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**SOUND TRANSMISSION
THROUGH A DOUBLE-LEAF WALL**

by
R. J. Donato

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LA TRANSMISSION DU SON A TRAVERS UN MUR A DOUBLE PANNEAU

SOMMAIRE

L'auteur présente une nouvelle formulation de la méthode traditionnelle de calcul de la perte de transmission du son à travers les murs à double panneau. On a recours à diverses méthodes visant à réduire les intégrales résultantes à une forme qui permet de trouver une solution analytique. On donne une approximation à de basses fréquences permettant d'inclure, au moyen d'une méthode de transformation, la grandeur finie du mur. Des comparaisons avec les mesures expérimentales indiquent qu'il y a accord avec les prédictions théoriques.

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Sound Transmission through a Double-Leaf Wall

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The classical approach to the calculation of sound transmission loss through double-leaf walls is reformulated and various devices employed to reduce the resulting integrals to a tractable form for analytical solution. An approximation is made at low frequencies whereby, using a transform technique, the finite size of the wall may be included. Comparisons with experimental measurements show good agreement with theoretical predictions.

INTRODUCTION

There are essentially two methods for calculating the sound transmission loss through a double-leaf wall. The classical approach chosen by London¹ and Beranek and Work² uses the easily derived formulas for transmission at a fixed angle of incidence and then sums over all possible incident angles. These authors found the resulting integrals to be too intractable for analytical solution and computed the transmission loss numerically. The other method is the statistical mechanical approach adopted by White and Powell,³ who use the ideas of Lyon and Maidanik⁴ in considering energy flows between two or more coupled multiresonant systems as an analog of a heat transmission problem; the analog of temperature is the modal energy. This technique has the advantage of considering finite radiators and vibrators, whereas the classical approach deals with infinite area components.

From a practical point of view, the requirement is for an analysis that will show the contributions of the various parameters to the transmission loss. For instance, the effect of wall spacing is not readily seen from the analysis of White and Powell. This is partly due to a doubtful feature in the analysis. These authors take the power flow between systems r and s to be $R_r^s(S_r^v - S_s^v)$, where R_r^s is the nondissipative (gyroscopic) coupling resistance between the systems, and S^v is the velocity spectral density. This is at variance with Lyon and Scharton⁵ and Eichler,⁶ who take the power flow to be proportional to

$$(\omega \eta_s^r S_r^E - \omega \eta_r^s S_s^E),$$

where

$$\eta_s^r = R_r^s / \omega M_s, \quad \eta_r^s = R_r^s / \omega M_r,$$

and S^E is the energy spectral density. White⁷ assumes that $R_s^r = R_r^s$, which allows $R_r^s S_r^v - R_s^r S_s^v$ to be written as $R_r^s (S_r^v - S_s^v)$. On the other hand Eichler writes the equation of power flow between the two systems as

$$P_r^s - P_s^r = \omega S_r^E \eta_r^s - \omega S_s^E \eta_s^r = \frac{1}{2} [R_r^s V_r^2 - R_s^r V_s^2],$$

where V is velocity. If we take systems of equal modal density, then because $n_s \eta_s^r = n_r \eta_r^s$, where n_r , n_s are the modal densities (in this case equal), there is no power flow when $S_r^E = S_s^E$ or $M_r V_r^2 = M_s V_s^2$, whereas White and Powell say there is no power flow when $V_r^2 = V_s^2$. Further, if $\eta_r^s = \eta_s^r$, then $R_r^s / R_s^r = M_r / M_s$. Clearly, then, the relationship between R_r^s and R_s^r must include the modal masses as well as modal densities as $n_r \eta_r^s = n_s \eta_s^r$, where n is the modal density.^{5,6} This possibly accounts for the absence of any term representing the panel spacing in the derivation of White and Powell.³

Even without this objection one still has to make some guesses as to the value of radiation resistance to use. For the externally radiated power one may use Maidanik's expression,⁸ but there is still doubt about the correct value to use for energy radiated into the cavity. White and Powell use a value equal to twice the external radiation impedance, but this is probably only true below coincidence.

A later paper by Price and Crocker applies these techniques to the double-wall problem.⁹ Although the results are impressive, there still remain a few peculiarities. While the results agree well with our own experimental ones for a double wall having an absorbing layer, the results do not agree with those for an absorption-free space, the discrepancies being around 15-20 dB. Further, these authors show the effect of varying

FIG. 1. Model used in the analysis: $Z_1 = j\rho c \tan \theta$; $Z_2 = j\rho c / \sin 2\theta$; $\theta = (\omega L / 2c) \cos \varphi$.

the wall space from 1 to 40 cm to be negligible—a result certainly at variance with usual experience. There still remains the doubt as to the amount of the contribution from the nonresonant modes. Price and Crocker take it to be basically that given by applying the mass law to the two leaves. It is difficult to see how this can be so, especially at frequencies just below coincidence. For a very large wall, the resonant mode at such a frequency will be very sharply peaked in the k domain, the radiation impedance very low, and the radiation given predominantly by the nonresonant modes. Here the modes will also be narrow but more densely packed. Then, on the above argument, we should expect the transmission to be given by the mass law. However, classical theory shows that at this frequency there will be the well-known departures from the mass law given by a $1 - (f/f_c)^2 \sin^4 \varphi$ term in the denominator of the transmission loss equation.¹⁰

The following method reverts to classical procedures, but simplifies the integrals obtained so that they may be evaluated analytically. We have attempted to compensate for the finite size of the wall at low frequencies by using a wavenumber approach. This is at best an approximation, for at the low frequencies the mode spacing is too wide for either the statistical method or the infinite-area method to be used. One should sum the individual modes, which would make both methods rather unwieldy.

I. THEORETICAL ANALYSIS OF TRANSMISSION THROUGH A DOUBLE WALL

Let us assume first of all that the wall is infinite in area. The modifications needed to correct for a finite area wall are discussed in Sec. IV. In the analysis the angle of incidence φ of the incoming sound is used as the variable although the use of k , the wavenumber along the wall, would be equally acceptable.

The air space is represented by a T network as shown in Fig. 1. Z_1 is the series arm of the network and Z_2 the parallel. The impedances of the two leaves of the wall are assumed to be equal and denoted by Z_p , where this term for the moment is taken to include the radiation impedance of each leaf as well as a reactive component.

From the equivalent circuit of Fig. 1,

$$u_0/2p = [Z_p + Z_1 + Z_2(Z_1 + Z_p)/(Z_1 + Z_2 + Z_p)]^{-1}, \quad (1)$$

$$u_1/2p = (u_0/2p)[Z_2/(Z_2 + Z_1 + Z_p)], \quad (2)$$

where u_0 and u_1 represent the velocity of the two leaves, and p is the incident sound pressure.

The impedances Z_1 , Z_2 given in Fig. 1 should strictly be multiplied by $(\cos \varphi)^{-1}$ to conform to the subsequent use of impedances. However, this creates problems. It is well known that the range of incident angles present in the incident field must not spread over a full quadrant, for then the transmission loss is calculated to be lower than the measured values. The effect is worsened with a double wall because there is an extra source of angular limitation. Putting in the $(\cos \varphi)^{-1}$ factor gives results highly dependent on the limiting angle chosen, which of course will in turn depend on the wall size and spacing. Any modification is to a certain extent arbitrary, but in the present analysis we have chosen to take the free-field impedance of the cavity to be multiplied by $\cos \varphi$. The modification affects the subsequent integrand in such a way that the resulting integration carried out between the limits 0 and $\pi/2$ gives the same result as the more rigorous expression integrated between 0 and an angle less than $\pi/2$.

Upon simplification we find

$$\frac{u_1}{2p} = -\frac{j\rho c}{2} (Z_p \cos \theta + j\rho c \sin \theta)^{-1} (Z_p \sin \theta - j\rho c \cos \theta)^{-1}. \quad (3)$$

The plate impedance term $Z_p = j\omega M + \rho c / \cos \varphi$, where M is the mass of the wall per unit area and ρc is the characteristic impedance of the ambient medium.

Substituting for Z_p in Eq. 3 gives

$$\frac{u_1}{2p} = -\frac{1}{2} j\rho c \left(j\omega M \cos \theta + \frac{\rho c \sin \theta}{\cos \varphi} + j\rho c \sin \theta \right)^{-1} \times \left(j\omega M \sin \theta + \frac{\rho c \cos \theta}{\cos \varphi} - j\rho c \cos \theta \right)^{-1}. \quad (4)$$

It is interesting to note that equating the imaginary part of the second term in the denominator to zero gives both the zero-order resonance when the masses and the air cavity resonate as a lumped system, and the higher-order modes when path differences in the air space are multiples of a wavelength.

If the spacing L between the two leaves of the wall is zero, Eq. 4 reduces to

$$(u_1/2p)_{L=0} = \frac{1}{2} \cos \varphi \cdot (j\omega M \cos \varphi + \rho c)^{-1}, \quad (5)$$

which is the well-known form for a wall of surface density $2M$.

It may be shown that the second term in the denominator is the predominant factor in determining the

maximum amplitude of $u_1/2p$. Approximating θ for $\sin\theta$, then

$$\begin{aligned} \frac{u_1}{2p} &= -\frac{1}{2}j\rho c \left[\left(j\omega M + \frac{\rho c}{\cos\varphi} + \frac{j\rho c\omega L \cos\varphi}{2c} \right) \right. \\ &\quad \times \left. \left(\frac{j\omega M\omega L \cos\varphi}{2c} + \frac{\rho c\omega L}{2c} - j\rho c \right) \right]^{-1} \\ &\simeq -\frac{1}{2}j\rho c \left(\frac{\omega L}{2c} \right)^{-1} \\ &\quad \times \left[\left(j\omega M + \frac{\rho c}{\cos\varphi} \right) \left(j\omega M \cos\varphi - j\rho c \frac{2c}{\omega L} + \rho c \right) \right]^{-1}. \quad (6) \end{aligned}$$

Thus, we may write for Eq. 6

$$\begin{aligned} \left| \frac{u_1}{2p} \right|^2 &= \left(\frac{\cos\varphi}{\rho c} \right)^2 \left(\frac{\omega L}{c} \right)^{-2} \left[\left(\frac{\omega M \cos\varphi}{\rho c} \right)^2 + 1 \right]^{-1} \\ &\quad \times \left[\left(\frac{\omega M \cos\varphi}{\rho c} - \frac{2c}{\omega L} \right)^2 + 1 \right]^{-1}. \quad (7) \end{aligned}$$

The radiation impedance associated with the second leaf of the double wall is $\rho c/\cos\varphi$ per unit area, and so for an incident diffuse sound field the radiated power W from this unit area becomes

$$\begin{aligned} W &= \frac{8\pi p^2}{\rho c} \left(\frac{\omega L}{c} \right)^{-2} \int_0^{\pi/2} \cos\varphi \sin\varphi \left[\left(\frac{\omega M \cos\varphi}{\rho c} \right)^2 + 1 \right]^{-1} \\ &\quad \times \left[\left(\frac{\omega M \cos\varphi}{\rho c} - \frac{2c}{\omega L} \right)^2 + 1 \right]^{-1} d\varphi. \quad (8) \end{aligned}$$

In the extreme case $L=0$, $M=0$,

$$W_0 = \frac{2\pi p^2}{\rho c} \int_0^{\pi/2} \cos\varphi \sin\varphi d\varphi = \frac{\pi p^2}{\rho c}, \quad (9)$$

which is the power transmitted in the absence of the wall. Thus, if τ is the transmission factor of the wall,

$$\begin{aligned} \tau &= 8 \left(\frac{\omega L}{c} \right)^{-2} \int_0^{\pi/2} \cos\varphi \sin\varphi d\varphi \left[\left(\frac{\omega M \cos\varphi}{\rho c} \right)^2 + 1 \right]^{-1} \\ &\quad \times \left[\left(\frac{\omega M \cos\varphi}{\rho c} - \frac{2c}{\omega L} \right)^2 + 1 \right]^{-1}. \quad (10) \end{aligned}$$

As a check, we see that for $L \rightarrow 0$ Eq. 10 reduces to

$$\tau = 2 \int_0^{\pi/2} \cos\varphi \sin\varphi d\varphi \left[\left(\frac{\omega 2M \cos\varphi}{2\rho c} \right)^2 + 1 \right]^{-1}. \quad (10')$$

To simplify the integration involved in Eq. 10, we substitute

$$A = \omega M / \rho c, \quad D = 2c / \omega L, \quad x = \cos\varphi;$$

$$\begin{aligned} \tau &= 2D^2 \int_0^1 x dx (A^2 x^2 + 1)^{-1} [(Ax - D)^2 + 1]^{-1} \\ &= -\frac{2D^2}{(D^2 + 4)} \\ &\quad \times \int_0^1 \left[\frac{x - 2/AD}{(A^2 x^2 + 1)} - \frac{x - 2(D^2 + 1)/AD}{(Ax - D)^2 + 1} \right] dx. \quad (11) \end{aligned}$$

When the integration of Eq. 11 is performed we obtain

$$\begin{aligned} \tau &= \frac{2D^2}{(D^2 + 4)} \left\{ \frac{1}{2A^2} \ln A^2 - \frac{2}{A^2 D} \tan^{-1} A \right. \\ &\quad - \frac{1}{2A^2} \ln \left[\frac{(A - D)^2}{D^2 + 1} + 1 \right] - \frac{1}{A^2} \left[D - \frac{2(D^2 + 1)}{D} \right] \\ &\quad \times [\tan^{-1}(A - D) + \tan^{-1} D] \left. \right\}, \quad (12) \end{aligned}$$

and for large A this approaches

$$\begin{aligned} \tau &\simeq \frac{D^2}{A^2(D^2 + 4)} \left\{ \ln A^2 - \frac{2\pi}{D} - \ln \left[\frac{(A - D)^2}{D^2 + 1} + 1 \right] \right. \\ &\quad \left. + 2 \left(D + \frac{2}{D} \right) \left(\frac{\pi}{2} + \tan^{-1} D \right) \right\}. \quad (12') \end{aligned}$$

As previously stated, the lower limit of $\cos\varphi$ used in the determination of Eq. 12 has been taken to be zero. The integrand has a maximum amplitude at $\varphi = \varphi_{\max}$ fixed by the second term in the denominator. Typically, for the lower frequencies this term peaks at around 60° to 70° from the vertical, and the lower limit of the integration is of no great concern. However, when the frequency is raised, $\varphi_{\max}$ approaches 90° and the value of this limit assumes greater importance if the rigorous formula is used. By modifying the free-field impedance in the cavity by the $\cos\varphi$ factor, it is found that this dependence is removed and the results obtained are close to those observed experimentally.

The solution given by Eq. 12 is valid only as long as θ approximates $\sin\theta$. This in turn depends upon the value of D . When D becomes equal to unity we assume that the second term in the denominator of Eq. 6 no longer exhibits its peak. This is more or less justified by the previous discussion, where we showed $\cos\varphi_{\max}$ to approach zero progressively. In the region $D \simeq 1$, we calculate the transmission loss to be

$$\tau = 4(\rho c / \omega M)^2. \quad (13)$$

The point where we change over from one formula to the other is arbitrary to a certain extent; the criterion chosen here is when $D=1$. Around this point, it may be shown that the two formulas agree.

For frequencies below and approaching the coincidence frequency f_c , M is replaced by $M[1 - (f/f_c)^2 \times \sin^4\varphi]$.¹⁰ Normally this correction is only applied at

the two third-octaves nearest to coincidence, and for this range we take $\sin\varphi \simeq 1$.

II. FREQUENCIES ABOVE COINCIDENCE

For the frequencies above coincidence (f_c), we neglect Z_1, Z_2 in the denominator of Eqs. 1 and 2. We replace ωM by $\omega M[1 - (f/f_c)^2 \sin^4\varphi]$ and ρc by $\rho c + \omega M\eta \times (f/f_c)^2 \sin^4\varphi \cos\varphi$. These modifications arise from the lower impedance owing to waves traveling along the plate where η is the damping factor.

Thus Eq. 3 becomes

$$u_1/2\dot{p} = -jZ_0(2\sin\theta\cos\theta Z_p^2)^{-1}, \quad (14)$$

with

$$Z_p = \frac{\rho c}{\cos\varphi} + \omega M\eta \left(\frac{f}{f_c}\right)^2 \sin^4\varphi + j\omega M \left[1 - \left(\frac{f}{f_c}\right)^2 \sin^4\varphi\right]$$

and with $Z_0 = \rho c$.

When the necessary substitutions are made, Eq. 10 is modified to

$$\tau = 2 \int_0^{\pi/2} \cos^3\varphi \sin\varphi d\varphi (\sin\theta\cos\theta)^{-2} \times \left\{ \left[1 + \frac{\omega M}{\rho c} \eta \left(\frac{f}{f_c}\right)^2 \sin^4\varphi \cos\varphi\right]^2 + \left[\frac{\omega M}{\rho c} \cos\varphi \left(1 - \frac{f^2}{f_c^2} \sin^4\varphi\right)\right]^2 \right\}^{-2}. \quad (15)$$

The integral in Eq. 15 clearly has a maximum when $\sin^4\varphi_0 = (f_c/f)^2$. If we expand the denominator in the integral around the angle φ_0 , Eq. 15 reduces to

$$\tau = 2 \int_{-\infty}^{\infty} \cos^3\varphi_0 \sin\varphi_0 d\varphi' (\cos\theta\sin\theta)^{-2} \times \left[\left(1 + \frac{\omega M}{\rho c} \eta \left(\frac{f}{f_c}\right)^2 \sin^4\varphi_0 \cos\varphi_0\right)^2 + 16 \left(\frac{\omega M}{\rho c}\right)^2 \cos^4\varphi_0 \sin^6\varphi_0 \left(\frac{f}{f_c}\right)^4 (\varphi')^2 \right]^{-2}. \quad (16)$$

If we make the substitutions

$$A = 1 + (\omega M/\rho c)\eta(f/f_c)^2 \sin^4\varphi_0 \cos\varphi_0, \\ B = 4(\omega M/\rho c)\cos^2\varphi_0 \sin^3\varphi_0 (f^2/f_c^2),$$

Eq. 16 reduces to

$$\tau = 4\pi \cos^3\varphi_0 \sin\varphi_0 (A^3 B)^{-1}, \quad (17)$$

where we have replaced $\cos^2\theta \sin^2\theta$ by its mean value $= \frac{1}{8}$. Substituting back again for A, B, φ_0 , we finally obtain

$$\tau = 2\pi \left(1 - \frac{f_c}{f}\right)^{\frac{1}{2}} \left(\frac{\omega M}{\rho c} \frac{f_c}{f}\right)^{-1} \left[1 + \frac{\omega M}{\rho c} \left(1 - \frac{f_c}{f}\right)^{\frac{1}{2}}\right]^{-1}. \quad (18)$$

III. COINCIDENCE FREQUENCY

The peak of the integrand now occurs near one of the limits of integration, and so we must modify the procedure of Sec. II. In fact, we restore the Z_1, Z_2 terms removed from the derivation of Eq. 16 and obtain

$$\left(\frac{u_1}{2\dot{p}}\right) = Z_0 \cos\varphi \left(\frac{\omega L}{c}\right)^{-1} \times \left[j \left(\frac{2\omega M}{\rho c} \cos^3\varphi + \frac{\omega L}{2c} \cos^2\varphi \right) + \left(1 + \frac{\omega M}{\rho c} \eta \cos\varphi \right) \right]^{-1} \times \left[j \left(\frac{2\omega M}{\rho c} \cos^3\varphi - \frac{2c}{\omega L} \right) + \left(1 + \frac{\omega M}{\rho c} \eta \cos\varphi \right) \right]^{-1}. \quad (19)$$

Over the range of integration where the integrand peaks,

$$(\omega L/2c)\cos^2\varphi \ll (2\omega M/\rho c)\cos^3\varphi$$

and so will be neglected. Thus the transmission factor τ is given by

$$\tau = 8 \left(\frac{\omega L}{c}\right)^{-2} \int_0^1 x dx \left\{ \left[\left(\frac{2\omega M}{\rho c} x^3 \right)^2 + \left(1 + \frac{\omega M}{\rho c} \eta x \right)^2 \right] \times \left[\left(\frac{2\omega M}{\rho c} x^3 - \frac{2c}{\omega L} \right)^2 + \left(1 + \frac{\omega M}{\rho c} \eta x \right)^2 \right] \right\}^{-1}. \quad (20)$$

A fairly good approximation may be made to Eq. 20 by writing the integrand in the form $\alpha x \exp(-\beta x^4)$, which is immediately integrable if β is large enough for us to write the limits as 0 and ∞ . We fit the approximate curve at the maximum of the integrand. If we write $A = 2\omega M/\rho c$, $B = \omega M\eta/\rho c$, and if we assume that the maximum is not far distant from the maximum of $x/(1+Bx)^4$, we find the maximum x_0 to occur at $x_0 \simeq (3B)^{-1}$. The maximum of $x \exp(-\beta x^4)$ occurs when $4\beta x_0^4 = 1$, and so $\beta = (3B)^4/4$. Now

$$\alpha^{-1}e^{\frac{1}{4}} = \left[\frac{A^2}{(3B)^6} + (1.33)^2 \right] \left[\left(\frac{2A}{3B^3} - \frac{2c}{\omega L} \right)^2 + (1.33)^2 \right]. \quad (21)$$

For the particular case shown in Fig. 2, $\alpha = 1/2.9$ and

$$\tau = 8 \left(\frac{\omega L}{c}\right)^{-2} \int_0^{\infty} \frac{x}{2.9} \exp(-2.1 \times 10^4 x^4) dx \\ = \left(\frac{8}{5.8}\right) \left(\frac{\omega L}{c}\right)^{-2} \int_0^{\infty} \exp(-2.1 \times 10^4 y^2) dy.$$

Thus

$$\tau = \left(\frac{8}{5.8}\right) \left(\frac{\omega L}{c}\right)^{-2} \left(\frac{\pi}{4.2}\right)^{\frac{1}{2}} \times 10^{-2}. \quad (22)$$

With our particular value of $\omega L/c = 2.8$ (coincidence frequency is 1600 Hz, $L = 3\frac{5}{8}$ in., $M = 4.6$ lb/ft², and $\eta = 0.01$), this reduces to $-10 \log_{10}\tau = 28.2$ dB.

FIG. 2. Curve fit around coincidence.
(—): theoretical curve, Eq. 20;
(---): $x \exp(-\beta x^4)$ approximation.

IV. CORRECTION TO BE APPLIED TO LOW-FREQUENCY CALCULATIONS

We assumed at the beginning of the analysis that the wall surface could be considered to be infinite in extent. In practical situations this condition is of course violated, especially for the low frequencies. To see what kind of correction is necessary to allow for a finite wall, we shall restate the problem using the spatial Fourier representation. It is well known that, using a statistical representation for the incident sound field,¹¹ the mean amplitude power spectrum of the vibrating wall surface is given by

$$\langle \xi^2(\omega; \mathbf{r}) \rangle = \sum_m \sum_n \frac{S}{|X_{mn}|^2} \int_S \int_S W_p(\omega; \mathbf{r}_1, \mathbf{r}_2) \times \varphi_{mn}(\mathbf{r}_1) \varphi_{mn}(\mathbf{r}_2) d\mathbf{r}_1 d\mathbf{r}_2, \quad (23)$$

where ξ is the vibration amplitude at radian frequency ω and point $\mathbf{r}$.

$W_p(\omega; \mathbf{r}_1, \mathbf{r}_2)$ is the pressure correlation at ω between points $\mathbf{r}_1, \mathbf{r}_2$ on the wall surface. The quantity φ_{mn} is an orthogonal characteristic function satisfying the boundary conditions at the edges of the wall and fulfilling $S(\varphi_{mn}^2) = 1$. S is the area of the wall and X_{mn} is the modal impedance of the wall. Two subscripts m, n are used to satisfy the general case of a two-dimensional vibrator.

If we now consider the practical case of an 80-ft² wall, we find that there are around 0.32 modes per unit bandwidth, and in a third-octave band centered on 125 Hz there will be about 5 modes. The number per third-octave band will increase proportionally to the frequency so that at 1250 Hz there will be 50 modes. We shall, however, assume that the number of modes is always sufficiently numerous that we may replace the summation by an integration process. We shall further

assume that the correlation function W_p is dependent solely upon $(\mathbf{r}_1 - \mathbf{r}_2)$; in other words we have circular symmetry. Equation 23 can now be expressed in a simpler form,

$$\langle \xi^2(\omega; \mathbf{r}) \rangle = \int |p_0|^2 R(\omega; |\mathbf{r}_1 - \mathbf{r}_2|) h(\mathbf{r}, \mathbf{r}_1) \times h(\mathbf{r}, \mathbf{r}_2) d\mathbf{r}_1 d\mathbf{r}_2, \quad (24)$$

where $h(\mathbf{r}, \mathbf{r}_1)$ is the impulse response of the wall, and $R(\omega; |\mathbf{r}_1 - \mathbf{r}_2|)$ is the incident sound-field correlation function. Then

$$\langle \xi^2(\omega; \mathbf{r}) \rangle = |p_0|^2 \int F(\omega; k) |H(\omega; k)|^2 k dk 2\pi, \quad (25)$$

where F is the Hankel transform in the k domain of R , and H is that of h . We can replace $H(\omega; k)$ by $[Z(\omega; k)]^{-1}$, the wall displacement impedance.

Up to now we have said nothing about the form of $R(\omega; \mathbf{r}_1 - \mathbf{r}_2)$. For an infinite area this term will be the autocorrelation of the incident sound field. In a restricted area, however, a better approximation would be given by $R(\omega; \mathbf{r}_1 - \mathbf{r}_2) \cdot (1 - r^2/4R^2)$, where circular symmetry is maintained and the area of the wall is given by $\pi R^2 = S$, and $r = |\mathbf{r}_1 - \mathbf{r}_2|$. To simplify the calculations, we find the Hankel transform of the modulating function $(1 - r^2/4R^2)$ and then use this transform to convolve the infinite area solution. Thus, we end up with a modified k -domain spectrum. Finally, we integrate this modified spectrum between 0 and k_α ($k_\alpha = \omega/c$, where c is velocity of sound in air) to find the fraction of the incident energy being radiated. The Hankel transform

$$I(k) = \int_0^R (1 - r^2/4R^2) J_0(kr) r dr$$

FIG. 3. Low-frequency corrections to transmission loss.

can be shown to be given by

$$I(k) = J_1(kR) \left[(k^2 R)^{-1} + \frac{3}{4} (R/k) \right] - (2k^2)^{-1} J_0(kR) \quad (26)$$

and

$$I(0) = 3R^2/8.$$

As a further simplification we shall find the effective bandwidth of $I(k)$ and use a rectangular wave $I(0)$ for the convolution process. Now

$$\begin{aligned} \int_0^\infty I(k) k dk &= \int_0^\infty \left[\frac{J_1(kR)}{k^3 R} - \frac{J_0(kR)}{2k^2} + \frac{3RJ_1(kR)}{4k} \right] k dk \\ &= \frac{1}{2} |J_1(kR)/kR|_0^\infty + \frac{3}{4} |J_0(kR)|_0^\infty \\ &= 1. \end{aligned} \quad (27)$$

Then the effective bandwidth k_0 of $I(k)$ is given by

$$\int_0^{k_0} I(0) k dk = 1$$

or

$$k_0 R = (16/3)^{1/2} = 2.3. \quad (27')$$

To find the effective R we may either equate πR^2 to

the actual wall area or—which is possibly a better procedure—find the radius of the largest circle we can fit into the rectangular area.

We must now convolve $I(k)$ with the k spectrum derived for the infinite area case. The value of $F(\omega; k)$ for the infinite area case is given by

$$F(\omega; k) = [k_\alpha (k_\alpha^2 - k^2)^{1/2}]^{-1}, \quad (28)$$

where we have taken the incident sound field to be three-dimensionally diffuse with the resulting

$$R(\omega; r) = (\sin k_\alpha r) / k_\alpha r.$$

For the frequencies we shall be considering $k_0 < k_\alpha$, and so we can split the convolution into three parts

$$\begin{aligned} &\int_0^k && \text{for } k < k_0, \\ &\int_0^{k_0} && \text{for } k_0 < k < k_\alpha, \\ &\int_{k-k_\alpha}^{k_0} && \text{for } k_\alpha < k < k_\alpha + k_0. \end{aligned} \quad (29)$$

SOUND TRANSMISSION THROUGH A DOUBLE-LEAF WALL

FIG. 4. Theoretical and experimental transmission losses for studless double-leaf walls. (—): experimental curve; ($\circ$): theoretical $\eta=0.015$; ($\blacksquare$): theoretical $\eta=0.010$; ($\bullet$ and $\square$): as above but with $\cos\varphi$ correction at higher frequencies. (a) Each leaf 2.7 lb/ft², spacing 3 $\frac{1}{8}$ in.; (b) each leaf 2.1 lb/ft², spacing 2 $\frac{1}{2}$ in.; (c) each leaf 2.1 lb/ft², spacing 3 $\frac{5}{8}$ in.; and (d) each leaf 4.6 lb/ft², spacing 3 $\frac{5}{8}$ in.

If we denote $F(k)*I(k)=G(k)$, we find

$$\begin{aligned} k_\alpha(8/3R^2)G(k) &= k \sin^{-1}(k/k_\alpha) + (k_\alpha^2 - k^2)^{1/2} - k_\alpha, \quad k \leq k_0; \\ &= k \sin^{-1}(k/k_\alpha) - k \sin^{-1}[(k - k_0)/k_\alpha] + (k_\alpha^2 - k^2)^{1/2} \\ &\quad - [k_\alpha^2 - (k - k_0)^2]^{1/2}, \quad k_0 \leq k \leq k_\alpha; \\ &= k \sin^{-1}(k_\alpha/k_\alpha) - k \sin^{-1}[(k - k_0)/k_\alpha] \\ &\quad - [k_\alpha^2 - (k - k_0)^2]^{1/2}, \quad k_\alpha \leq k \leq k_0 + k_\alpha; \\ &= 0, \quad k > k_0 + k_\alpha. \end{aligned} \quad (30)$$

By carrying out the integrals outlined in Eq. 29 with the values of $G(k)$ calculated from Eq. 30, we may derive values of the total normalized k spectrum E_T and the part of the k spectrum E_{k_α} which takes part in sound radiation. If $k_\alpha = \beta k_0$ is written $8k_\alpha E_T/3R^2 = k_0^3 \times (\pi/6 + \beta/2)$, then

$$\begin{aligned} \frac{8k_\alpha E_{k_\alpha}}{3R^2} &= k_0^3 \left(\frac{1 - \beta^3}{3} \right) \sin^{-1} \frac{\beta - 1}{\beta} + \frac{\pi \beta^3}{6} + \frac{\beta}{2} \\ &\quad - [\beta^2 - (\beta - 1)^2]^{1/2} \left(\frac{\beta^2}{3} + \frac{\beta}{9} + \frac{4}{9} \right). \end{aligned} \quad (31)$$

We also note that

$$E_T = \left(\frac{\pi}{6} \right) (k_0 R)^2 \left[(\pi/6) (k_0/k_\alpha) + \frac{1}{2} \right]. \quad (32)$$

Substituting into Eq. 32 from Eq. 28,

$$E_T = (\pi/3) (k_0/k_\alpha) + 1. \quad (33)$$

For an infinitely large panel, $k_0 \rightarrow 0$ and

$$E_{T_\infty} = 1. \quad (34)$$

This result can, of course, be obtained directly from

$$\int_0^{k_\alpha} (k/k_\alpha) (k_\alpha^2 - k^2)^{-1/2} dk = 1.$$

As an illustration of the above procedure let us consider the case of a 10×8-ft wall. The radius of the largest circle fitting into this area is 4 ft and from Eq. 27' k becomes 0.57 ft⁻¹. From Eqs. 31 and 33 we may prepare the correction factors of Fig. 3 which are to be added to the results of the transmission loss calculation from Eq. 12.

V. COMPARISON WITH EXPERIMENTAL RESULTS

Figures 4(a)–4(d) show the comparison between the calculated and observed results for four studless gypsum board walls. Throughout, the damping factor has been chosen to be 0.010, which agrees with laboratory measurements made on the vibration of strips and panels of the wall material.

An extra correction has been added at the coincidence frequency itself. The calculated results apply for a particular frequency, whereas the experimental points are those obtained using a third-octave filter. Thus, if

the curve is strongly peaked in the coincident region, one would expect the calculated loss to be too low. A computer study was made of this region and it was concluded that a fair allowance for the third-octave containing the coincidence frequency was to add 4 dB to the coincidence loss figure. With this adjustment the agreement is good over most of the range. On the heavier wall (4.6 lb/ft² on each side) two calculated values are shown for the 1000-Hz point ($D \simeq 1$). The lower one, which is closer to the experimental point, is obtained by using Eq. 13 at this frequency.

The worst fit is in the high-frequency region. It appears that Eq. 18 is not a very good description for frequencies close to coincidence and that in this region a modification of Eq. 19 would be more suitable. For frequencies above coincidence, it seems that $\eta = 0.01$ is the best value to use. At coincidence, on the other hand, $\eta = 0.015$ gives better results. It might be that in the high-frequency region where the transmission loss changes rapidly with slope a correction should be applied for the third-octave bandwidth. This would tend to bring the theoretical results down. It could also be that the original assumption of a $\cos \varphi$ multiplier in the cavity no longer applies at the high frequencies. Removing this multiplier brings these calculated figures closer to those observed.

It is heartening that the fit is as good as it is in the midfrequency range. One would expect a good fit at the lowest frequencies, but at the middle frequencies one might expect to run into trouble with the integration limits in Eq. 12. The good fit goes some way towards justifying the cavity model we have chosen.

VI. COMPARISON WITH PRACTICAL CASE

The analytical representation, while introducing some simplification, does represent the transmission loss fairly accurately. It must be emphasized, though, that the problem solved is somewhat artificial—as are indeed most solutions to this problem. In a practical problem we would have to include the effect of the supporting studs. This effect may be twofold in that (1) the angular distribution of waves through the cavity may be distorted so that, in general, the effect would be to increase the transmission loss, and (2) the studs themselves might be acting as a coupling medium between the two panels. The effect of any absorbing medium placed in the cavity would also have to be included in any comprehensive theory. This should not be too much of a problem, however, as we could replace the real wavenumber for the waves in the cavity by a complex one. The resulting equations then become more complex but remain of the same general form.

An empirical attack on these problems using experimental data is given in Appendix A.

One very important parameter is the internal loss in the panel. In the present analysis we have taken

$\eta=0.010$ to give a good fit, and this agrees with experimental evidence based on vibration studies, where η was found to be 0.012.

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APPENDIX A. EMPIRICAL APPROACH TO THE EFFECTS OF STUDS AND CAVITY ABSORPTION

One may carry out a very crude analysis of experimental results to get some idea of the relative contributions made by stud type, stud spacing, and an absorbing cavity. We shall take as our measures of the transmission factor the sound transmission class (STC) of the wall. We have results on various walls for the STC of the walls with (a) no stud supports, (b) wooden studs 16 in. apart, (c) wooden studs 24 in. apart, and (d) metal channel studs 24 in. apart. These results are available both with and without 2 in. of mineral wool absorptive material in the cavities. We assume that the metal channels remove the effect of stud coupling. If the effect of the studs modifies the air transmission path by α_1 for the 16-in. spacing and α_2 for the 24-in. spacing, if the effect of stud coupling for the wooden studs is γ_1 for the 16-in. spacing and γ_2 for the 24-in. spacing, and, finally, if β_0 , β_1 , β_2 are the effects of the absorbing materials for the studless case and for the two different stud separations, we may write the following equations for the STC values compared to the studless no-absorption case τ_0 as

$$\left. \begin{aligned} \alpha_1 + \gamma_1 / \tau_0 &= B, \\ \alpha_2 + \gamma_2 / \tau_0 &= C, \\ \alpha_2 &= D, \\ \beta_0 &= A', \text{ typically} \\ \beta_1 \alpha_1 + \gamma_1 / \tau_0 &= B', \\ \beta_2 \alpha_2 + \gamma_2 / \tau_0 &= C', \\ \beta_2 \alpha_2 &= D', \end{aligned} \right\} \begin{aligned} &0 \text{ dB} \\ &-2 \text{ dB} \\ &-4 \text{ dB} \\ &-12 \text{ dB} \\ &-3 \text{ dB} \\ &-7 \text{ dB} \\ &-13 \text{ dB}. \end{aligned} \quad (35)$$

As an example we take the case of absorption present

TABLE A-I. Air, stud, and absorbent transmission factors.

β_0	β_1	β_2	α_1	α_2	τ_1/τ_0	τ_2/τ_0
0.06	0.07	0.07	0.54	0.42	0.46	0.19
0.12	0.11	0.11	0.67	0.33	0.33	0.29
0.03		0.15		0.20		
0.08		0.16		0.32		0.09

with the 16-in. stud arrangement. Then the transmission factor of the wall will be $\gamma_1 + \beta_1 \alpha_1 \tau_0$. For the studless absorption-free case, the corresponding factor will be τ_0 . The ratio is thus $\beta_1 \alpha_1 + \gamma_1 / \tau_0$, and $-10 \log_{10}(\beta_1 \alpha_1 + \gamma_1 / \tau_0)$ is taken to be the difference of the STC values for the two cases. At first sight it might seem that we have seven equations with seven unknowns. This is not so, however, as there are four equations with three unknowns α_2 , τ_2/τ_0 , β_2 and only two equations to determine β_1 , α_1 , τ_1/τ_0 . A least-squares fit is first made to the four equations. It is found that β_2 is not too far from β_0 , and since $\beta_2 > \beta_1 > \beta_0$, we take $\beta_1 = \beta_2$. The results are shown in Table A-I. As we should expect, $\tau_1 > \tau_2$ (the stud contribution is greater for 16-in. separation than for 24-in. separation). Similarly $\beta_2 < \beta_1$. The puzzling feature is that $\alpha_1 > \alpha_2$, whereas simple reasoning would lead one to expect the reverse. To make $\alpha_1 < \alpha_2$ one would need τ_1/τ_0 to be larger and τ_2/τ_0 smaller, which is not very likely. Another possibility is that there is some coupling through the resilient channels. This would make α_2 smaller, which is a change in the wrong direction. Yet another possibility is that even for the studless case there may still be some contribution from the perimeter framework. The data show this to be small enough to be neglected. The most we can say is that both α_1 and α_2 are less than unity, but at the moment the dependence upon stud spacing seems to be opposite to what one would expect. It might also be that the STC measure is too blunt a tool to separate the various effects.

¹ A. London, "Transmission of Reverberant Sound through Double Walls," J. Acoust. Soc. Amer. 22, 270-279 (1950).

² L. L. Beranek and G. A. Work, "Sound Transmission through Multiple Structures Containing Flexible Blankets," J. Acoust. Soc. Amer. 21, 419-428 (1949).

³ P. H. White and A. Powell, "Transmission of Sound Vibration through a Rectangular Double Wall," J. Acoust. Soc. Amer. 40, 821-832 (1965).

⁴ R. H. Lyon and G. Maidanik, "Power Flow Between Linearly Coupled Oscillators," J. Acoust. Soc. Amer. 34, 623-639 (1962).

⁵ R. H. Lyon and T. D. Scharton, "Vibrational Energy Transmission in a Three Element Structure," J. Acoust. Soc. Amer. 38, 253-261 (1965).

⁶ E. Eichler, "Thermal Circuit Approach to Vibrations in Coupled Systems and the Noise Reduction of a Rectangular Box," J. Acoust. Soc. Amer. 37, 995-1007 (1965).

⁷ P. H. White, "Sound Transmission of Double Flexible Walls Excited by Random Pressure Fields," PhD thesis, Univ. California, Los Angeles (1965), 157 pp.

⁸ G. Maidanik, "Response of Ribbed Panels to a Reverberant Sound Field," J. Acoust. Soc. Amer. 34, 809-826 (1962).

⁹ A. J. Price and M. J. Crocker, "Sound Transmission through Double Panels Using Statistical Energy Analysis," J. Acoust. Soc. Amer. 47, 683-693 (1970).

¹⁰ L. L. Beranek, Ed., *Noise Reduction* (McGraw-Hill, New York, 1960), p. 298.

¹¹ E. Skudrzyk, *Simple and Complex Vibrating Systems* (McGraw-Hill, New York, 1968), p. 321.

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