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Three-Dimensional Boundary Element Reservoir Model for Seismic Analysis of Arch and Gravity Dams

by A.M. Jablonski and J.L. Humar

ANALYZED

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Résumé

La méthode des équations intégrales a été utilisée avec succès, dans le passé, pour l'analyse des forces hydrodynamiques dans les réservoirs infinis bidimensionnels ainsi que dans les réservoirs finis bi- et tridimensionnels soumis à des mouvements sismiques du sol. Ce document fait état des résultats de recherches récentes concernant l'application de la méthode des équations intégrales constantes à l'analyse 3D de la vibration des réservoirs. On a incorporé à cette formulation des conditions limites spéciales, auparavant utilisées dans le cas 2D, afin de traiter l'amortissement dû au rayonnement infini et l'amortissement attribuable au sol de fondation et aux rives. Les auteurs présentent des données numériques rendant compte de la vibration d'un réservoir rectangulaire infini 3D, ainsi que d'un réservoir infini 3D formé par un barrage-voûte, et ils les comparent à certains résultats obtenus par d'autres chercheurs.

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THREE-DIMENSIONAL BOUNDARY ELEMENT RESERVOIR MODEL FOR SEISMIC ANALYSIS OF ARCH AND GRAVITY DAMS

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SUMMARY

The boundary element method has been successfully applied in the past to the analysis of hydrodynamic forces in two-dimensional infinite as well as two- and three-dimensional finite reservoirs subjected to seismic ground motions. This paper presents the results of more recent research on the application of the constant boundary element method to the 3D analysis of reservoir vibration. Special boundary conditions, previously used in the 2D case, to treat infinite radiation damping and damping from foundation soil and banks have been incorporated in this formulation. Numerical results for vibration of a 3D infinite rectangular reservoir as well as of a 3D infinite reservoir impounded by an arch dam are presented and compared with some existing results obtained by other researchers.

INTRODUCTION

The seismic analysis and design of an arch dam has not been studied in as much detail as that of a concrete gravity dam because of the complex nature of the problem. The hydrodynamic forces caused by the vibration of the impounded water add to the complexity. Recently, closed form solutions have been developed by Porter and Chopra for the seismic vibrations of arch dam-reservoir systems of regular geometry.^{1,2} For the systems of irregular geometry the finite element technique has been applied successfully. A three-dimensional model for the analysis of a gravity dam-reservoir-foundation system was first proposed by Hall and Chopra.³ In this model the infinite water reservoir was divided into two parts: a finite irregular region and an infinite channel of constant cross-section. The finite element discretization was limited to the irregular region of the reservoir. For the infinite channel a two-dimensional finite element solution was employed. Compatibility of pressures and pressure gradients was then enforced at the interface of the regular and irregular regions of the reservoir to complete the solution.

Additional studies on the finite element analysis of arch dams have been carried out by Fok and Chopra.⁴⁻⁹ They developed an analytical procedure to account for both the effect of radiation damping and water reservoir-foundation interaction. The analysis of foundation interaction was based on a simplified model, first introduced by Hall and Chopra.³

The considerable computational effort involved in the application of the FEM and the difficulty in modelling infinite domains have given impetus to studies based on the application of the boundary element method (BEM) to the response analysis of dam-reservoir-foundation systems. The boundary element method has been successfully applied to the analysis of hydrodynamic pressures in two-dimensional finite and infinite reservoirs.¹⁰⁻¹⁷ The three-dimensional finite arch dam-reservoir vibration problem was solved by Elkhodary¹⁸ using the direct boundary element method and by Humar and Jablonski¹⁹ using the so-called boundary element superposition method (BESM). The three-dimensional boundary element model has also been used by Bičkovski²⁰ in the treatment of seismic vibrations of a regular, rectangular, finite

reservoir. Soong and Murtha²¹ have recently reported the application of a three-dimensional boundary element model to the analysis of a dam-reservoir system.

This paper presents the results of the application of the boundary element method to the seismic analysis of a 3D infinite reservoir impounded by a simple arch dam.²² In a recent paper, Tsai and Lee²³ have studied the hydrodynamic pressures in a 3D infinite reservoir based on a BEM model and omitting the far boundary. The present authors have found that the technique of omitting the far boundary is useful only for incompressible water or for excitation frequencies less than the first natural frequency of vibrations of the water reservoir. In the study reported here, the radiation damping due to outgoing waves in an infinite reservoir is accounted for by the application of a special boundary at the far end. This condition is similar to that developed earlier for finite element solutions.^{2,5} The damping introduced by absorption of energy in a flexible foundation and banks is accounted for by another boundary condition originally developed by other researchers.³

Because BEM reduces the dimensionality of the problem by one, converting a three-dimensional domain problem to that of a surface, the discretization effort is considerably reduced. Thus BEM may serve as an effective alternative to FEM based procedures in the seismic analysis of a 3D dam-reservoir system. The objective of the present study was to demonstrate the effectiveness of BEM in modelling the reservoir. The dam was therefore treated as rigid. However, the procedure is extended easily to the case of a flexible dam. Additional work is required to develop the procedure into an effective tool for design. With this in view, conclusions and recommendations for future research are included.

ASSUMPTIONS

An arch dam-reservoir-foundation system consists of three parts: the dam, the impounded water reservoir, the foundation and banks (Figure 1). The arch dam is usually a curved thick shell (cylindrical, conical or paraboloidal). The ground motion to be considered in the analysis may be in any of the following three directions: upstream-downstream (x), cross-stream (y) and vertical (z).

It is assumed that, even though the ground motion varies with time, at any one instant it is identical throughout the reservoir bottom and banks.

The following assumptions are made in the derivation of the equations of motion for water in the 3D reservoir.

1. Water is compressible but non-viscous.
2. Effects of surface waves are negligible.
3. Only small amplitude motion is considered.

The reservoir may have a short length or may extend to a large distance. In the first case, the reservoir can be treated as finite; in the latter, it is considered to be infinite for the purpose of analysis.

EQUATIONS OF MOTION AND BOUNDARY CONDITIONS

Equations of motion

With the assumptions outlined in the foregoing paragraph the motion of the water in a 3D reservoir is governed by linearized Navier-Stokes equations. For a harmonic excitation $e^{i\omega t}$, these equations can be reduced to the known 3D Helmholtz equation

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} + k^2 p = 0 \quad (1)$$

where x , y and z = the Cartesian coordinates shown in Figure 1; $k = \omega/c$ is the wave number; $\bar{p} = p e^{i\omega t}$, the hydrodynamic pressure; ω = the exciting frequency; and c = the velocity of sound in water.

Figure 1. Constant boundary element discretization of a 3D finite reservoir impounded by an arch dam

Boundary conditions

The boundary conditions applicable to a general three-dimensional geometry are listed below.

Along the dam-fluid interface

$$\frac{\partial p}{\partial n}(s, r, \omega) = -\frac{w}{g}a_n(s, r) \quad (2)$$

where w = specific weight of water; s, r = coordinates over the dam-reservoir interface; $a_n(s, r)$ = normal component of dam acceleration; n = outward normal to the interface.

Along the surface of the reservoir. Since the effect of surface waves is neglected, the hydrodynamic pressure should vanish at the free surface, giving

$$p(x, y, H, \omega) = 0 \quad (3)$$

Along the reservoir bottom and banks. The boundary condition depends on whether the foundation is flexible or rigid. For a flexible foundation the boundary condition is derived from a simplified model in which one-dimensional seismic waves are assumed to propagate in a direction normal to the boundary and are reflected there. The detailed derivation has been presented by Hall and Chopra³ and leads to

$$\frac{\partial p}{\partial n}(s', r', \omega) = -\frac{w}{g}a_n(s', r') - i\omega\gamma p(s', r', \omega) \quad (4)$$

where s', r' = coordinates over the reservoir bottom and banks; γ = the damping coefficient; $a_n(s', r')$ = the acceleration of the reservoir bottom and banks; ω = the excitation frequency; and n = the outward normal to the reservoir bottom.

The second term on the R.H.S. of equation (4) represents the effect of foundation damping. The foundation damping coefficient is given by the following expression:³

$$\gamma = \frac{1}{c} \left(\frac{1 - \alpha_R}{1 + \alpha_R} \right) \quad (5)$$

where α_R = the coefficient of reflection. For a rigid foundation, $\alpha_R = 1.0$ and hence $\gamma = 0$, while for full absorption $\alpha_R = 0$, $\gamma = 1/c$.

The boundary condition along the banks is similar to that along the foundation.

Along the far boundary. For a 3D finite reservoir, the condition at the far end depends on whether that boundary is fixed or free to move. An infinite reservoir is modelled as consisting of two regions, a finite region which may be irregular, and an infinite channel of uniform cross-section. The interface between these two regions is called the far boundary or transmitting boundary. The energy loss due to radiation damping in the infinite region is accounted for through a special boundary condition similar to one used in the 2D case^{14, 15} and based on that derived by Hall and Chopra.³ In this derivation, the infinite channel of regular section is modelled by 2D finite elements along the interface cross-section with the variation in the longitudinal direction being represented by an exponentially decaying function. The problem ultimately reduces to the solution of a 2D eigenvalue problem. As an alternative a 2D boundary element representation can be employed to model the infinite channel. Details of the alternative procedure have been presented by Kandasamy.²⁴ The FEM discretization is used in the present case. Details of the derivation are similar to those in the 2D case and only a brief discussion of the eigenvalue problem is presented here.

The 3D Helmholtz equation [equation (1)] governs the vibration in the infinite channel as well, and can be solved by separation of variables:

$$p = p_x p_{yz} \quad (6)$$

in which p_x satisfies the equation

$$\frac{d^2 p_x}{dx^2} - \kappa^2 p_x = 0 \quad (7)$$

and p_{yz} satisfies the equation

$$\frac{\partial^2 p_{yz}}{\partial y^2} + \frac{\partial^2 p_{yz}}{\partial z^2} + \lambda^2 p_{yz} = 0 \quad (8)$$

where $\lambda^2 = \omega^2/c^2 + \kappa^2$ and κ is the separation constant.

Equation (8) is solved by a two-dimensional finite element discretization. The FEM solution leads to the following eigenvalue problem:

$$[\mathbf{A} + i\omega\gamma\mathbf{B} - \lambda^2\mathbf{F}]\mathbf{p}_{yz} = \frac{w}{g}\mathbf{d} \quad (9)$$

where $\mathbf{p}_{yz}$ is the vector of pressure values at nodes; and matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{F}$, $\mathbf{d}$ are obtained by assembling the corresponding element matrices. Details of the derivation may be found in References 3 and 22.

Equation (9) is valid for both horizontal excitations (downstream-upstream and cross-stream) and also for a vertical excitation, which is represented by the R.H.S. of that equation. Accordingly, the only non-zero terms in vector $\mathbf{d}$ are those corresponding to normal accelerations of the foundation and the banks.

For a horizontal excitation equation (9) reduces to

$$[\mathbf{A} + i\omega\gamma\mathbf{B}]\mathbf{p}_{yz} = \lambda^2\mathbf{F}\mathbf{p}_{yz} \quad (10)$$

Equation (10) represents an eigenvalue problem which for non-zero values of γ leads to complex eigenvalues and eigenvectors. If the matrix of M eigenvectors obtained from equation (10) is represented by Ψ , the following relationship must hold:

$$\Psi^T [A + i\omega\gamma B] \Psi = \Lambda \quad (11)$$

in which Λ = a diagonal matrix of the M eigenvalues $\lambda_1^2, \lambda_2^2, \dots, \lambda_M^2$ (real or complex), and the eigenvectors have been normalized to be orthonormal w.r.t. F so that

$$\Psi^T F \Psi = I \quad (12)$$

For a horizontal excitation the general solution for pressure vector p_{yz} relating to the infinite region of the reservoir can be expressed in terms of computed eigenvalues λ_n and eigenvectors ψ_n as

$$p_{yz} = \sum_{n=1}^M \eta_n \psi_n e^{-\kappa_n x} \quad (13)$$

or

$$p_{yz} = \Psi E \eta \quad (14)$$

in which η = a vector of modal coordinates, E = a diagonal matrix of terms $e^{-\kappa_n x}$ and $\kappa_n = \sqrt{\lambda_n^2 - k^2}$.

Differentiating equation (14) w.r.t. the outward normal to the finite part of the reservoir and noting that the normal is parallel to the x axis,

$$q_{yz} = -\Psi K \eta \quad (15)$$

where K = a diagonal matrix with elements $\kappa_1, \kappa_2, \dots, \kappa_M$, and $q_{yz} = \partial p_{yz} / \partial n$.

For the case of vertical excitation, taking the origin at the far boundary ($x = 0$), the vector of the nodal pressures can be expressed as

$$p_{yz} = \frac{w}{g} \Psi K^{-2} \Psi^T d \quad (16)$$

The total pressure is obtained by adding equations (14) and (16):

$$p_{yz} = \Psi \eta + \frac{w}{g} \Psi K^{-2} \Psi^T d \quad (17)$$

A modified mesh of the constant boundary elements at the transmitting boundary is required in order to match the boundary element pressure values with the nodal values of pressure in 4-node quadrilateral finite elements. This is discussed in a later section of this paper.

FORMULATION OF BOUNDARY ELEMENT EQUATIONS

For a numerical solution of the problem governed by equation (1), a weighted residual technique is used in which the governing equation is satisfied in an average sense over the domain V . Accordingly, equation (1) is multiplied by a weighting function p^* and the integral of the product over the domain is equated to zero. Repeated partial integration and the application of Green's theorem, leads to the following boundary integral equation:^{15, 18, 22}

$$\begin{aligned} & \iiint_V \left(\frac{\partial^2 p^*}{\partial x^2} + \frac{\partial^2 p^*}{\partial y^2} + \frac{\partial^2 p^*}{\partial z^2} + k^2 p^* \right) p \, dx \, dy \, dz \\ &= \iint_S p \frac{\partial p^*}{\partial n} \, dS - \iint_S p^* \frac{\partial p}{\partial n} \, dS \end{aligned} \quad (18)$$

The R.H.S. of equation (18) contains surface integrals over the boundary S only while the L.H.S. contains a volume integral. By an appropriate choice of the function p^* , the integration over domain V may be avoided. The desired weighting function p^* is chosen to satisfy the following equation:

$$\frac{\partial^2 p^*}{\partial x^2} + \frac{\partial^2 p^*}{\partial y^2} + \frac{\partial^2 p^*}{\partial z^2} + k^2 p^* = \Delta^i \quad (19)$$

in which Δ^i = a Dirac delta function centred at point i having coordinates (x_i, y_i, z_i) .

The solution of equation (19) is called the fundamental solution. It can be obtained by the so-called Green's function approach^{18, 25, 26} and is given by

$$p^* = -\frac{1}{4\pi r} e^{-ikr} \quad (20)$$

where r = the radial distance given by $r = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}$, and the point with the coordinates (x_i, y_i, z_i) is referred to as the source point.

Other possibilities exist for the selection of a fundamental solution. For example, Tsai and Lee²³ have used the following fundamental solution which satisfies implicitly the free surface condition:

$$p^* = \frac{1}{4\pi} \left(\frac{e^{-ikr}}{r} - \frac{e^{-ikr'}}{r'} \right) \quad (21)$$

where r' = the distance between the field point and the mirror image point. Research on the most appropriate and effective fundamental solution for both 2D and 3D cases is still continuing.^{17, 23}

Substitution of equation (19) into equation (18) gives

$$c_i p_i = \iint_S p q_i^* dS - \iint_S p_i^* q dS \quad (22)$$

in which $c_i = 1$ for a point inside the water reservoir, $c_i = \frac{1}{2}$ for a point on the smooth boundary of the water reservoir, $c_i = 0$ for a point outside the water reservoir, p_i = the hydrodynamic pressure at point i , $q = \partial p / \partial n$ and $q^* = \partial p^* / \partial n$. The subscripts on p_i^* and q_i^* indicate that the latter are produced by a source at point i .

Equation (22) provides a means of determining the unknown pressure at point i within the domain in terms of the integrals of pressure and pressure derivative values on the boundary. While some of these values are specified, others must be determined before equation (22) can be used. In a boundary element formulation, equation (22) itself is used to determine the unknowns on the boundary. To achieve this the boundary surface is divided into a series of elements and the parameters p and q are assumed to vary in a prescribed manner over a given element.

For example, in a 2D case, individual elements are straight lines and the pressure and pressure derivatives may be assumed to be constant over an element, leading to the so-called constant boundary element formulation. As an alternative, the parameters p and q may be assumed to vary linearly over an element. They are expressed as linear functions of the p and q values at the end points of the element. This gives a linear element formulation. In fact, the variation may be expressed as a superposition of shape functions of a coordinate along the boundary, each function being weighted by a nodal parameter. These nodal parameters become the unknowns in the formulation. This procedure is exactly parallel to the shape function approach used in the FEM.

In the present work, the boundary elements are quadrilaterals and p and q are assumed to be constant over an element and equal to their values at the midpoints of the element. Boundary elements of this type are referred to as constant elements. Investigations conducted by the authors for the 2D case^{15, 22} showed that the constant boundary elements were quite effective and that acceptable accuracy was achieved with a reasonable number of such elements.

The constant boundary element formulation for the three-dimensional Helmholtz equation has previously been used by Elkhodary¹⁸ in his analysis of the 3D finite reservoir subjected to small harmonic vibrations. Therefore only the essential points of the derivation are presented in this paper.

Equation (22) for a given point i on the boundary S has the following discretized form:

$$-\frac{1}{2}p_i + \sum_{j=1}^N p_j \iint_{S_j} q_i^* dS_j = \sum_{j=1}^N q_j \iint_{S_j} p_i^* dS_j \quad (23)$$

where p_j and q_j are the pressure and pressure gradient, respectively, over the centre of the j th element and are assumed to be constant over that element, while S_j represents the surface of the element.

A constant boundary element model of the three-dimensional infinite water reservoir is presented in Figure 1.

Equation (23) can be also expressed in a matrix form as follows:

$$\mathbf{H}\mathbf{p} = \mathbf{G}\mathbf{q} \quad (24)$$

where $\mathbf{p}$ and $\mathbf{q}$ are respectively the vectors of N unknown pressures and pressure derivatives at the boundary element nodes; and $\mathbf{H}$ and $\mathbf{G}$ are matrices of order N whose elements are given by

$$H_{ij} = \iint_{S_j} q_i^* dS_j \quad i \neq j \quad (25a)$$

$$H_{ii} = -\frac{1}{2} + \iint_{S_i} q_i^* dS_i \quad (25b)$$

$$G_{ij} = \iint_{S_j} p_i^* dS_j \quad (25c)$$

Defining a local coordinate system ξ_1 and ξ_2 and using a Gauss quadrature formula for evaluating the integrals, an expression for H_{ij} is obtained in the following form:

$$H_{ij} = \frac{1}{4\pi} \sum_{n=1}^4 \sum_{m=1}^4 \left(\frac{\cos \theta}{r^2} \right)_{nm} [(\cos kr + kr \sin kr) + i(-\sin kr + kr \cos kr)]_{nm} |J| \omega_{nm} \quad (26)$$

where ω_{nm} = weighting function used in the Gauss quadrature; $|J|$ = the Jacobian of transformation between the (x, y, z) coordinates system and the (ξ_1, ξ_2) local coordinates system; and θ = the angle between the outward normal n and the radius vector r , as shown in Figure 1. When $i = j$ equation (26) reduces to $H_{ii} = -\frac{1}{2}$.

The term G_{ij} is also determined by a numerical integration and is given by

$$G_{ij} = -\frac{1}{4\pi} \sum_{n=1}^4 \sum_{m=1}^4 \left[(\cos kr - i \sin kr) \frac{1}{r} \right]_{nm} |J| \omega_{nm} \quad (27)$$

The term G_{ii} is calculated by excluding the singular point i . Details have been presented by Elkhodary.¹⁸

INCORPORATION OF INFINITE RADIATION CONDITION AND DAMPING ALONG FOUNDATION AND BANKS

To match the solution at the interface of the finite and infinite region, and to apply the condition along the reservoir bottom and banks, equation (24) is expressed in a partitioned matrix form as follows:

$$\begin{pmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} & \mathbf{H}_{13} \\ \mathbf{H}_{21} & \mathbf{H}_{22} & \mathbf{H}_{23} \\ \mathbf{H}_{31} & \mathbf{H}_{32} & \mathbf{H}_{33} \end{pmatrix} \begin{pmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ \mathbf{p}_3 \end{pmatrix} = \begin{pmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} & \mathbf{G}_{13} \\ \mathbf{G}_{21} & \mathbf{G}_{22} & \mathbf{G}_{23} \\ \mathbf{G}_{31} & \mathbf{G}_{32} & \mathbf{G}_{33} \end{pmatrix} \begin{pmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \\ \mathbf{q}_3 \end{pmatrix} \quad (28)$$

in which $\mathbf{p}_1, \mathbf{q}_1$ represent, respectively, the pressures and pressure derivatives at the nodes on the reservoir surface and along the dam face, $\mathbf{p}_2, \mathbf{q}_2$ represent the same quantities on the reservoir bottom and banks, while $\mathbf{p}_3, \mathbf{q}_3$ represent them at the nodes on the transmitting boundary.

When a constant boundary element formulation is used in the 3D infinite case, difficulty arises in the evaluation of $\mathbf{p}_3$, because, while the finite element nodes are located at the vertices of a 4-node quadrilateral element, the constant boundary element nodes are at the geometric centres of these elements. One possible way to resolve it is to use different meshes for the boundary discretization and the finite element discretization. By proper selection of the meshes it is possible to match most of the nodes in the two formulations. An example of such a mesh division is presented here in Figure 2. Nodes 6, 7, 10 and 11 are identical in two meshes. Nodes 1 to 4, 5, 8, 9, 12 and 13 to 16 do not match exactly, but the $\mathbf{p}$ and $\mathbf{q}$ values for the BEM and FEM sets are assumed to be identical. Also the pressure at nodes 4, 8, 12 and 16 is assumed to be zero in both cases. Using equations (4), (15) and (17) equation (28) can be expressed as

$$\begin{pmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} + i\omega\gamma\mathbf{G}_{12} & \mathbf{H}_{13}\Psi + \mathbf{G}_{13}\Psi\mathbf{K} \\ \mathbf{H}_{21} & \mathbf{H}_{22} + i\omega\gamma\mathbf{G}_{22} & \mathbf{H}_{23}\Psi + \mathbf{G}_{23}\Psi\mathbf{K} \\ \mathbf{H}_{31} & \mathbf{H}_{32} + i\omega\gamma\mathbf{G}_{32} & \mathbf{H}_{33}\Psi + \mathbf{G}_{33}\Psi\mathbf{K} \end{pmatrix} \times \begin{pmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ \eta \end{pmatrix} = \begin{pmatrix} \mathbf{G}_{11}\mathbf{q}_1 - \frac{w}{g}\mathbf{G}_{12}\mathbf{a}_n - \frac{w}{g}\mathbf{H}_{13}\Psi\mathbf{K}^{-2}\Psi^T\mathbf{d} \\ \mathbf{G}_{21}\mathbf{q}_1 - \frac{w}{g}\mathbf{G}_{22}\mathbf{a}_n - \frac{w}{g}\mathbf{H}_{23}\Psi\mathbf{K}^{-2}\Psi^T\mathbf{d} \\ \mathbf{G}_{31}\mathbf{q}_1 - \frac{w}{g}\mathbf{G}_{32}\mathbf{a}_n - \frac{w}{g}\mathbf{H}_{33}\Psi\mathbf{K}^{-2}\Psi^T\mathbf{d} \end{pmatrix} \quad (29)$$

Equation (29) represents the constant boundary element formulation for the 3D case, including the infinite radiation condition and damping along the foundation and banks. It contains N unknown values of $\mathbf{p}$, $\mathbf{q}$ and η .

ANALYTICAL RESULTS

A computer program was developed to calculate the hydrodynamic pressures in a 3D reservoir due to prescribed excitation of the dam face and reservoir bottom and banks. The program uses the constant boundary elements. An infinite reservoir is divided into the two previously described regions, one irregular but finite and the other in the form of an infinite channel of constant cross-section. The eigenvalue problem associated with the infinite region is solved by two-dimensional finite element discretization using the QZ algorithm originally developed by Moler and Stewart.²⁷

Selected results of analytical studies carried out by using the formulation and the computer program mentioned in the preceding paragraphs are presented here. A BEM model of the three-dimensional rectangular reservoir is considered first in order to compare the analytical results with classical results obtained from a corresponding 2D model. A 3D reservoir impounded by a circular arch dam is considered next.

Figure 2. Regular (FEM) and modified (BEM) meshes at the far boundary

Rectangular three-dimensional infinite reservoir

The rectangular reservoir studied is shown in Figure 3. The following data are used in the computations: height of the reservoir, $H = 100$ m; velocity of sound in water, $c = 1440$ m/s; the mass density of water, $\rho = 1$ tonne/m³. For the data given, the fundamental frequency of the reservoir $\omega_1 = \pi c/2H$ becomes 22.62 rad/s (3.6 Hz). The transmitting boundary is placed at a distance of 100 m from the upstream dam face.

The constant boundary element model of the reservoir being studied consists of 119 boundary elements. In the finite element model of the infinite channel, 15 4-node linear isoparametric quadrilateral elements are used.

The results presented here show the variations of hydrodynamic pressure along the dam face for two types of harmonic motion: upstream-downstream and vertical motion. Four different foundation damping levels are considered with the following values of coefficient of reflection: $\alpha_R = 0.0, 0.5, 0.75$ and 1.0 , the last one corresponding to full reflection or the rigid case.

The results for upstream-downstream harmonic motion are presented in Figures 4 and 5. The real and imaginary components of the hydrodynamic pressure are shown separately. Results are presented for two excitation frequencies given by $\omega/\omega_1 = 0.4$ and 1.2 . The results for a rigid case ($\alpha_R = 1.0$) show very good agreement with 2D classical³ and BEM results,¹⁵ especially for $\omega < \omega_1$. In this case, the imaginary part of the hydrodynamic pressure is negligible; theoretically it ought to be zero. For $\omega > \omega_1$ ($\omega/\omega_1 = 1.2$), the imaginary part is predominant. The results for a rigid case ($\alpha_R = 1.0$) are still in reasonably good agreement with 2D classical³ and BEM solutions.²² In both cases, the introduction of damping results in a decrease in the response. Evidently the decrease is most significant for $\alpha_R = 0$ (full damping case).

The results for vertical harmonic motion are presented in Figure 6. The real and imaginary parts of the hydrodynamic pressure are shown separately. Results are presented for an excitation frequency $\omega = 1.2\omega_1$. Four different levels of damping with coefficient of reflection values of 1.0 (rigid case), 0.75 , 0.50 and 0.0 (full damping case) are considered. Again, the results for a rigid case are in good agreement with the 2D classical and BEM results.^{3, 22} Introduction of damping results in a decrease in the response. The good comparison between the 3D BEM results and the 2D classical and BEM results provides a partial verification of the formulation and the computer algorithm.

Three-dimensional reservoir impounded by an arch dam

Results of analytical studies of hydrodynamic pressures in a 3D infinite reservoir impounded by a circular arch dam are presented next. The effects of infinite radiation damping and foundation and banks damping are also considered. The following data are used in the computation: height of the reservoir, $H = 60$ m; radius of the circular section of the arch dam, $R = 90$ m; length of the finite part of the reservoir, $L = 110$ m; velocity of sound in water, $c = 1440$ m/s; the mass density of water, $\rho = 1$ tonne/m³. Beyond $L = 110$ m the reservoir is represented by an infinite channel of rectangular cross-section. For the given data the fundamental frequency of the reservoir $\omega_1 = \pi c/2H$ becomes 37.7 rad/s (6 Hz).

Figure 3. A 3D rectangular infinite reservoir

Figure 4(a). Real part of the hydrodynamic pressure on a gravity dam due to upstream-downstream harmonic motion in a 3D infinite rectangular reservoir— $\omega/\omega_1 = 0.4$

Figure 4(b). Imaginary part of the hydrodynamic pressure on a gravity dam due to upstream-downstream harmonic motion in a 3D infinite rectangular reservoir— $\omega/\omega_1 = 0.4$

Figure 5(a). Real part of the hydrodynamic pressure on a gravity dam due to upstream-downstream harmonic motion in a 3D infinite rectangular reservoir— $\omega/\omega_1 = 1.2$

Figure 5(b). Imaginary part of the hydrodynamic pressure on a gravity dam due to upstream-downstream harmonic motion in a 3D infinite rectangular reservoir— $\omega/\omega_1 = 1.2$

Figure 6(a). Real part of the hydrodynamic pressure on a gravity dam due to vertical harmonic motion in a 3D infinite rectangular reservoir— $\omega/\omega_1 = 1.2$

Figure 6(b). Imaginary part of the hydrodynamic pressure on a gravity dam due to vertical harmonic motion in a 3D infinite rectangular reservoir— $\omega/\omega_1 = 1.2$

The constant boundary element model of the reservoir consists of 260 4-node constant quadrilateral boundary elements. In addition 30 4-node linear isoparametric quadrilateral finite elements are used to model the infinite channel.

The results presented here show the variation of hydrodynamic pressure along the upstream dam face for three types of harmonic motion: upstream-downstream, cross-stream and vertical motion.

The results for upstream-downstream harmonic motion are presented in Figures 7 and 8 for $\omega/\omega_1 = 0.4$ and $\omega/\omega_1 = 1.2$ respectively. Two different values of coefficient of reflection $\alpha_R = 0.75$ (25 per cent absorption) and $\alpha_R = 1.0$ (full reflection, rigid case) are considered. The 3D classical results obtained by Porter and Chopra^{1,2} for a radially infinite reservoir are also shown for comparison. The classical results should be compared with the 3D BEM results for a rigid case, $\alpha_R = 1.0$. Most of the results show qualitative similarity in the two sets of response; however, significant differences exist, presumably because of the difference in the geometry of the infinite part. The classical solution is based on radial banks while in the BEM solution the banks in the infinite channel are parallel to the x axis.

Figure 7. Absolute value of the hydrodynamic pressure on an arch dam due to upstream-downstream harmonic motion in a 3D infinite reservoir— $\omega/\omega_1 = 0.4$

Figure 8. Absolute value of the hydrodynamic pressure on an arch dam due to upstream-downstream harmonic motion in a 3D infinite reservoir— $\omega/\omega_1 = 1.2$

Figure 9. Absolute value of the hydrodynamic pressure on an arch dam due to cross-stream harmonic motion in a 3D infinite reservoir— $\omega/\omega_1 = 0.4$

Figure 10. Absolute value of the hydrodynamic pressure on an arch dam due to cross-stream harmonic motion in a 3D infinite reservoir— $\omega/\omega_1 = 1.2$

Figure 11. Absolute value of the hydrodynamic pressure on an arch dam due to vertical harmonic motion in a 3D infinite reservoir— $\omega/\omega_1 = 0.4$

Figure 12. Absolute value of the hydrodynamic pressure on an arch dam due to vertical harmonic motion in a 3D infinite reservoir— $\omega/\omega_1 = 1.2$

The results of the analysis for cross-stream harmonic motion are presented in Figures 9 and 10 for $\omega/\omega_1 = 0.4$ and $\omega/\omega_1 = 1.2$ respectively. The results of 3D BEM are different from the classical infinite case reported by Porter and Chopra.^{1,2} Apparently, the geometry of the infinite part of the reservoir has a significant influence on the results.

Finally the hydrodynamic pressures due to a vertical harmonic motion are presented in Figures 11 and 12 for $\omega/\omega_1 = 0.4$ and $\omega/\omega_1 = 1.2$ respectively. For $\omega < \omega_1$ ($\omega = 0.4\omega_1$) the results of BEM analysis are quite close to the classical solution.^{1,2} Evidently, the difference in the geometry of the infinite channel does not affect the results. For $\omega > \omega_1$ ($\omega = 1.2\omega_1$) the two sets of results are different even though a qualitative similarity exists. In this case the difference in the geometry of the infinite channel has a pronounced effect.

SUMMARY AND CONCLUSIONS

This paper describes the boundary element technique for the linear analysis of hydrodynamic pressures in a 3D reservoir for a harmonic motion of the dam and/or the reservoir bottom and banks. The constant boundary element formulation is used. The energy loss in the waves travelling to infinity is represented by a

special boundary condition to be applied to the truncated far end of the reservoir (transmitting boundary). This boundary condition has been successfully used in a finite element analysis and also in a 2D boundary element analysis of dam-reservoir systems. In addition, a special foundation damping condition developed by other researchers for an FEM analysis has been incorporated in the BEM formulation presented here. Although the results presented are for harmonic motion only, the analytical procedure can be extended easily to the case of the random earthquake motion by using the fast Fourier transformation technique.

Several observations can be made as a result of the present research.

1. The boundary element method (BEM) is an effective procedure for evaluating hydrodynamic pressures in a 3D water reservoir of arbitrary shape.
2. When compared to the finite element method (FEM), BEM offers the advantage of reducing the dimensionality of the problem by one.
3. In comparison to the 2D case, the fundamental solution for a 3D BEM formulation is of simpler nature.
4. The analysis of a flexible dam for an arbitrary ground motion taking into account the foundation-dam interaction can be achieved by a substructuring technique in which the dam is discretized by FEM or BEM and the reservoir is modelled by BEM.
5. A drawback of the BEM is the frequency dependence of the fundamental solution which requires the computation of the elements of boundary element matrices for each excitation frequency. The use of a frequency independent fundamental solution is therefore of interest and needs to be pursued.
6. The effect of the location of the far end transmitting boundary needs additional studies.
7. The effect of damping introduced by a flexible foundation and banks could be evaluated by a more detailed representation using the BEM to model a semi-infinite medium and coupling the model with that for the reservoir.

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