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# A Fuzzy Logic-based Approach to Uncertainty Treatment in the Rule Interchange Format: From Encoding to Extension

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**Abstract.** The Rule Interchange Format (RIF) is a W3C recommendation that allows rules to be exchanged between rule systems. Uncertainty is an intrinsic feature of real world knowledge, hence it is important to take it into account when building logic rule formalisms. However, the set of truth values in the RIF Basic Logic Dialect (RIF-BLD) currently consists of only two values (t and f), although the RIF Framework for Logic Dialects (RIF-FLD) allows for more. In this paper, we first present two techniques of encoding uncertain knowledge and its fuzzy semantics in RIF-BLD presentation syntax. We then propose an extension leading to an Uncertainty Rule Dialect (RIF-URD) to support a direct representation of uncertain knowledge. In addition, rules in Logic Programs (LP) are often used in combination with the other widely-used knowledge representation formalism of the Semantic Web, namely Description Logics (DL), in many application scenarios of the Semantic Web. To prepare DL as well as LP extensions, we present a fuzzy extension to Description Logic Programs (DLP), called Fuzzy DLP, and discuss its mapping to RIF. Such a formalism not only combines DL with LP, as in DLP, but also supports uncertain knowledge representation.[4]

**Keywords:** Rule Interchange Format, Uncertainty, Fuzzy Logic

## 1 Introduction

Description Logics (DL) and Logic Programs (LP)[5] are the two main paradigms of knowledge representation formalisms for the Semantic Web, both of which are based on subsets of first-order logic [19]. DL and LP cover different but overlapping areas of knowledge representation. They are complementary to some degree;

[4] This work was done mainly while the first author was affiliated with the University of New Brunswick, Fredericton, Canada.

[5] In this paper, we only consider the Horn Logic subset of LP, without negation-as-failure.

for example, typical DL cannot directly express LP's n-ary function applications (complex terms) while classic LP cannot express DL's disjunctions (in the head). Combining DL with LP in order to "build rules on top of ontologies" or, "build ontologies on top of rules" has become an emerging topic for various applications of the Semantic Web. It is therefore important to research the combination of DL and LP with different strategies. There have been various achievements in this area, including several proposed combination frameworks [11, 9, 17, 22, 23]. As a minimal approach in this area, the Description Logic Program (DLP) 'intersection' of DL and LP has been studied, along with mappings from DL to LP [11]. Both [9] and [22] studied the combination of standard Datalog inference procedures with intermediate DL satisfiability checking.

On the other hand, as evidenced by Fuzzy RuleML [6] and W3C's Uncertainty Reasoning for the World Wide Web (URW3) Incubator Group [20], handling uncertain knowledge is becoming a critical research direction for the (Semantic) Web. For example, many concepts needed in business ontology modeling lack well-defined boundaries or, precisely defined criteria of relationships with other concepts. To take care of these knowledge representation needs, different approaches for integrating uncertain knowledge into traditional rule languages and DL languages have been studied [19, 18, 27–29, 26, 21, 31, 34, 7].

The Rule Interchange Format (RIF) has been developed by W3C's Rule Inter-change Format (RIF) Working Group to support the exchange of rules between rule systems [4]. In particular, the Basic Logic Dialect (RIF-BLD) corresponds to the language of definite Horn rules with equality and a first-order semantics. While RIF's Framework for Logic-based Dialects (RIF-FLD) [5] permits multi-valued logics, RIF-BLD instantiates RIF-FLD with the set of truth values consisting of only two values, $t$ and $f$, hence is not designed for expressing uncertain knowledge.

According to the final report from the URW3 Incubator group, uncertainty is a term intended to include different types of uncertain knowledge, including incompleteness, vagueness, ambiguity, randomness, and inconsistency [20]. Mathematical theories for representing uncertain knowledge include, but are not limited to, Probability, Fuzzy Sets, Belief Functions, Random Sets, Rough Sets, and combinations of several models (Hybrid). The uncertain knowledge representations and interpretations discussed in this paper are limited to Fuzzy Sets and Fuzzy Logic (a multi-valued logic based on Fuzzy set theory); other approaches should be studied in future work.

The main contributions of this paper are: (1) two techniques of encoding uncertain information in RIF as well as an uncertainty extension to RIF; (2) an extension of DLP to Fuzzy DLP and the mapping of Fuzzy DLP to RIF.

Two earlier uncertainty extensions to the combination of DL and LP that we can expand on are [30] and [32]. While our approach emphasizes the interoperation in the intersection of fuzzy DL and fuzzy LP allows DL atoms in the head of hybrid rules and DL subsumption axioms in hybrid rules, the approach of [30] does not allow the expressiveness. Our approach deals with fuzzy subsumption of fuzzy concepts of the form $C \sqsubseteq D \ = c$ whereas [32] deals with crisp subsumption of fuzzy concepts of the form $C \sqsubseteq D$. Also, we do not limit hybrid knowledge bases to the intersection of (fuzzy) DL and (fuzzy) LP. We extend [32] and study the decidable union of DL and LP.

The rest of this paper is organized as follows. Section 2 reviews earlier work on the interoperation between DL and LP in the intersection of these two formalisms (known as DLP) and represents the DL-LP mappings in RIF. Section 3 addresses the syntax and semantics of fuzzy Logic Programs, and then presents two techniques of bringing uncertainty into the RIF presentation syntax (and then into its semantics and XML syntax), using encodings as RIF functions and RIF predicates. Section 4 adapts the definition of the set of truth values in RIF-FLD for the purpose of representing uncertain knowledge directly, and proposes the new Uncertainty Rule Dialect (RIF-URD), extending RIF-BLD. Section 5 extends DLP to Fuzzy DLP, supporting mappings between fuzzy DL and fuzzy LP, and gives representations of Fuzzy DLP in RIF and RIF-URD. Finally, Section 6 summarizes our main results and gives an outlook on future research.

## 2 Description Logic Programs and Their Representation in RIF

In this section, we summarize the work on Description Logic Programs (DLP) [11] and then show how to represent the mappings between DL and LP in RIF presentation syntax.

The paper [11] studied the intersection between the leading Semantic Web approaches to rules in LP and ontologies in DL, and showed how to interoperate between DL and LP in the intersection known as DLP. A DLP knowledge base permits:

1. stating that a class $C$ is a *Subclass* of a class $D$, $C \sqsubseteq D$;
2. stating that the *Domain* of a property $R$ is a class $C$, $\top \sqsubseteq \forall R^{-}.C$;
3. stating that the *Range* of a property $R$ is a class $C$, $\top \sqsubseteq \forall R.C$;
4. stating that a property $R$ is a *Subproperty* of a property $P$,$R \sqsubseteq P$;
5. stating that an individual $a$ is an Instance of a class $C$,$C(a)$;
6. stating that a pair of individuals $(a, b)$ is an Instance of a property $R$, $R(a, b)$;
7. using the *Intersection* connective (conjunction) within class descriptions, $C_1 \sqcap C_2$;
8. using the *Union* connective (disjunction) within subclass descriptions, $C_1 \sqcup C_2 \sqsubseteq D$;
9. using *Universal quantification* within superclass descriptions, $C \sqsubseteq \forall R.D$;
10. using *Existential quantification* within subclass descriptions$\exists R.C \sqsubseteq D$;
11. stating that a property $R$ is *Transitive*, $R^{+} \sqsubseteq R$;
12. stating that a property $R$ is the Inverse of a property $P$.

Here $C$,$D$,$C_1$, $C_2$ are concepts, $\top$ is the universal concept, $R$,$P$ are roles, $R^{-}$ and $R^{+}$ are the inverse role and the transitive role of $R$, respectively, and $a$,$b$ are individuals.

In RIF presentation syntax, the quantifiers Exists and Forall are made explicit, rules are written with a ":-" infix, variables start with a "?" prefix, and whitespace is used as a separator. Table 1 summarizes the mappings in [11] between DL and LP in the DLP intersection, and shows its representation in RIF. Note that in DLP, a complex concept expression which is a disjunction (e.g. $C_1 \sqcup C_2$) or an existential (e.g. $\exists R.C$) is not allowed in the right side of a concept subsumption axiom (superclass).

**Table 1.** Mapping between LP and DL

| LP Syntax | DL Syntax | RIF |
|---|---|---|
| $D(x) \leftarrow C(x)$ | $C \sqsubseteq D$ | Forall ?x (D(?x) :- C(?x)) |
| $C(y) \leftarrow R(x, y)$ | $\top \sqsubseteq \forall R.C$ | Forall ?x ?y (C(?y) :- R(?x ?y)) |
| $C(x) \leftarrow R(x, y)$ | $\top \sqsubseteq \forall R^{-}.C$ | Forall ?x ?y (C(?x) :- R(?x ?y)) |
| $P(x, y) \leftarrow R(x, y)$ | $R \sqsubseteq P$ | Forall ?x ?y (P(?x ?y) :- R(?x ?y)) |
| $C(a)$ | $C(a)$ | C(a) |
| $R(a, b)$ | $R(a, b)$ | R(a,b) |
| $D(x) \leftarrow C_1(x) \wedge C_2(x)$ | $C_1 \sqcap C_2 \sqsubseteq D$ | Forall ?x (D(?x) :- And($C_1$(?x) $C_2$(?x))) |
| $D_1(x) \leftarrow C(x)$,<br>$D_2(x) \leftarrow C(x)$, | $C \sqsubseteq D_1 \sqcap D_2$ | Forall ?x ($D_1$(?x) :- C(?x))<br>Forall ?x ($D_2$(?x) :- C(?x)) |
| $D(x) \leftarrow C_1(x)$,<br>$D(x) \leftarrow C_2(x)$ | $C_1 \sqcup C_2 \sqsubseteq D$ | Forall ?x (D(?x) :- $C_1$(?x))<br>Forall ?x (D(?x) :- $C_2$(?x)) |
| $D(y) \leftarrow C(x), R(x, y)$ | $C \sqsubseteq \forall R.D$ | Forall ?x ?y (D(?y) :- And(C(?x) R(?x ?y))) |
| $D(x) \leftarrow C(y), R(x, y)$ | $\exists R.C \sqsubseteq D$ | Forall ?x ?y (D(?x) :- And(C(?y) R(?x ?y))) |
| $R(x, z) \leftarrow R(x, y), R(y, z)$ | $R^{+} \sqsubseteq R$ | Forall ?x ?y ?z(R(?x ?z) :- And(R(?x ?y) R(?y ?z))) |
| $R(x, y) \leftarrow P(y, x)$,<br>$P(y, x) \leftarrow R(x, y)$ | $P \equiv R^{-}$ | Forall ?x ?y(R(?x ?y) :- P(?y ?x))<br>Forall ?x ?y(P(?y ?x) :- R(?x ?y)) |

## 3 Encoding Uncertainty in RIF

Fuzzy set theory was introduced in [37] as an extension of the classical notion of sets to capture the inherent vagueness (the lack of crisp boundaries) of real-world sets. Formally, a fuzzy set $A$ with respect to a set of elements $X$ (also called a universe) is characterized by a membership function $\mu_A(x)$ which assigns a value in the real unit interval [0,1] to each element $x \in X$. $\mu_A(x)$ gives the degree to which an element $x$ belongs to the set $A$. Fuzzy logic is a form of multi-valued logic derived from fuzzy set theory to deal with reasoning that is approximate rather than precise. In Fuzzy Logic the degree of truth of a statement can range between 0 and 1 and is not constrained to the two truth values, $t$ and $f$, as in classic predicate logic [24]. Such degrees can be computed based on various specific membership functions, for example, a trapezoidal function.

Fuzzy Logic extends the Boolean operations defined on crisp sets and relations for fuzzy sets and fuzzy relations. Basic operations in Fuzzy Logic apply to fuzzy sets include negation, intersection, union, and implication. Today, in the broader sense, Fuzzy Logic is actually a family of fuzzy operations [35] [13] divided into different classes, among which, the most widely known include Zadeh Logic [37], Lukasiewicz Logic [16], Product Logic [13], Gödel Logic [10, 1], and Yager Logic [36]. For example, in Zadeh Logic, the membership function of the union of two fuzzy sets is defined as the *maximum* of the two membership functions for the two fuzzy sets (the *maximum* criterion); the membership function of the intersection of two fuzzy sets is defined as the *minimum* of the two membership functions (the *minimum* criterion); while the membership function of the complement of a fuzzy set is defined as the negation of the specified membership function (the *negation* criterion).

In this section, we first present the syntax and semantics for fuzzy Logic Programs based on Fuzzy Sets and Fuzzy Logic [37] and on previous work on fuzzy LP [31, 34, 33], and then propose two techniques of encoding the semantics of uncertain knowledge based on Fuzzy Logic in the presentation syntax of RIF-BLD using BLD functions and BLD predicates respectively.

### 3.1 Fuzzy Logic Programs

Rules in van Emden's formalism for fuzzy LP have the syntactic form

$$H \leftarrow_c B_1, \cdots, B_n \tag{1}$$

where $H$,$B_i$ are atoms, $n \geq 0$, and the factor $c$ is a real number in the interval [0,1] [31]. For $n = 0$, such fuzzy rules degenerate to fuzzy facts.

The fuzzy LP language proposed by [34, 33] is a generalization of van Emden's work [31]. Rules are constructed from an implication ($\leftarrow$) with a corresponding t-norm adjunction operator ($f_1$), and another t-norm operator denoted by $f_2$. A t-norm is a generalization to the many-valued setting of the conjunction connective. In their setting, a rule is of the form $H \leftarrow_{f_1} f_2(B_1, \cdots, B_n) \;\; withCF \;\; c$, where the confidence factor $c$ is a real number in the unit interval [0,1] and $H, B_i$ are atoms with truth values in (0, 1]. If we take the operator $f_1$ as the product following Goguen implication and the operator $f_2$ as the Gödel t-norm (minimum), this is exactly of the form by van Emden [31].

In [40], we presented norm-parameterized fuzzy Description Logics. In this paper, we follow this norm-parameterized approach when considering the DL counterpart of the DLP and propose a corresponding norm-parameterized fuzzy extension to Logic Programs, more precisely, to the Horn Logic subset of Logic Programs. We call it norm-parameterized as we integrate different norms from the Fuzzy Logic family into the fuzzy extension. A fuzzy LP knowledge base consists of these norm parameters and a finite set of fuzzy rules. The norm parameters, $F_{IN}$, $F_U$, and $F_{IM}$, define the intersection, union, and implication operators respectively. Since only Horn Logic is considered, we can ignore the negation operation for now. A fuzzy rule has the following form:

$$H(\overrightarrow{x}) \leftarrow B_1(\overrightarrow{x_1}), \cdots, B_n(\overrightarrow{x_n}) \quad /c \tag{2}$$

Here $H(\overrightarrow{x})$, $B_i(\overrightarrow{x_i})$ are atoms, $\overrightarrow{x}$, $\overrightarrow{x_i}$ are vectors of variables or constants, $n \geq 0$ and the confidence factor $c$ (also called certainty degree) is a real number in the interval [0,1]. For the special case of fuzzy facts this becomes $H \quad /c$. These forms with a "/" symbol have the advantages of avoiding possible confusion with the equality symbol usually used for functions in logics with equality, as well as using a unified and compact format to represent fuzzy rules and fuzzy facts.

The semantics of such fuzzy LP is an extension of classical LP semantics. Let $B_R$ stand for the Herbrand base of a fuzzy knowledge base $KB_{LP}$. A fuzzy Herbrand interpretation $H_I$ for $KB_{LP}$ is defined as a mapping $B_R \to [0,1]$. It is a fuzzy subset of $B_R$ under fuzzy semantics and can be specified by a function $val$ with two arguments: a variable-free atom $H$(or $B_1, \cdots, B_n$) and a fuzzy Herbrand interpretation $H_I$. The returned result of the function $val$ is the membership value of $H$(or $B_1, \cdots, B_n$) under $H_I$, denoted as $val(H, H_I)$ (or $val(B_i, H_I)$).

Therefore, if min is specified as the intersection operator and $\times$ is as the implication operator, a variable-free instance of a rule 2 is true under $H_I$ iff $val(H, H_I) \geq c \times \min\{val(B_i, H_I)|i \in \{1, \cdots, n\}\}$ (min{}=1 if $n = 0$). In other words, such an interpretation can be separated into the following two parts [12–14].

- The body of the rule consists of $n$ atoms. Our confidence that all these atoms are true is interpreted under Gödel's semantics for fuzzy logic:
  $val((B_1, \cdots, B_n), H_I) = \min\{val(B_i, H_I)|i \in \{1, \cdots, n\}\}$
- The implication is interpreted as the product:
  $val(H, H_I) = c \times val((B_1, \cdots, B_n), H_I)$

Furthermore, a rule is true under $H_I$ iff each variable-free instance of this rule is true under $H_I$ and a fuzzy knowledge base $KB_{LP}$ is true under $H_I$ iff every rule in $KB_{LP}$ is true under $H_I$. Such a Herbrand interpretation $H_I$ is called a Herbrand model of $KB_{LP}$.

For a fuzzy knowledge base $KB_{LP}$, the reasoning task is a fuzzy entailment problem written as $KB_{LP} \models H \quad /c$ ($H \in B_R$, $c \in [0,1]$).

For simplicity, we take the min and $\times$ operators as default specifications in the examples presented hereafter.

*Example 1.* Consider the following fuzzy LP knowledge base:
$cheapFlight(x, y) \leftarrow affordableFlight(x, y) \quad /0.9 \qquad (1)$
$affordableFlight(x, y) \quad /left\_shoulder0k4k1k3k(y) \qquad (2)$

Figure 1 shows the left_shoulder membership function $left\_shoulder(0, 4000, 1000, 3000)$. We use the name $left\_shoulder0k4k1k3k$ for this parameterization. The function has the mathematical form

$$left\_shoulder0k4k1k3k(y) = \begin{cases} 1 & 0 \leq y \leq 1000 \\ -0.0005y + 1.5 & 1000 < y \leq 3000 \\ 0 & 3000 < y \leq 4000 \end{cases}$$

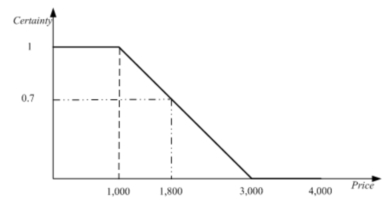


**Fig. 1.** A Left_shoulder Membership Function

For example, the certainty degree computed by this function for the fact $affordableFlight(flight0001, 1800)$ is 0.7.

Applying the semantics we discussed, $val(cheapFlight(flight0001, 1800), H_I)$=$0.9 * 0.7 = 0.63$, so we have that $KB_{LP} \models cheapFlight(flight0001, 1800) \quad /0.63$.

*Example 2.* Consider the following fuzzy LP knowledge base:
$A(x) \leftarrow B(x), C(x) \quad /0.5 \qquad (1)$
$C(x) \leftarrow D(x) \quad /0.5 \qquad (2)$
$B(d) \quad /0.5 \qquad (3)$
$D(d) \quad /0.8 \qquad (4)$

We have that $KB_{LP} \models A(d) \quad /0.2$. The reasoning steps of example 2 are described as follows:

$$\begin{aligned} val(A(d), H_I) &= 0.5 \times \min(val(B(d), H_I), val(C(d), H_I)) \quad accordingto(1) \\ &= 0.5 \times \min(val(B(d), H_I), 0.5 \times val(D(d), H_I)) \quad accordingto(2) \\ &= 0.5 \times \min(0.5, 0.5 \times val(D(d), H_I)) \quad accordingto(3) \\ &= 0.5 \times \min(0.5, 0.5 \times 0.8) \quad accordingto(4) \\ &= 0.5 \times 0.4 \\ &= 0.2 \end{aligned}$$

### 3.2 Encoding Uncertainty Using RIF Functions

RIFs main logic dialect is RIF-BLD [3]. RIF-BLD corresponds to the language of definite Horn rules with equality and a standard first-order semantics. Syntactically, RIF-BLD has a number of extensions to support features such as objects and frames as in F-logic, internationalized resource identifiers (IRIs) as identifiers for concepts, and a rich set of datatypes and built-ins. RIF-BLD uses a standard first-order semantics. For example, there is a rule in English describes that *A buyer buys an item from a seller if the seller sells the item to the buyer* and a fact *John sells LeRif to Mary.* Assuming Web IRIs for the predicates buy and sell, as well as for the individuals John, Mary, and LeRif, the above English text can be represented in the RIF-BLD Presentation Syntax as follows.

```
Document(
 Base(<http://example.com/people#>)
 Prefix(cpt <http://example.com/concepts#>)
 Prefix(bks <http://example.com/books#>)
 Group
 ( Forall ?Buyer ?Item ?Seller (
     cpt:buy(?Buyer ?Item ?Seller) :- cpt:sell(?Seller ?Item ?Buyer)
)
 cpt:sell(<John> bks:LeRif "Mary" 'rif:iri)
))
```

One technique to encode uncertainty in logics with equality such as the current RIF-BLD (where equality in the head is "At Risk") is mapping all predicates to functions and using equality for letting them return uncertainty values [15]. We assume that $H, B_i$ of the fuzzy rule of equation 2 contain variables in $\{?x_1, \cdots, ?x_k\}$ and that the head and body predicates are applied to terms $t_1, \cdots, t_r$ and $t_{j,1}, \cdots, t_{j,sj}$ $(1 \leq j \leq n)$ respectively, which can all be variables, constants or complex terms. A fuzzy rule in the form of equation 2 can then be represented in RIF-BLD as (for simplicity, we will omit prefix declarations)

```
Document(
 Group
 ( Forall ?x_1 ... ?x_k (
     h(t_1 ... t_r)=?c_h :- And(b_1(t_{1,1} ... t_{1,s1})=?c_1 ... b_n(t_{n,1} ... t_{n,sn})=?c_n
        ?c_t =External(F_IN(?c_1 ... ?c_n))
        ?c_h=External(F_IM(c ?c_t)))  ))
```

Each predicate in the fuzzy rule thus becomes a function. Body predicates $b_i$ $(1 \leq i \leq n)$ in the fuzzy rule has uncertainty values between 0 and 1 by definition. The semantics of the fuzzy rules is then defined by the norm parameters: the intersection operator $F_{IN}$ and the implication operator $F_{IM}$. For example, if $F_{IN}$ and $F_{IM}$ are specified using the minimum membership function and the multiply membership function respectively, the semantics of the fuzzy rules can be encoded in RIF-BLD using the built-in functions numeric-multiply from RIF Datatypes and Built-Ins (RIF-DTB) [25] and an aggregate function numeric-minimum proposed here as an addition to RIF-DTB (this could also be defined using rules). Based on the properties of the functions, it is fairly obvious that the uncertainty value for the variable $?c_t$ is a positive number less than 1 and the value for the variable $?c_h$ (i.e., the value returned for the head predicate function)is between 0 and 1. Therefore, each predicate in the fuzzy rule returns a uncertainty value between 0 and 1.

A fact of the form $H \ \ /c$ can be represented in RIF-BLD presentation syntax as

```
h(t_1 ... t_r)=c
```

*Example 3.* We can rewrite example 1 using RIF functions as follows:

```
(* <http://example.org/fuzzy/membershipfunction > *)
Document(
 Group
 ( (* "Definition of membership function left_shoulder(0, 4000, 1000, 3000)"[] *)
   Forall ?y(
      left_shoulder0k4k1k3k(?y)=1 :- And(External(numeric-less-than-or-equal(0 ?y))
         External(numeric-less-than-or-equal(?y 1000))))
   Forall ?y(
      left_shoulder0k4k1k3k(?y)=External(numeric-add(External(numeric-multiply(-0.0005?y))
1.5))
         :- and(External(numeric-less-than(1000 ?y))
         External(numeric-less-than-or-equal(?y 3000))))
   Forall ?y(
      left_shoulder0k4k1e3k(?y)=0 :- And(External(numeric-less-than(3000 ?y))
         External(numeric-less-than-or-equal(?y 4000))))
. .
) )
```

Note that membership function $left\_shoulder(0, 4000, 1000, 3000)$ is encoded as three rules.

```
Document(
 Import(<http://example.org/fuzzy/membershipfunction >)
 Group
 ( Forall ?x ?y(
      cheapFlight(?x ?y)=?c_h :- And(affordableFlight(?x ?y)=?c_1
         ?c_h=External(numeric-multiply(0.9 ?c_1))))
  Forall ?x ?y(affordableFlight(?x ?y)=left_shaulder0k4k1k3k(?y))
) )
```

The Import statement loads the left_shoulder0k4k1k3k function defined at the given "< . . . >" IRI.

*Example 4.* We can rewrite example 2 in RIF functions as follows:

```
Document(
 Group
 ( Forall ?x(
     A(?x)=?x_h :- And(B(?c)=?c_1 C(?x)=?c_2
        ?c_t =External(numeric-minimum(?c_1 ?c_2))
        ?c_h=External(numeric-multiply(0.5 ?c_t))))
   Forall ?x(
     C(?x)= ?c_h :- And(D(?x)=?c_1 ?c_h=External(numeric-multiply(0.5 ?c_1)))
   )
   B(d)=0.5
   D(d)=0.8
) )
```

### 3.3 Encoding Uncertainty Using RIF Predicates

Another encoding technique is making all n-ary predicates into (1+n)-ary predicates, each being functional in the first argument which captures the certainty factor of predicate applications. A fuzzy rule in the form of equation 2 can then be represented in RIF-BLD as

```
Document(
 Group
 ( Forall ?x_1 ... ?x_k (
     h(?c_h t_1 ... t_r) :- And(b_1(?c_1 t_{1,1} ... t_{1,s1}) ... b_n(?c_n t_{n,1} ... t_{c,sn})
        ?c_t =Exetrnal(F_{IN}(?c_1 ... ?c_n))
        ?c_h=Exetrnal(F_{IM}(c ?c_t))    )
) )
```

Likewise, a fact of the form $H \quad /c$ can be represented in RIF-BLD as

```
h(c t_1 ... t_r)
```

*Example 5.* We can rewrite example 1 in RIF predicates as follows:

```
Document(
 Import (<http://example.org/fuzzy/membershipfunction>)
 Group
 ( Forall ?x ?y(
     cheapFlight(?c_h ?x ?y) :- And(affordabldFlight(?c_1 ?x ?y)
        ?c_h=External(numberic-multiply(0.9 ?c_1)))
        )
   Forall ?x ?y(affordableFlight(?c_1?x ?y) :- ?c_1=left_shoulder0k4k1k3k(?y))
) )
```

# 4 Uncertainty Extension of RIF

In this section, we adapt the definition of the set of truth values from RIF-FLD and its semantic structure. We then propose a RIF extension for directly representing uncertain knowledge.

## 4.1 Definition of Truth Values and Truth Valuation

In previous sections, we showed how to represent the semantics of fuzzy LP with RIF functions and predicates in RIF presentation syntax. We now propose to introduce a new dialect for RIF, RIF Uncertainty Rule Dialect (RIF-URD), so as to directly represent uncertain knowledge and extend the expressive power of RIF.

The set $TV$ of truth values in RIF-BLD consists of just two values, $t$ and $f$. This set forms a two-element Boolean algebra with $t = 1$ and $f = 0$. However, in order to represent uncertain knowledge, all intermediate truth values must be allowed. Therefore, the set $TV$ of truth values is extended to a set with infinitely many truth values ranging between 0 and 1. Our uncertain knowledge representation is specifically based on Fuzzy Logic, thus a member function maps a variable to a truth value in the 0 to 1 range.

**Definition 1.** *(Set of truth values as a specialization of the set in RIF-FLD) In RIF-FLD, $\leq_t$ denotes the truth order, a binary relation on the set of truth values $TV$. Instantiating RIF-FLD, which just requires a partial order, the set of truth values in RIF-URD is equipped with a total order over the 0 to 1 range. In RIF-URD, we specialize $\leq_t$ to $\leq$, denoting the numerical truth order. Thus, we observe that the following statements hold for any element $e_i$, $e_j$ or $e_k$ in the set of truth values $TV$ in the 0 to 1 range, justifying to write it as the interval [0,1].*

1. *The set $TV$ is a complete lattice with respect to $\leq$, i.e., the least upper bound (lub) and the greatest lower bound (glb) exist for any subset of $\leq$.*
2. *Antisymmetry. If $e_i \leq e_j$ and $e_j \leq e_i$ then $e_i = e_j$.*
3. *Transitivity. If $e_i \leq e_j$ and $e_j \leq e_k$ then $e_i \leq e_k$.*
4. *Totality. Any two elements should satisfy one of these two relations: $e_i \leq e_j$ or $e_j \leq e_i$.*
5. *The set $TV$ has an operator of negation, $\sim: TV \rightarrow TV$, such that*
   *(a) $\sim e_i$=1-$e_i$*
   *(b) $\sim$ is self-inverse, i.e., $\sim\sim e_i$=$e_i$.*

*Let $TVal(\varphi)$ denote the truth value of a non-document formula, $\varphi$, in RIF-BLD. Here a non-document formula could be a well-formed term whose signature is formula, or a group formula, but not a document formula. $TVal(\varphi)$ is a mapping from the set of all non-document formulas to $TV$, $I$ denotes an interpretation, and c is a real number in the interval [0,1].*

**Definition 2.** *(Truth valuation adapted from RIF-FLD) Truth valuation for well-formed formulas in RIF-URD is determined as in RIF-FLD, adapting the following cases.*

*(1) Conjunction ($glb_t$ becomes $F_{IN}$): $TVal_I(And(B_1 \cdots B_n)) = F_{IN}(TVal(B_1) \cdots TVal(B_n))$.*
*(2) Disjunction ($lub_t$ becomes $F_U$): $TVal_I(Or(B_1 \cdots B_n)) = F_U(TVal(B_1) \cdots TVal(B_n))$.*
*(3) Rule implication (t becomes 1, f becomes 0, condition valuation is multiplied with c):*
$TVal_I(conclusion : -condition\ /c) = 1\ if\ \ TVal_I(conclusion) \geq F_{IM}(c, TVal_I(condition))$
$TVal_I(conclusion : -condition\ /c) = 0\ if\ \ TVal_I(conclusion) < F_{IM}(c, TVal_I(condition))$

### 4.2 Using RIF-URD to Represent Uncertain Knowledge

A fuzzy rule in the form of equation 2 can be directly represented in RIF-URD as

```
Document(
 Group
 ( Forall ?x1 ... ?xk (
      h(t1 ... tr) :- And(b1(t1,1 ... t1,s1) ... bn(tn,1 ... tn,sn))
 ) / c
)
```

Likewise, a fact of the form $H\ \ /c$ can be represented in RIF-URD as

```
h(t1 ... tr) / c
```

Such a RIF-URD document of course cannot be executed by an ordinary RIF-compliant reasoner. RIF-URD-compliant reasoners will need to be extended to support the above semantics and the reasoning process shown in Section 3.

*Example 6.* We can directly represent example 1 in RIF predicates as follows:

```
Document(
 Import (<http://example.hog/fuzzy/membershipfunction >)
 Group
 ( Forall ?x ?y(
      cheapFlight(?x ?y) :- affordableFlight(?x ?y)
      ) / 0.9
   Forall ?x ?y(affordableFlight(?x ?y)) / left_shoulder0k4k1k3k(?y)
 ) )
```

## 5 Fuzzy Description Logic Programs and Their Representation in RIF

In this section, we extend Description Logic Programs (DLP) [11] to Fuzzy DLP by fuzzifizing each axiom in DLP and studying the semantics and the mappings in Fuzzy DLP; we also show how to represent such mappings in RIF-BLD and RIF-URD based on the three uncertainty treatment methods addressed in previous sections.

Since DL is a subset of FOL, it can also be seen in terms of that subset of FOL, where individuals are equivalent to FOL constants, concepts and concept descriptions are equivalent to FOL formulas with one free variable, and roles and role descriptions are equivalent to FOL formulas with two free variables.

A concept inclusion axiom of the form $C \sqsubseteq D$ is equivalent to an FOL sentence of the form $\forall x.C(x) \rightarrow D(x)$, i.e. an FOL implication. In uncertainty representation and reasoning, it is important to represent and compute the degree of subsumption between two fuzzy concepts, i.e., the degree of overlap, in addition to crisp subsumption. Therefore, we consider fuzzy axioms of the form $C \sqsubseteq D \quad = c$ generalizing the crisp $C \sqsubseteq D$. The above equivalence leads to a straightforward mapping from a fuzzy concept inclusion axiom of the form $C \sqsubseteq D \quad = c (c \in [0, 1])$ to an LP rule as follows: $D(x) \leftarrow C(x) \ \ /c$.

Similarly, a role inclusion axiom of the form $R \sqsubseteq P$ is equivalent to an FOL sentence consisting of an implication between two roles. Thus we map a fuzzy role inclusion axiom of the form $R \sqsubseteq P = c(c \in [0, 1])$ to a fuzzy LP rule as $P(x, y) \leftarrow R(x, y) \ /c$. Moreover, $\cap_{i=1}^{n} R_i \sqsubseteq P = c$ can be transformed to $P(x, y) \leftarrow R_1(x, y), \cdots, R_n(x, y) \ /c$.

A DL assertion $C(a)$ (respectively, $R(a, b)$) is equivalent to an FOL atom of the form $C(a)$ (respectively, $R(a, b)$), where $a$ and $b$ are individuals. Therefore, a fuzzy DL concept-individual assertion of the form corresponds to a ground fuzzy atom $C(a) \ /c$ in fuzzy LP, while a fuzzy DL role-individual assertion of the form $R(a, b) = c$ corresponds to a ground fuzzy fact $R(a, b) \ /c$.

The intersection of two fuzzy concepts in fuzzy DL is defined as $(C_1 \sqcap C_2)^I(x) = F_{IN}(C_1^I(x), C_2^I(x))$. Therefore, a fuzzy concept inclusion axiom of the form $C_1 \sqcap C_2 \sqsubseteq D \quad = c$ including the intersection of $C_1$ and $C_2$ can be transformed to an LP rule $D(x) \leftarrow C_1(x), C_2(x) \ \ /c$. Here the certainty degree of (variable-free) instantiations of the atom $D(x)$ is defined by the valuation $val(D, H_I) \geq F_{IM}(c, F_{IN}(val(C_i, H_I)|i \in \{1, 2\}))$. If the intersection connective is within the *Superclass* description, that is, $C \sqsubseteq D_1 \sqcap D_2 \quad = c$, it can be transformed to LP rules $D_1(x) \leftarrow C(x) \ \ /c$ and $D_2(x) \leftarrow C(x) \ \ /c$. Instantiations of the atoms $D_1$ and $D_2$ as well as the conjunctive query of the two atoms have a certainty degree defined by the valuation $F_{IM}(c, val(C, H_I))$. It is easy to see that such fuzzy concept inclusion axioms can be extended to include the intersection of $n$ concepts ($n > 2$). Furthermore, when the *Union* connective is adopted in the subclass descriptions of a fuzzy concept inclusion axiom, $C_1 \sqcup C_2 \sqsubseteq D \quad = c$, it can be transformed to two LP rules $D(x) \leftarrow C_1(x) \ \ /c$ and $D(x) \leftarrow C_2(x) \ \ /c$. Semantically, the certainty degree of the atom $D(x)$ is defined by the valuation $val(D, H_I) \geq F_{IM}(c, F_U(val(C_i, H_I)|i \in \{1, 2\}) = F_U(F_{IM}(c, val(C_1, H_I)), F_{IM}(c, val(C_2, H_I)))$.

For an axiom stating that the *Domain* of a property $R$ is a class $C$ is true to some degree, $\top \sqsubseteq \forall R^{-}.C \quad = c$, it can be mapped to a fuzzy LP rule $C(x) \leftarrow R(x, y) \ \ /c$ with the valuation $val(C, H_I) \geq F_{IM}(c, val(R, H_I))$; an axiom stating that the *Range* of a property $R$ is a class $C$, $\top \sqsubseteq \forall R.C \quad = c$, can be mapped to a fuzzy LP rule $C(y) \leftarrow R(x,y) \quad /c$ with the valuation $val(C, H_I) \geq F_{IM}(c, val(R, H_I))$. As in DLP, Fuzzy DLP allows the *Universal quantification* within superclass descriptions, $C \sqsubseteq \forall R.D \quad = c$. Such an axiom is mapped to the following fuzzy LP rule $D(y) \leftarrow C(x), R(x,y) \quad /c$. Next, a fuzzy axiom using the *Existential quantification* within subclass descriptions in the form of $\exists R.C \sqsubseteq D \quad = c$ can be mapped to the fuzzy LP rule $D(x) \leftarrow C(y), R(x,y) \quad /c$.

In classic logics, a role $R$ is symmetric iff for all $x, y \in H_I$, $val(R^-, H_I) = val(R, H_I)$, where $R^-$ defines the inverse of a role. The same property holds for a fuzzy symmetric role. Therefore, in Fuzzy DLP, the axiom stating that a property $R$ is the Inverse of a property $P$ has the same syntax as in DLP.

In classic logics, a role $R$ is transitive iff for all $x, y, z \in H_I$, $R(x,y)$ and $R(y,z)$ imply $R(x,z)$. While in Fuzzy Logic, a fuzzy role $R$ is transitive iff for all $x, y, z \in H_I$, it satisfies the following inequality [8]:

$$R(x,z) \geq \sup_{y \in H_I} F_{IN}(R(x,y), R(y,z)) \tag{3}$$

where $F_{IN}$ denotes the intersection operator. For example, in the case of Zadeh Logic, a transitive role satisfies:

$$R(x,z) \geq \sup_{y \in H_I} \min(R(x,y), R(y,z)) \tag{4}$$

Therefore, in Fuzzy DLP, we define the axiom stating that a property $R$ is *Transitive* use the following syntax $R^+ \sqsubseteq R$. Table 2 summarizes all the mappings in Fuzzy DLP. In summary, Fuzzy DLP is an extension of Description Logic Programs supporting the following concept and role inclusion axioms, range and domain axioms, concept and role assertion axioms to build a knowledge base: $\cap_{i=1}^n C_i \sqsubseteq D = c$, $\top \sqsubseteq \forall R.C = c$, $\top \sqsubseteq \forall R^-.C = c$, $\cap_{i=1}^n R_i \sqsubseteq P = c$, $P \equiv R^-$, $R^+ \sqsubseteq R$, $C(a) = c$, and $R(a,b) = c$, where $C, D, C_1, \cdots, C_n$ are concepts, $P, R$ are roles, $a, b$ are individuals, $c \in [0,1]$ and $n \geq 1$. Notice that the crisp DLP axioms in DLP are special cases of their counterparts in Fuzzy DLP. For example, $C \sqsubseteq D$ is equal to its fuzzy version $\cap_{i=1}^n C_i \sqsubseteq D = c$ for $n = 1$ and $c = 1$.

Table 2: Representing Fuzzy DLP in RIF

| | |
|---|---|
| LP syntax | $D(x) \leftarrow C_1(x), \cdots, C_n(x) \quad /c$ |
| DL syntax | $\cap_{i=1}^n C_i \sqsubseteq D = c$ |
| RIF function | $Forall\ ?x(\ D(?x) = ?c_h \quad :-$<br>$And(C_1(?x) = ?c_1 \cdots C_n(?x) = ?c_n$<br>$?c_t = External(F_{IN}(?c_1 \cdots ?c_n))$<br>$?c_h = External(F_{IM}(c\ \ ?c_t)))$ |
| RIF predicate | $Forall\ ?x(\ D(?c_h ?x) \quad :-$<br>$And(C_1(?c_1\ ?x) \cdots C_n(?c_n\ ?x)$<br>$?c_t = External(F_{IN}(?c_1 \cdots ?c_n))$<br>$?c_h = External(F_{IM}(c\ \ ?c_t)))$ |

Continued on next page

| | |
|---|---|
| RIF-URD | $Forall\ ?x($<br>$D(?x)\ \ :-\ And(C_1(?x)\cdots C_n(?x))$<br>$)\ \ /c$ |
| LP syntax | $P(x,y) \leftarrow R_1(x,y),\cdots,R_n(x,y)\quad /c$ |
| DL syntax | $\cap_{i=1}^{n} R_i \sqsubseteq P = c$ |
| RIF function | $Forall\ ?x\ ?y(\ P(?x\ ?y) = ?c_h\ \ :-$<br>$And(R_1(?x\ ?y) = ?c_1 \cdots R_n(?x\ ?y) = ?c_n$<br>$?c_t = External(F_{IN}(?c_1 \cdots ?c_n))$<br>$?c_h = External(F_{IM}(c\ \ ?c_t)))$ |
| RIF predicate | $Forall\ ?x\ ?y(\ P(?c_h\ ?x\ ?y)\ \ :-$<br>$And(R_1(?c_1\ ?x\ ?y) \cdots R_n(?c_n\ ?x\ ?y)$<br>$?c_t = External(F_{IN}(?c_1 \cdots ?c_n))$<br>$?c_h = External(F_{IM}(c\ \ ?c_t)))$ |
| RIF-URD | $Forall\ ?x\ ?y($<br>$P(?x\ ?y)\ \ :-\ And(R_1(?x\ ?y) \cdots R_n(?x\ ?y))$<br>$)\ \ /c$ |
| LP syntax | $C(y) \leftarrow R(x,y)\ \ /c$ |
| DL syntax | $\top \sqsubseteq \forall R.C\ \ = c$ |
| RIF function | $Forall\ ?x?y(\ C(?y) = ?c_h\ \ :-$<br>$And(R(?x\ ?y) = ?c_1\ \ ?c_h = External(F_{IM}(c\ \ ?c_1)))$ |
| RIF predicate | $Forall\ ?x?y(\ C(?c_h\ ?y)\ :-$<br>$And(R(?c_1\ ?x\ ?y)\ \ ?c_h = External(F_{IM}(c\ \ ?c_1)))$ |
| RIF-URD | $Forall\ ?x?y(\ \ C(?y)\ \ :-\ R(?x\ ?y))\ /c$ |
| LP syntax | $C(x) \leftarrow R(x,y)\ \ /c$ |
| DL syntax | $\top \sqsubseteq \forall R^{-}.C\ \ = c$ |
| RIF function | $Forall\ ?x?y(\ C(?x) = ?c_h\ \ :-$<br>$And(R(?x\ ?y) = ?c_1\ \ ?c_h = External(F_{IM}(c\ \ ?c_1)))$ |
| RIF predicate | $Forall\ ?x?y(\ C(?c_h\ ?x)\ :-$<br>$And(R(?c_1\ ?x\ ?y)\ \ ?c_h = External(F_{IM}(c\ \ ?c_1)))$ |
| RIF-URD | $Forall\ ?x?y(\ \ C(?x)\ \ :-\ R(?x\ ?y))\ /c$ |
| LP syntax | $C(x) \leftarrow R(x,y)\ \ /c$ |
| DL syntax | $\top \sqsubseteq \forall R^{-}.C\ \ = c$ |
| RIF function | $Forall\ ?x?y(\ C(?x) = ?c_h\ \ :-$<br>$And(R(?x\ ?y) = ?c_1\ \ ?c_h = External(F_{IM}(c\ \ ?c_1)))$ |
| RIF predicate | $Forall\ ?x?y(\ C(?c_h\ ?x)\ :-$<br>$And(R(?c_1\ ?x\ ?y)\ \ ?c_h = External(F_{IM}(c\ \ ?c_1)))$ |
| RIF-URD | $Forall\ ?x?y(\ \ C(?x)\ \ :-\ R(?x\ ?y))\ /c$ |
| LP syntax | $D_1(x) \leftarrow C(x)\ /c, D_2(x) \leftarrow C(x)\ /c$ |
| DL syntax | $C \sqsubseteq D_1 \sqcap D_2\ \ = c$ |
| RIF function | $Forall\ ?x(\ D_1(?x) = ?c_h\ \ :-$<br>$And(C(?x) = ?c_1\ \ ?c_h = External(F_{IM}(c\ \ ?c_1)))$<br>$Forall\ ?x(\ D_2(?x) = ?c_h\ \ :-$<br>$And(C(?x) = ?c_1\ \ ?c_h = External(F_{IM}(c\ \ ?c_1)))$ |
| RIF predicate | $Forall\ ?x(\ D_1(?c_h\ ?x)\ :-$<br>$And(C(?c_1\ ?x)\ ?c_h = External(F_{IM}(c\ \ ?c_1)))$<br>$Forall\ ?x(\ D_2(?c_h\ ?x)\ :-$ |

Continued on next page

| | |
|---|---|
| | $And(C(?c_1\ ?x)\ ?c_h = External(F_{IM}(c\ \ ?c_1)))$ |
| RIF-URD | $Forall\ ?x(\ \ D_1(?x)\ \ :-C(?x))\ /c$<br>$Forall\ ?x(\ \ D_2(?x)\ \ :-C(?x))\ /c$ |
| LP syntax | $D(x) \leftarrow C_1(x)\ /c, D(x) \leftarrow C_2(x)\ /c$ |
| DL syntax | $C_1 \sqcup C_2 \sqsubseteq D\ \ = c$ |
| RIF function | $Forall\ ?x(\ D(?x) = ?c_h\ \ :-$<br>$And(C_1(?x) = ?c_1\ \ C_2(?x) = ?c_2$<br>$?c_h = External(F_U(F_{IM}(c\ \ ?c_1), F_{IM}(c\ \ ?c_2)))$ |
| RIF predicate | $Forall\ ?x(\ D(?c_h\ ?x)\ :-$<br>$And(C_1(?c_1\ ?x)\ \ C_2(?c_2\ ?x)$<br>$?c_h = External(F_U(F_{IM}(c\ \ ?c_1), F_{IM}(c\ \ ?c_2)))$ |
| RIF-URD | $Forall\ ?x(\ \ D(?x)\ \ :-C_1(?x))\ /c$<br>$Forall\ ?x(\ \ D(?x)\ \ :-C_2(?x))\ /c$ |
| LP syntax | $D(y) \leftarrow C(x), R(x,y)\ \ /c$ |
| DL syntax | $C \sqsubseteq \forall R.D\ \ = c$ |
| RIF function | $Forall\ ?x?y(\ D(?y) = ?c_h\ \ :-$<br>$And(C(?x) = ?c_1\ \ R(?x\ ?y) = ?c_2\ \ ?c_h = External(F_{IM}(c\ \ F_{IN}(?c_1\ ?c_2))))$ |
| RIF predicate | $Forall\ ?x?y(\ D(?c_h\ ?y)\ :-$<br>$And(C(?c_1\ ?x)\ \ R(?c_2\ ?x\ ?y)\ ?c_h = External(F_{IM}(c\ \ F_{IN}(?c_1\ ?c_2))))$ |
| RIF-URD | $Forall\ ?x?y(\ \ D(?y)\ \ :-And(C(?x)\ R(?x\ ?y)))\ /c$ |
| LP syntax | $D(x) \leftarrow C(y), R(x,y)\ \ /c$ |
| DL syntax | $\exists R.C \sqsubseteq D\ \ = c$ |
| RIF function | $Forall\ ?x?y(\ D(?x) = ?c_h\ \ :-$<br>$And(C(?y) = ?c_1\ \ R(?x\ ?y) = ?c_2\ \ ?c_h = External(F_{IM}(c\ \ F_{IN}(?c_1\ ?c_2))))$ |
| RIF predicate | $Forall\ ?x?y(\ D(?c_h\ ?x)\ :-$<br>$And(C(?c_1\ ?y)\ \ R(?c_2\ ?x\ ?y)\ ?c_h = External(F_{IM}(c\ \ F_{IN}(?c_1\ ?c_2))))$ |
| RIF-URD | $Forall\ ?x?y(\ \ D(?x)\ \ :-And(C(?y)\ R(?x\ ?y)))\ /c$ |
| LP syntax | $R(x,y) \leftarrow P(y,x), P(y,x) \leftarrow R(x,y)$ |
| DL syntax | $R^- \equiv P$ |
| RIF function | $Forall\ ?x\ ?y(\ R(?x\ ?y) = ?c_h\ \ :-$<br>$And(P(?y\ ?x) = ?c_1\ \ ?c_h = ?c_1)$ |
| RIF predicate | $Forall\ ?x\ ?y(\ R(?c_h\ ?x\ ?y)\ :-$<br>$And(P(?c_1\ ?y\ ?x)\ ?c_h = ?c_1)$ |
| RIF-URD | $Forall\ ?x\ ?y(\ \ R(?x\ ?y)\ \ :-P(?y\ ?x))$ |

| | | |
|---|---|---|
| LP syntax | $C(a)\ /c$ | $R(a,b)\ /c$ |
| DL syntax | $C(a)\ \ = c$ | $R(a,b)\ \ = c$ |
| RIF function | $C(a)\ \ = c$ | $R(a\ b)\ \ = c$ |
| RIF predicate | $C(c\ a)$ | $R(c\ a\ b)$ |
| RIF-URD | $C(a)\ /c$ | $R(a\ b)\ /c$ |

In previous sections, we presented two techniques of encoding uncertainty in RIF and proposed a method based on an extension of RIF for uncertainty representation. Subsequently, we also showed how to represent Fuzzy DLP in RIF-BLD and RIF-URD in Table 2.

Layered on Fuzzy DLP, we can build fuzzy hybrid knowledge bases in order to build fuzzy rules on top of ontologies for the Semantic Web and reason on such KBs.

**Definition 3.** *A fuzzy hybrid knowledge base $KB_{hf}$ is a pair $< K_{DL}, K_{LP} >$, where $K_{DL}$ is the finite set of (fuzzy) concept inclusion axioms, role inclusion axioms, and concept and role assertions of a decidable DL defining an ontology. $K_{LP}$ consists of a finite set of (fuzzy) hybrid rules and (fuzzy) facts.*

A hybrid rule $r$ in $K_{LP}$ is of the following generalized form (we use the BNF choice bar, $|$):

$$(H(\overrightarrow{y})|\&H(\overrightarrow{z})) \leftarrow B_1(\overrightarrow{y1}), \cdots, B_l(\overrightarrow{yl}), \&Q_1(\overrightarrow{z1}), \cdots, \&Q_n(\overrightarrow{zn}) \quad /c \qquad (5)$$

Here, $H(\overrightarrow{y}), H(\overrightarrow{z}), B_i(\overrightarrow{yi}), Q_j(\overrightarrow{zj})$ are atoms, & precedes a DL atom, $\overrightarrow{y}, \overrightarrow{z}, \overrightarrow{yi}, \overrightarrow{zj}$ are vectors of variables or constants, where $\overrightarrow{y}$ and each $\overrightarrow{yi}$ have arbitrary lengths, $\overrightarrow{z}$ and each $\overrightarrow{zj}$ have length 1 or 2, and $c \in [0, 1]$. Also, & atoms and $/c$ degrees are optional (if all & atoms and $/c$ degrees are missing from a rule, it becomes a classical rule of Horn Logic).

Such a fuzzy hybrid rule must satisfy the following constraints:

(1) $H$ is either a DL predicate or a rule predicate ($H \in \sum T \bigcup \sum R$). $H$ is a DL predicate with the form $\&H$, while it is a rule predicate without the & operator.

(2) Each $B_i$ ($1 < i \leq l$) is a rule predicate ($B_i \in \sum R$), and $B_i(y_i)$ is an LP atom.

(3) Each $Q_j$ ($1 < j \leq n$) is a DL predicate ($Q_j \in \sum T$), and $Q_j(z_j)$ is a DL atom.

(4, pure DL rule) If a hybrid rule has head $\&H$, then each atom in the body must be of the form $\&Q_j$ ($1 < j \leq n$); in other words, there is no $B_i$ ($l = 0$). A head $\&H$ without a body ($l = 0$, $n = 0$) constitutes the special case of a pure DL fact.

*Example 7.* The rule $\&CheapFlight(x, y) \leftarrow AffordableFlight(x, y) \ \ /c$ is not a pure DL rule according to (4), hence not allowed in our hybrid knowledge base, while $CheapFlight(x, y) \leftarrow \&AffordableFlight(x, y) \ \ /c$ is allowed.

A hybrid rule of the form $\&CheapFlight(x, y) \leftarrow \&AffordableFlight(x, y) \ /c$ can be mapped to a fuzzy DL role subsumption axiom $AffordableFlight \sqsubseteq CheapFlight = c$.

Our approach thus allows DL atoms in the head of hybrid rules which satisfy the constraint (4, pure DL rule), supporting the mapping of DL subsumption axioms to rules. We also deal with fuzzy subsumption of fuzzy concepts of the form $C \sqsubseteq D = c$ as shown in Example 7.

An arbitrary hybrid knowledge base cannot be fully embedded into the knowledge representation formalism of RIF with uncertainty extensions. However, in the proposed Fuzzy DLP subset, DL components (DL axioms in LP syntax) can be mapped to LP rules and facts in RIF. A RIF-compliant reasoning engine can be extended to do reasoning on a hybrid knowledge base on top of Fuzzy DLP by adding a module that first maps atoms in rules to DL atoms, and then derives the reasoning answers with a DL reasoner, e.g. Racer or Pellet, or with a fuzzy DL reasoner, e.g. fuzzyDL [2]. The specification of such a reasoning algorithm for a fuzzy hybrid knowledge base $KB_{hf}$ based on Fuzzy DLP and a query $q$ is treated in a companion paper [38].

## 6 Conclusion

In this paper, we propose two different principles of representing uncertain knowledge, encodings in RIF-BLD and an extension leading to RIF-URD. We also present a fuzzy extension to Description Logic Programs, namely Fuzzy DLP. We address the mappings between fuzzy DL and fuzzy LP within Fuzzy DLP, and give Fuzzy DLP representations in RIF. Since handling uncertain information, such as with fuzzy logic, was listed as a RIF extension in the RIF Working Group Charter [3] and RIF-URD is a manageable extension to RIF-BLD, we propose here a version of URD as a RIF dialect, realizing a fuzzy rule sublanguage for the RIF standard.

The paper is an extended version of our previous work with the same title [39]. Here we presented a unified framework for uncertainty representation in RIF. Our fuzzy extension directly relates to the semantics of fuzzy sets and fuzzy logic, allowing the parameterization of RIF-URD to support Lotfi Zadeh's, Jan Lukasiewicz's, and other classes in the family of fuzzy logics. We do not yet cover here cover other uncertainty formalisms, based on probability theory, possibilities, or rough sets. Future work will include generalizing our fuzzy extension of hybrid knowledge bases to some of these different kinds of uncertainty.

The combination strategy presented in this paper is based on resolving some atoms in the hybrid knowledge base to DL queries. Therefore, another direction of future work would be the extension of uncertain knowledge to various combination strategies of DL and LP without being limited to DL queries.

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