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Zaman, M. Hasanat; Peng, Heather; Baddour, Emile; Spencer, Don; McKay, Shane

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**IDENTIFICATIONS OF SPURIOUS WAVES IN THE WAVE TANK WITH
SHALLOW WATER**

M. Hasanat Zaman

National Research Council – Canada
Institute for Ocean Technology
Arctic Ave., P.O.Box 12093, Station A
St. John's, NL, A1B 3T5, Canada.
Hasanat.Zaman@nrc-cnrc.gc.ca

Heather Peng

Memorial University of Newfoundland,
Faculty of Applied Science and Engineering
St. John's, NL, Canada
hpeng@mun.ca

Emile Baddour

National Research Council - Canada
Institute for Ocean Technology
Arctic Ave., P.O.Box 12093, Station A
St. John's, NL, A1B 3T5, Canada
Emile.Baddour@nrc-cnrc.gc.ca

Don Spencer

Oceanic Consultant Corporation
95 Bonaventure Ave., Suite 401
St. John's, NL, A1B 2X5 Canada
Don_Spencer@oceaniccorp.com

Shane McKay

National Research Council - Canada
Institute for Ocean Technology
Arctic Ave., P.O.Box 12093, Station A
St. John's, NL, A1B 3T5, Canada
Shane.Mckay@nrc-cnrc.gc.ca

ABSTRACT

Accurate generation of the primary waves and the reproduction of the group-induced second-order low and high frequency waves have been considered essential for physical model test in the laboratory. In the laboratory when bi-chromatic primary waves are generated the required bounded waves will be generated naturally at the difference frequency. In addition to that several unwanted free waves are also generated. The free waves, having the same frequency of the bounded wave are reproduced due to mismatch of the boundary conditions at the wave paddle. The other two types of free waves are due to the wave paddle displacement and the local disturbances.

We carried out an extensive experimental program to identify the second order spurious waves in shallow water in the Offshore Engineering Basin (OEB) at the Institute for Ocean Technology (IOT) of National Research Council (NRC) Canada. In the experiments water depths are used in the range of 0.3m to 0.8m. The wave periods also have varied from 0.9s to 2.22s. In the experiments mono- and bi-chromatic waves are used. The drive signals of the wave-maker are generated using first-order and second-order wave generation techniques. Total 14 wave probes are used to capture the data in the wave tank. A NRC-IOT code is used to isolate the primary waves, the bounded waves and the unwanted free waves from the measured data at each wave probe. The measured data are analyzed in this paper to illustrate the differences in the waves generated by two different generation techniques.

KEYWORD

Physical experiments, shallow water waves, second order waves, Bounded waves, second-order spurious waves, data comparisons.

INTRODUCTION

When bi-chromatic primary waves are generated in the wave basin the required bounded waves will also be generated naturally at the difference frequency. In addition to that several unwanted free waves are also generated. The Free wave-1, having the same frequency of the bounded wave is reproduced due to mismatch of the boundary conditions at the wave paddle. The Free wave-2 is due to the wave paddle displacement. There is another type of free wave due to the local disturbances. The local disturbances usually disappear at some distance from the wave-makers and thus are not discussed here. If the wave basin is not unconditionally flat then there might be some other types of unwanted free waves in the basin in addition to the aforesaid three categories.

Proper understanding of the effects of the wave-action and consequent loading pattern of the primary waves along with their bounded waves are very important factors to design, implement and operation of any ocean structures, mooring system, floating vessels, harbor resonance, etc.. In an accurate physical model test in the laboratory it is crucial to choose the precise design parameters of such structures and/or vessels, etc. So the generation of spurious waves free primary waves and the reproduction of the group-induced second-order low frequency and high frequency components are essential for physical model tests in the laboratory. Please see Zaman and Mak (2007) for high frequency second-order wave components, not be discussed here.

When there are two waves of frequencies f_1 and f_2 form a group, the group-induced second-order low and high frequency waves (bounded) are generated along with other unwanted free waves. A low frequency wave or bounded wave will be produced due to the difference ($f_1 - f_2$) of the frequencies and a high frequency or short wave would be generated due to the summation ($f_1 + f_2$) of the frequencies. The profile of the bounded wave having frequency ($f_1 - f_2$) is generally termed as the set-down in the larger wave zone and the set-up in the smaller wave zone. These set-down and set-up phenomena were first investigated and reported by Longuet-Higgins and Stewart (1961, 1962, 1963, 1964 and 1977) in a series of papers. They introduced the radiation stress concept, which explained that in a wave group individual wave components exerted an internal compressive force in the direction of the wave propagation. To balance this force the mean water level goes down in the region of larger waves known as set-down and goes up in the region of smaller waves known as set-up. Bowen et al (1968) later explained the set-up and set-down phenomena with experimental data.

The theoretical and experimental descriptions of such natural high and low frequency waves along with their various unwanted free waves were given by many researchers. Hansen (1978), Sand (1982), Barthel et al (1983), Sand and Mansard (1986), Mansard (1991), Mansard et al (1987), Schaffer (1993), Stansberg (2006), Zaman and Mak (2007), Spinneken and Swan (2009) are a few who described the methods to curtailing

these unavoidable free waves from the resulting surface elevations in the wave basin.

In our present experiments the first-order and the second-order wave generation techniques for the mono- and bi-chromatic waves were used. In the experiment water depths are varied in the range of 0.3m to 0.8m. The wave periods also have varied from 0.9s to 2.22s. However in this paper several cases for bi-chromatic waves in the water depth of 0.4m are shown and discussed. Comparisons are made between data obtained from the first-order wave generation technique and the second-order wave generation technique.

FOMULATION OF THE FIRST-ORDER AND SECOND-ORDER GENERATION METHODS

A wave group would be generated with the presence of at least two frequencies (bi-chromatic wave). The difference of these two frequencies would generate a long period bounded wave with a period equal to the period of the wave group. This long wave is also known as 'set-up and set-down'.

A pair of regular waves with frequencies f_n and f_m and, surface elevations η_n and η_m respectively would constitute a wave group as follows (Sand 1982):

$$\begin{aligned}\eta_{nm}(t) &= \eta_n(t) + \eta_m(t) \\ &= a_n \cos(\omega_n t - k_n x) + b_n \sin(\omega_n t - k_n x) \\ &\quad + a_m \cos(\omega_m t - k_m x) + b_m \sin(\omega_m t - k_m x)\end{aligned}\quad (1)$$

where, ω_n and ω_m are the wave angular frequencies, k_n and k_m the wave numbers, x the spatial distance, t the time and a_n , b_n , a_m and b_m are the Fourier coefficients.

The first-order control signal for only two frequencies can be derived as follows [see also Barthel et al (1983)]:

$$\begin{aligned}X_1 &= \frac{\cosh(k_1 h) \sinh(k_1 h) + k_1 h}{2 \sinh^2(k_1 h)} (a_1 \sin \omega_1 t - b_1 \cos \omega_1 t) + \\ &\quad \frac{\cosh(k_2 h) \sinh(k_2 h) + k_2 h}{2 \sinh^2(k_2 h)} (a_2 \sin \omega_2 t - b_2 \cos \omega_2 t)\end{aligned}\quad (2)$$

By means of Laplace equations, the second order contributions due to difference frequency can be given as follows:

$$\begin{aligned}\eta_2^-(x, t) &= G_{mn}^-(a_m a_n - b_m b_n) \cos(\Delta \omega_{mn}^- t - \Delta k_{mn}^- x) + \\ &\quad G_{mn}^-(a_m b_n + a_n b_m) \sin(\Delta \omega_{mn}^- t - \Delta k_{mn}^- x)\end{aligned}\quad (3)$$

where G_{mn}^- is a second order quadratic transfer function, $\Delta \omega_{mn}^- = \omega_n - \omega_m$ and $\Delta k_{mn}^- = k_n - k_m$.

The quadratic transfer function is given by the following equation:

$$G_{mn}^- = \left(\frac{A_{mn}^-(\omega_n - \omega_m)}{2g} - \frac{k_m k_n g}{4\omega_m \omega_n} (1 + \tanh k_m h \tanh k_n h) + \frac{1}{4} (k_m \tanh k_m h + k_n \tanh k_n h) \right) \delta \quad (4)$$

where,

$$A_{mn}^- = \frac{1}{2} \frac{B_{mn}^- + C_{mn}^-}{(\omega_n - \omega_m)^2 - (k_n - k_m)g \tanh(k_n - k_m)h} g^2$$

$$B_{mn}^- = \frac{k_m^2}{\omega_m \cosh^2 k_m h} - \frac{k_n^2}{\omega_n \cosh^2 k_n h}$$

$$C_{mn}^- = \frac{2k_m k_n (\omega_n - \omega_m) (1 + \tanh k_m h \tanh k_n h)}{\omega_m \omega_n}$$

where, h is the water depth and g is the acceleration due to gravity and

$$\delta = \begin{cases} 1 & n \neq m \\ 0.5 & n = m \end{cases}$$

The second order control signal for a correct reproduction of the wave train up to second order takes the following form:

$$X_2^-(t) = ((a_n b_m - a_m b_n) F_1^- + (a_n a_m + b_m b_n) F_{23}^-) * \cos(\Delta \omega_{mn}^- t) + ((a_n a_m + b_m b_n) F_1^- + (a_m b_n - a_n b_m) F_{23}^-) \sin(\Delta \omega_{mn}^- t) \quad (5)$$

The function, F_1^- is written as:

$$F_1^- = F_{11}^- + F_{12}^- \quad (6)$$

$$F_{11}^- = \left[G_{mn}^- \Delta k_f^- h \left\{ (\Delta k_{mn}^- h - \Delta k_f^- h) \sinh(\Delta k_{mn}^- h + \Delta k_f^- h) + (\Delta k_{mn}^- h + \Delta k_f^- h) \sinh(\Delta k_{mn}^- h - \Delta k_f^- h) \right\} \right] / \left[2((\Delta k_{mn}^- h)^2 - (\Delta k_f^- h)^2) \sinh(\Delta k_{mn}^- h) \sinh(\Delta k_f^- h) \right] \quad (7)$$

$$F_{12}^- = \frac{f_m \Delta k_f^- k_m h (1 + H_n) [\delta k_m^- h \sinh(\delta k_m^+ h) + \delta k_m^+ h \sinh(\delta k_m^- h)]}{8(f_n - f_m)((k_m h)^2 - (\Delta k_f^- h)^2) \sinh(\Delta k_f^- h) \sinh(k_m h) \tanh(k_n h)} + \frac{f_n \Delta k_f^- k_n h (1 + H_m) [\delta k_n^- h \sinh(\delta k_n^+ h) + \delta k_n^+ h \sinh(\delta k_n^- h)]}{8(f_n - f_m)((k_n h)^2 - (\Delta k_f^- h)^2) \sinh(\Delta k_f^- h) \sinh(k_n h) \tanh(k_m h)} \quad (8)$$

The free long wave number, Δk_f^- is computed from the dispersion relation given as:

$$(\Delta \omega_{mn}^-)^2 = g \Delta k_f^- \tanh(\Delta k_f^- h) \quad (9)$$

where,

$$\delta k_m^\pm = k_m \pm \Delta k_f^-$$

$$\delta k_n^\pm = k_n \pm \Delta k_f^-; H_m = \frac{2k_m h}{\sinh(2k_m h)}; H_n = \frac{2k_n h}{\sinh(2k_n h)}$$

$$F_{23}^- = F_2^- (F_{3,m}^- - F_{3,n}^-) \quad (10)$$

$$F_2^- = \frac{\Delta k_f^- (1 + H_m)(1 + H_n)}{8 \tanh(k_m h) \tanh(k_n h)} \quad (11)$$

$$F_{3,m}^- = \frac{f_m}{(f_n - f_m)} * \sum_{j=1}^{\infty} \frac{2k_j h \sin(k_j h) [k_j h \sin(k_j h) \coth(\Delta k_f^- h) + \Delta k_f^- h \cos(k_j h)]}{((k_j h)^2 + (\Delta k_f^- h)^2) (\sin(k_j h) \cos(k_j h) + k_j h)} \quad (12)$$

where, $k_j h$ is computed from the following expression:

$$\frac{4\pi^2 h f_m^2}{g} = -k_j h \tan(k_j h), \text{ with } (j - \frac{1}{2})\pi < k_j h < j\pi \quad (13)$$

DESCRIPTION OF THE EXPERIMENTAL SETUP

The experiment was run at the Offshore Engineering Basin of National Research Council Canada, Institute for Ocean Technology. The top view of the basin is shown in Fig. 1. The Offshore Engineering Basin is 75 m long x 32 m wide. 56 independently controlled segmented wave generators installed on the west wall generated the waves. Each segmented wave generator is 2 m high and 0.5 m wide. Passive absorbers, made of expanded metal sheets with varying porosities and spacing, are installed on the east wall. A solid metal wall is used to cover the north side of the basin. The water depths for the experiments are 0.3m, 0.4m, 0.5m, 0.6m, 0.7m and 0.8m. However, the cases presented and discussed in this paper are confined to the 0.4m water depth experiments. All other data would be published elsewhere later.

During the experiment, 14 wave probes installed as shown in Fig. 1 and Table-1 measured the wave elevation throughout the basin. All the wave probes are capacitance type. All the data was acquired using GDAC (GEDAP Data Acquisition and Control) client-server acquisition system, developed by National Research Council Canada, Institute for Ocean Technology.

Table 1 Location of the wave probes in the OEB

No of the probe	Distance from the east wave paddle (m)	Distance from the south wall (m)
1	26.891	13.475
2	27.221	13.475
3	27.731	13.475
4	27.731	12.955
5	27.731	12.635
6	27.731	14.825
7	27.731	18.365
8	29.081	13.475
9	32.621	13.475
10	41.621	13.475
11	2.0	12.635
12	2.0	13.475
13	2.0	18.365
14	10.744	13.475

RESULTS AND DISCUSSIONS

As we already stated above, several cases of the 0.4m water depth experiments are reported. Table 2 summarizes the incident wave parameters of the three different wave conditions examined in this paper.

Table 2 Incident wave parameters

	T ₁ (s)	H ₁ (m)	T ₂ (s)	H ₂ (m)	h/L
Case-1	1.25	0.06	1.17	0.06	0.195
Case-2	1.55	0.06	1.45	0.06	0.146
Case-3	2.22	0.06	2.00	0.06	0.096

For each case, there were two runs, one used first-order generation technique and the other one used the second-order wave generation technique. Figs. 2 and 3 show the comparisons of the spectrums at 6 different locations in the wave basin between first-order and second-order wave generation techniques for Case-1 and Case-2, respectively. These 6 locations are at Probe-14, Probe-1, Probe-2, Probe-3, Probe-8 and Probe-9. These probes are on the same line, see Fig. 1. No big differences in the results are observed in these comparisons, especially in the components of the low frequency second order waves.

Fig. 4 shows the same comparisons for Case-3. In this figure one can perceive the differences in low frequency second order wave components between two different generation techniques. So from now on we will concentrate on Case-3 only to identify the spurious components. A NRC-IOT computer code that can split a surface elevation data set into its component waves is used to isolate the primary waves, bounded second order waves and unwanted free waves from the raw measured data at every probe location.

For Case-3, Figs. 5 to Figs. 8 show the comparisons of the measured raw bi-chromatic waves and separated, primary waves, corresponding bounded waves and unwanted free waves between the first-order and the second-order wave generation techniques. The comparisons are done at Probe-1, Probe-2, Probe-3 and Probe-8 locations.

Figs. 5a, 5b, 5c and 5d, respectively, show the comparisons of the measured raw waves, comparisons of the separated primary waves, comparisons of the separated bounded second order waves and the comparisons of the isolated unwanted free waves at Probe-1. Similar comparisons are shown in Figs. 6, 7 and 8 for Probe-2, Probe-3 and Probe-8.

From Figs. 5d, 6d, 7d and 8d, it may be observed that Free wave-2 which is due to the wave paddle displacement differ insignificantly for both generation techniques. On the other hand, the Free wave-1 which is due to mismatch of the boundary conditions at the wave paddle differ at a low magnitude.

Table 3 shows the comparisons of the bounded waves and free waves at different probe locations for both first-order and second-order generation techniques. The comparisons [% = (FW1 / MWH1)*100] or [% = (FW2 / MWH1)*100] are done with respect to the measured wave height at Probe-1 (MWH1=0.0864m) obtained in first-order wave generation method.

Table 3 Comparisons of Free waves and Bounded waves

	FW1 (FOG) %	FW1 (SOG) %	FW2 (FOG) %	FW2 (SOG) %	BW (FOG) %	BW (SOG) %
P-1	9.50	7.70	2.30	2.30	11.30	9.68
P-2	8.64	7.84	2.13	2.19	10.38	9.49
P-3	9.28	9.68	2.60	2.61	10.62	11.17
P-8	10.36	9.88	2.90	2.89	12.30	11.74

P-1: Probe-1, P-2: Probe-2, P-3: Probe-3 and P-8: Probe-8

FW1: Free wave-1, FW2: Free wave-2 and BW: Bounded wave

FOG: first-order generation technique

SOG: second-order generation technique

Figs. 9 show the comparisons of different wave spectrums at Probe-14 and at Probe-1 after when the reflection analysis is carried over. Again Case-3 is only used for this analysis. Fig. 9a shows the comparisons of the primary waves at Probe-1 and at Probe-14 for the first-order wave generation method. On the other hand, Fig. 9b shows the same comparisons for the second-order wave generation method. Fig. 9c shows the comparisons of the reflected waves at Probe-1 and at Probe-14 for the first-order wave generation method. Fig. 9d shows the similar comparisons for the second-order wave generation method. Figs. 9e and 9f show the comparisons of the reflected waves at Probe-1 and at Probe-14 for the first-order and the second-order wave generation technique. It may be observed from Figs. 9 that the primary and the reflected waves are not exactly the same at Probe-1 and at Probe-14. The reflected low frequency wave components obtained from the first-order method are a little larger than that obtained from the second-order method at Probe-1 and at Probe-14.

CONCLUSION

First-order and second-order wave generation techniques are used to study the propagation of the primary waves, bounded waves and unwanted free waves in the Offshore Engineering Basin of NRC-IOT. It is observed that for moderately shallow

water for Case-1 and Case-2 as shown in Figs. 2 and 3, the differences in low frequency wave components are insignificant. This difference becomes moderate for Case-3 where the wave is extremely shallow. Comparing first-order and second-order wave generation techniques, it is observed that the second-order wave generation technique reduces the unwanted free waves in the wave tank trivially.

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Fig. 1: Layout of the experimental tank (not to scale)

Fig. 2: Comparisons between corrected and uncorrected spectrums at Probe-14, Probe-1, Probe-2, Probe-3, Probe-8 and Probe-9 ($h=0.4\text{m}$, $T_1=1.25\text{s}$, $H_1=0.06\text{m}$, $T_2=1.17\text{s}$ and $H_2=0.06\text{m}$). Black line is 1st-order and Red line is 2nd order generation method.

Fig. 3: Comparisons between corrected and uncorrected spectrums at Probe-14, Probe-1, Probe-2, Probe-3, Probe-8 and Probe-9 ($h=0.4\text{m}$, $T_1=1.55\text{s}$, $H_1=0.06\text{m}$, $T_2=1.45\text{s}$ and $H_2=0.06\text{m}$). Black line is 1st-order and Red line is 2nd order generation method.

Fig. 4: Comparisons between corrected and uncorrected spectrums at Probe-14, Probe-1, Probe-2, Probe-3, Probe-8 and Probe-9 ($h=0.4\text{m}$, $T_1=2.22\text{s}$, $H_1=0.06\text{m}$, $T_2=2.0\text{s}$ and $H_2=0.06\text{m}$). Black line is 1st-order and Red line is 2nd order generation method.

Fig. 5a: Comparisons of the measured waves amplitudes at Probe-1
($h=0.4m$, $T_1=2.22s$, $H_1=0.06m$, $T_2=2.0s$ and $H_2=0.06m$)

Fig. 5b: Comparisons of the primary waves amplitudes at Probe-1
($h=0.4m$, $T_1=2.22s$, $H_1=0.06m$, $T_2=2.0s$ and $H_2=0.06m$)

Fig. 5c: Comparisons of the bounded waves amplitudes at Probe-1
($h=0.4m$, $T_1=2.22s$, $H_1=0.06m$, $T_2=2.0s$ and $H_2=0.06m$)

Fig. 5d: Comparisons of the free waves amplitudes at Probe-1
($h=0.4m$, $T_1=2.22s$, $H_1=0.06m$, $T_2=2.0s$ and $H_2=0.06m$)

Fig. 6a: Comparisons of the measured waves amplitudes at Probe-2
($h=0.4m$, $T_1=2.22s$, $H_1=0.06m$, $T_2=2.0s$ and $H_2=0.06m$)

Fig. 6b: Comparisons of the primary waves amplitudes at Probe-2
($h=0.4m$, $T_1=2.22s$, $H_1=0.06m$, $T_2=2.0s$ and $H_2=0.06m$)

Fig. 6c: Comparisons of the bounded waves amplitudes at Probe-2
($h=0.4m$, $T_1=2.22s$, $H_1=0.06m$, $T_2=2.0s$ and $H_2=0.06m$)

Fig. 6d: Comparisons of the free waves amplitudes at Probe-2
($h=0.4m$, $T_1=2.22s$, $H_1=0.06m$, $T_2=2.0s$ and $H_2=0.06m$)

Fig. 7a: Comparisons of the measured waves amplitudes at Probe-3
($h=0.4m$, $T_1=2.22s$, $H_1=0.06m$, $T_2=2.0s$ and $H_2=0.06m$)

Fig. 7b: Comparisons of the primary waves amplitudes at Probe-3
($h=0.4m$, $T_1=2.22s$, $H_1=0.06m$, $T_2=2.0s$ and $H_2=0.06m$)

Fig. 7c: Comparisons of the bounded waves amplitudes at Probe-3
($h=0.4m$, $T_1=2.22s$, $H_1=0.06m$, $T_2=2.0s$ and $H_2=0.06m$)

Fig. 7d: Comparisons of the free waves amplitudes at Probe-3
($h=0.4m$, $T_1=2.22s$, $H_1=0.06m$, $T_2=2.0s$ and $H_2=0.06m$)

Fig. 8a: Comparisons of the measured waves amplitudes at Probe-8
($h=0.4m$, $T_1=2.22s$, $H_1=0.06m$, $T_2=2.0s$ and $H_2=0.06m$)

Fig. 8b: Comparisons of the primary waves amplitudes at Probe-8
($h=0.4m$, $T_1=2.22s$, $H_1=0.06m$, $T_2=2.0s$ and $H_2=0.06m$)

Fig. 8c: Comparisons of the bounded waves amplitudes at Probe-8
($h=0.4m$, $T_1=2.22s$, $H_1=0.06m$, $T_2=2.0s$ and $H_2=0.06m$)

Fig. 8d: Comparisons of the free waves amplitudes at Probe-8
($h=0.4m$, $T_1=2.22s$, $H_1=0.06m$, $T_2=2.0s$ and $H_2=0.06m$)

Fig. 9: Comparisons of the primary and reflected waves from first-order and second-order generation at Probe 14 and at Probe-1 ($h=0.4m$, $T_1=2.22s$, $H_1=0.06m$, $T_2=2.0s$ and $H_2=0.06m$). Legend starts with F1_BIP4_ or F14_BIP4_ are from first-order and starts with F1_WD04_ or F14_WD04_ are from second-order generation method.