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CREEP OF STRUCTURAL STEEL IN FIRE: ANALYTICAL EXPRESSIONS

by G. Williams-Leir

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Deux modèles de fluage, l'un proposé par Harmathy et l'autre empirique, ont été adaptés aux mesures de fluage connues pour sept types de structures, de façon à permettre de déterminer si le fluage est protégé ou en béton armé.

RéSUMÉ
Creep of Structural Steel in Fire: Analytical Expressions

G. Williams-Leir
Division of Building Research, National Research Council of Canada, Ottawa, Canada

Two models of creep, one proposed by Harmathy, the other empirical, have been fitted to published measurements of creep in seven steels. The data have been selected so as to cover the range needed for predicting the survival of protected steel or reinforced concrete buildings exposed to fire.

When steel is repeatedly subjected to moderate loads at normal temperatures, whether in tension or compression, it returns to its original dimensions on removal of the load, and while loaded, or stressed, its elongation or strain is proportional to the stress. The frame of a steel building can be expected to suffer no permanent harm unless its temperature or stress exceeds certain levels, and the same observation applies to the reinforcement in a reinforced concrete building. When these levels are passed, damage to the building is not immediate, but a sufficient stress can then produce a time-dependent non-reversible strain. This strain is known as creep, and the survival of the building will depend on its magnitude.

The engineer responsible for the structural design of any major building can discharge his responsibilities only by estimating the fuel load, the heat balance and the temperature and duration of a potential fire, and ensuring that the steelwork is sufficiently protected that it will not suffer an unacceptable strain. The actual strain developed will depend upon the stress and temperature history in the fire and on the creep properties of the steel.

It is customary to recognize three stages or phases of creep once temperature and stress are high enough for creep to be significant: primary, where rate of strain at constant stress and temperature decreases with time from first application of stress; secondary, where rate is constant; and tertiary, where rate increases until rupture (see Fig. 1). It is considered that strains unacceptable in the steel of a building will develop no later than during the secondary stage, so that it is not essential to the design to predict the creep behaviour of the steel into the tertiary stage.

The composition, forging procedure and heat treatment of steels differ, resulting in products with very different mechanical properties. Data already published on creep of steel is not readily usable by building designers. For each steel, strain can, in principle, be predicted by interpolation in a table of strain measurements. Interpolation would not be difficult if all the points used were free from stochastic variations. In fact, observations of creep at fire temperatures are not very repeatable, and the most probable strain can be estimated only by taking many observations into account. Prediction is most conveniently done by using expressions that relate strain to stress, temperature and duration of stress, and the purpose of this paper is to supply suitable expressions, with a firm experimental base.

The expressions should describe the creep behaviour of each material in terms of the observable determining factors. All other effects have necessarily to be treated as random variation.

No reliable model of creep is yet available that predicts all three stages, but two models will be described that were found acceptable for both primary and secondary creep. These models each depend on several constants; to determine them, it is necessary to make use of published observations of creep in the steel that relate four of the following five quantities:

- $T(\text{K})$: temperature
- $o(\text{lb} \text{f in}^{-2})$: stress ($1 \text{ lbf in}^{-2} = 6.897 \text{ KPa}$)
- $\varepsilon$ (dimensionless): creep strain
- $\dot{\varepsilon} \text{ min}^{-1}$: rate of creep strain
- $t \text{ (min)}$: duration of application of constant stress

The composition, forging procedure and heat treatment of steels differ, resulting in products with very different mechanical properties. Data already published on creep of steel is not readily usable by building designers. For each steel, strain can, in principle, be predicted by interpolation in a table of strain measurements. Interpolation would not be difficult if all the points used were free from stochastic variations. In fact, observations of creep at fire temperatures are not very repeatable, and the most probable strain can be estimated only by taking many observations into account. Prediction is most conveniently done by using expressions that relate strain to

\[ \varepsilon = b_1 \coth^2 (b_2 \varepsilon) \] (1)
Again, when log t, is plotted against log t,, the value of log t, at a specific stress may be inferred. Finally, when log t,,, is plotted against T, a third linear relation is found.

The most successful empirical model so far found will be referred to in this paper as model E. As previously observed, log ε is roughly linear with log t at constant σ and T. From this relation, values of log t, the value of log t at any specific strain ε, may be inferred. Again, when log t, is plotted against σ at constant T, the relationship is found to be nearly linear, and log t,, the value of log t at a specific stress σ, may be inferred. Finally, when log t,, is plotted against T, a third linear relation is found.

where:

\[ b_1 = c_1 \exp \left( c_2 \ln \sigma - c_3/T \right), \quad \sigma < \sigma_t \]  
\[ b_1 = c_6 \exp \left( c_1 \sigma - c_5/T \right), \quad \sigma > \sigma_t \]  
\[ b_2 = 1/(c_3 \sigma^2) \]  

σ is the value of stress at which transition is made between Eqns (2) and (3) for \( b_1 \) (it is taken as 15 000 lbf in\(^{-2}\) (103 MPa)) and \( c_1 \) to \( c_7 \) are constants.

It is easily shown that continuity between the regimes represented by Eqns (2) and (3) requires that:

\[ c_6 = c_1 (\sigma_t/\sigma)^2 \]  
\[ c_7 = c_2/\sigma_t \]  

There are thus only five independent constants, \( c_1 \) to \( c_5 \), in this model.

It will be useful to examine some of the implications of these expressions. Equation (1) may be integrated at constant stress to obtain a relationship between strain at constant stress, temperature and time:

\[ b_2 \varepsilon - \tanh \left( b_2 \varepsilon \right) = b_1 b_3 t \]  

When the group \( b_2 \varepsilon \) is positive but small compared with unity, the Maclaurin–Taylor expansion

\[ \tanh x = x - x^3/3 + \cdots \]  

leads, when terms above \( x^3 \) are neglected, to the expression

\[ \varepsilon \sim (3b_2/2b_2)^{1/3} \]  

whereas for \( b_2 \varepsilon \) large, the tanh term tends toward unity, and

\[ \varepsilon \sim 1/b_2 + b_1 t \]  

Graphs of log ε against log t normally have a linear section, and their gradients should thus lie within the range \( \frac{1}{3} \) to 1. Examination of the observations bears this out in general, though some gradients are found outside each end of the range. The reasons for this have not been determined, but the gradients observed are thought to lend support to Eqns (1) to (4).

The five coefficients \( c_1 \) to \( c_5 \) were determined by non-linear regression analysis (see below).

The H model is found to fit some of the sets of observations well, as will be discussed later. There are exceptions, however, and these have influenced the decision to search for alternative, totally empirical, models.

**THE EMPirical MODEL**

The most successful empirical model so far found will be referred to in this paper as model E. As previously observed, log ε is roughly linear with log t at constant σ and T. From this relation, values of log t, the value of log t at any specific strain ε, may be inferred. Again, when log t, is plotted against σ at constant T, the relationship is found to be nearly linear, and log t,, the value of log t at a specific stress σ, may be inferred. Finally, when log t,, is plotted against T, a third linear relation is found.

If these relationships were exact, it would follow that:

\[ \log t = a_1 T_C + a_2 \log \varepsilon + a_3 \sigma + a_4 T_C \log \varepsilon + a_5 T_C \log \sigma + a_6 \]  

where \( T_C \) is the Celsius temperature, and in fact this expression is found to fit the data quite well. It has the additional advantage of permitting the use of multiple linear regression analysis to determine the coefficients for the prediction of log t. Once these coefficients are known, Eqn (11) is directly soluble for any of the variables, \( t, \varepsilon, \sigma \) or \( T_C \); thus for example:

\[ \log \varepsilon = \frac{\log t - T_C (a_1 + a_4 \sigma) - a_3 \sigma - a_5 T_C}{a_2 + \sigma (a_0 + a_7 T_C) + a_5 T_C} \]  

To use the same coefficients as before in Eqn (12) would provide a rough estimate of strain, but not the best possible estimate. For predicting \( \varepsilon, \sigma \) or \( T \), the coefficients need to be optimized by non-linear regression analysis, a more complex mathematical procedure, to be described below.

**SELECTION OF OBSERVATIONS**

To develop the most reliable expressions for the prediction of creep in the protected structural steel of burning buildings it is necessary to make certain choices within the wide range of observations available. Below about 500 °C protected steel creeps too slowly to prejudice building survival over the time scale of a fire. (If the frame is unprotected and the building has fuel and openings, early failure can be predicted without detailed analysis.) Observations earlier than 4 min tend to be inconsistent with those for longer periods, and are scarcely relevant to building failure. When strains as large as 2 per cent develop, it is probable that the building will have been for practical purposes already destroyed, so larger strains are of less immediate concern.

Accordingly, only observations at 500 °C or hotter at times of 4 min or greater have been used, and only a few strains exceeding 2 per cent. The strain rates used in Harmathy’s experiments on creep in steel\(^2\) were such that strain exceeded 2 per cent in less than 4 min, so no use was made of the results of this work.

The data that have been used come from two sources: the work of Fujimoto, Furumura, Ave and Shinohara\(^3\) on three steels, SS41, SM50 and SM58 (which will be collectively referred to as the J data) and the work of Knight, Skinner and Lay\(^4\) on four steels, A135, A149, Austen 50 (a ‘weathering steel’) and X-60 (a ‘pipeline steel’) (the A data). A149 is evidently identical with Australian steel 37.\(^5\)

The J results were presented in the form of graphs of strain to 1.5 per cent against time to 6 h for temperatures at 25 degree intervals up to 600 °C and for stresses that are multiples of 2.5 kg mm\(^{-2}\). For the present work, a number of points, typically four, have been chosen that characterize each strain–time curve, and all the points have been used in the calculations.
Table 1. Elastic modulus and temperature coefficient (see Eqn (13)) (calculated from data in references 3 and 4)

<table>
<thead>
<tr>
<th>Steel type</th>
<th>( c_8 \times 10^7 ) (lb in(^{-2} ))</th>
<th>( c_9 \times 10^{-5} ) K(^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A135</td>
<td>3.00</td>
<td>1.51</td>
</tr>
<tr>
<td>A149</td>
<td>3.00</td>
<td>1.51</td>
</tr>
<tr>
<td>X-60</td>
<td>3.00</td>
<td>1.51</td>
</tr>
<tr>
<td>Austen 50</td>
<td>3.00</td>
<td>1.51</td>
</tr>
<tr>
<td>SS41</td>
<td>2.98</td>
<td>0.91</td>
</tr>
<tr>
<td>SM50</td>
<td>3.07</td>
<td>1.04</td>
</tr>
<tr>
<td>SM58</td>
<td>3.04</td>
<td>1.50</td>
</tr>
</tbody>
</table>

In contrast, the A results are given in the form of tables showing strains at times from 2 min to 16 h, at temperature intervals of 50 degrees up to 650°C, and at various stresses. The values are not all consistent with smooth curves, and a selection procedure was necessary. The first step was to plot log \( \varepsilon \) against log \( t \) for each \( T \) and \( \sigma \), to identify the straight section, and discard points outside it. Using only the points that passed this first test, log \( \varepsilon \) was plotted against log \( \sigma \) for each \( T \) and \( t \). This typically produced a smooth curve, convex upwards, which was sometimes followed by a concave-upwards section, interpreted as an indication of incipient failure. This section was therefore discarded, together with any other points not consistent with a smooth convex curve.

**ELASTIC STRAIN**

Both Refs 3 and 4 give elastic modulus as a function of temperature for each steel. The following relation, developed by Lie,5 has been fitted to the values given:

\[
E = c_8(1 - c_9(T - 293)) \quad (13)
\]

and the constants \( c_8 \) and \( c_9 \) determined by linear regression analysis. The results are given in Table 1.

**NON-LINEAR REGRESSION ANALYSIS**

From each observation of total strain, the elastic strain

\[
\varepsilon_0 = \sigma/\text{modulus} \quad (14)
\]

was deducted, leaving creep strain. For each model, the constants were optimized, using the program ZXSSQ7 for non-linear regression analysis. The program searches for the set of values of the constants that minimize the sum of squares of residuals, i.e. of differences between each observation and the corresponding prediction. For this purpose, ‘observation’ can mean either the crude measurement or some function of it, and the choice of function will influence the properties of the solution found.

The best prediction of large strains would be obtained by minimizing the residual between observed strain and its predicted value \( \bar{\varepsilon} \):

\[
\sum (\varepsilon - \bar{\varepsilon})^2 \quad (15)
\]

but this will in practice lead to large errors relative to small observed strains. If the residual between the logarithm of strain and that of the prediction is used, the quantity minimized is

\[
\sum (\ln (\bar{\varepsilon}/\varepsilon))^2 \quad (16)
\]

This produces minimal relative error, which may mean large errors where large strains were observed.

Since the choice is arbitrary, a compromise may be sought between these extremes. It may be shown that

\[
\sum ((\varepsilon - \bar{\varepsilon})^2/\varepsilon) \quad (17)
\]

is such a compromise, and this expression has been used throughout the present work.

**COMPARISON OF PREDICTIONS WITH OBSERVATIONS**

It was not feasible to present here a comprehensive comparison of predictions with observations for seven steels and two models. Representative comparisons are given in Figs 2 and 3. Coefficients for all the steels are given in Tables 2-4. (Table 3 represents a linear regression of log \( \varepsilon \) on \( \sigma \), \( T \), and log \( \varepsilon \), while the coefficients in Table 4 have been optimized for prediction of strain by non-linear regression analysis, as described above.) SI units are not exclusively used because the format, intervals and units have been chosen for compatibility with those of the reports containing the original observations.

The model \( H \) equations ((2) to (7)) can be solved directly for \( t \). To solve for other variables, strain for example, demands numerical or graphical methods such as ZBRENT2 or the ‘rule of false position’8,9. Those who wish to keep the programming simple, or who do not have access to computers, should solve for \( t \) and then interpolate the other variables, relying on linearity between any two of log \( \varepsilon \), \( \sigma \), \( T \), and log \( \varepsilon \). A simple program to evaluate both predictions and compare them with data points is available from the author. As mentioned above, the model \( E \) equation can be directly solved for any of the four variables.

Figure 2. Creep plus elastic strain for A135 steel against tensile stress for four temperatures from 500 to 650°C and nine times from 4 min to 8 h. Model E fitted to observations.
RANGE OF APPLICABILITY

The models provide predictions that are of utility within a space whose bounds lie inside the following ranges: 500–650 °C, 4 min to 16 h, and strains up to 2 per cent. It would be unwise to rely on either model outside the range of the original data. For total strain, elastic strain based on Eqns (13) and (14) and Table 1 should be added to creep calculated as described.

COMPARISON OF THE SEVEN STEELS

Equations have been supplied for predicting creep in seven steels that differ significantly in their performance. Which of the seven has the lowest creep depends on the conditions; as an average over the ranges examined in this study, SM58, Austen 50 and X-60 creep least, and SS41 and A149 most. This leaves A135 or SM50 in the median or typical position. If the extreme case is desired, SS41 should be used.

Strains of 2 per cent indicate the probability of serious damage. Stresses of 124 MPa (18 000 lbf in⁻²) will serve as an example comparable with typical design practice. Fixing these two variables at these values makes it possible to estimate duration of

Table 2. Coefficients for model H, calculated as described in text

<table>
<thead>
<tr>
<th>Steel type</th>
<th>$c_1$ (min)</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
<th>$c_5$ (°K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A135</td>
<td>1.143 × 10⁻²</td>
<td>4.721</td>
<td>1.261 × 10⁻¹</td>
<td>1.695</td>
<td>43 250</td>
</tr>
<tr>
<td>X-60</td>
<td>2.987 × 10⁻³</td>
<td>4.624</td>
<td>1.696 × 10⁻²</td>
<td>4.661</td>
<td>41 390</td>
</tr>
<tr>
<td>Austen 50</td>
<td>1.264 × 10⁻⁷</td>
<td>4.858</td>
<td>8.564 × 10⁻¹</td>
<td>1.731</td>
<td>44 210</td>
</tr>
<tr>
<td>SM50</td>
<td>3.175 × 10⁻⁷</td>
<td>6.460</td>
<td>4.940 × 10⁻¹</td>
<td>1.843</td>
<td>48 970</td>
</tr>
<tr>
<td>SM58</td>
<td>3.080 × 10⁻³</td>
<td>4.206</td>
<td>9.032 × 10⁻⁵</td>
<td>0.302</td>
<td>40 510</td>
</tr>
<tr>
<td>SS41</td>
<td>7.991 × 10⁻²⁰</td>
<td>3.225</td>
<td>3.485 × 10⁻⁶</td>
<td>6.701</td>
<td>77 390</td>
</tr>
<tr>
<td>A149</td>
<td>1.120 × 10⁻²</td>
<td>4.948</td>
<td>4.914 × 10⁻¹⁰</td>
<td>1.638</td>
<td>44 960</td>
</tr>
</tbody>
</table>

The coefficients $c_2$, $c_3$ and $c_4$ are dimensionless.

Table 3. Coefficients in model E for predicting time to reach a specified strain (calculated)

<table>
<thead>
<tr>
<th>Steel type</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$a_5$</th>
<th>$a_6$</th>
<th>$a_7$</th>
<th>$a_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A149</td>
<td>-1.958 × 10⁻²</td>
<td>9.528 × 10⁻⁵</td>
<td>5.009 × 10⁻⁴</td>
<td>-1.719 × 10⁻⁷</td>
<td>-9.732 × 10⁻⁵</td>
<td>-2.601 × 10⁻⁷</td>
<td>1.430 × 10⁻⁶</td>
<td>16.53</td>
</tr>
<tr>
<td>Au50</td>
<td>-1.937 × 10⁻²</td>
<td>3.802</td>
<td>2.420 × 10⁻⁴</td>
<td>-7.624 × 10⁻⁷</td>
<td>-4.734 × 10⁻³</td>
<td>-1.197 × 10⁻⁶</td>
<td>2.196 × 10⁻⁶</td>
<td>19.20</td>
</tr>
<tr>
<td>X-60</td>
<td>-2.757 × 10⁻²</td>
<td>3.573</td>
<td>1.981 × 10⁻⁴</td>
<td>-6.918 × 10⁻⁷</td>
<td>-3.918 × 10⁻⁶</td>
<td>7.191 × 10⁻⁵</td>
<td>-1.453 × 10⁻⁷</td>
<td>23.59</td>
</tr>
<tr>
<td>SS41</td>
<td>-9.929 × 10⁻³</td>
<td>-1.781</td>
<td>7.592 × 10⁻⁴</td>
<td>-1.829 × 10⁻⁵</td>
<td>5.301 × 10⁻⁴</td>
<td>2.109 × 10⁻⁴</td>
<td>-2.831 × 10⁻⁷</td>
<td>15.69</td>
</tr>
<tr>
<td>SM50</td>
<td>-3.788 × 10⁻²</td>
<td>7.580</td>
<td>-1.236 × 10⁻⁴</td>
<td>-1.493 × 10⁻⁷</td>
<td>-1.112 × 10⁻⁵</td>
<td>-1.634 × 10⁻⁷</td>
<td>3.087 × 10⁻⁷</td>
<td>30.18</td>
</tr>
<tr>
<td>SM58</td>
<td>-5.376 × 10⁻²</td>
<td>9.434</td>
<td>-4.330 × 10⁻⁴</td>
<td>-4.622 × 10⁻⁷</td>
<td>-1.358 × 10⁻⁵</td>
<td>-1.677 × 10⁻⁷</td>
<td>2.647 × 10⁻⁷</td>
<td>38.90</td>
</tr>
</tbody>
</table>

Table 4. Coefficients in Model E for predicting strain at a specified time (calculated)

<table>
<thead>
<tr>
<th>Steel type</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$b_3$</th>
<th>$b_4$</th>
<th>$b_5$</th>
<th>$b_6$</th>
<th>$b_7$</th>
<th>$b_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A135</td>
<td>-1.663 × 10⁻²</td>
<td>-0.8271</td>
<td>5.144 × 10⁻⁴</td>
<td>-1.217 × 10⁻⁶</td>
<td>3.100 × 10⁻³</td>
<td>2.101 × 10⁻⁴</td>
<td>-3.276 × 10⁻⁷</td>
<td>17.26</td>
</tr>
<tr>
<td>A149</td>
<td>-2.204 × 10⁻²</td>
<td>1.050</td>
<td>2.214 × 10⁻⁴</td>
<td>-5.449 × 10⁻⁷</td>
<td>-1.374 × 10⁻³</td>
<td>5.744 × 10⁻⁵</td>
<td>2.013 × 10⁻⁸</td>
<td>18.68</td>
</tr>
<tr>
<td>Au50</td>
<td>-2.742 × 10⁻²</td>
<td>5.733</td>
<td>1.141 × 10⁻⁴</td>
<td>-5.659 × 10⁻⁷</td>
<td>-7.677 × 10⁻³</td>
<td>-5.702 × 10⁻⁵</td>
<td>9.051 × 10⁻⁵</td>
<td>24.44</td>
</tr>
<tr>
<td>X-60</td>
<td>-3.039 × 10⁻²</td>
<td>4.262</td>
<td>2.526 × 10⁻⁴</td>
<td>-8.294 × 10⁻⁷</td>
<td>-4.709 × 10⁻³</td>
<td>1.034 × 10⁻⁴</td>
<td>-2.165 × 10⁻⁷</td>
<td>25.83</td>
</tr>
<tr>
<td>SS41</td>
<td>-2.156 × 10⁻²</td>
<td>0.5973</td>
<td>1.859 × 10⁻⁴</td>
<td>-4.452 × 10⁻⁷</td>
<td>1.035 × 10⁻³</td>
<td>-5.658 × 10⁻⁵</td>
<td>3.031 × 10⁻⁷</td>
<td>20.00</td>
</tr>
<tr>
<td>SM50</td>
<td>-3.571 × 10⁻²</td>
<td>6.918</td>
<td>-1.866 × 10⁻⁵</td>
<td>-3.199 × 10⁻⁷</td>
<td>-1.012 × 10⁻²</td>
<td>-1.158 × 10⁻⁴</td>
<td>2.347 × 10⁻⁷</td>
<td>28.78</td>
</tr>
<tr>
<td>SM58</td>
<td>-5.196 × 10⁻²</td>
<td>9.118</td>
<td>-4.794 × 10⁻⁴</td>
<td>5.674 × 10⁻⁷</td>
<td>-1.322 × 10⁻²</td>
<td>-1.929 × 10⁻⁴</td>
<td>3.162 × 10⁻⁷</td>
<td>37.55</td>
</tr>
</tbody>
</table>
survival of a building as a function of the temperature of its steel structure; this is shown in Fig. 4, as calculated on Model E.

It is a familiar rule of thumb that structural steel loses all strength at about 1000 °F, i.e. 538 °C. Under these conditions survival varies from 7 h for SS41 to 4 days for SM58.

**PREDICTING CREEP WHEN TEMPERATURE VARIES WITH TIME**

All creep observations used in this paper were at constant temperature and stress; within each series of observations, strain depended on time alone. This is the appropriate way to study creep behaviour, but it does not closely reproduce the effects of building fires, where temperature starts from cold. If the steel is well protected by insulation, its temperature will lag far behind the fire gas temperature, and treating its temperature rise as linear with time will be a reasonable approximation. The variation of stress will be complicated, as unequal strains may redistribute stresses. Such problems must be treated individually.

It will, however, be useful to examine an idealized situation where stress is held constant, but temperature rises linearly, starting from room temperature. For this purpose, model $H$ has the advantage over model $E$ that it is readily integrable. Substituting:

\[ T = T_0 + at \]  

in Eqn (3), and integrating as before, gives a result analogous to Eqn (7):

\[ b_2 \varepsilon - \tanh (b_2 \varepsilon) = b_2 T^2 (b_1 - b_0) c_s a t \]  

where

\[ b_0 = c_6 \exp (c_7 \sigma - c_8 / T_0), \quad \sigma > \sigma_t \]  

\[ b_0 = c_1 \exp (c_2 \ln \sigma - c_3 / T_0), \quad \sigma < \sigma_t \]

Witteveen and Twilt describe experiments in which a small beam specimen of FeE 24 steel was, initially, loaded at room temperature until its central deflection was 1/30 of the span. The deflection corresponds to a strain of 0.218 per cent in the extreme fibres. The load that caused this deflection was called $P_{bar}$.

In subsequent runs, a series of smaller loads was applied, and for each the beam was heated from room temperature at various constant rates. The critical temperature $T_f$ at which deflection reached 1/30 of span was observed. The loads, normalized to $P_{0.20}$, were plotted against $T_f$ for each heating rate.

These experiments have been simulated by calculation, using the model $H$ coefficients for A135 steel. Stress was calculated on the assumption that strain followed Eqn (19) while temperature increased linearly from $T_0$ to $T_f$, at which point strain reached 0.218 per cent. The results are shown in Fig. 5. Within the range 500–650 °C there is good agreement, assuming the two steels to have comparable properties. In particular, the observation of Witteveen and Twilt that rate of heating does not significantly influence the critical temperature is found consistent with the model.

**CONCLUSION**

The equations and coefficients presented make possible the prediction of total strain in structural steel at temperatures and stresses met with in building fires over a time scale appropriate to such fires.

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**NOMENCLATURE**

\[ a_1 - a_8 \]  

Coefficients in model $E$ ($a_2$ and $a_6$ are dimensionless; $a_1$ and $a_5$ have dimension
\[ K^{-1}, \ a_3 \text{ and } a_6, \ \text{in}^2\ \text{lbf}^{-1}; \ a_4 \text{ and } a_7, \ \text{in}^2\ \text{lbf}^{-1}\ \text{K}^{-1} \]

\[ b_0 - b_2 \quad \text{Functions of } \sigma \text{ and } T \quad (\text{see Eqns (2), (3) and (20)}) \]

\[ c_1 - c_7 \quad \text{Coefficients in model } H \quad (c_1 \text{ in min}^{-1}, \ c_5 \text{ in K}, \ c_2 \text{ to } c_4 \text{ dimensionless}) \]

\[ c_8 \quad \text{Elastic modulus at 20 } \text{C}, \ \text{lbf in}^{-2} \]

\[ c_9 \quad \text{Temperature coefficient of } c_8, \ \text{K}^{-2} \quad \text{(see Eqn (13))} \]

\[ t \quad \text{Time (min)} \]

\[ T \quad \text{Temperature (K)} \]

\[ T_C \quad \text{Temperature (°C)} \]

\[ T_c \quad \text{Critical temperature (°C)} \]

\[ T_0 \quad \text{Room temperature (K)} \]

\[ \varepsilon \quad \text{Strain (m m}^{-1} \]

\[ \sigma \quad \text{Stress (lbf in}^{-2} \]

REFERENCES


7. IMSLIB: International Mathematical and Statistical Libraries, 7500 Bellaire Blvd, Houston, TX 77036.


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