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Analytic Model for Low Earth Orbit Satellite Solar Power

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Abstract

An analytic model for calculating the instantaneous and average orbital power available to a satellite in a circular low-Earth orbit is presented. By accounting for the effects of angle of solar incidence and eclipsing, for both stabilized and tumbling satellites, this model allows for fast, simple, accurate calculations of available power without the use of complex, often expensive, numerical simulation software. In addition, this method provides the value of maximum and minimum solar power available.

Index Terms

Aerospace electronics, Low earth orbit satellites, Solar power generation, circular orbits, solar incidence.

I. Introduction

Available solar power is an important quantity to know in both the design phase and throughout the lifetime of a satellite. Most methods for estimating available power fall into one of two categories: complex simulation software [1] [2] [3] [4], and crude approximations [5]. Complex simulation software has many challenges. It is time consuming to use, or develop, is often challenging to learn and operate, and most software will only simulate a single case at a time. Because of this, many satellite design teams, especially in the early design phases, will resort to crude approximations. The model proposed in this paper bridges the gap between these two methods, by providing a sufficiently accurate method of calculating the instantaneous and average solar power available to a satellite. The ease of use and low computational power required through this proposed method also makes it feasible to consider all orbital configurations.

We begin by defining a model for power available to a solar cell. We then use this model to calculate power to a satellite in sample orbits. These orbits, noon-midnight and dawn-dusk, serve as intuitive cases, from which we then derive a generalized model including all possible low-Earth orbits. To finish the model, we then derive a conversion

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between the sun-fixed coordinate system, used in the previously mentioned model, and Keplerian elements. Finally, we demonstrate how to use the model and compare its results against other simulation methods.

To make an analytic solution possible, it is necessary to constrain the orientation of solar cells on the satellite. The orientation we consider in this paper are cells that are pointed along the zenith, nadir, ram, or anti-ram directions, as well as the two faces orthogonal to these (see Fig. 1). This configuration of cells is shared by many Earth observation satellites and in particular by many attitude stabilized CubeSats [6] [7] [8]. CubeSats accounted for 49 percent of satellites launched in 2015 [9], and the teams supporting these satellites generally have far fewer resources than other satellite providers. Uninformed decisions in the early design cycle for these teams will lead to unexpected consequences on power constraints later in the mission. Our analytic model permits teams to make informed decisions regarding load budgeting and to make immediate appraisal of design alternatives.

II. Power for a Solar Cell

The first step in developing our model for how a satellite produces solar power is defining a model for solar cell power production. The power generation system on a satellite generally consists of solar cells connected together to form solar panels and then connected to circuitry which draws the power from the solar panels and converts it to a form usable by the rest of the satellite. This paper considers the power available from the solar cells and not the power ultimately available to run the satellite after efficiency losses in other circuitry (e.g., maximum power point tracking, regulators, battery, etc).

The power produced by a solar cell (P_{cell}) is a function of three main parameters: the area of the cell (A), the efficiency of the cell (η), and the intensity of the incident light (I).

$$P_{cell} = A \times \eta \times I \tag{1}$$

Further, the intensity of light incident on the cell is only equal to the intensity of incoming light (I_{max}) at normal incidence; otherwise, the intensity of light incident on the cell is reduced by the cosine of the angle of incidence between the cell normal and the incident light ray, ϕ.

$$I = I_{max} \times \cos\phi \tag{2}$$

Ignoring the Earth's albedo, when a satellite is normally illuminated I_{max} is approximately equal to the solar constant. The solar constant is the flux density of incoming light at a distance of one astronomical unit from the sun [10]. Since the Earth is in an elliptical orbit around the sun, and therefore not always one astronomical unit from the sun, the solar constant is not a perfect measure of incoming sunlight; however, it serves as a reasonable approximation.

Rewriting (1) to include light that is not at normal incidence and grouping the terms that define the power produced at normal incidence into the variable P_{max}, we can write an equation for the power produced by the cell in terms of just P_{max} and ϕ.

$$P_{cell} = (A \times \eta \times I_{max}) \times \cos\phi = P_{max} \cos\phi \tag{3}$$

At the beginning of life for the satellite, P_{max} is the manufacturer provided wattage for the solar cell. This value will decrease throughout the mission lifetime as the solar cells are bombarded by high energy particles, ultraviolet radiation, micro meteorites, and atomic oxygen, all of which decrease cell performance over time. Predicting the degree to which solar cell degradation will occur can be an involved process and many publications exist describing different methods for estimating this degradation occurence [11] [12] [13] [14].

To calculate the end of life performance of solar cells, the beginning of life efficiency or manufacturer provided wattage can be multiplied by a degradation factor. [12] suggests that for low Earth orbit, GaAs cell power production degrades by 2.75% per year.

Equation (3) assumes that the reflection from a solar cell is accounted for in the solar cell efficiency and that reflection does not vary with ϕ. This assumption is valid since most solar cells only exhibit significant reflectivity far from normal incidence [15]. At these angles (i.e., as ϕ approaches $\pi/2$), the radiant flux through the solar cell is small and in turn so is power production regardless of reflection. Only in extreme scenarios will the majority of power production come from cells at glancing angles to the incoming sunlight where reflections have a dominating effect. For example, it would not be appropriate to use this model on a satellite with solar cells only pointed away from the Earth in an orbit with a beta angle near $\pm 90°$.

Using these relations, by calculating ϕ for each solar cell, and excluding the times when the satellite is in eclipse, we can calculate what power will be generated by the satellite as described in (4).

$$P = \sum P_{cell} = \sum P_{max} \cos\phi \tag{4}$$

III. Simple Cases

We start by examining simple cases for calculating both the eclipse times and ϕ throughout an orbit. These cases provide intuitive results in-which to build and test a more comprehensive model that includes all possible circular orbits.

In order to define the incoming power, we will adopt the coordinate system as shown in Fig. 1. This coordinate system is a Cartesian coordinate system centred on a satellite, with the positive y direction pointed away from the Earth, the negative z direction pointed in the direction of travel of the satellite, and the x direction creating a right handed coordinate system. Using these coordinates we will define $P_{max_{x+}}$ as the power absorbed by the satellite when the positive x axis is pointed towards the sun. We will use the same convention to define $P_{max_{x-}}$, $P_{max_{y+}}$, $P_{max_{y-}}$, $P_{max_{z+}}$, and $P_{max_{z-}}$.

A simple model for instantaneous solar power while illuminated has been presented. Whole orbits including noon-midnight and dawn-dusk will now be derived in order to build our proposed model.

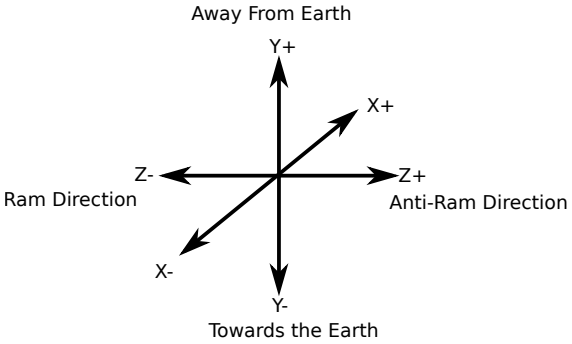

Fig. 1. Defined satellite-centred coordinate system.

A. Tumbling Satellites

The first case we will analyze is that of a tumbling satellite. If the satellite is tumbling at a known rate, along a known axis, then a numerical simulation can be employed; however, if these values are not known, an average value over all possibilities is more valuable. We can generate an expectation value for power generated by a solar cell by averaging the power generated by that solar cell across all possible directions incoming sunlight could come from. This includes ϕ angles from 0° to 90° where the sunlight is illuminating the front of the solar cell, and angles from 90° to 180° where the sunlight does not illuminate the front of the solar cell and therefore no power is generated. We can calculate this expectation angle for power generated by integrating $P_{max}\cos\phi$ over a hemisphere and then dividing by the surface area of that sphere with the same radius. A hemisphere is used since power can only be generated when light is incident on the front face of a solar cell. This leaves us with the power generated by each cell as $P_{max}/4$.

$$P = \frac{\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\left(P_{max}\cos\phi\right)\sin\phi\, d\phi d\theta'}{4\pi} = \frac{P_{max}}{4} \tag{5}$$

In Section IV-B, a derivation is shown for what portion of an orbit is sunlit for any orbit.

B. Dawn-Dusk Orbit

The first specific orbit we will analyze is a dawn-dusk orbit. This is a sun-synchronous orbit, which means the satellite's orbit will not change from the perspective of the sun. In a dawn-dusk orbit, the normal to the orbit's plane of motion points towards the sun. For this case the satellite never enters eclipse, and the power generated by the satellite is equal to $P_{max_{x+}}$ or $P_{max_{x-}}$ depending on the direction of the orbit.

C. Noon-Midnight Orbit

The second specific orbit we will consider, is called a noon-midnight orbit. This is also a sun-synchronous orbit; however, unlike the dawn-dusk orbit, where the plane of motion is normal to the incoming sun, in this orbit the plane of motion is parallel to the incoming sun. In this orbit a satellite experiences the maximum amount of eclipse.

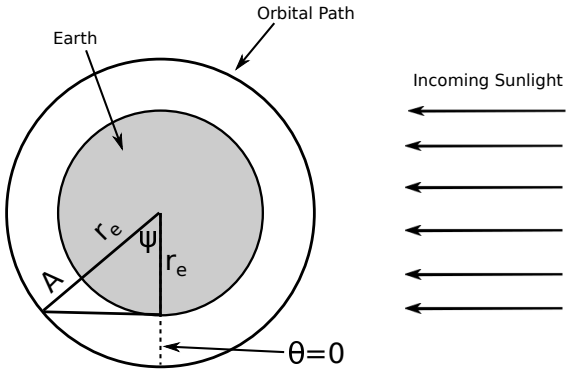

Fig. 2. 2D representation of a noon-midnight orbit. θ is measured counter-clockwise from the $\theta = 0$ mark.

We will start by calculating when the satellite eclipses. Since the plane of motion is parallel to the incoming sunlight, we can do this with a 2D representation of the orbit. This 2D representation is shown in Fig. 2 where r_e is the radius of the Earth, A is the altitude of the satellite, θ is the position of the satellite in the orbit, and ψ is the angle defining the position of the satellite when it enters eclipse. In Fig. 2, θ is measured counter-clockwise from the $\theta = 0$ mark.

As shown in equation (6), we can solve for ψ using basic trigonometry. This allows us to state that the satellite is in eclipse for $\pi + \psi < \theta < 2\pi - \psi$.

$$\cos(\psi) = \frac{r_e}{r_e + A} \tag{6}$$

$$\psi = \cos^{-1}\left(\frac{r_e}{r_e + A}\right) \tag{7}$$

As the satellite travels across the boundary between sunlit and eclipse it passes through a region known as the penumbra. In this region, the satellite is still illuminated but at a lower intensity than when it is illuminated by direct sunlight. For low Earth orbit satellites, this region is travelled through so quickly that it can be ignored.

We can determine the available power throughout the orbit by calculating the angle between the incoming sunlight and the satellite. Solar cells on the $\pm$x faces of the satellite will be perpendicular to the sunlight and generate no power. Power contributions from the $\pm$y and $\pm$z faces as a function of angle theta are shown in Table I.

Next we convert from an orbit defined by angle (θ) to an orbit defined by time (t). Analysing power generation in terms of time is not necessary to establish the overall satellite power balance, but is necessary to determine the maximum battery depth of discharge in an orbit. To convert from θ to time, we equate the centripetal acceleration of the satellite to the gravity of the Earth and use this to determine the angular frequency of the orbit (ω). This is shown in equation (8)-(10) where g is acceleration due to gravity, M is the mass of the Earth, and G is the gravitational constant.

TABLE I

THE POWER AVAILABLE TO THE SATELLITE AS IT PROGRESSES THROUGH A NOON-MIDNIGHT ORBIT

Section of Orbit	**Power** (P)
$0 < \theta < \frac{\pi}{2}$	$P_{max_{z-}} \cos\theta + P_{max_{y+}} \sin\theta$
$\frac{\pi}{2} < \theta < \pi$	$-P_{max_{z+}} \cos\theta + P_{max_{y+}} \sin\theta$
$\pi < \theta < \pi + \psi$	$-P_{max_{z+}} \cos\theta - P_{max_{y-}} \sin\theta$
$\pi + \psi < \theta < 2\pi - \psi$	0
$2\pi - \psi < \theta < 2\pi$	$P_{max_{z-}} \cos\theta - P_{max_{y-}} \sin\theta$

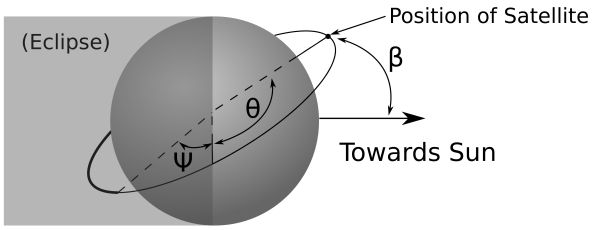

Fig. 3. (β,θ) Coordinate System

$$\begin{aligned} g =& \frac{GM}{r_e + A} \\ =& \omega^2 (r_e + A) \end{aligned} \tag{8}$$

$$\omega = \frac{\sqrt{GM}}{r_e + A} \tag{9}$$

$$\theta = \omega t = \frac{\sqrt{GM}}{r_e + A} t \tag{10}$$

IV. GENERAL CASE

To calculate the available power for all circular low-Earth orbits, we generalize the model to include all possible angles, including dawn-dusk and noon-midnight orbits.

A. Coordinate System

To model this general case we will use a coordinate system, shown in Fig. 3, that has several unspecified degrees of freedom. Many possible sets of Keplerian elements will correspond to an orbit defined using this coordinate system.

This coordinate system is built around an Earth centred Cartesian frame. The Cartesian frame is independent of the previously discussed satellite centred coordinate system. In this Cartesian frame we define the y axis as always pointing towards the sun. To define an orbit, we start with a circle in the x-y plane defined by polar coordinates.

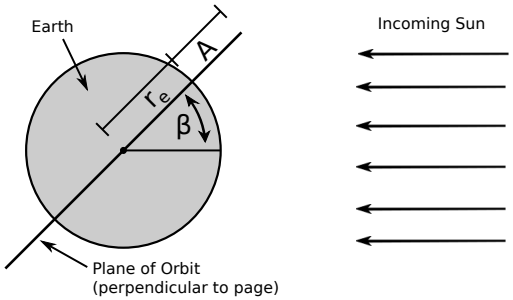

Fig. 4. Illustration of how β determines whether a satellite eclipses.

We then rotate this circle, about the x axis, by an angle β. This allows us to move from a noon-midnight orbit to a dawn-dusk orbit. The mathematical definition of this coordinate system is shown in equation (11) and (12).

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = (r_e + A) \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & \sin\beta \\ 0 & -\sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = (r_e + A) \begin{bmatrix} \cos\theta \\ \cos\beta\sin\theta \\ \sin\beta\sin\theta \end{bmatrix} \tag{12}$$

B. Eclipse Calculation

When calculating the eclipse positions we first need to account for orbits which do not eclipse. As Fig. 4 shows, for β near $\pi/2$ or $-\pi/2$ there is no eclipse. Included in this is the dawn-dusk case discussed earlier.

Based on Fig. 4, equation (13) shows the condition for an orbit to have no eclipse [16].

$$(r_e + A)\,|\sin\beta| > r_e \tag{13}$$

In orbits where the satellite enters eclipse behind the Earth, we can find the eclipse times by projecting the position of the satellite onto the $y = 0$ plane and then solving for when the length of the position vector is equal to r_e [16]. This will produce four positions in the orbit that represent, from the point of view of the sun, when the satellite passes by the outline of the Earth.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ 0 \\ z \end{bmatrix} \tag{14}$$

$$(r_e + A) \begin{bmatrix} \cos\theta \\ \cos\beta\sin\theta \\ \sin\beta\sin\theta \end{bmatrix} \Rightarrow (r_e + A) \begin{bmatrix} \cos\theta \\ 0 \\ \sin\beta\sin\theta \end{bmatrix} = f(\beta, \theta) \tag{15}$$

$$||f(\beta,\theta)|| = (r_e + A)\sqrt{\cos^2\theta + \sin^2\beta\sin^2\theta} = r_e \tag{16}$$

$$\theta = \tan^{-1}\sqrt{\frac{(r_e+A)^2 - {r_e}^2}{{r_e}^2 - (r_e+A)^2\sin^2\beta}} \tag{17}$$

Two of these four positions, where the satellite passes by the outline of the Earth from the perspective of the sun, are where the satellite enters and leaves eclipse. These can be determined based on the value of θ. Defining ψ the same way we did for the noon midnight orbit (the satellite is in eclipse for: $\psi + \pi < \theta < 2\pi - \psi$), we are left with the definition shown in (18).

$$\psi = \begin{cases} \tan^{-1}\sqrt{\frac{(r_e+A)^2-{r_e}^2}{{r_e}^2-(r_e+A)^2\sin^2\beta}}, & \text{if } (r_e+A)\,|\sin\beta| < r_e \\ \frac{\pi}{2}, & \text{otherwise} \end{cases} \tag{18}$$

C. Instantaneous Power

Now that we determined when a satellite will be in eclipse, we calculate the power available throughout the non-eclipsing part of the orbit.

We start by looking at the component of incoming sunlight perpendicular to the satellite's plane of motion. These values will be given as a fraction of $P_{max_{x+}}$ and $P_{max_{x-}}$. As the satellite moves from a dawn-dusk to a noon-midnight orbit, the value will decrease from its maximum value to 0. Using (3), we see that power generation along this direction is proportional to $|\sin\beta|$. This leaves us with $P_{max_{x+}}|\sin\beta|$ for $-\pi < \beta < 0$ and $P_{max_{x-}}|\sin\beta|$ for $0 < \beta < \pi$. For convenience, we combine these two cases using the variable P_{max_x} which is defined in (19).

$$P_{max_x} = \begin{cases} P_{max_{x+}}, & \text{if } -\pi < \beta < 0 \\ P_{max_{x-}}, & \text{if } 0 < \beta < \pi \end{cases} \tag{19}$$

Next we consider the power from incoming sunlight parallel to the satellite's plane of motion. This will have the same form as for the noon-midnight orbit (Table I), but with the addition of a scaling factor which accounts for the rotation by β. Since in the dawn-dusk orbit, power generation from the x directions goes to 0, using (3), we determine that the factor is $\cos\beta$.

Summing the instantaneous absorbed power from the three linearly independent directions in the satellite frame together (Fig. 1), we produce Table II. Table II contains details of the combined power absorption from all sides of a satellite throughout a full orbit.

D. Average Orbital Power

For most applications, an average, rather than instantaneous, model of power is more applicable. To calculate the average orbital power ($\bar{P}$), we will integrate over one full orbital period and divide this by the length of that orbital period. For simplicity we will do this in terms of θ.

TABLE II

THE POWER AVAILABLE TO THE SATELLITE AS IT PROGRESSES THROUGH ANY POSSIBLE ORBIT DEFINED BY β.

Section of Orbit	**Power** (P)
$\theta < \frac{\pi}{2}$	$\left(P_{max_{z-}} \cos\theta + P_{max_{y+}} \sin\theta\right) \cos\beta + P_{max_x} \lvert \sin\beta \rvert$
$\frac{\pi}{2} < \theta < \pi$	$\left(-P_{max_{z+}} \cos\theta + P_{max_{y+}} \sin\theta\right) \cos\beta + P_{max_x} \lvert \sin\beta \rvert$
$\pi < \theta < \pi + \psi$	$\left(-P_{max_{z+}} \cos\theta - P_{max_{y-}} \sin\theta\right) \cos\beta + P_{max_x} \lvert \sin\beta \rvert$
$\pi + \psi < \theta < 2\pi - \psi$	0
$2\pi - \psi < \theta < 2\pi$	$\left(P_{max_{z-}} \cos\theta - P_{max_{y-}} \sin\theta\right) \cos\beta + P_{max_x} \lvert \sin\beta \rvert$

$$\bar{P} = \frac{1}{2\pi} \int_0^{2\pi} P d\theta \tag{20}$$

To simplify the calculations, we solve for $\bar{P}$ coming from each direction, x, y, and z, in the satellite reference frame independently (Fig. 1) independently, and then sum the three directions to get the total power generation. Let the components of incoming sunlight from the x, y, and z directions be defined as P_x, P_y, and P_z, respectively. Let the components of the average orbital power from the x, y, and z directions be defined as $\bar{P}_x$, $\bar{P}_y$, and $\bar{P}_z$.

Since $\bar{P}_x$ is the easiest to calculate, we will start here. The power produced from the x component of incoming sunlight is a constant value of $P_{max_x} \lvert \sin\beta \rvert$ when the satellite is not in eclipse, and 0 otherwise. We will write the eclipse bounds in terms of ψ as defined by (18).

$$\bar{P}_x = \frac{1}{2\pi} \int_0^{2\pi} P_x d\theta \tag{21}$$

$$\bar{P}_x = \frac{1}{2\pi} \left(\int_{-\psi}^{\pi+\psi} P_{max_x} \lvert \sin\beta \rvert d\theta + \int_{\pi+\psi}^{2\pi-\psi} (0)\, d\theta \right) \tag{22}$$

$$\bar{P}_x = \frac{\pi + 2\psi}{2\pi} P_{max_x} \lvert \sin\beta \rvert \tag{23}$$

Next we solve for $\bar{P}_y$. Since P_y switches between three states, namely, eclipse, $y+$ component of sunlight, and $y-$ component of sunlight, we break the integral into the different orbital portions and then solve and simplify the result.

$$\bar{P}_y = \frac{1}{2\pi} \left(\int_0^{\pi} P_{max_{y+}} \sin\theta \cos\beta \; d\theta + \int_{\pi}^{\pi+\psi} -P_{max_{y-}} \sin\theta \cos\beta \; d\theta + \int_{\pi+\psi}^{2\pi-\psi} (0)\, d\theta + \int_{2\pi-\psi}^{2\pi} -P_{max_{y-}} \sin\theta \cos\beta \; d\theta \right) \tag{25}$$

$$\bar{P}_y = \frac{\cos\beta}{\pi}\left(P_{max_{y-}}\left(1-\cos\psi\right)+P_{max_{y+}}\right) \tag{26}$$

Solving for $\bar{P}_z$ is similar to $\bar{P}_y$ with three cases to integrate over, namely, eclipse, $z+$ component of sunlight, and $z-$ component of sunlight.

$$\bar{P}_z = \frac{1}{2\pi}\int_0^{2\pi} P_z d\theta \tag{27}$$

$$\begin{aligned}\bar{P}_z =& \frac{1}{2\pi}\left(\int_{-\psi}^{\pi/2} P_{max_{z-}}\cos\theta\ d\theta\right.\\ &\left.+\int_{\pi/2}^{\pi+\psi} -P_{max_{z+}}\cos\theta\ d\theta+\int_{\pi+\psi}^{2\pi-\psi}(0)\,d\theta\right)\end{aligned} \tag{28}$$

$$\bar{P}_z = \frac{(1+\sin\psi)\cos\beta}{2\pi}\left(P_{max_{z-}}+P_{max_{z+}}\right) \tag{29}$$

Summing $\bar{P}_x$, $\bar{P}_y$, and $\bar{P}_z$, we find the total average solar power for a satellite as defined in (30).

$$\bar{P} = \bar{P}_x + \bar{P}_y + \bar{P}_z \tag{30}$$

$$\begin{aligned}\bar{P} =& \frac{\pi+2\psi}{2\pi}P_{max_x}|\sin\beta|\\ &+\frac{\cos\beta}{\pi}\left(P_{max_{y-}}\left(1-\cos\psi\right)+P_{max_{y+}}\right)\\ &+\frac{(1+\sin\psi)\cos\beta}{2\pi}\left(P_{max_{z-}}+P_{max_{z+}}\right)\end{aligned} \tag{31}$$

$$\begin{aligned}\bar{P} =& \frac{1}{2\pi}\Bigg(|\sin\beta|\left(P_{max_x}\right)(\pi+2\psi)\\ &+\cos\beta\Big(2P_{max_{y-}}\left(1-\cos\psi\right)+2P_{max_{y+}}\\ &+\left(1+\sin\psi\right)\left(P_{max_{z-}}+P_{max_{z+}}\right)\Big)\Bigg)\end{aligned} \tag{32}$$

V. Power Calculations using Keplerian Elements

While having a model for incoming power to a satellite in terms of the beta angle of the orbit is useful (Fig. 3), satellites are not generally tracked using a sun fixed coordinate system; instead, satellites are tracked using Earth fixed coordinate systems. The most common coordinate system of this form is Keplerian Elements. This coordinate system fully describes a specific orbit with reference to the equator and a moving point on the Earth called the First Point of Aries. In the following section, we outline how to convert the variables in Keplerian Elements such that they can be input into our previous model. In order to do this, we build a simplified Keplerian coordinate system which only accounts for circular orbits and then find equivalent parameters between the simplified Keplerian Element coordinate system and the beta angle defined coordinate system.

TABLE III
DEFINITION OF ORBIT VARIABLES

Variable	Definition
u	Argument of Latitude
i	Inclination
Ω	RAAN
λ	$360^\circ \times$ Time Since March Equinox / 1 year

A. Coordinate System

Just like with the β defined coordinate system (Fig. 3), we start with a Cartesian coordinate system and then build on top of it. Let the Cartesian coordinate system be Earth centred. We define the y axis as always pointing towards the sun, and the x axis as perpendicular to the y axis and parallel to the equatorial plane. Since the y axis of this coordinate system is the same as the β defined coordinate system, we use this to find an equivalence between these two systems.

Also, with the β-θ coordinate system (Fig. 3), we use a parametrically defined circle in the x-y plane to represent an orbit and then apply rotation matrices to reposition it relative to the Cartesian frame. This circle will have radius $r_e + A$ and the position of the satellite along it will be given by u. The first rotation will be about the x axis. The angle rotates by i, the inclination of a orbit. We then rotate this orbit by Ω about the z axis. Ω is the right ascension of the ascending node (RAAN) of a orbit. For the y axis to continue pointing towards the sun, we need to add two more matrices to correct for the tilt of the Earth. The first is a rotation by an angle 23.5° about the y axis. The second is a rotation about the z axis by an angle λ. The angle λ is time dependent. Since this variable represents the rotation of the Earth about the sun, we know its period is 1 year. By design, let $\lambda = 0$ which is the March Equinox.

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = (r_e + A) \begin{bmatrix} \cos\lambda & -\sin\lambda & 0 \\ \sin\lambda & \cos\lambda & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 23.5^\circ & 0 & -\sin 23.5^\circ \\ 0 & 1 & 0 \\ \sin 23.5^\circ & 0 & \sin 23.5^\circ \end{bmatrix} \begin{bmatrix} -\sin\Omega & -\cos\Omega & 0 \\ \cos\Omega & -\sin\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & -\sin i \\ 0 & \sin i & \cos i \end{bmatrix} \begin{bmatrix} \cos u \\ \sin u \\ 0 \end{bmatrix} \tag{33}$$

$$\begin{aligned} y' =&(r_e + A)((-\sin\lambda\cos 23.5^\circ \sin\Omega - \cos\lambda\cos\Omega)\cos u \\ &+ ((\sin\lambda\cos 23.5^\circ \cos\Omega - \cos\lambda\sin\Omega)\cos i \\ &+ \sin\lambda\sin 23.5^\circ \sin i)\sin u) \end{aligned} \tag{34}$$

B. Conversion to β-θ Coordinate System

Next we convert our calculated power generation from the β-θ coordinate system to the simplified Keplerian coordinate system. Since the y axis in both coordinate systems is defined the same way, we can equate them, and use this to find the value of $\cos\beta$, $\sin\beta$, and θ in terms of our new variables.

$$y = \cos\beta\sin\theta = y' \tag{35}$$

To make this process more manageable, we use the identity shown in (36), to divide y' into its u dependent and u independent parts.

$$A\cos u + B\sin u = \sqrt{A^2 + B^2}\cos\left(u + \tan^{-1}\frac{B}{A}\right) \tag{36}$$

Equating the u independent parts of y and y' we can solve for $\cos\beta$.

$$\begin{aligned} \cos\beta = &\big((-\sin\lambda\cos 23.5^\circ \sin\Omega - \cos\lambda\cos\Omega)^2 + \\ &((\sin\lambda\cos 23.5^\circ \cos\Omega - \cos\lambda\sin\Omega)\cos i \\ &+ \sin\lambda\sin 23.5^\circ \sin i)^2\big)^{\frac{1}{2}} \end{aligned} \tag{37}$$

To solve for $\sin\beta$ we use the identity shown in (38).

$$\sin\left(\cos^{-1}x\right) = \sqrt{1 - x^2} \tag{38}$$

$$|\sin\beta| = \sqrt{1 - \cos^2\beta} \tag{39}$$

We do not need to explicitly solve for β as most of the equations we solved for in Section IV used $\cos\beta$ or $\sin\beta$ but never β on its own. The one exception is our definition of P_{max_x}, which is shown in (19), and requires us to know the sign of β.

To determine this, we can look at the direction the satellite is revolving around the y axis as defined in (33). If the revolution is clockwise, as seen by an observer on the positive side of the y axis, then β must be negative; conversely, if the revolution is counter-clockwise, then β must be positive.

The way we determine the direction the satellite is revolving is using the cross product of two different points in the satellite's revolution (two different values of u). We chose $u = 0^\circ$ and $u = 90^\circ$ since these helped simplify the result. To determine the direction the satellite is revolving around the y axis, we only need the sign of the y component of this cross product. The formula for this is shown in (40):

$$
\begin{aligned}
&sign\big(\beta\big)\\
=&sign\big(x(u=0^\circ)z(u=90^\circ)-x(u=90^\circ)z(u=0^\circ)\big)\\
=&sign\big(\sin 23.5^\circ\cos^2\Omega\cos i\cos\lambda\\
&-\sin 23.5^\circ\sin\Omega\cos i\sin\lambda\cos 23.5^\circ\cos\Omega\\
&-\sin^2 23.5^\circ\sin\Omega\sin\lambda\sin i\\
&-\sin\lambda\cos^2\Omega\sin 23.5^\circ\cos i\\
&+\sin\lambda\cos\Omega\cos 23.5^\circ\sin i\\
&+\cos\lambda\cos 23.5^\circ\sin\Omega\sin 23.5^\circ\cos\Omega\cos i\\
&-\cos\lambda\cos^2 23.5^\circ\sin\Omega\sin i\big)
\end{aligned}
\tag{40}
$$

We now substitute the values of $\cos\beta$ and $\sin\beta$, calculated from (37) and (39) respectively, into (18) to calculate ψ. We then input the values of $\cos\beta$, $\sin\beta$, and ψ into (32) to calculate the average power for any set of Keplerian elements.

To calculate the instantaneous incoming power, we solve the u dependent part of (35). This gives us an equivalence between θ and the orbital elements in Table III.

$$
\begin{aligned}
&\sin\theta=\\
&\cos\left(u+\tan^{-1}\left(\frac{-\sin\lambda\sin 23.5^\circ\sin i}{\sin\lambda\cos 23.5^\circ\sin\Omega+\cos\lambda\cos\Omega}\right.\right.\\
&\left.\left.+\frac{(\cos\lambda\sin\Omega-\sin\lambda\cos 23.5^\circ\cos\Omega)\cos i}{\sin\lambda\cos 23.5^\circ\sin\Omega+\cos\lambda\cos\Omega}\right)\right)
\end{aligned}
\tag{41}
$$

$$
\begin{aligned}
&\theta=u+\tan^{-1}\left(\frac{-\sin\lambda\sin 23.5^\circ\sin i}{\sin\lambda\cos 23.5^\circ\sin\Omega+\cos\lambda\cos\Omega}\right.\\
&\left.+\frac{(\cos\lambda\sin\Omega-\sin\lambda\cos 23.5^\circ\cos\Omega)\cos i}{\sin\lambda\cos 23.5^\circ\sin\Omega+\cos\lambda\cos\Omega}\right)+\frac{\pi}{2}
\end{aligned}
\tag{42}
$$

VI. Summary of Assumptions

For clarity, we now restate the main assumptions that our analytical model uses:

- The Earth remains at a distance of one astronomical unit from the sun such that the intensity of light is always given by the solar constant (1370 W/m^2)
- The Earth's albedo is ignored
- Power loss due to angle dependent reflection from solar cells is negligible
- The satellite does not cast any shadows on the solar cells
- The orbit is circular
- The time to pass through the penumbra is negligible
- Light from the sun is collimated
- The Earth is spherical

TABLE IV
EX-ALTA 1 MAXIMUM POWER FOR EACH FACE

Face	Maximum Power
$P_{max_{x-}}$	$1.2\text{ W} \times 6 = 7.2\text{ W}$
$P_{max_{x+}}$	$1.2\text{ W} \times 6 = 7.2\text{ W}$
$P_{max_{y-}}$	$1.2\text{ W} \times 2 = 2.4\text{ W}$
$P_{max_{y+}}$	$1.2\text{ W} \times 6 = 7.2\text{ W}$
$P_{max_{z-}}$	$1.2\text{ W} \times 0 = 0\text{ W}$
$P_{max_{z+}}$	$1.2\text{ W} \times 0 = 0\text{ W}$

VII. APPLICATION EXAMPLE

We now show how to apply this model to a satellite. To do this, we use the example of a CubeSat. This is a satellite which is made of $10\,\text{cm} \times 10$ cm $\times\, 10\,\text{cm}$ modular sections. CubeSats are often produced by groups with limited resources at a low cost. A big mission constraint for this type of satellite is power. Incoming power constrains what payloads can be used, how much data can be transmitted, and how many solar cells need to be integrated.

The example CubeSat we will use is the Ex-Alta 1 (the first University of Alberta satellite) which was launched in 2017 as part of the QB50 mission [17]. This CubeSat is a 3U, which means it has three $10\,\text{cm} \times 10\,\text{cm} \times 10\,\text{cm}$ modular sections connected together. Once stabilized, the satellite will travel with one of its $10\,\text{cm} \times 10\,\text{cm}$ faces pointing in the direction of travel, and one of its $10\,\text{cm} \times 30\,\text{cm}$ faces always pointing towards the Earth. The satellite will have no solar cells on the ram or anti-ram faces, and six solar cells on three of the four $10\,\text{cm} \times 30\,\text{cm}$ faces. The final face of the satellite, which will point towards the earth, will have two solar cells. Each of these solar cells can produce 1.2 W of power when exposed to direct sunlight.

A. Average Orbital Power

In the initial design phase of the satellite, when a launch provider has not been decided, it is useful to know what the power would be over all possible orbits. To determine the available power over all possible β, we will use (18) and (32). The results of this are shown in Fig. 5 for a range of altitudes.

From Fig. 5 we can see that the maximum power for our satellite does not happen in a dawn-dusk orbit, but instead at $\beta \approx 70°$. At this beta angle, two of the satellites 7.2 W faces are exposed to sunlight, and the orbit is non-eclipsing. We also now have constraints on what the maximum and minimum power our satellite will produce once stabilized.

Next, it is useful to repeat these calculations for when the satellite in not attitude stabilized. If we assume the satellite is tumbling with a rotational period much faster than the orbital period, we can use (5) to approximate the available power in sunlight; we do this by summing the maximum power for all the faces, and then dividing the

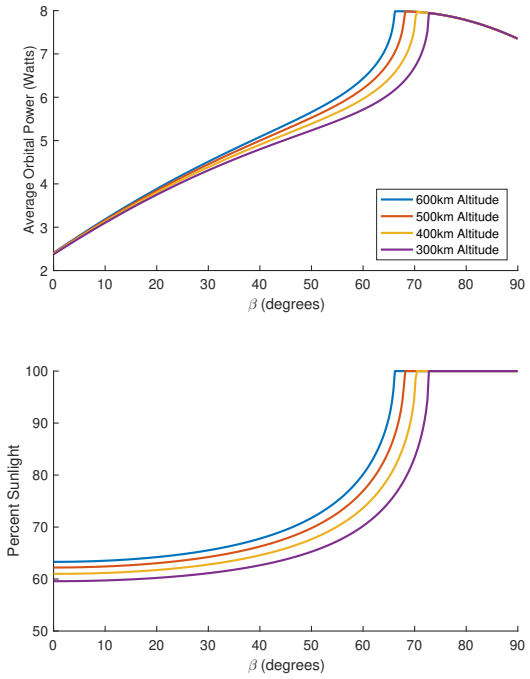

Fig. 5. The average power per orbit and the percentage of time spent not eclipsing for the Ex-Alta 1 cube satellite in all possible orbits.

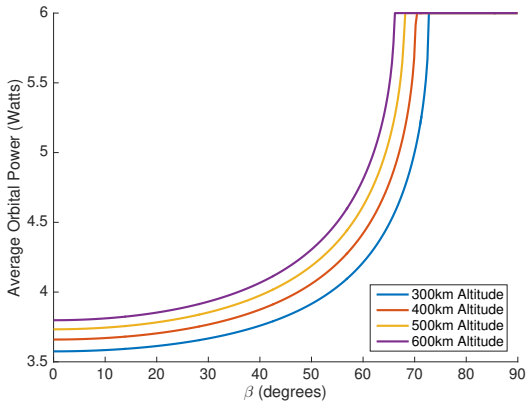

Fig. 6. The average power for the Ex-Alta 1 satellite when tumbling at various altitudes and for various β.

result by 4. We then multiply the previous result by the percentage of time the satellite is sunlit to find the average orbital power when tumbling, shown in Fig. 6.

B. Instantaneous Power

Once we know more about the satellite's orbit, we can calculate the instantaneous power for the satellite in specific cases. The instantaneous power can provide more insight into the available power for the satellite than the

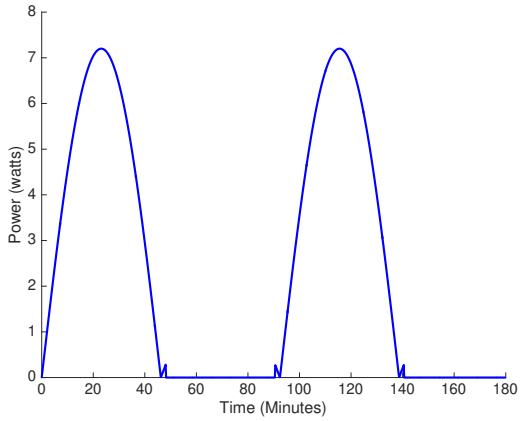

Fig. 7. The power produced by Ex-Alta 1 in a 400km altitude, noon-midnight orbit. Data is shown for two consecutive orbits.

TABLE V
DATA FROM TWO LINE ELEMENT SET

Variable	Value
Mean Motion	14.252886
Eccentricity	0.000861
Inclination	98.679°
RAAN	190.148°
Mean Anomaly	154.493°
Argument of Perigee	328.530°
Year	2015
Days into the Year	182.000

average can. As an example, if the satellite is switching between high and low incoming power throughout an orbit, more power will be lost to inefficiencies with the battery.

First, we will look at the worst case scenario: noon-midnight. We can calculate this using Table I. To do this we will use the actual altitude for the Ex-Alta 1 satellite, 400km, at beginning of life.

C. Power Calculations Using Two-Line Elements

Next we will use a set of two-line elements (TLEs), a format for expressing Keplerian elements, to demonstrate how instantaneous power could be computed while the satellite is orbiting. While we do not know the exact orbit that Ex-Alta 1 will have, we can test some possibilities. Table V gives some values from the two-line element set for a satellite in low-Earth-orbit.

We first need to convert the values from the TLE into the variables in Table III. The argument of latitude (u) is approximately equal to the sum of the mean anomaly and the argument of perigee. The inclination (i) and RAAN

TABLE VI
ORBIT VARIABLES FOR SIMULATION

Variable	Value
u	$154.493^\circ + 328.530^\circ = 483.023^\circ \Rightarrow 123.023^\circ$
i	98.679°
Ω	190.148°
λ	$360^\circ\,(182.000 - 79.9479)\,/365 = 101.574^\circ$
A	826.0488 km

(Ω) are already values listed in the TLE. The ecliptic longitude λ can be calculated using the epoch, which is also a value in the TLE. Finally the altitude, A, can be calculated from the mean motion using (43), where n is the mean motion in revolutions per day, G is the gravitational constant, and M is the mass of the Earth.

$$A = \left(\frac{GM}{((2\pi n)/(86400s))^2}\right)^{1/3} - r_e \tag{43}$$

We now need to input the values from Table VI into (37), (39), (40), and (42) to convert these values into the β based coordinate system. From there, we can determine the average power using (19) and (32), or the instantaneous power, using (19) and Table 2. The results of calculating the instantaneous power are shown in Fig. 8-9 and compared with the Systems Toolkit (STK) software [4] produced by AGI.

The J4 Perturbation propagator was used, which is an analytic propagator that does not account for atmospheric drag, solar radiation pressure or third body gravity, but does account for asymmetry in the gravitational field. Discrepancies between the analytic model and STK are a result of assumptions in the analytic model, such as the orbit being perfectly circular and the Earth being perfectly spherical. In this orbit, the largest discrepancy is the shorter eclipse time generated by the analytic model.

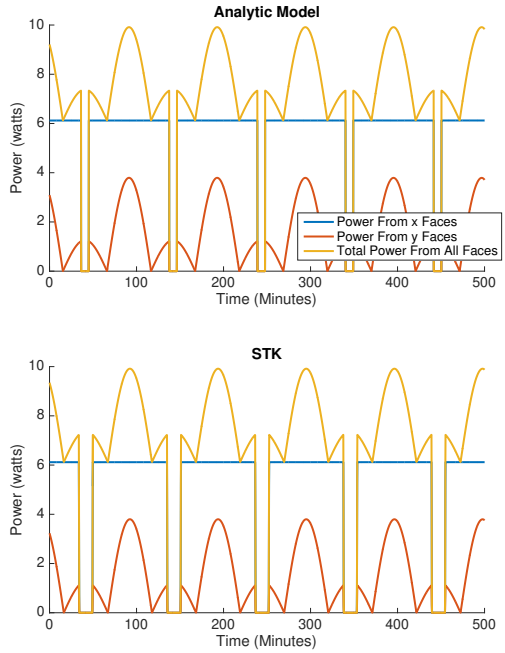

Fig. 8. The estimated incoming power using two separate models. The top plot is the model described in this paper, and the bottom is generated using data from the Systems Tool Kit (STK) software package [4].

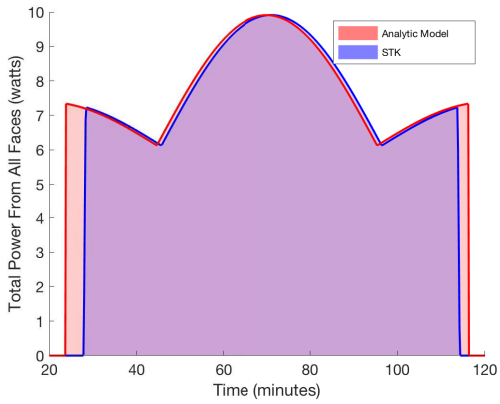

Fig. 9. The first complete orbit from Fig. 8 with the total power from each model superimposed. The largest discrepancy is the eclipse timings; the analytic model predicts a shorter eclipse as compared to STK. Otherwise, the two models are in very close agreement.

TABLE VII
PERCENT DIFFERENCE BETWEEN ANALYTIC MODEL AND STK

		Inclination (degrees)				
		0	**22.5**	**45**	**67.5**	**90**
Right Ascension (degrees)	**0**	6.74	2.06	6.79	8.04	3.38
	45	6.63	4.31	4.06	6.10	2.98
	90	6.89	7.16	7.56	2.43	0.48
	135	6.84	8.76	9.81	10.13	10.04
	180	6.61	7.83	3.14	0.64	3.56
	225	6.39	6.97	1.27	8.53	2.91
	270	6.34	5.74	4.67	3.11	0.42
	315	6.66	3.33	6.26	8.78	9.65

D. Sensitivity Analysis

As a final test for the model, forty orbits were simulated for a full day (January 1, 2015) in both STK and the analytic model. Each orbit was circular with a 500 km altitude but different right ascension and inclination.

The percentage difference in total power generation between the two models is shown in Table VII. The average percentage difference between STK and the analytic model was 5.60 percent and in all cases the discrepancy is less than approximately 10 percent.

In these simulations, orbital parameters were given to the analytic model, from STK, after each time step (30s). STK was used for the sake of simplicity; other orbital simulators are available [18]. This compensated for orbital perturbations which have a non-negligible impact on power generation over the full day simulated. Directly importing the orbital elements additionally corrected slight discrepancies between the orbital elements provided to the orbit wizard tool in STK and the actual orbit object built in STK. For example, when defining an orbit with a right ascension of 135 degrees using the orbit wizard tool, the actual orbit simulated may have a right ascension of 134.8 degrees. This is part of why some orbits, which would be expected to have identical power generation, do not. The slight variations between orbits, in Table VII, with a zero degree inclination are an example of this.

After visually comparing the power generation vs. time for the set of orbits in the sensitivity analysis, a significant source of error was found to be the eclipse times just as seen in Fig. 9.

VIII. CONCLUSION

By using a geometric representation of an orbit, we were able to calculate the angle of solar incidence, eclipsing, and available solar power for satellites in circular low-Earth orbits. This includes both the instantaneous and average orbital power for tumbling and stabilized satellites. The model accepts orbits defined by either beta angle, or Keplerian elements and shows agreement with commercial orbital simulation software. The model is also very

powerful in that it provides the maximum and minimum power across all possible circular low-Earth orbits for attitude stabilized satellites with solar cells in the defined directions.

For ease of use, the equations and model presented are also available as Matlab open-source code [19].

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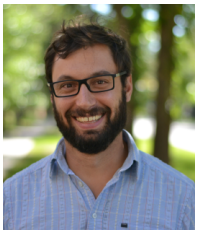

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