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03838 *Calculation of the Fire
Resistance of Composite
Concrete Floor and
Roof Slabs*

by T. T. Lie

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SOMMAIRE

La publication présente une méthode de calcul de la résistance au feu des planchers et des dalles de toits composites. À partir de cette méthode, on a établi des formules simples pour le calcul de la résistance au feu et de l'épaisseur minimale du béton recouvrant l'armature dans les dalles de béton armé et précontraint composées d'une couche de béton léger et d'une couche de béton ordinaire. La comparaison avec les résultats d'essais démontre la similarité entre la résistance au feu calculée et celle obtenue à partir d'essais.

Calculation of the Fire Resistance of Composite Concrete Floor and Roof Slabs

T. T. LIE

Division of Building Research

National Research Council of Canada

A calculation method is described for the determination of the fire resistance of composite floor and roof slabs. Using this method, simple approximate formulas are derived for the calculation of the thermal fire resistance and minimum cover thickness to the steel of reinforced and prestressed concrete slabs, consisting of two layers of concrete, one lightweight and one normal weight. Comparison with test results shows good agreement between calculated and experimental fire resistances.

IN RECENT years, methods have been developed that have made it possible to solve many fire performance problems by calculation. In order to calculate the fire performance of a building element, it is necessary to know the temperature history of the element during exposure to fire. Methods exist for the calculation of the temperature history in various building elements, such as protected and unprotected steel columns, solid columns, and slabs. If the relevant material properties are known, accurate prediction of the temperatures in these elements during fire exposure is possible.

In this study, a method is described for the calculation of the temperature distribution in composite floor and roof slabs exposed to heating from below, under the conditions specified in ASTM E119.¹ As a particular case, slabs composed of two layers of concrete of various thicknesses, one lightweight and the other normal weight concrete, are considered. Their fire resistance, defined as the time to reach specific critical temperatures on the unexposed side of the slabs or on the steel in the slabs, is evaluated.

CALCULATION METHOD

To calculate the temperature history of the slab, a finite difference method is used.² In this method, a cross-section of the slab is divided into a number of elementary layers (Figure 1). The thickness of the layers is

Δx with the exception of the boundary layers, which are $\frac{1}{2}\Delta x$ thick. Each layer is represented by a point P_m . The temperature in each elementary layer is assumed to be uniform and equal to that of the representative point. At the top, the slab is covered by an asbestos pad of prescribed thickness.¹ In Figure 1, a composite slab is shown consisting of two laminar concrete slabs, the lower made of concrete type n_1 and the upper of concrete type n_2 . The thickness of the (concrete) $_{n_1}$ slab is $(M_1 - 1)\Delta x$, that of the (concrete) $_{n_2}$ slab is $(M_2 - M_1)\Delta x$.

For each elementary layer a heat transfer equation is written for the time $t = j\Delta t$, where $j = 0, 1, 2, \dots$ and Δt is an appropriate time increment. With the aid of these equations, the temperature of each layer can be successively evaluated for any time $t = (j + 1)\Delta t$ if the temperature at the time $t = j\Delta t$ is known.

The procedure of deriving the heat transfer equations for fire-exposed building elements is described in detail in previous studies.³ Therefore, the method of deriving these equations for the composite slab will not be discussed in this study; only the equations used for the calculation of the slab temperatures and information not yet dealt with in previous studies will be given.

EQUATIONS FOR THE BOUNDARY FIRE AND SLAB

In the derivation of the equations for the boundary fire and slab, it is assumed that the slab is exposed from below to a fire, the temperature of which increases according to the temperature-time curve specified in ASTM E119. This curve can be described approximately by the following expression:

$$T_f = T_o + 1,350 [1 - \exp(-3.79553\sqrt{t})] + 306.74\sqrt{t} \quad (1)$$

The temperature at the time $t = (j + 1)\Delta t$ of the boundary elementary layer of the slab, represented by the point p_1 can be given by:

$$\begin{aligned} T_1^{j+1} = T_1^j &+ \frac{2\sigma\epsilon_{n_1}}{(\rho_{n_1}c_{n_1})_1^j\Delta x} \left[(T_f^j)^4 - (T_1^j)^4 \right] \Delta t \\ &- \frac{(k_1^j + k_2^j)}{(\rho_{n_1}c_{n_1})_1^j(\Delta x)^2} (T_1^j - T_2^j) \Delta t \end{aligned} \quad (2)$$

EQUATIONS FOR THE INSIDE OF THE SLAB

For an elementary layer represented by a point P_m , located inside the slab but not at the boundary of two layers of different material, the temperature at the time $t = (j + 1)\Delta t$ is given by:

$$T_m^{j+1} = T_m^j + \frac{\Delta t}{(\rho_n c_n)_m^j (\Delta x)^2} - \left[\left(\frac{k_{m-1}^j + k_m^j}{2} \right) (T_{m-1}^j - T_m^j) - \left(\frac{k_m^j + k_{m+1}^j}{2} \right) (T_m^j - T_{m+1}^j) \right] \quad (3)$$

For a boundary elementary layer inside the slab, represented by the point P_{M_1} and composed partly of concrete type n_1 and partly of concrete type n_2 (Figure 1), the temperature is given by:

$$T_{M_1}^{j+1} = T_{M_1}^j + \frac{\Delta t}{((\rho_{n_1} c_{n_1})_{M_1}^j + (\rho_{n_2} c_{n_2})_{M_1}^j) (\Delta x)^2} \left[(k_{M_1-1}^j + (k_{n_1})_{M_1}^j) (T_{M_1}^j - T_{M_1}^j) - ((k_{n_2})_{M_1}^j + k_{M_1+1}^j) (T_{M_1}^j - T_{M_1+1}^j) \right] \quad (4)$$

EQUATION FOR THE BOUNDARY SLAB AND ASBESTOS PAD

According to the specifications in ASTM E119, temperatures of the unexposed face of the slab should be measured under an asbestos pad of prescribed dimensions and properties. In the calculation of these tem-

Figure 1. Arrangement of elementary layers in composite slab.

peratures, it is assumed that the heat flow through the slab and asbestos pad is one-dimensional. The equation that determines the temperature at the time $t = (j + 1)\Delta t$ of the unexposed face of the concrete slab, i.e., the boundary slab and asbestos pad, is in this case:

$$T_{M_2}^{j+1} = T_{M_2}^j + \frac{\Delta t}{((\rho_n c_n)_{M_2}^j + (\rho_a c_a)_{M_2}^j) (\Delta x)^2} \left[((k_n)_{M_2-1}^j + (k_n)_{M_2}^j) (T_{M_2-1}^j - T_{M_2}^j) - 2k_a^j (T_{M_2}^j - T_{M_2+1}^j) \right] \quad (5)$$

EQUATION FOR THE INSIDE OF THE ASBESTOS PAD

For the inside of the asbestos pad the temperature at the time $t = (j + 1)\Delta t$ of an elementary layer represented by a point P_m is:

$$T_m^{j+1} = T_m^j + \frac{\Delta t}{(\rho_a c_a)^j (\Delta x)^2} \left[k_a^j (T_{m-1}^j - T_m^j) - k_a^j (T_m^j - T_{m+1}^j) \right] \quad (6)$$

EQUATION FOR THE BOUNDARY ASBESTOS PAD AND AIR

At the boundary of the asbestos pad and air, heat is transferred from the pad to the air by convection and radiation. For the heat transferred by convection from the asbestos pad to the ambient air, the conventional expression given in Reference 4 has been used in the derivation of the heat transfer equations. It follows that the temperature of the asbestos pad at the boundary pad and ambient air at the time $t = (j + 1)\Delta t$ is given by:

$$T_{M_3}^{j+1} = T_{M_3}^j + \frac{2k_a^j}{(\rho_a c_a)_{M_3}^j (\Delta x)^2} (T_{M_3-1}^j - T_{M_3}^j) \Delta t - \frac{2\sigma \epsilon_a}{(\rho_a c_a)_{M_3}^j \Delta x} ((T_{M_3}^j)^4 - (T_o^j)^4) \Delta t - \frac{2\gamma (T_{M_3}^j - T_o^j)^{1.25}}{(\rho_a c_a)_{M_3}^j \Delta x} \Delta t \quad (7)$$

where

ρ_a^j = density of asbestos:¹ 31.2 lb ft⁻³

c_a^j = specific heat of asbestos:⁵ 0.25 Btu lb⁻¹ °R⁻¹

k_a^j = thermal conductivity of asbestos:¹ 0.0316 Btu ft⁻¹ h⁻¹ °R⁻¹

γ = coefficient expressing convective heat transfer from pad to air:⁴ 0.1823 Btu ft⁻² h⁻¹ °R^{-1.25}

STABILITY CRITERION OF THE NUMERICAL SOLUTION

In order to ensure that any error existing in the solution at some time level will not be amplified in the subsequent calculations, a stability criterion must be satisfied, which, for a selected value of Δx , limits the maximum value of Δt . For fire-exposed composite slabs made of concrete, this criterion is:

$$\Delta t \leq \frac{(\Delta x)^2 (\rho c)_{\min}}{2k_{\max} + 2\Delta x h_{\max}} \quad (8)$$

where

$(\rho c)_{\min}$ = the minimum value of the volumetric specific heat of concrete met in practice: 13 Btu ft⁻³ °R⁻¹

$k_{\max}$ = the maximum value of the thermal conductivity of concrete met in practice: 1.6 Btu ft⁻¹ h⁻¹ °R⁻¹

$h_{\max}$ = the maximum value of the coefficient of heat transfer attained in practice at fire-exposed concrete surfaces: 147 Btu ft⁻² h⁻¹ °R⁻¹.

CALCULATION PROCEDURE

With the aid of Equations 1 to 8, the temperatures at any point of the composite slab can be calculated in successive steps for any time $t = j\Delta t$. Initially, at time $t = 0$, the slab and asbestos pad are at room temperature, here assumed to be 68° F. The first step is to calculate the temperatures in the various layers of the slab and asbestos pad for the time $t = \Delta t$. These are now used as initial temperatures for the calculation of the temperatures at the time $t = 2\Delta t$. This process is repeated until the critical temperatures are exceeded. As critical temperatures, those specified in ASTM E119 were selected. These are a temperature rise of 250° F at the unexposed face of the concrete slab, a temperature of 800° F for prestressing steel, and a temperature of 1,100° F for reinforcing steel.

As can be seen in the equations, in order to calculate the temperatures of the slab, it is necessary to know the thermal properties of the concretes of which the slab is composed.

THERMAL PROPERTIES

The thermal properties that mainly determine the temperature rise in the concrete are thermal conductivity and thermal capacity. In addition, there is an influence of the absorptivity of the material on the temperature.

The thermal properties of concrete depend on the thermal properties of both the cement and the aggregate. Investigations⁶ show that the thermal properties of cement paste are not subject to large variations; the thermal

properties of aggregates, however, can vary over a wide range and therefore have substantial influence.

An important factor in determining the thermal properties of concrete is the molecular structure of the aggregate. For example, crystalline materials have higher conductivity than amorphous materials. It is probable that the amount of material in the aggregate that undergoes endothermic reactions upon heating, such as dehydration, decomposition, and transformation, also has great influence on the thermal properties of the aggregate. In siliceous aggregates, for example, the presence of quartz, which transforms from α -quartz into β -quartz at about 1,000° F, will cause an increase in the specific heat of the aggregate. In calcareous aggregates, the presence of magnesite and dolomite, which dissociate at temperatures of 650 and 1,350° F respectively, will affect the thermal properties of the aggregate. During dissociation, heat is absorbed so that the presence of magnesite and dolomite should be beneficial for the fire resistance of the slab. Their effect on thermal properties, however, is not yet known precisely.

Because the composition of aggregates varies widely, a single set of thermal properties representative of those for all concretes cannot be given. It is possible, however, to indicate extreme values for the thermal properties, one high and one low value, and to select with the aid of existing data^{6, 7} properties that will likely produce conservative results if used in fire resistance calculations. A study of these data shows that such values are obtained by averaging the extreme values. The values of the thermal conductivity and thermal capacity, which have been selected in this way for use in the calculations, are given in Tables 1 and 2 for normal weight concrete and lightweight concrete, respectively. For the absorptivity, a value of 0.9 derived from the data⁸ will be used.

COMPARISON OF CALCULATED AND TEST RESULTS

To verify the correctness of the heat transfer model and the values for the material properties, calculations were performed for a number of cases for which experimental information⁹ was available. A comparison between the calculated fire resistances, i.e., the time to reach a temperature rise of 250° F at the unexposed face of the slab, and the experimental fire resistances is given in Figure 2. It is seen that the calculated results are consistently lower than the experimental fire resistances. Lower values for the calculated fire resistances, however, are to be expected because a somewhat conservative method was used and the calculations were carried out for a dry slab, whereas the tested specimens contained moisture.

It is possible to make corrections in the fire resistance for the moisture in the slab. According to ASTM E119, the standard moisture content of a test specimen is that corresponding to the equilibrium state of drying in air of 50 percent relative humidity at 73° F. If drying is difficult because of specimen size, it is permissible to test the specimen when, at mid-depth

TABLE 1. *Thermal Properties of Lightweight Concrete*

<i>Temperature, (°R)</i>	<i>Volumetric Heat Capacity, ρc (Btu ft⁻³ °R⁻¹)</i>	<i>Thermal Conductivity, k (Btu ft⁻¹ h⁻¹ °R⁻¹)</i>
530	14.900	0.2380
550	15.275	0.2390
600	16.150	0.2425
650	16.900	0.2455
700	18.600	0.2490
750	21.200	0.2515
800	22.150	0.2530
850	22.350	0.2550
900	22.475	0.2570
950	22.550	0.2595
1000	22.550	0.2620
1050	22.450	0.2640
1100	22.425	0.2665
1150	22.400	0.2690
1200	22.575	0.2705
1250	23.925	0.2710
1300	29.650	0.2715
1350	35.875	0.2690
1400	37.650	0.2670
1450	31.350	0.2655
1500	24.800	0.2650
1550	21.800	0.2655
1600	21.300	0.2660
1650	21.800	0.2670
1700	22.725	0.2680
1750	23.375	0.2685
1800	23.400	0.2690
1850	22.950	0.2695
1900	22.350	0.2705
1950	21.950	0.2725
2000	21.700	0.2750
2100	21.875	0.2805
2200	22.125	0.2865
2300	22.350	0.2930
2400	22.575	0.2995
2500	22.800	0.3060
2600	23.050	0.3125
2700	23.250	0.3190
2800	23.475	0.3255

of the slab, the moisture content reaches a level corresponding to that in equilibrium with air in the range of 50 to 75 percent relative humidity. The standard describes a procedure for deriving the fire resistance for one moisture condition from that for other moisture conditions. Following this procedure, it can be derived that, depending on the thickness of the slab and its composition, the fire resistance increases by approximately 5 to 15 percent if the moisture content is increased from zero to that corresponding to 50 percent relative humidity at mid-depth of the slab. If the moisture content is increased at mid-depth from zero to that corresponding to 75 percent relative humidity, the fire resistance increases by approximately 10 to 25 percent.

The moisture content at mid-depth of the test specimens was that corresponding to 75 percent relative humidity. To compare the calculated

TABLE 2. *Thermal Properties of Normal Weight Concrete*

<i>Temperature, (°R)</i>	<i>Volumetric Heat Capacity, ρc (Btu ft⁻³ °R⁻¹)</i>	<i>Thermal Conductivity, k (Btu⁻¹ ft⁻¹ h⁻¹ °R⁻¹)</i>
530	26.450	1.1045
550	27.100	1.0955
600	28.775	1.0750
650	30.175	1.0570
700	32.850	1.0420
750	36.400	1.0265
800	37.850	1.0090
850	38.450	0.9915
900	38.900	0.9770
950	39.225	0.9655
1000	39.500	0.9570
1050	39.700	0.9515
1100	39.900	0.9430
1150	40.125	0.9335
1200	40.400	0.9215
1250	42.350	0.9055
1300	49.750	0.8850
1350	57.325	0.8595
1400	60.250	0.8290
1450	53.300	0.8010
1500	46.000	0.8170
1550	43.075	0.7675
1600	40.500	0.7555
1650	40.000	0.7475
1700	40.500	0.7695
1750	41.200	0.7275
1800	41.200	0.7155
1850	40.700	0.7040
1900	40.025	0.6990
1950	39.600	0.6965
2000	39.300	0.6980
2100	39.400	0.7055
2200	39.775	0.7085
2300	40.125	0.7145
2400	40.425	0.7210
2500	40.725	0.7285
2600	41.075	0.7365
2700	41.400	0.7455
2800	41.725	0.7540

and experimental fire resistances, those calculated for the dry condition should be increased by an amount of the order of 15 to 20 percent. Figure 3 shows this comparison, with the calculated results corrected by 20 percent to take into account the influence of moisture on fire resistance. It is seen that, although most of the calculated fire resistances are still somewhat lower than the experimental, there is good agreement between theoretical and experimental results.

APPROXIMATE FORMULAS

The fire resistance, in this case the time to reach the critical temperature on the unexposed face of the slab or on the steel in the slab, can be calculated from Equations 1 to 8. The calculation procedure, however, is not

Figure 2. Comparison of calculated and experimental fire resistances (not corrected for moisture).

Figure 3. Comparison of calculated and experimental fire resistances (corrected for moisture).

suitable for general use in practice because it is elaborate and requires programming of the equations and computer processing. Therefore, an attempt was made to derive simplified expressions that would be sufficiently accurate and allow the determination of fire resistance by hand calculation.

For this purpose, the fire resistance of composite slabs was calculated for a wide range of practical cases. Calculated results are given in Figures 4 to 9. In Figure 4, the fire resistance, based on attainment of a temperature rise of 250° F at the unexposed surface between concrete and asbestos pad, is plotted as a function of the ratio d/l , where d is the thickness of the lower layer and l the total thickness of the slab. The curves in this figure are for slab thicknesses of 2, 4, 6, and 8 in., respectively. It is assumed that the lower layer of the slab is composed of normal weight concrete and the upper layer of lightweight concrete. The solid lines give the fire resistances derived from the basic Equations 1 to 8. These lines can be described approximately by the expression:

$$R = 0.05 \left(2l^2 - dl + \frac{6}{l} \right) \quad (9)$$

Figure 4. Fire resistance of composite slabs (lower layer of normal weight concrete).

where

R = fire resistance of the slab, h

l = thickness of the slab, in.

d = thickness of the lower layer, in.

Results obtained from this expression are given by the dashed lines in Figure 4.

In Figure 5, similar curves are given for the case where the lower layer is composed of lightweight concrete and the upper layer of normal weight concrete. In this case, the fire resistances calculated from the basic equations can be approximated by the expression:

$$R = 0.055 (l^2 + 2dl - d^2 + \frac{4}{l}) \quad (10)$$

where R , l , and d have the same meaning as in Equation 9.

In addition to exceeding a temperature rise of 250° F at the unexposed surface, floor and roof assemblies may also fail because of excessive temperature rise of the prestressing or reinforcing steel in the slab. The critical steel temperature, i.e., the temperature at which the steel can no longer support the loading, depends on several factors, such as loading and end conditions.¹⁰ In this paper, the values specified by ASTM E119 for unrestrained floor and roof slabs are selected as critical temperatures. These are 800° F for prestressing steel and 1,100° F for reinforcing steel.

It is assumed that the slab fails structurally at the time that the temperature of the steel in the slab exceeds one of the previously mentioned

Figure 5. Fire resistance of composite slabs (lower layer of lightweight concrete).

Figure 6. Minimum cover thickness to prestressing steel as a function of fire resistance (lower layer of normal weight concrete — unrestrained assembly classification).

critical temperatures. In fire tests, the steel temperatures at which the specimen actually fails may differ from the selected critical temperatures. In particular, if the specimen is restrained against thermal expansion, which will be beneficial for its fire resistance, the failure temperatures will likely be higher. The chosen critical temperatures are expected to be representative of those of unrestrained slabs, but conservative in the case of restrained slabs.

By calculating the temperature distribution in the slab, the depth at which critical temperatures are attained when the unexposed surface temperature reaches 250° F can be determined. By providing the steel with a concrete cover that is at least equal in thickness to this depth, the attainment of steel temperatures in excess of the critical temperatures can be prevented.

The minimum cover thickness to obtain a certain fire resistance is shown for a large number of cases in Figures 6 to 9. Cover thickness is here defined as the distance between the fire-exposed surface and the cen-

Figure 7. Minimum cover thickness to reinforcing steel as a function of fire resistance (lower layer of normal weight concrete — unrestrained assembly classification).

ter of the prestressing or reinforcing steel. The curves in Figure 6 give the minimum cover thickness for prestressed concrete slabs of which the lower layer is composed of normal weight concrete. For such slabs the dependence of minimum cover thickness on fire resistance desired is given approximately by the relation:

$$C = \sqrt{4R - 1} \quad (11)$$

where

C = minimum cover thickness, in.

R = fire resistance desired, h.

Figure 7 shows the minimum cover thickness to reinforcing steel when the lower layer is composed of normal weight concrete. It is given approximately by the relation:

$$C = 0.75 R \quad (12)$$

where C and R have the same meaning as that in Equation 11.

In Figures 8 and 9, the calculated minimum cover thicknesses are shown for slabs of which the lower layer is composed of lightweight concrete.

Figure 8. Minimum cover thickness to prestressing steel as a function of fire resistance (lower layer of lightweight concrete — unrestrained assembly classification).

For prestressed concrete slabs (Figure 8), the relation between minimum cover thickness C and desired fire resistance R is given approximately by the expression:

$$C = 0.6 R + 0.4 \quad (13)$$

and for reinforced concrete slabs (Figure 9), by the expression:

$$C = 0.5 R \quad (14)$$

EXAMPLE

It might be useful to illustrate in an example the use of the approximate formulas for the determination of the fire resistance and minimum cover thickness of composite slabs. A reinforced concrete composite slab is considered, consisting of a lower layer of $3\frac{1}{2}$ -in. thick normal weight concrete and an upper layer of lightweight concrete $2\frac{1}{2}$ in. thick. The moisture content of the slab is equal to that corresponding to equilibrium with air of 75 percent relative humidity at room temperature (approximately 70°F).

Figure 9. Minimum cover thickness to reinforcing steel as a function of fire resistance (lower layer of lightweight concrete — unrestrained assembly classification).

The task is to calculate the thermal fire resistance and the required minimum cover thickness to prevent excessive temperature rise and failure of the steel before the slab fails thermally on the unexposed surface.

According to Equation 9, the thermal fire resistance of the slab is:

$$R = 0.05(2l^2 - dl + \frac{6}{l})$$

where

$l = 6$ in. (thickness of slab)

$d = 3.5$ in. (thickness of lower layer).

Substitution in the equation gives:

$$R = 0.05 \left[2 \times (6)^2 - (3.5 \times 6) + \frac{6}{6} \right] = 2.6 h$$

This is the thermal fire resistance for the dry slab. As suggested previously, for a moisture condition corresponding to equilibrium with air of 75 percent relative humidity at room temperature, the fire resistance may be derived by increasing the fire resistance for the dry slab by 20 percent. Thus, the fire resistance of the slab under consideration is $2.6 + (0.2 \times 2.6) = 3.12$ h or 3 h and 7 min. In a test, a fire resistance of 3 h and 24 min was found.

The minimum cover thickness to prevent failure of the reinforcing steel before the temperature of the unexposed face reaches the failure temperature (i.e., before the elapse of an exposure time of 3.12 h) can be calculated from Equation 13. According to this equation, the minimum cover thickness is:

$$C = 0.75 R$$

$$\text{or } C = 0.75 \times 3.12 = 2.34 \text{ in.}$$

Thus the required minimum thickness of concrete between the exposed face of the slab and the center of the steel is 2.34 in.

SUMMARY

A numerical calculation method is described for the determination of the fire resistance of composite floor and roof slabs. Using this method, simple approximate formulas are derived for the calculation of the thermal fire resistance and minimum cover thickness to the steel of reinforced and prestressed concrete slabs, consisting of two layers of concrete, one light-weight and one normal weight.

Comparison with test results shows good agreement between calculated and experimental fire resistances, although calculated values are on the average somewhat more conservative.

REFERENCES

- ¹"Standard Methods of Fire Tests of Building Construction and Materials," American Society for Testing and Materials, Designation E119-73, Part 18, 1974, pp. 610-628.
- ²Dusinberre, G. M., *Heat Transfer Calculations by Finite Differences* (International Textbook Company, Scranton, Pennsylvania, 1961), p. 293.
- ³Lie, T. T., "Temperature Distributions in Fire-Exposed Building Columns," *Journal of Heat Transfer, ASME*, Vol. 99, No. 1 (February 1977), pp. 113-119.
- ⁴Spiers, H. M., *Technical Data on Fuel* (6th Edition, World Press Conference, London, 1961), p. 66.
- ⁵Liley, P. E., "Physical and Chemical Data" in *Chemical Engineer's Handbook* (McGraw-Hill, New York, 1963), p. 133.
- ⁶Harmathy, T. Z., "Thermal Properties of Concrete at Elevated Temperatures," *ASTM Journal of Materials*, Vol. 5, No. 1 (March 1970), pp. 47-74.
- ⁷Harmathy, T. Z., "Thermal Properties of Selected Masonry Unit Concretes," *Journal of the American Concrete Institute*, Vol. 70, No. 2 (February 1973), pp. 132-142.
- ⁸Gilmour, M. S. et al., "Heat Transmission" in *Chemical Engineer's Handbook* (McGraw-Hill, New York, 1963), pp. 35-36.

⁹Abrams, M. S. and Gustaferro, A. H., "Fire Endurance of Two-Course Floors and Roofs," *Journal of the American Concrete Institute*, Vol. 66, No. 2 (February 1969), pp. 92-102.

¹⁰Lie, T. T., *Fire and Buildings* (Applied Science Publishers Ltd., London, 1972), pp. 168-176.

APPENDIX A — APPROXIMATE FORMULAS

This appendix gives a summary of the approximate formulas for the calculation of the thermal fire resistance and minimum cover thickness to the steel of composite floor and roof slabs. Considered are reinforced and prestressed concrete slabs consisting of two layers of concrete. One layer is lightweight concrete, the other layer is normal weight concrete. Formulas are given for both cases, i.e., the lower layer is made of lightweight concrete or it is made of normal weight concrete.

The thermal fire resistance is defined as the time to reach a temperature rise at the unexposed face of the slab of 250° F. The minimum cover thickness is the thickness of concrete between the exposed face and the center of the steel necessary to prevent the steel temperature from exceeding 800° F for prestressing steel and 1,100° F for reinforcing steel. The fire resistance values are for dry slabs or those close to the dry condition. For a moisture condition corresponding to that in equilibrium with air of 50 percent or more but less than 75 percent relative humidity at 73° F, it is suggested that 10 percent be added to the fire resistance for the dry condition and 20 percent for a moisture condition corresponding to 75 percent or higher relative humidity.

The formulas are valid for fire resistances in the range of 20 min to 5 h.

LOWER LAYER OF NORMAL WEIGHT CONCRETE

Fire Resistance

$$R = 0.05(2l^2 - dl + \frac{6}{l})$$

where

R = fire resistance of slab, h

l = thickness of slab, in.

d = thickness of lower layer, in.

Moisture Correction

Add 10 percent for a moisture condition corresponding to 50 to <75 percent relative humidity. Add 20 percent for a moisture condition corresponding to ≥75 percent relative humidity.

Minimum Cover Thickness

Prestressed concrete: $C = \sqrt{4R - 1}$

Reinforced concrete: $C = 0.75R$

where

C = minimum cover thickness, in.

R = fire resistance of slab, h.

LOWER LAYER OF LIGHTWEIGHT CONCRETE

Fire Resistance

$$R = 0.055(l^2 + 2dl - d^2 + \frac{4}{l})$$

where

R = fire resistance of slab, h

l = thickness of slab, in.

d = thickness of lower layer, in.

Moisture Correction

Add 10 percent for a moisture condition corresponding to 50 to <75 percent relative humidity. Add 20 percent for a moisture condition corresponding to ≥ 75 percent relative humidity.

Minimum Cover Thickness

Prestressed concrete: $C = 0.6R + 0.4$

Reinforced concrete: $C = 0.5R$

where

C = minimum cover thickness, in.

R = fire resistance of slab, h.

APPENDIX B — NOMENCLATURE

c = specific heat, Btu lb⁻¹ °R⁻¹

C = minimum cover thickness, in.

d = thickness of lower layer, in.

h = coefficient of heat transfer at fire-exposed surface, Btu ft⁻² h⁻¹ °R⁻¹

j = 0, 1, 2, . . .

k = thermal conductivity, Btu h⁻¹ ft⁻¹ °R⁻¹

l = thickness of slab, in.

p = point

R = fire resistance of slab, h

t = time, h

T = temperature, °R

x = coordinate, ft

GREEK LETTERS

γ = coefficient expressing convective heat transfer from pad to air: 0.1823 Btu ft⁻² h⁻¹ °R^{-1.25}

Δ = increment

ϵ = emissivity

ρ = density

σ = Stefan-Boltzmann constant, 0.1713×10^{-8} Btu h^{-1} ft^{-2} $^{\circ}\text{R}^{-4}$

SUBSCRIPTS

a = of asbestos

f = of the fire

$m, M, M_1, M_2, \dots$ = at a point in the m -th, M -th, M_1 -th, M_2 -th $\dots$ = elementary layer

$_{\max}$ = maximum value of

$_{\min}$ = minimum value of

$n, n_1, n_2, \dots$ = concrete type

$_0$ = initial

SUPERSCRIPT

j at $t = j\Delta t$

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