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WATER VAPOR PERMEABILITY OF POROUS MEDIA

BY

WILLIAM WOODSIDE

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WATER VAPOR PERMEABILITY OF POROUS MEDIA¹

WILLIAM WOODSIDE

ABSTRACT

Following the analogy between the laws of heat conduction and vapor diffusion, two theoretical expressions for the thermal conductivity of a composite medium are applied to the water vapor permeability coefficient of certain porous media. It is shown that both expressions reduce to a form very similar to the empirical relationships found by Penman and Edenhorn for soils, glass spheres, charcoal, and cellular concrete. The calculation of the variation of water vapor permeability with density for a cellular lightweight concrete is illustrated.

granular material is made up of one or more types of particles surrounded homogeneous medium (usually air). All granular materials have associated them values of thermal conductivity, electrical conductivity, dielectric constant, magnetic permeability, and other diffusion-type coefficients. In many instances, it would be advantageous to be able to calculate the values of these coefficients from the corresponding values for the various constituents and the volume fractions of the constituents.

Since the mathematical theories of steady-state thermal and electrical conduction, electric and magnetic fields, and diffusion are all similar, a formula derived for the thermal conductivity of a granular material will also be applicable to the calculation of electrical conductivity, dielectric constant, magnetic permeability, and diffusion coefficient (de Vries 1952). Two of these coefficients are of main interest here, namely thermal conductivity and water vapor diffusion coefficient. Several equations have already been applied by the author to the calculation of the thermal conductivity of porous media (Woodside 1958). It is the object of this note to describe the application of similar equations to the calculation of the water vapor diffusion coefficient or water vapor permeability of certain porous media.

The following equation of de Vries (1952) was derived by Maxwell for electrical conductivity and applied by Eucken to the calculation of thermal conductivity.

$$\frac{k}{k_g} = \frac{1+2(1-P) [(1-k_g/k_s)/(1+2k_g/k_s)]}{1-(1-P) [(1-k_g/k_s)/(1+2k_g/k_s)]}$$

where P is the fractional porosity of the porous medium, and k , k_g , and k_s are the thermal conductivities of the porous medium, the gas phase (air), and solid respectively. This equation applies to a granular porous material, that is, one in which discrete solid particles are distributed in a continuous gas phase. For a cellular porous material, one in which discrete gas cells are distributed in a continuous solid phase, the equation must be modified by interchanging k_g and k_s and by replacing P by $(1-P)$.

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For application of equation (1) to calculation of vapor permeability of porous media, the thermal conductivities k , k_g , and k_s must be replaced by the corresponding vapor permeabilities μ , μ_g , and μ_s . Thus the equation for the vapor permeability μ of a granular material reads

$$(2) \quad \frac{\mu}{\mu_g} = \frac{1+2(1-P) [(1-\mu_g/\mu_s)/(1+2\mu_g/\mu_s)]}{1-(1-P) [(1-\mu_g/\mu_s)/(1+2\mu_g/\mu_s)]},$$

where P is again the porosity, and μ_g and μ_s are the vapor permeabilities of the gas (air) and solid respectively.

For most granular materials, the solid particles are impervious to water vapor, and therefore $\mu_s = 0$. Also since air is the gas phase in most granular materials, $\mu_g = \mu_{\text{air}}$. Therefore,

$$\frac{1-\mu_g/\mu_s}{1+2\mu_g/\mu_s} = \frac{\mu_s - \mu_g}{\mu_s + 2\mu_g} = -\frac{1}{2}$$

and equation (2) reads

$$(3) \quad \mu = \frac{2P}{3-P} \mu_{\text{air}}.$$

Maxwell's equation (de Vries 1952) was derived for a random distribution of spherical inclusions. Russell (1935) has derived a thermal conductivity equation for a cubic lattice of cubic inclusions. Russell's equation for a granular material is

$$(4) \quad k/k_g = \frac{(1-P)^{2/3} k_s/k_g + 1 - (1-P)^{2/3}}{[(1-P)^{2/3} - 1 + P] k_s/k_g + 2 - (1-P)^{2/3} - P}.$$

Again, replacing k , k_g , and k_s by the corresponding vapor permeabilities, μ_g , and μ_s , and also placing $\mu_s = 0$ and $\mu_g = \mu_{\text{air}}$, this becomes

$$\mu/\mu_{\text{air}} = \frac{1 - (1-P)^{2/3}}{2 - (1-P)^{2/3} - P}.$$

By expanding the terms in brackets, this reduces to

$$\mu/\mu_{\text{air}} = \frac{\frac{2}{3}P + \frac{1}{9}P^2 + \frac{4}{81}P^3 + \dots}{1 - \frac{1}{3}P + \frac{1}{9}P^2 + \frac{4}{81}P^3 + \dots}.$$

For small values of the porosity P , the terms of order two and higher may be neglected, and the equation is

$$\mu/\mu_{\text{air}} = 2P/(3-P).$$

This is identical with equation (3), which was derived from Maxwell's equation.

This simple relationship between the vapor permeability of a granular material and its porosity is very similar to the relationship found experimentally by Penman (1940):

$$(5) \quad \mu = 0.66P\mu_{\text{air}}.$$

Penman observed that the diffusion of several different gases through

and moist soils and other granular materials satisfied this relationship. In the case of moist soils, P represents the air-filled porosity, that is, the total porosity less the volumetric water content.

The dry-cup method for water vapor permeability determination involves a symmetric measurement of the steady-state water vapor transmission through a plate of the test material under isothermal conditions (73° F) with a relative humidity of 0% maintained on one side (by means of a desiccant) and a relative humidity of 50% maintained on the other. For most porous media the equilibrium moisture content corresponding to a mean relative humidity of 25% is low so that moisture transmission by mechanisms other than vapor diffusion is relatively small. At higher mean relative humidities, the apparent vapor permeability may be several times larger than the dry-cup value, owing to other mechanisms of moisture transfer such as capillarity. The true vapor permeability must decrease at increasing mean relative humidities because of the decrease in air-filled pore space available for diffusion. Since vapor permeabilities determined by the dry-cup method correspond most closely to those for vapor diffusion, only dry-cup values will be compared with calculated or permeabilities.

Edenholm (1945) observed experimentally that the 'dry-cup' water vapor permeability μ is given by

$$\mu = k_r \times P \times \mu_{\text{air}},$$

where k_r is a constant for any given material (named reduction factor by Edenholm) and P is the coarse-capillary porosity of the material. Edenholm found a value of 0.65 for k_r for materials such as sand, glass spheres, charcoal, and cellular concrete. The value of μ_{air} , the water vapor permeability of air, at room temperature is 120 perm. in., the perm. in. being the presently accepted unit for vapor permeability, which is grains in./hr ft² (in. Hg). In Edenholm's units, $\mu_{\text{air}} = 0.069 \text{ kg/m atm hr}$.

The analysis given above results in a porosity-dependent reduction factor, k_r , from equation (3)

$$k_r = 2/(3 - P),$$

whereas both Penman and Edenholm found k_r to be constant. However, the reasonable agreement between the theoretical and measured values of vapor permeability is illustrated by the following calculation. Edenholm measured vapor permeabilities ranging between 0.018 and 0.022 kg/m atm hr for a bed of glass spheres of porosity 0.395. Substituting $P = 0.395$ and $\mu_{\text{air}} = 0.069$ in equation (3), gives a value for $\mu = 0.0209 \text{ kg/m atm hr}$, which is between the two measured values.

The further application of the theory to the calculation of the variation of vapor permeability with density for a cellular concrete is now described. Since cellular material is essentially a continuous solid with distributed air cells, equation to be used is the converse of equation (2):

$$\frac{\mu}{\mu_s} = \frac{1 + 2P[(1 - \mu_s/\mu_g)/(1 + 2\mu_s/\mu_g)]}{1 - P[(1 - \mu_s/\mu_g)/(1 + 2\mu_s/\mu_g)]}.$$

Hallen (6) reports a value of 5.6 perm. in. at a density of 80 lb/ft³ for particular cellular concrete being considered. It is assumed that at this density there are no 'large' air cells in the material (Edenholm's coarse lary porosity equal to zero). Thus $\mu_s = 5.6$. The vapor permeability of cellular concrete at 32 lb/ft³ density will now be calculated. At this density the 'coarse' porosity, P , is

$$P = 1 - (32/80) = 0.60.$$

Taking $\mu_{air} = 120$ perm. in. as the value for μ_g , equation (7) results in a for μ of 24.0 perm. in. This compares favorably with laboratory 'dry' test results for material of density 32 lb/ft³ which show a value of 22.3 in.

The application of a thermal conductivity equation for porous media to calculation of water vapor permeability of certain porous materials is warranted by the agreement with the empirical equations of Penman, Edenholm and by the agreement of predicted and experimental values.

It would appear that, because of the analogy between Darcy's law for flow in porous media and the heat conduction equation, the same technique as described above could be applied to the calculation of the saturated permeability (of porous media) for liquid water. The difficulty in this application would appear to be the assignment of a value for k_g (k_s in most cases is zero), since the value of k_g must obviously depend upon the mean pore size and hence upon the pore size distribution.

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