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3-D ANALYSIS OF THERMAL RESISTANCE OF EXTERIOR BASEMENT INSULATION SYSTEMS (EIBS)

W. Maref¹, M.C. Swinton¹, M.K. Kumaran¹ and M.T. Bomberg¹

(1) National Research Council of Canada
Institute of Research in Construction, M-24 Montreal Road, Ottawa K1A 0R6

Wahid Maref
Research Associate
Tel. 1 (613) 993 5709
Fax. 1 (613) 954 3733
E-mail: Wahid.Maref@nrc.ca

Mike Swinton
Senior Research Officer
Tel. 1 (613) 993 9708
Fax. 1 (613) 954 3733
E-mail: Mike.Swinton@nrc.ca

Kumar Kumaran
Senior Research Officer
Tel. 1 (613) 993 9611
Fax. 1 (613) 954 3733
E-mail: Kumar.Kumaran@nrc.ca

Mark Bomberg
Senior Research Officer
Tel. 1 (613) 993 3662
Fax. 1 (613) 954 3733
E-mail: Mark.Bomberg@nrc.ca

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Proof should be sent to Dr. Wahid Maref at the above address.

Abstract

A consortium¹ research project was initiated to determine the field performance of various thermal insulation products as applied in the exterior insulation basement system (EIBS). Initially a two-dimensional analytical tool was used to derive the thermal transmission characteristics from an array of temperature measurements performed over a period of two years. Results immediately showed the influence of lateral heat flux between various products that differed in thermal transmission properties. Therefore, development of a three dimensional model became imperative.

This paper gives the theoretical and numerical approaches adopted to develop a three-dimensional computer model of heat transfer. The implicit Spline Method was selected for the problem solver. The applicability of the model was verified using measured data on temperature distributions at several material interfaces. Then the model was used to estimate the effect of lateral heat flows.

The paper also briefly reports the experimental details and presents results on model verification.

¹ The consortium included Expanded Polystyrene Association of Canada, Canadian Urethane Foam Contractors Association, Owens Corning Inc. and Roxul Inc.

Nomenclature

C Specific heat [J/kg.K]

K_i Thermal conductivity in x direction [W/m.K]

K_j Thermal conductivity in y direction [W/m.K]

K_k Thermal conductivity in z direction [W/m.K]

Q Heat generation [W/m³]

t Time [s]

ρ Density [kg/m³]

T_0 Initial temperature[°C]

S_1, S_2 and S_3 Boundary surfaces

Δt Time step [s]

$\Delta x, \Delta y, \text{ and } \Delta z$ Step of spatial discretisation
in x, y and z direction [m]

Subscripts

i Nodal point value

j Nodal point value

k Nodal point value

s On boundary surface

BACKGROUND

Over the recent decades, a large part of basements in newly built Canadian houses became used as a habitable space. This, combined with increased requirements for energy conservation, resulted in the development of many insulated basement systems. While thermal insulation may be placed either on the inside or on the outside of the basement wall, the material choice and the insulation placement may affect overall performance of the basement wall. Some external insulation systems, in addition to controlling heat losses, may also increase durability of the basement wall.

In this context, the Canadian thermal insulation industry, working together with the National Research Council, decided to revisit² the design and performance of external insulation basement systems (EIBS). Over the two-year period from June 1996 to June 1998, ten expanded polystyrene (EPS), two spray polyurethane foam (SPF), two mineral fibre insulation (MFI) and two glass fibre insulation (GFI) specimens were placed on the exterior of the basement wall in an experimental building located on NRC Campus in Ottawa.

This research project involved a number of material and system issues. Material considerations involved selection of existing and new thermal insulation products (being under development). These products were placed side-by-side on the basement wall to create virtual test compartments. The system considerations involved two different technical solutions to protect the above-grade part of the EIBS. Furthermore, different

² See previous publications: Tao et al [1], Bomberg [2]

conditions for surface water drainage were implemented on East and West walls of the NRC experimental basement facility.

Because of the wide scope of the basement research, this project is reported in four parts:

1. Development of analytical tools to increase confidence in the experimental results and facilitate the analysis of the field data
2. Reporting and analyzing results obtained from the experimental basement
3. Placing the thermal and moisture performance of expanded polystyrene in below-grade application in context of other research (state-of-the-art review to identity future research needs on the material side)
4. Reporting and analyzing the system effects in context of other research (state-of-the-art review to identity future research needs for development of basement systems)

The first part of the EIBS project is addressed in this paper, which presents theoretical and numerical aspects of 3-D computer model developed as a part of integrated approach (modelling and testing). The capabilities of this model to enhance the analysis of experimental results will be demonstrated in the next paper (Swinton et al) [3].

1. EXPERIMENTAL APPROACH

To evaluate performance of exterior basement insulation, different materials were installed on the exterior surface of concrete basement walls. As shown in Figure 1, eight test specimens were placed side-by-side, on each of two basement walls (east and west) insulating a whole wall with approximately 76-mm thick and 2.4 m high specimens. On the interior of the wall, a 25-mm layer of expanded polystyrene (EPS) board was installed

over the entire surface. At 3 vertical locations on each test wall, cut-outs were made and calibrated EPS specimens, with identical thickness, were tightly inserted. These specimens were used for determination of transient heat flux entering the wall (Bomberg et al) [4].

Thermocouples were placed at the boundary surfaces as well as the interfaces of each layer in the wall, in an array consisting of 16 points per tested compartment. All sensors placed on the west wall are shown in Figure 2.

The parameters monitored in the EIBS are:

1. Surface temperatures, on both sides of the calibrated specimen, the concrete and the test specimens
2. Heat flux across the calibrated insulation specimens
3. Soil temperatures from 1 to 2 m away from the specimens, and at 5 depths
4. Interior basement air temperature (average of 4 readings)
5. Exterior air temperature (at north face, shielded from sun)
6. Relative humidity (RH) and other parameters of indoor and outdoor environments.

The instrumentation package consisted of approximately 145 thermocouples, 2 humidity sensors, 21 calibrated insulation specimens (heat flux transducers), 4 junction boxes, and a data acquisition unit operated by a computer.

1.1 Data acquisition

The collection of data was done with an automated data acquisition and scanning system, and a high precision multi-meter to measure separate thermocouples, serial thermopiles

and relative humidity sensors. All thermocouples and power signals were routed through a HP command module (HP E1406) connected to a PC 486/50. Measurements were taken every 2 minutes and averaged by the software package (HP_VEE) for 10-minute intervals for the wall thermocouples (5 readings), and 30-minute intervals for the soil thermocouples (15 readings).

1.2 Duration of the experimental program

The data acquisition system was commissioned in the spring of 1996 and monitoring started after adjusting the air conditioning system on June 5, 1996. The monitoring ended on June 5, 1998, exactly 2 fully years of monitoring.

Figure 3 shows measured temperatures of indoor air, calibrated insulation specimens, concrete, and soil surface in the “mid-position” of the wall over a period of two years. The spikes in the interface between soil and the EIBS correspond to thaw periods with a heavy rainfall. Observe that these effects do not appear to affect the temperature on the concrete surface, i.e., behind the external insulation.

1.3. Preliminary results

Examination of preliminary results highlighted difficulties in establishing the precision of thermal testing without addressing 3-D effects. This is best illustrated by comparing W5 and W6 tested wall sections.

Table 1 shows average temperature difference across the insulation layer and heat fluxes entering each of these two tested wall sections. These results are averaged over a period of one week to approximate the steady state and use the ratio between the temperature

difference and the heat flux as an indicator of thermal resistance. When thermal resistance of W5 is taken as a benchmark (100%), the ratio between W6 and W5 specimens is 122%. This result is, however, much lower than the ratio of 164 percent obtained from the initial, laboratory measurements of thermal resistance performed on specimens W5 and W6. Even if the R-value of W6 specimen was corrected for the effect of foam ageing, the difference between one-dimensional estimate and the measured results is significant. This difference highlights the need for 3-D model application.

1.5. Discussion

As the test set-up involved diverse thermal insulating materials placed next to each other, the presence of lateral heat flow was inevitable. In the case of specimen W6 (closed-cell, gas-filled thermal insulating foam), some heat would be conducted laterally through the concrete wall and pass through the adjacent specimen, which has a lower thermal resistance. The lateral heat flow may explain that the ratio of estimated thermal resistance of two adjacent specimens W5 and W6, shown in Table 1, is smaller than the ratio indicated by the laboratory comparison.

To evaluate effects of lateral heat flow on the precision of the field measurements, a new task was added to the EIBS project, namely development of a 3-D model. The first question was then, how to discretise the continuous equations to arrive at the system, which is easy to solve with numerical techniques.

2 SOLUTION OF THE DIFFERENTIAL EQUATION OF HEAT CONDUCTION

2.1 State of the art of analytical techniques

Several different techniques of numerical analysis for solving transient heat conduction problems, such as the finite-difference, finite-element, and boundary integral equation methods have been presented in the technical literature.

Bhattacharya [5] and Lick [6] applied the improved finite-difference method (FDM) to time-dependent heat conduction problems with step-by step computation in the time domain. The finite-element method (FEM) based on variational principle, was used by Gurtin [7] to analyze the unsteady problem of heat transfer. Emery and Carson [8] as well as Visser [9] applied variational formulations in their finite-element solutions of nonstationary temperature distribution problems. Bruch and Zyvoloski [10] solved the transient linear and non-linear two-dimensional heat conduction problems using the finite-element weighted residual process. Rources and Alarcon [11] presented a formulation for a two-dimensional isotropic continuous solid using the boundary integral equation method (BIEM) with a finite-difference approach in the time domain. Combined application of BIEM and the Laplace transform to transient heat conduction problems started in 1970 with a paper by Rizzo and Shippy [12]. Later, Liggett and Liu [13] and Cheng and Liggett [14] applied this scheme to the linear time-dependent problems. Chen et al [15] successfully applied a hybrid method based on the Laplace transform and the FDM to transient heat conduction problems.

The disadvantages of these methods are the complicated procedure, need for large storage and long computation time. Wang et al [16] used the implicit spline method of splitting to solve the two and three-dimensional transient heat conduction problems. The method is applied to homogeneous and isotropic solid.

A cubic spline method has been developed in the numerical integration of partial differential equations since the pioneering work of Rubin and Graves [17], and Rubin and Khosla [18]. This method provides a simple procedure, small storage, short computation time, and a high order of accuracy. Furthermore, the spline method has a direct representation of gradient boundary condition.

In this paper, an implicit spline method is used for solution of three-dimensional transient heat conduction problems for a non-homogeneous medium. This method, for each computation step, treats the problem as a one-dimensional case in the implicit form and only one tridiagonal system is evaluated. In section 2, a spline scheme of splitting is presented. In section 3, the stability of the method is assessed.

2.2 Mathematical formulation

The governing differential equation for transient heat conduction in a homogeneous and isotropic solid is given by:

$$\rho C \frac{\partial T}{\partial t} = \left(\frac{\partial}{\partial x} \left(K_i \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(K_j \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(K_k \frac{\partial T}{\partial z} \right) \right) + Q \quad (2.1)$$

This equation, with suitable boundary and initial conditions, represents the problem of temperature distribution at any time and at any point of the volume, under consideration.

Different density ρ but constants for each type were used. They were determined as the average of several specimens. Discussion on EPS variability in physical properties and the method that has been used for their homogenisation is currently published [19].

The specific heat C which appears in the eqs.(2.1) and consistently further was from the Handbook of Heat Transfer.

The above quoted paper [19] deals with thermal conductivity coefficient as it varies with material thickness, temperature, density of specimen and specific mass extinction coefficient. As far as Q is concerned, it is a result of gradient T and K .

Thermal conductivity K_i in eqs.(2.1) does not include any effect of latent heat transfer that may be induced by the humidity changes.

2.2.1 Special treatment at the interface

Assuming the perfect contact, i.e., when heat fluxes ϕ_{ic} and ϕ_{ci} are equal, the following equation is obtained:

$$K_{i+1/2} = \frac{K_i K_{i+1} (X_{i,c} + X_{c,i+1})}{K_i X_{c,i+1} + K_{i+1} X_{i,c}}$$

and the interface temperature is given by:

$$T_c = \frac{K_i X_{c,i+1} T_i + K_{i+1} X_{i,c} T_{i+1}}{K_i X_{c,i+1} + K_{i+1} X_{i,c}}$$

2.2.2 Boundary and initial conditions

The temperatures are imposed at the surfaces.

$$T = T_{S_1} \quad \text{on surface } S_1$$

$$T = T_{S_2} \quad \text{on surface } S_2$$

$$T = T_{S_3} \quad \text{on surface } S_3$$

and the initial condition is

$$T(x, y, z, 0) = T_0(x, y, z)$$

where the union of the surfaces S_1 , S_2 and S_3 forms the complete boundary of the solid of volume V ; S_1 , S_2 and S_3 are part of boundary on which the temperatures T are prescribed.

2.2.3 Implicit Spline Schemes

The implicit spline schemes of splitting for the numerical solution of equation (2.1) are (Wang) [20]

$$T_{ijk}^{n+1/3} = T_{ijk}^n + \Delta t \cdot \frac{1}{\rho C} \left(P_{ijk}^{n+1/3} + \frac{1}{3} Q \right) \quad (2.2a)$$

$$T_{ijk}^{n+2/3} = T_{ijk}^{n+1/3} + \Delta t \cdot \frac{1}{\rho C} \left(R_{ijk}^{n+2/3} + \frac{1}{3} Q \right) \quad (2.2b)$$

$$T_{ijk}^{n+1} = T_{ijk}^{n+2/3} + \Delta t \cdot \frac{1}{\rho C} \left(S_{ijk}^{n+1} + \frac{1}{3} Q \right) \quad (2.2c)$$

where

T_{ijk} , P_{ijk} , R_{ijk} , S_{ijk} are the spline approximation of $(T)_{ijk}$, $\left(\frac{\partial}{\partial x} \left(K_i \frac{\partial T}{\partial x} \right) \right)_{ijk}$,

$\left(\frac{\partial}{\partial y} \left(K_j \frac{\partial T}{\partial y} \right) \right)_{ijk}$, $\left(\frac{\partial}{\partial z} \left(K_k \frac{\partial T}{\partial z} \right) \right)_{ijk}$ respectively, and there are spline relations as follows

$$\frac{\Delta x_{ijk}}{6} \cdot P_{i-1,jk} + \frac{(\Delta x_{ijk} + \Delta x_{i+1,jk})}{3} P_{ijk} + \frac{\Delta x_{i+1,jk}}{6} \cdot P_{i+1,jk} = K_{i+1/2} \frac{(T_{i+1,jk} - T_{ijk})}{\Delta x_{i+1,jk}} - K_{i-1/2} \frac{(T_{ijk} - T_{i-1,jk})}{\Delta x_{i,jk}} \quad (2.3a)$$

$$\frac{\Delta y_{ijk}}{6} \cdot R_{i,j-1,k} + \frac{(\Delta y_{ijk} + \Delta y_{i,j+1,k})}{3} R_{ijk} + \frac{\Delta y_{i,j+1,k}}{6} \cdot R_{i,j+1,k} = K_{j+1/2} \frac{(T_{i,j+1,k} - T_{ijk})}{\Delta y_{i,j+1,k}} - K_{j-1/2} \frac{(T_{ijk} - T_{i,j-1,k})}{\Delta y_{i,jk}} \quad (2.3b)$$

$$\frac{\Delta z_{ijk}}{6} \cdot S_{i,j,k-1} + \frac{(\Delta z_{ijk} + \Delta z_{ij,k+1})}{3} S_{ijk} + \frac{\Delta z_{ij,k+1}}{6} \cdot S_{i,j,k+1} = K_{k+1/2} \frac{(T_{ij,k+1} - T_{ijk})}{\Delta z_{ij,k+1}} - K_{k-1/2} \frac{(T_{ijk} - T_{ij,k-1})}{\Delta z_{i,jk}} \quad (2.3c)$$

where

$$\Delta x_{ijk} = x_{ijk} - x_{i-1,jk} > 0,$$

$$\Delta y_{ijk} = y_{ijk} - y_{i,j-1,k} > 0,$$

$$\Delta z_{ijk} = z_{ijk} - z_{ij,k-1} > 0$$

If boundary conditions are given, we obtain spline solutions of equation (2.1) from equations (2.2) and (2.3).

The equation (2.2a) can be written in the following form

$$T_{ijk}^{n+1/3} = E_{ijk} + F_{ijk} P_{ijk}^{n+1/3} \quad (2.4)$$

From Equation (2.2a) and (2.4), we find

$$A_i T_{i-1,jk}^{n+1/3} + B_i T_{ijk}^{n+1/3} + C_i T_{i+1,jk}^{n+1/3} = D_i, \quad i=2, N-1 \quad (2.5)$$

where

$$A_i = \frac{\Delta x_{ijk}}{6 F_{i-1,jk}} - \frac{K_{i-1/2}}{\Delta x_{ijk}}$$

$$B_i = \frac{\Delta x_{ijk} + \Delta x_{i+1,jk}}{3 F_{ijk}} + \frac{K_{i-1/2}}{\Delta x_{ijk}} + \frac{K_{i+1/2}}{\Delta x_{i+1,jk}}$$

$$C_i = \frac{\Delta x_{i+1,jk}}{6 F_{i+1,jk}} - \frac{K_{i+1/2}}{\Delta x_{i+1,jk}}$$

$$D_i = \frac{\Delta x_{ijk} E_{i-1,jk}}{6 F_{i-1,jk}} + \frac{(\Delta x_{ijk} + \Delta x_{i+1,jk}) E_{ijk}}{3 F_{ijk}} + \frac{\Delta x_{i+1,jk} E_{i+1,jk}}{6 F_{i+1,jk}}$$

Under proper conditions this system can be solved with Thomas Algorithm (A.D.I Method) [21].

The same techniques are used for solving equations (2.2b) and (2.2c) for determining $T_{ijk}^{n+2/3}$ and T_{ijk}^{n+1} .

2.2.4 Truncation error and stability

The spatial accuracy of the spline approximation for interior points has been considered in the work of Rubin and Khosla [18].

For diffusion terms, *cubic spline approximation* has second-order accuracy, which can be maintained even with rather large nonuniformities in mesh width.

For the linear heat conduction equation (2.1) with $Q=0$, the interior point stability of scheme (2.2) can be assessed with the Von Neumann Fourier decomposition for

$$\Delta x_{ijk} = \Delta y_{ijk} = \Delta z_{ijk} = h = \text{const} \tan t.$$

Let

$$T_{ijk}^{n+1/3} = \bar{T}_{k_1 k_2 k_3}^{n+1/3} . e^{I(k_1 i h + k_2 j h + k_3 k h)},$$

$$T_{ijk}^{n+2/3} = \bar{T}_{k_1 k_2 k_3}^{n+2/3} . e^{I(k_1 i h + k_2 j h + k_3 k h)}$$

$$T_{ijk}^{n+1} = \bar{T}_{k_1 k_2 k_3}^{n+1} . e^{I(k_1 i h + k_2 j h + k_3 k h)}$$

$$T_{i\pm 1, j k}^{n+1/3} = \bar{T}_{k_1 k_2 k_3}^{n+1/3} . e^{I(k_1(i\pm 1)h + k_2 j h + k_3 k h)}$$

$$T_{i, j\pm 1, k}^{n+2/3} = \bar{T}_{k_1 k_2 k_3}^{n+2/3} . e^{I(k_1 i h + k_2(j\pm 1)h + k_3 k h)}$$

$$T_{i j, k\pm 1}^{n+1} = \bar{T}_{k_1 k_2 k_3}^{n+1} . e^{I(k_1 i h + k_2 j h + k_3(k\pm 1)h)}$$

Let

$$\xi_1 = \frac{\bar{T}_{k_1 k_2 k_3}^{n+1/3}}{\bar{T}_{k_1 k_2 k_3}^n} \quad \text{Amplification Factor in x Direction}$$

$$\xi_2 = \frac{\bar{T}_{k_1 k_2 k_3}^{n+2/3}}{\bar{T}_{k_1 k_2 k_3}^{n+1/3}} \quad \text{Amplification Factor in y Direction}$$

$$\xi_3 = \frac{\bar{T}_{k_1 k_2 k_3}^{n+1}}{\bar{T}_{k_1 k_2 k_3}^{n+2/3}} \quad \text{Amplification Factor in z Direction}$$

$$\xi = \frac{\bar{T}_{k_1 k_2 k_3}^{n+1}}{\bar{T}_{k_1 k_2 k_3}^n} \quad \text{Global Amplification Factor, } I^2 = -1$$

From equation (2.2a) and (2.3a), we find

$$\xi_1 = \frac{1}{1 + \frac{3\alpha(1 - (\alpha_{i+1/2} e^{I(k_1 h)} + \alpha_{i-1/2} e^{-I(k_1 h)}))}{2 + \cos k_1 h}}$$

where $\alpha = \frac{\Delta t (K_{i+1/2} + K_{i-1/2})}{\rho C h^2} > 0$

$$\alpha_{i+1/2} = \frac{K_{i+1/2}}{K_{i+1/2} + K_{i-1/2}}$$

$$\alpha_{i-1/2} = \frac{K_{i-1/2}}{K_{i+1/2} + K_{i-1/2}}$$

In some manner ξ_2 and ξ_3 are calculated.

$$\xi_2 = \frac{1}{1 + \frac{3\alpha(1 - (\beta_{j+1/2} e^{I(k_2 h)} + \beta_{j-1/2} e^{-I(k_2 h)}))}{2 + \cos k_2 h}}$$

$$\xi_3 = \frac{1}{1 + \frac{3\alpha(1 - (\gamma_{k+1/2} e^{I(k_3 h)} + \gamma_{k-1/2} e^{-I(k_3 h)}))}{2 + \cos k_3 h}}$$

Where $\beta = \frac{\Delta t.(K_{j+1/2} + K_{j-1/2})}{\rho C.h^2} > 0$ and $\gamma = \frac{\Delta t.(K_{k+1/2} + K_{k-1/2})}{\rho C.h^2} > 0$

From $\xi = \xi_1 \cdot \xi_2 \cdot \xi_3$ we have $|\xi| \leq |\xi_1| \cdot |\xi_2| \cdot |\xi_3|$

ψ has to be bounded when t tends to 0, then $|\xi|$ have to be lower or equal to 1.

The Von Neuman's condition for this scheme necessary for the suppression of all error growth requires that the spectral radius $|\xi| \leq 1$.

With each of the coefficients, ξ_1 ξ_2 ξ_3 less than 1, their product is less than 1 and the scheme (2.2) is unconditionally stable with arbitrary values of $\Delta t, h, K_i, K_j$ and K_k .

2.3 Computational algorithm

The heat transfer through the basement wall does not involve heat generation. By using the equation (2.5), one can create the following system of linear equations:

$$AT = B$$

Where A is an (N x N) matrix and T and B are (N) vectors. The subscript of A, T and B range from 1 to N as follows:

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & & & & \\ a_{2,1} & a_{2,2} & a_{2,3} & & & \\ & \cdot & \cdot & \cdot & & \\ & & & & \cdot & \\ & & & & & \cdot \\ & & & & & & a_{n-1,n-2} & a_{n-1,n-1} & a_{n-1,n} \\ & & & & & & & a_{n,n-1} & a_{n,n} \end{bmatrix}$$

$$T = \begin{pmatrix} T_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ T_n \end{pmatrix}, \quad B = \begin{pmatrix} D_1 - A_1 T_0 \\ D_2 \\ \cdot \\ \cdot \\ \cdot \\ D_{n-1} \\ D_n - C_n T_{n+1} \end{pmatrix}$$

Where:

$$\begin{cases} a_{i+1,j} = A_i & \text{for } j = i \\ a_{i,j} = B_i & \text{for } j = i \\ a_{i,j} = C_i & \text{for } j = i + 1 \\ a_{i,j} = 0 & \text{else} \end{cases}$$

The temperatures T_0 and T_{n+1} are the known surface temperature.

The above system consists of a tridiagonal system of linear equations. To solve this system of equations, the Thomas algorithm is used [22].

3 RESULTS AND DISCUSSIONS

While the complete review of results from the Basement Consortium Research project is presented elsewhere (Swinton et al.) [3], the current paper elucidates and quantifies the effect of lateral heat flow with the help of the 3-D model.

3.1 Benchmarking of the model

This 3-D model is benchmarked with spatial and temporal temperature distributions. In the initial calculations, one forces the model to agree with the measured conditions by varying thermal conductivity of the material until finding the best fit. However, there are hundreds of subsequent calculations in which no further adjustment of material properties is allowed and where the model is used in a standard, predictive manner.

Figure 5 shows temperatures on both sides of the exterior insulation (soil and concrete). The results of 3-D calculations on the test wall W6 during the second year are compared with the measured values. The acceptable agreement between measured and calculated temperature fields permits application of the model to quantify the effect of lateral heat flow.

3.2 Application of the model

As shown in Figure 6, a reference calculation eliminating the lateral heat flow was called a 2-D solution. In this case, the model calculations were performed assuming that all

materials properties are identical to that of EPS placed in W5 position (i.e., material on the side of the tested specimen). As shown in Figure 6, except for area adjacent to the corners, temperature is uniformly distributed along the depth of the basement wall. In contrast to that uniformity, Figure 7 shows the distribution of temperature on the surface of the insulation, calculated for the actual temperature field which was measured by termocouples on the boundary of the control volume.

The model calculations, as previously discussed, were performed at three different positions where heat flux incoming into the concrete wall was continually measured. Within these three locations the top position of the wall was the most sensitive to lateral heat flow. Figure 8 shows the angle of heat flux vector relative to normal direction at the top position. It is evident that thermal insulating system tested at the W6 position has higher resistance to the heat flow than the adjacent materials W5 and W7. Moving from the centre towards the adjacent test specimen, one may observe that lines of heat flow start to deviate from the normal direction, reaching a maximum angle of about 10 degrees. Lack of the symmetry in Figure 8 implies that the thermal resistance of EIBS in W5 differs from that in W7 location. Also the performance of corner guards W8 and W2 is different, though Figure 8 shows that the width of these guards appear adequate to eliminate major disturbances from the adjacent test areas (W7 and W3).

Finally, Figures 9a, 9b, 10a and 10b show the average relative thermal resistance of the specimen W6 when applying different models to the measured data (temperatures and heat fluxes) in the control volume during the first and second year of testing.

The benchmark (100 percent) is the laboratory determined, initial value of thermal resistance of the specimen. The arbitrary nature of this benchmark will be discussed when discussing part 3 of this research project, nevertheless it is sufficient for comparative purposes. Figures 9a, 9b, 10a and 10b show such a comparison. Figure 9a and 9b show results obtained with the 3-D analysis during the first year and the second year of testing, while Figure 10a and 10b show the same calculations when the lateral heat flow component is eliminated (2-D model). These calculations are performed for the period during the second year of testing in which one-directional heat flow is prevailing (days 510 to 660).

Figures 9a, 9b, 10a and 10b show that, in the worst case of the tested insulation systems, the yearly difference between results obtained from 2-D and 3-D models is about 2 percentage points for the first year and seven percentage points for the second year.

4 CONCLUDING REMARKS

The basement consortium research project involved several different Exterior Basement Insulation Systems placed next to each other. To evaluate the effect of lateral heat flow in concrete wall that acted as heat sink or source for adjacent specimens with different thermal resistance one needed a 3-D model.

This model compared measured temperatures and heat fluxes in the test compartment with all tested compartments in the control element. The problem solver was based on the implicit Spline Method, which, for each computation step, stepwise reduces the 3-D conduction problem to one tri-diagonal matrix of implicit form solution.

Use of the 3-D model permitted a better assessment of the long-term thermal resistance of the thermal insulating systems used in the study. As shown in the above examples, the agreements between measured and calculated spatial and temporal temperature distributions are acceptable. Achieved in this manner benchmarking of the model gives a degree of confidence in evaluating the effect of lateral heat flow. In the discussed example, the yearly difference between results obtained from 2-D and 3-D models on west wall was seven percentage points.

This analysis increased the confidence in estimating the effect of lateral heat flow significantly.

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Specimen Position	W5		W6	
	Temperature Inside Specimen (°C)	Temperature Outside Specimen (°C)	Temperature Inside Specimen (°C)	Temperature Outside Specimen (°C)
Bottom	16.91	14.37	16.78	13.55
Low	17.38	12.30	17.44	11.67
Middle	16.82	9.60	17.14	8.49
High	15.98	5.24	16.47	4.37
Avg. temperature	16.77	10.38	16.96	9.52

Table 1a. Measured temperature, heat flux and temperature difference across the insulation specimen, averaged over the week 9 to 16 December 1996.

	W5	W6
Temp. difference across the wall, °K	6.39	7.44
Mean heat flux on inner surface, W/m ²	4.30	4.10
Apparent R-value, (m ² K)/W	1.49 (100%)	1.81(122%)

Table 1b. Apparent steady-state thermal resistance of specimens W5 and W6 calculated from data shown in Table 1a.

Figure 1. Schematic of placement of specimens on the west wall.

Figure 2. Thermocouples and calibrated insulation specimens mounted on the West wall.

Figure 3. Temperatures across the W6 specimen measured in the mid-height position.

Figure 4. Special treatment at the interface

Figure 5. Measured and simulated temperatures on both sides of the west wall, mid-height position.

Figure 6. Calculated temperatures on the soil-insulation interface of the west wall that would be there if all test specimens had properties equal to the W5 specimen (simulation). There is no lateral heat flow and therefore we call it a 2-D solution.

Figure 7. The measured temperatures on the soil-insulation interface of the west wall.

Figure 8. Angle of the heat flux vector calculated at the top position of the west. Positive sign indicates the lateral flow in the southward direction.

Figure 9a. Average relative thermal resistance of the specimen W-6 when applying 3-D model to the measured data (temperatures and heat fluxes) in the control volume during the first year of testing.

Figure 9b. Average relative thermal resistance of the specimen W-6 when applying 3-D model to the measured data (temperatures and heat fluxes) in the control volume during the second year of testing.

Figure 10a. Average relative thermal resistance of the specimen W-6 when applying 2-D model to the measured data (temperatures and heat fluxes) in the control volume during the first year of testing.

Figure 10b. Average relative thermal resistance of the specimen W-6 when applying 2-D model to the measured data (temperatures and heat fluxes) in the control volume during the second year of testing.