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ON THE IMPLEMENTATION OF STREAM-WISE PERIODIC BOUNDARY CONDITIONS

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ABSTRACT

Fully-developed periodic boundary conditions have frequently been employed to perform calculations on the performance of typical elements of heat exchangers. Many such calculations have been achieved by transforming the equations of motion to obtain a new set of state variables which are cyclic in the stream-wise direction. In others, primitive variables, based on substitution schemes are employed. In this paper; a review of existing procedures is provided, and a new method is proposed. The method is based on the use of primitive variables with periodic boundary conditions combined with the use of slip values. Either pressure difference or mass flow rate may be prescribed, and both constant wall temperature and constant heat flux wall conditions may be considered. The example of an offset-fin plate-fin heat exchanger is used to illustrate the application of the procedure. The scope and limitations of the method are discussed in detail, and the mathematical basis by which the method may be extended to the consideration of problems involving mass transfer, with associated continuity, momentum, and species source/sinks is proposed.

INTRODUCTION

In the application of computational fluid dynamics to heat exchanger design, it has long been recognized that much computational effort may be spared by considering elements deep within the design where the flow is 'fully-developed'; for example; Thom and Alpelt [1], LeFeuvre [2], and Massey [3] considered fully-developed flow in tube banks assuming a cyclical 2-D stream function/vorticity formulation. In [2,3] temperature was also solved-for.

More recently, the equations of motion are usually solved in terms of primitive variables, for which the steady-state form is typically written, subject to certain simplifications, as:

$$\mathrm{div}(\rho \boldsymbol{u}) = 0, \tag{1}$$

$$\mathrm{div}(\rho \boldsymbol{u}; \boldsymbol{u}) = -\mathrm{grad}\, p + \mathrm{div}(\mu\, \mathrm{grad}\ \boldsymbol{u}) \tag{2}$$

$$\mathrm{div}(\rho c_p \boldsymbol{u} T) = \mathrm{div}(k\, \mathrm{grad}\, T) \tag{3}$$

For convenience, and without loss of generality it is assumed that the region has been tessellated with a structured mesh with associated finite-volume equations [4]:

$$a_W(\phi_W - \phi_P) + a_E(\phi_E - \phi_P) + a_S(\phi_S - \phi_P) + a_N(\phi_N - \phi_P) + a_L(\phi_L - \phi_P) + a_H(\phi_H - \phi_P) + S = 0 \tag{4}$$

where $\phi = p, u, v, w, T$ is a general state-variable, and the well-known compass notation [4] has been employed. The source-term in Eq. (4) is frequently linearized according to

$$S = C(V - \phi_P) \tag{5}$$

where S is referred to as a source-term coefficient and V is a source-term value.

A problem frequently encountered is that it is difficult to construct a mesh large enough to describe the gross motion of the fluid within the entire heat exchanger, and yet fine enough to capture the boundary-layer detail around individual elements [5,6]. Often a single 'typical' module, or perhaps a small group of such modules, is considered, with the flow taken as being 'fully-developed' in the stream-wise direction,

$$\boldsymbol{u}(0, y, z) = \boldsymbol{u}(l, y, z) \tag{6}$$

$$p(0, y, z) = p(l, y, z) + \Delta p_0 \tag{7}$$

where Δp_0 is the pressure drop over characteristic length, l.

NOMENCLATURE

A	Area, m^2
a	Coefficient in finite-volume equation
B	Mass transfer driving force
b	Width, m
C	Value in linearized source term
c_p	Specific heat, J/kgK
f	Friction factor
j	Heat transfer factor
k	Thermal conductivity, W/mK
l	Length, m

T Temperature, K
u Stream-wise velocity component, m/s
v Cross-wise velocity component, m/s
w Cross-wise velocity component, m/s
p Pressure, Pa, pitch, m
V Value in linearized source term
V_p Volume of cell P, m^3
x Stream-wise displacement component, m
y Cross-wise displacement component, m
z Cross-wise displacement component, m

Greek Letters
β Volumetric term
δ Fin thickness
ϕ Generalized state variable
Γ Exchange coefficient, kg/ms
θ Non-dimensional temperature
ρ Density, kg/m^3
σ Volumetric term
τ Characteristic time, s
ω Weighting factor

REVIEW OF PREVIOUS WORK

Transformed variable approach

Patankar Liu and Sparrow [7] transformed the state-variables to a set of equations which were truly cyclic in the sense:

$$\phi(x,y,z) = \phi(x+l,y,z) \tag{8}$$

This was achieved in the momentum equations by defining a 'reduced' pressure, $\tilde{p}$, according to:

$$\tilde{p} = p - \beta x \tag{9}$$

where $\beta = \Delta p_0 / l$ and $\Delta p_0 = p_0(0) - p_0(l)$ is a reference pressure difference. The reduced pressure, $\tilde{p}$, is thus cyclic in the sense of Eq. (8); the pressure, p, may be obtained algebraically at the end of the computational cycle, if required. The momentum equation may readily be written in the form,

$$\mathrm{div}(\rho \boldsymbol{u}; \boldsymbol{u}) = \beta \hat{\mathbf{i}} - \mathrm{grad}\,\tilde{p} + \mathrm{div}(\mu\,\mathrm{grad}\,\boldsymbol{u}) \tag{10}$$

Thus when solving for $\tilde{p}$, a volumetric source term, $S = \beta V_p$ is introduced into the x-direction momentum equation. Pressure gradient, as opposed to mass flux or Reynolds number must be prescribed. Subsequently, Murthy and Mathur [8] suggested a rationale whereby β may be systematically adjusted until the desired mass flux is obtained.

For heat transfer at constant wall flux, $\dot{q}_{\mathrm{wall}}''$, an essentially similar situation exists with,

$$T(x,y,z) = T(x+l,y,z) + \Delta T_0 \tag{11}$$

A reduced temperature, $\tilde{T}$, may be defined as,

$$\tilde{T} = T - \gamma x \tag{12}$$

where $\gamma = \Delta T_0 / l$ and $\tilde{T}$ is cyclic. The transformed energy equation is:

$$\mathrm{div}(\rho c_p \boldsymbol{u} \tilde{T}) + u\gamma = \mathrm{div}(k\,\mathrm{grad}\,\tilde{T}) \tag{13}$$

and a source term $S = -u\gamma V_p$ is introduced in the energy equation.

For constant wall temperature T_{wall}, a non-dimensional temperature, θ, may be defined as;

$$\theta = \frac{T - T_{\mathrm{wall}}}{T_0 - T_{\mathrm{wall}}} \tag{14}$$

where $T_0(x)$ is some suitably-defined module reference temperature; for example, the local bulk temperature at x. The non-dimensional form of the energy equation with θ as state-variable is less straightforward

$$\mathrm{div}(\rho \boldsymbol{u} c_p \theta) = \mathrm{div}(k\,\mathrm{grad}\,\theta) + \sigma \tag{15}$$

where

$$\sigma = \left[2\frac{k}{\rho c_p}\left(\frac{\partial \theta}{\partial x}\right) - u\theta \right] \frac{dT_0/dx}{T_0 - T_{\mathrm{wall}}} - \frac{k}{\rho c_p}\theta \frac{d^2T_0/dx^2}{T_0 - T_{\mathrm{wall}}} \tag{16}$$

These various implementations have been adopted by numerous researchers, for example Patankar and Prakash [9], and at least one commercial CFD code has been modified to allow for solutions to equations of the form (10)-(16) to be incorporated for stream-wise periodic problems. The reader will note that it is necessary to modify wall boundary conditions in the reduced form of the transport equations, see [7] for details.

Because of the additional complexity associated with the solution of Eqs. (15) and (16); alternative formulations, Kelkar and Patankar [10], have been proposed based on a primitive-variable formulation, for constant T_{wall} problems. In this paper, all solved-for variables are primitive variables, regardless of the choice of wall boundary conditions.

Primitive variable approach

The present author did not adopt the methodology [7] but instead worked directly with the primitive variables, p, u, v, w, T, in previous work [11-13]. The reasons for this were as follows: (a) There is no need to introduce new state-variables: (b) Reynolds' number can be directly stipulated; (c) Constant T_{wall} boundary conditions may readily be prescribed; (d) Flow symmetry may be exploited for staggered or offset geometries, halving the required number of grid cells. In the primitive formulation, the temperature, T, is given by,

$$T(0,y,z) = c_1 T(l,y,z) + c_2 \tag{17}$$

where

$$c_1 = \begin{cases} \dfrac{T_0(0) - T_{\mathrm{wall}}}{T_0(L) - T_{\mathrm{wall}}} & \text{constant } T_{\mathrm{wall}} \\ 1 & \text{constant } \dot{q}''_{\mathrm{wall}} \end{cases}$$
$$c_2 = \begin{cases} T_{\mathrm{wall}}(1 - c_1) & \text{constant } T_{\mathrm{wall}} \\ T_0(L) - T_0(0) & \text{constant } \dot{q}''_{\mathrm{wall}} \end{cases} \tag{18}$$

The periodic boundary conditions were implemented by the addition of an additional line of 'halo' cells downstream, at $i = nx + 1$; a procedure which had been adopted previously by

Antonopoulos [14-16]. This approach was employed because non-standard grids, which could not readily be connected together in a structured manner, were employed.

Downstream velocity and temperature values were substituted upstream in the continuity, momentum, and energy equations in the normal sense as convected inlet boundary values; temperatures being scaled according to Eq. (17). The downstream pressure values at $i = nx + 1$ were fixed to the upstream values as $p(nx+1, j) = p(1, j) - p_0$. The mean pressure at $i = 1$ was chosen as the reference pressure, p_0; and thus ensured that the mean downstream pressure was zero, as the upstream values rose to some finite value. In addition the upstream velocity profile was scaled so as to render the desired overall Reynolds number. This scheme corresponded to the common practice of prescribing upstream boundary values and fixing the outlet pressure. The early methodology presumed no re-circulation to occur at the upstream/downstream boundaries; a situation sometimes referred to as 'locally parabolic' [4]. This requirement was subsequently relaxed by means of a double-substitution process whereby upstream values of velocity and temperature were also substituted downstream [12,13].

One advantage of the use of primitive variables is that transient problems may also be considered [17];

$$\boldsymbol{u}(0, y, z, t) = \boldsymbol{u}(l, y, z, t + \tau) \tag{19}$$

$$T(0, y, z, t) = c_1 T(l, y, z, t + \tau) + c_2 \tag{20}$$

In that case, downstream values were stored in a 'ring-buffer' and substituted upstream after time τ (substantially longer than the characteristic period of oscillation of the system) has elapsed. This allowed-for span-wise amplification of small periodic oscillations (vortex-shedding, wake/shear layer instabilities) to be successfully replicated.

The use of periodic boundary conditions is not confined to heat transfer problems. Comini and Croce [18] considered periodic mass transfer in tube-fin heat exchangers under conditions of prescribed wall value, based on local saturation conditions for an ideal gas mixture. Beale [19] considered fully-developed mass transfer in plane and square ducts. The velocity and scalar profiles were presumed similar according to

$$u(0) = cu(l) \tag{21}$$

with $v(0) = v(l)$ and $w(0) = w(l)$; the upstream bulk velocity being fixed to some value u_0.

For scalar transport, a constant transformed substance-state boundary condition, as opposed to constant wall flux or constant value, was presumed. Periodicity is imposed with

$$\phi(0) = c_1 \phi(l) + c_2 \tag{22}$$

by computing $c_1 = (\phi_b(0) - \phi_w(0))/(\phi_b(l) - \phi_w(l))$ and $c_2 = \phi_w(0) - c_1 \phi_w(l)$ in a manner analogous to Eq. (17). The upstream wall value must be computed, $\phi_w(0) = (\phi_b(0) + B(l)\phi_t)/(1 + B(l))$, where B is a mass transfer driving force [20].

PRESENT CONTRIBUTION

The problem addressed in this paper is the means whereby periodic boundary conditions may be reduced to cyclic conditions in the primitive-variable formulation, without the cumbersome step of introducing 'halo' cells. The solution is achieved by imposing 'slip' boundary conditions at the grid edges in the x-direction. Slip boundary conditions are naturally encountered, for example in the temperature field in radiative heat transfer problems, and elsewhere.

Momentum equation

No special treatment is required in the continuity (pressure correction) equation, or the cross-wise v and w momentum equations. For stream-wise u-momentum the situation is extremely simple: All that is required is that a constant step be imposed as a force corresponding to the overall pressure change

$$S = A_P \Delta p_0 \tag{23}$$

along a single y-z plane of cells. The pressure slip may be applied at any location in the domain, however if it is applied at the system boundary, $i = nx$, there is the superficial advantage that graphical post-processing software may not interpolate between the two values, producing spurious bands of iso-values within the computational domain. Figure 1(a) illustrates the notion schematically, for the case of a staggered scheme [21]. The in-cell pressure must also be fixed, elsewhere, to some reference value at one cell in the computational domain, in order to prevent the pressure field from wandering.

If it is desired that the bulk velocity, $\overline{u}$, or Reynolds' number, be prescribed; the imposed pressure difference, Δp_0, may be adjusted:

$$\Delta p_0 = \Delta p_0 * + \Delta p_0' \tag{24}$$

where $\Delta p_0 *$ is the value of the pressure difference at the previous iteration, and $\Delta p_0'$ is a pressure correction. Since in the SIMPLE algorithm [22] $u_e = b(p_P' - p_E')$ where $b = \partial u_e / \partial p_P = A_e / a_E$, it may be concluded that, $\Delta p_0' = \sum a_e(\overline{u} - u_e *) / \sum A_e$ or $\Delta p_0' = R(\overline{u} - u_e *)$. The choice of R is not critical, and need only be sufficiently accurate as to procure rapid convergence. Neglecting stream-wise diffusion and applying an order of magnitude analysis yields the following simplified relation;

$$\Delta p_0' = \rho |\overline{u}| (\overline{u} - \overline{u} *) \tag{25}$$

which is identically true in the lim $\overline{u} * \to \overline{u}$. This methodology is similar to, but simpler than that given in ref. [8].

Heat transfer

For the temperature field there is a slip; not in the flux or source, but in the actual value across the boundary faces: A different treatment is required: It is necessary to prescribe a source/sink pair. The reader will note that these are not necessarily equal and opposite, owing to the well-known non-linear property of the convection-diffusion system of equations [4].

Various means to code the slip temperatures at $x = 0$ and $x = l$. are available; (i) If the user has access to the neighbor values, ϕ_W and ϕ_E, in the finite-volume equation, Eq. (4); it will be possible to directly add/subtract the ΔT slip-values, from the neighbors, T_E and T_W. (ii) If, however, the user does not have access to the neighbor values, as is more often the case, then it is convenient to introduce a pair of linearized source terms:

$$S = a_W (\Delta T_W - T_P) \quad \text{at } i = 1 \tag{26}$$

$$S = a_E\left(\Delta T_E - T_P\right) \quad \text{at } i = nx \tag{27}$$

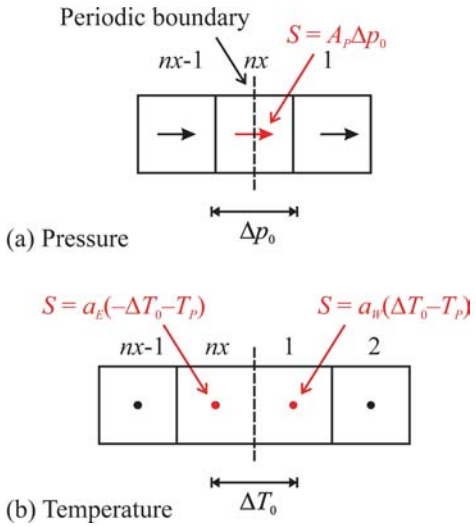

Figure 1 Slip boundary conditions for a staggered scheme, constant wall flux, .

where the slip values, ΔT_W and ΔT_E, are the differences between the actual temperatures and the 'apparent' temperatures that would arise if the field were truly periodic. If ΔT_W is positive then ΔT_E is negative, and vice-versa.

These source terms are easily coded simply by setting $C = a_{\text{nb}}$ and $V = \Delta T_{\text{nb}}$ in Eq. (5) where the subscript 'nb' refers to the east or west neighbor value. The linking coefficients a_E and a_W must be computed using exactly the same scheme employed in the CFD solver.

For the case of constant wall flux, $\dot{q}_{\text{wall}}''$, the temperature difference is constant, $\Delta T_W = \Delta T_0$, $\Delta T_E = -\Delta T_0$, and Fig. 1(b) illustrates schematically the coding of the source-sink pair. It is, in theory, possible to directly obtain $\Delta T_0 = \dot{q}/\dot{m}c_p$ [7], and code this as a fixed flux (source): However, the in-cell temperature must then be prescribed to a constant value at some chosen location, $T = T_{\text{ref}}$, as above. The temperature (enthalpy) slip must precisely balance the net heat flux at the wall. If there is even the slightest error in the value of the computed value of ΔT_0, e.g. for complex geometry, the error will be manifested as distortion in the region near the cell with fixed temperature. For this reason the author chooses to prescribe the temperature as a linearized source term; for example $C = a_W$ and $V = \pm\Delta T_0$, upstream in Eq. (5). The source-term value is adjusted iteratively until the upstream in-cell temperature reaches the desired reference value,

$$\Delta T_0 = T_0(0) - T_0(l) \tag{28}$$

In other words $T_0(l)$ is a fetched-value of T at some chosen location, or the mean-value downstream, during the iterative cycle, whereas $T_0(0)$ is a constant value prescribed by the user (e.g. $T_0 = 0$).

For the case of constant T_{wall}, $\Delta T_E \neq -\Delta T_W$, the implementation is similar to that in [10]. It is readily apparent that the temperature slip may be obtained from,

$$\frac{\Delta T_E}{T_E - T_{\text{wall}}} = \frac{T_0(l) - T_0(0)}{T_0(l) - T_{\text{wall}}} = -\frac{\Delta T_W}{T_W - T_{\text{wall}}} \tag{29}$$

From whence the source-term values for the general case may be written as;

$$V = (c_1 - 1)T_W + c_2 \quad \text{at } i = 1 \tag{30}$$

$$V = (1 - c_1)T_E - c_2 \quad \text{at } i = nx \tag{31}$$

where T_W is the west neighbor at $i = 1$ (i.e. T_P at $i = 1$) and T_E is the east neighbor at i = nx (i.e. T_P at $i = 1$). Once again, the source-term coefficients in Eq. (5) are just a_W and a_E. Eqs. (30) and (31) may be written in compact form as $V = \pm(1 - c_1)T_{\text{nb}} \mp c_2$.

CASE STUDY: OFFSET-FIN HEAT EXCHANGER

The problem considered to illustrate the methodology is that of fully-developed conjugate heat transfer in a 3-D offset-fin plate-fin heat exchanger under laminar flow conditions. Numerical solutions to this problem were previously presented in [11,12] using the 'halo' cell method. Figure 3 shows the geometry. The design selected was Core 105 from Shah and London [23,24]. Two plates separated by a distance b are considered to be at constant wall temperature T_{wall}. A rectangular strip-fin, length, l, thickness δ, is offset at pitch, p. A computational domain of $2l \times p/2 \times b$ corresponding to two complete modules was constructed. Geometry and properties are given in Tables 1 and 2, respectively.

The flow Reynolds number was prescribed and the resulting Δp_0 adjusted according to Eq. (25). Temperature slip was prescribed according to Eqs. (30) and (31) with the constants c_1 and c_2 computed according to Eq. (18). Figures 2, 4, and 6 display velocity vectors, pressure and temperature contours for plan and elevation views of the heat exchanger.

Overall friction and heat transfer factors are computed according to

$$f = \frac{D_h \Delta p_0}{2l\rho u_b^2} \tag{32}$$

$$j = \frac{A_c}{A}\,\text{Pr}^{2/3}\ln(c_1) \tag{33}$$

where $A_c = (b - 2\delta)(p - \delta)$ is the minimum free-flow area and $A = 2l(b - \delta) + 2l(p - \delta) + 2b\delta + (p - \delta)\delta$ is the area for heat transfer. These are compared to experimental values in Fig. 5. It can be seen that agreement in the laminar flow regime is quite good.

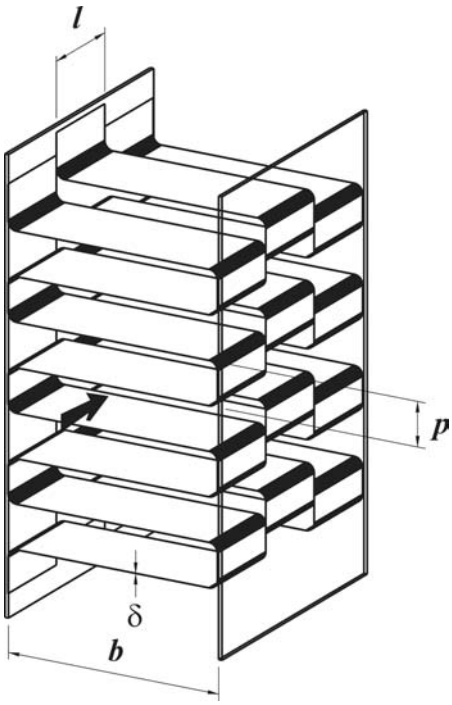

Figure 3 Offset-fin plate-fin geometry.

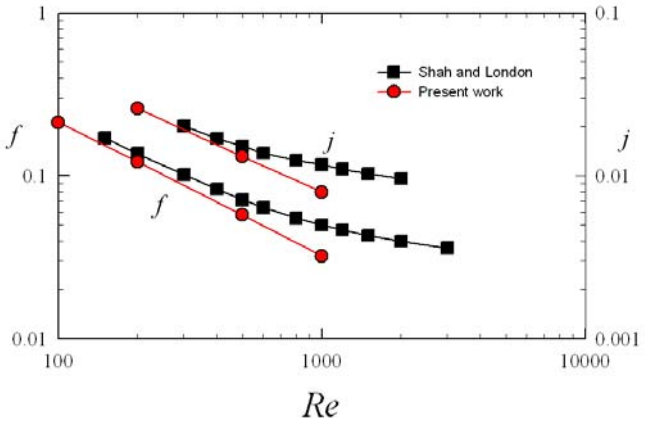

Figure 5 Friction and heat transfer factors compared with available experimental data.

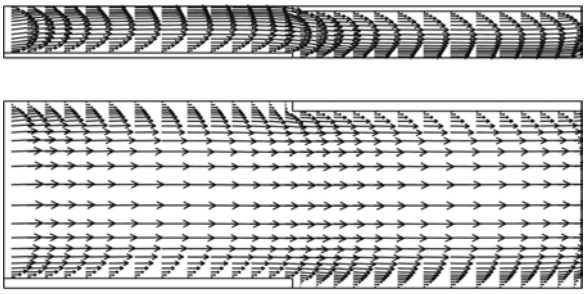

Figure 2 Velocity vectors, Re = 500.

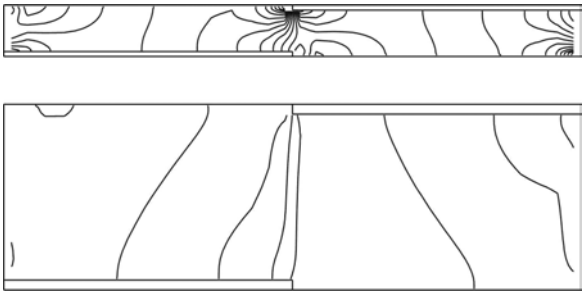

Figure 4 Pressure contours, Re = 500.

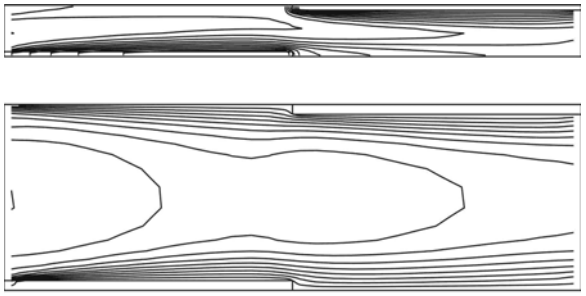

Figure 6 Temperature contours, Re = 500.

Table 1. Dimensions for Core 105.

Quantity	Value (mm)
b	1.905
p	1.054
l	2.822
δ	0.1016

Table 2. Fluid properties.

Quantity	Value	
ρ_f	0.9944	(kg/m^3)
μ_f	20.82x10^{-6}	Pa.s
k	0.03035	W/mK
c_p	1.016 x10^{-3}	J/kg.K

DISCUSSION

The results show that it is possible to code stream-wise periodic problems in primitive form with cyclic boundary conditions, by the selective introduction of slip boundaries. For problems involving fluid mechanics and heat transfer, the approach proposed in this work offers the advantages that storage does not require to be allocated for additional 'reduced' variables. Moreover the implementation of the slip boundary value is essentially replicating, in a mathematical manner, that which occurs naturally. It is true that the slip conditions must be coded, but this is offset by the fact that the wall boundary conditions are in the standard form, i.e. do not require modification.

Of course it might be argued that even if Eqs. (15) and (16) are somewhat inelegant; provided the implementation of reduced variables is hidden from the user, the means is inconsequential to the engineer; so long as a correct solution is obtained. Notwithstanding the above, the primitive variable formulation is easy to code and offers clear advantages, not only for the constant wall temperature problem, but potentially for fully-developed mass transfer problems.

The reader will note for the case of locally-parabolic flow at sufficiently high Reynolds number; upstream $S = \rho c_P u A \Delta T_w$, and downstream $S = 0$ as expected. For the case of pure conduction; $S = \Gamma A\left(\Delta T - T_p\right)\big/\delta x$, both upstream and downstream. The reader will note that for constant T_{wall} the ΔT slip values are not necessary equal on either side of the boundary.

The simple method devised to adjust the pressure jump based on a desired reference bulk velocity or Reynolds number, as given in Eq. (25), was found to work well for the problem under consideration, and others (not shown).

Choice of reference values

The choice of reference value, ϕ_o, is not exclusive; One obvious possibility is that of the weighted mean value at $x = L$,

$$\phi_0 = \frac{\int_A \omega\phi dA}{\int_A \omega dA} \tag{34}$$

where ω is a weighting factor. The most commonly-used weighting factor is the velocity, $\omega = u$, i.e., the so-called bulk value, $\phi_0 = \overline{\phi}$. It has also been suggested [7] that for re-circulating flows a more appropriate weighting in Eq. (34) is velocity magnitude $\omega = |u|$; a discussion of some of the issues may be found in [12]. For computing a reference pressure, p_0, the weighting factor would normally be unity.

Another basis for the reference value, ϕ_o, is a module average value,

$$\phi_0 = \frac{\int_0^l \int_A u\phi dA dx}{\int_0^l \int_A u dA dx} \tag{35}$$

Another very convenient reference ϕ_o, is the in-cell value at some particular location, say $j = j_0$, $k = k_0$. Finally it is worth noting that ϕ_o, need not correspond to any particular x-location; i.e., it could be at some offset $i = i_0+n$. Provided a consistent practice is adopted, there is a great deal of flexibility in the choice of ϕ_0. In other words, the choice of weighting factor is rather arbitrary, but is usually chosen to correspond to some well-established norm in fluid mechanics or heat transfer.

Mass transfer

There are numerous problems involving mass transfer with finite wall velocity, v_{wall}. A fully-developed periodic condition with a similarity velocity profile, $u(0) = cu(l)$ can be presumed, where $c = u_0(0)/u_0(l)$. Putting

$$\Delta u_0 = u_0(0) - u_0(l) = \frac{A_{\text{wall}}}{A_{\text{flow}}} v_{\text{wall}} \tag{36}$$

In the continuity (pressure-correction) equation, a single constant sink equal and opposite to the overall mass source (or vice-versa) at the wall is required:

$$S = -\rho A_w \Delta u = -\rho A_w (c-1) u(l) \tag{37}$$

In the u-momentum equations it is also necessary to introduce a velocity slip, analogous to the temperature slip, on either side of the continuity cells for which Eq. (37) is applied,

$$V = \pm(c-1) u_{\text{nb}} \tag{38}$$

The momentum equations may further be complicated by the fact that the source-term coefficients may also need to be corrected.

With these modifications; future work will include examples of 3-D fully-developed periodic mass transfer problems: Previous works on this subject by other authors [18,25] were primarily for low mass flow rates; where no continuity modifications were implemented. At higher mass flow rates injection/suction rates have a significant impact on the v-velocity, pressure gradients and scalar transport, which cannot be captured merely by neglecting the impact on continuity of mass transfer. While it might, in theory, be possible to incorporate these effects by defining a 'reduced velocity' satisfying a cyclic form, Eq. (8), for continuity; the resulting momentum equation will contain quadratic terms and may not therefore be readily amenable to such a treatment.

CONCLUSIONS AND FUTURE WORK

Periodic boundary conditions may readily be coded, based on a primitive variable formulation for fluid mechanics and heat transfer problems. This is achieved by combining cyclic boundary conditions with the use of slip values. The latter are introduced as linearized source terms.

The methodology has the advantages that no new variables need be introduced, and that standard wall boundary conditions may be used. Moreover it corresponds physically to the actual situation at hand. It may readily be implemented in existing CFD codes with only minor modifications. Either a mean pressure gradient, or a reference velocity/flow Reynolds number may be prescribed using a simple algorithm.

Although the work presented here was based on a finite-volume method, with a structured mesh, and employing a staggered scheme; the technique is quite general and may

readily be applied to; unstructured meshes, schemes employing co-located variables, and other methodologies such as finite-element analysis. The technique will be extended in the future, to consideration of engineering problems involving mass transfer under constant transformed substance state with associated variation in the continuity, momentum, and species fields.

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