruled out because the two installations gave consistent readings in summer. In this regard, a study by Brown (1963) indicates that, for depths greater than 1 m, errors due to heat conduction along the probe should be negligible in the present circumstance.

Because the probes in the shallow installation were inserted horizontally into undisturbed soil, it was assumed that temperatures measured by them were correct. It was also assumed that the temperature measured at the 610-cm depth at the deep installation was correct; that the temperature measured 213 cm below the surface was in error by an amount equal to that of the 100-cm depth, as shown by profiles of average daily temperature; and that the temperature measured 366 cm below the surface was in error by one-half this amount. The average annual temperatures for these depths were adjusted according to this assumption. This adjustment was not made for the calculation of the Fourier components.

PRESENTATION OF RESULTS

Average Annual Temperatures

Figure 1 shows the dependence on time of the average annual temperatures for each depth. A marked fluctuation is present that has a period of about 2 years and an amplitude at the 10-cm depth of almost 1 C degree. The amplitude of this fluctuation decreases and the date at which it is a maximum occurs later with increasing depth. For example, the amplitude of the fluctuation at the 610-cm depth (20 ft) was almost 0.2 C degree, and its maximum value occurred about 3/4 of a year later than at the 10-cm depth.

Profiles of the modified average annual temperature for the period 1 November to 31 October are given for the five years in Fig. 2 as well as the average temperature profile for the 5-year period. It appears that the modified annual averages for the 213- and 366-cm depths are still not consistent with those

FIG. 1. Change with time in the average annual ground temperature for period Nov. 1/57 to Oct. 31/62.

for shallower depths. Some of this apparent distortion is due to the phase change that occurs with depth in the fluctuation in the average annual temperature. For example, the decrease in the average annual temperature that is apparent at the 10-cm depth at the end of the 1961-1962 period had only propagated to about the 2-m depth for the 1 November to 31 October calculation.

Fourier Component of Period 1 Year

Figure 3 shows for each depth the amplitude of the Fourier component of period 1 year. A fluctuation is present that corresponds to that associated with the average annual temperatures. The amplitude of the fluctuation is about 1.5 C degree at the 10-cm depth and decreases to almost zero at the 610-cm depth. In Fig. 4 the logarithms of the amplitudes calculated from the 5-year

FIG. 2. Average annual ground temperature for period Nov. 1 to Oct. 31.

FIG. 3. Change with time in the amplitude of Fourier component of period 1 year.

sums for the sine and cosine products are plotted against depth. According to diffusion theory, if the soil is behaving as an ideal, semi-infinite medium for a harmonic disturbance of period 1 year, the points should lie along a straight line. This is observed to occur, except for small deviations at the 40- and 610-cm depths. The thermal diffusivity of the soil can be calculated from the slope of the line and was found to be $10.8 \text{ cm}^2/\text{h}$, in good agreement with the value observed by Pearce and Gold (1959) for the same site.

The phase angle was found to be constant at each depth and fluctuations that did occur increased with depth. The 5-year and extreme values for the phase are given in Table I. In Fig. 5 the value obtained from the 5-year sums

TABLE I
Five-year average and extremes of phase angles for Fourier components
of period 1 year and $\frac{1}{2}$ year

Depth	Phase angle in degrees					
	1 year			$\frac{1}{2}$ year		
	5-year value	Extremes		5-year value	Extremes	
10	184.4	181.7	186.3	-67.9	-99.8	-36.5
40	170.4	164.9	172.9	-87.4	105.4	-46.4
90	156.5	151.5	159.6	-106.8	-130.6	-61.0
213	115.9	106.4	118.9	+187.5	128.0	-128.8
366	58.2	47.3	67.1	95.4	22.9	146.1
610	-22.6	-34.7	1.7	128.8	162.8	-127.8

for the sine and cosine products are plotted against depth. As for the logarithm of the amplitudes, the plotted points should lie along a straight line if diffusion theory is valid. In the present case, the phases at the 40- and 366-cm depth deviate slightly from a straight line. The diffusivity obtained from the slope of the line is $10.1 \text{ cm}^2/\text{h}$, which is about 7% lower than that obtained from the amplitudes.

Fourier Component of Period $\frac{1}{2}$ Year

Figure 6 shows the time-dependence of the Fourier component of period $\frac{1}{2}$ year. The amplitude of this component at the 10-cm depth, about 1 C degree, is roughly 1/10th the amplitude of the component of period 1 year, and the fluctuations that occur in it are similar to those that occurred in the average annual temperature.

In Figs. 4 and 5 are plotted the phase and logarithm of the average amplitudes calculated from the 5-year sums for the sine and cosine products. Lines corresponding to a diffusivity of $10.8 \text{ cm}^2/\text{h}$ for the logarithm of the amplitude and $10.1 \text{ cm}^2/\text{h}$ for the phase are drawn through the points. It is of interest that the respective lines fit the plotted points satisfactorily for the shallow installation but significant deviations occur in the results from the deep hole site, particularly for the phase. The deviation in the phase shift at the 213- and 366-cm depths are consistent with the suggested possible influence of ground-

FIG. 4. Dependence on depth of the logarithms of the amplitudes of the Fourier components.

water during the winter period. The 5-year and extreme values for the phase are given in Table I.

Fourier Component with Period of 2 Years

Amplitudes and phases of Fourier components with period of 2 years were determined from the observations. The change in the amplitude with time is shown in Fig. 7 for the 10-, 213-, 366-, and 610-cm depths and the amplitude and phase, calculated from the sums of the sine and cosine products for the 4-year period, 1 November 1957 to 31 October 1961, are shown in Figs. 4 and 5, respectively. The lines obtained using the diffusivities $\lambda = 10.8 \text{ cm}^2/\text{h}$ for the amplitude and $\lambda = 10.1 \text{ cm}^2/\text{h}$ for the phase are also shown. The plotted lines are consistent with the observations to about the 2-m depth, and significant deviations occur at greater depth.

FIG. 5. Dependence on depth of the phase of the Fourier components.

DISCUSSION

Ground temperatures are necessary information for much agricultural and engineering activity, as well as for the definition of the lower boundary conditions of the atmosphere. Because of its harmonic character, a useful description of ground temperature can often be obtained by simple Fourier analysis. In the present study, attention was directed to the annual temperature wave at a particular site and to its Fourier components of period $\frac{1}{2}$, 1, and 2 years.

In Fig. 8 are plotted average monthly values of ground temperature at the 10-cm depth for the 5-year period. Although the Fourier component of period 1 year is a reasonable approximation of the observations, it gives temperatures that are too low in winter and too high in spring and fall. A study by Gold (1963) of factors influencing the temperature at the 10-cm depth had shown that snow cover was the cause of the flattened portion of the temperature curve in winter. It would be expected that this influence would induce a

FIG. 6. Change with time in the amplitude of Fourier component of period $\frac{1}{2}$ year.

FIG. 7. Change with time in the amplitude of Fourier component of period 2 years.

FIG. 8. Five-year average temperature, Fourier components with period 1 year and $\frac{1}{2}$ year, and their sum for the 10 cm depth.

Fourier component of period $\frac{1}{2}$ year, but since it is present only in winter, the component calculated from the observations overcorrected the first Fourier component approximation in summer, as shown in Fig. 8. A better measure of the influence of snow cover would probably have been the Fourier component of period $\frac{1}{2}$ year calculated from observations made in the winter period only (i.e. about 1 November to 30 April).

It would have been possible to determine the amplitudes and phases of the additional Fourier components required to yield an accurate analytical expression for the annual ground temperature, as was done by Carson (1963) for average monthly ground temperatures at Argonne, Illinois. It was considered, however, that these components would have limited physical significance in the present case in comparison with the components of period $\frac{1}{2}$ and 1 year, their main contribution probably being to adjust for the overcorrection in summer caused by the calculated Fourier component with period $\frac{1}{2}$ year.

A change in average annual ground temperature or amplitudes of the Fourier components indicates a difference between the sums for the new 1/7 year of observations in a calculation and those for the corresponding period 1 year earlier. For example, the increase in the amplitude of the Fourier component associated with point 4, 1958–1959 (Fig. 3), was caused by an increase in ground temperatures during the period 29 May to 19 July 1959, over those for the corresponding period in 1958. An earlier study showed that the fluctuations in the average annual ground temperature at the 10-cm depth correlated

qualitatively with year-to-year fluctuations in average snow depth and average summer air temperature (Gold 1963). This correlation, as well as the decrease in amplitude and shift in phase with depth, shows that the non-periodic fluctuations in the average annual ground temperature and in the amplitudes of the Fourier components are due to real fluctuations in the ground thermal regime, and not to the characteristics of the recording apparatus. In the present case, the fluctuations induced had a period of about 2 years during the 5 years of observations as shown by calculation of the Fourier components, but this periodicity is probably fortuitous.

The observations at the 610-cm depth indicate a gradual warming of the soil amounting to about 0.2 C degree in 5 years, suggesting the possibility of a fluctuation with a period of several years. The observed small decrease with depth of about 3.0×10^{-4} °C/cm in the 5-year average temperature shown in Fig. 2 supports this possibility. It is not considered that the unexplained behavior at the 210- and 366-cm depths is responsible for the gradual increase, as it should have an influence similar to that of snow cover. The gradual increase could be due to a systematic error in the observations, but this is considered improbable.

It is of interest that there is a small discrepancy between the diffusivity calculated from the change with depth of the phase and of the logarithm of the amplitudes. The discrepancy is such that the phase is retarded more than would be expected from the amplitude change. This is shown by the broken lines in Figs. 4 and 5; these correspond to diffusivities of 10.1 and 10.8 cm²/h, respectively. A difference between the diffusivities calculated by the two methods indicates that the observations do not completely satisfy the assumptions underlying the simple theory. This may be because the thermal conductivity, specific heat, and density of the soil are not independent of depth or time, and that mathematical models such as that given by Lettau (1954) are required for an accurate analysis. For calculations under the variable conditions encountered in the field, the implications of a discrepancy of 7% in the diffusivities can usually be ignored, however, and simple theory assumed valid.

The non-periodic fluctuations in the annual ground temperature and amplitude of Fourier components of period $\frac{1}{2}$ and 1 year are caused primarily by variations that occur from year to year in the heat transferred between the soil surface and the atmosphere, with a possible contribution from variations in the thermal properties of the soil. If it is assumed that the soil thermal properties determined for the site by Pearce and Gold (1959) are constant, the fluctuation of about 1 C degree at the 10-cm depth in the average annual ground temperature and Fourier component of period 1 year corresponds to a change in total heat content of the ground of less than 400 cal/cm², which is about 20% of the net heat flow into the ground and about 1% of the net radiation absorbed by the ground surface during the period 1 April to 30 September. The gradient of 3.0×10^{-4} °C/cm in the average annual ground temperature corresponds to an annual heat flux into the ground of about 20 cal/cm² year, of the same order as the heat flow associated with the geothermal gradient.

Such small changes and fluctuations in the ground thermal regime would not have a significant effect on soil properties in many areas. In areas where the average annual ground temperature is close to 0 °C, however, they could result in the progressive freezing or thawing of soil water. In such circumstances, the effective thermal diffusivity would be considerably larger than that for the Ottawa site, and consequently fluctuations of period about 2 years would be confined to a shallower depth. A fluctuation of period of several years, however, would propagate to an appreciable depth and could result in the degradation or formation of permafrost. Such an occurrence would be of considerable practical significance at many locations.

CONCLUSIONS

The ground temperatures observed at the site in Ottawa had significant Fourier components with period of $\frac{1}{2}$ and 2 years. The average annual ground temperatures and amplitudes of the Fourier components of period 1 year and $\frac{1}{2}$ year underwent non-periodic fluctuations of about 1 C degree at the 10-cm level. These fluctuations decreased with depth, and had an amplitude of about 0.2 C degree 6 m below the surface. There was evidence of a gradual increase in average annual ground temperature of about 0.2 C degree in the 5-year period at the 610-cm depth. It is possible that groundwater influenced the observations made at the deep installation.

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