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Investigating evidential reasoning for the interpretation of microbial water quality in distribution network

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Abstract

Total coliforms are used as indicators for evaluating microbial water quality in distribution network. However, total coliform provides only a weak ‘evidence’ of possible fecal contamination because pathogens are subset of total coliform and therefore their presence in drinking water do not necessarily mean fecal contamination. Heterotrophic plate counts, covers even a wider range of organisms and are also used commonly to evaluate microbial water quality in the distribution network. Both of these indicators provide incomplete and highly uncertain evidences individually, but the combination of evidence using data fusion may provide improved insight for interpreting microbial water quality in distribution network.

The term data fusion refers to the synergistic aggregation of observations and measurements. Different attributes and inputs (e.g. various water quality indicators) can provide information on various aspects of a system or process by complementing each other. Complementary information and redundant data sets form the basis of data fusion applications in water quality monitoring and for condition assessment of infrastructure systems.

Approximate reasoning methods like fuzzy logic and probabilistic reasoning are commonly employed for data fusion, where knowledge is uncertain (i.e., ambiguous, incomplete or vague). Within a probabilistic framework, traditionally inferencing is done through conditioning (based on *a priori* probabilities) using Bayesian analysis. The Dempster-Shafer (DS) theory generalizes this approach. The DS theory can efficiently deal with the difficulties related to the host of indicators describing water quality, with spatial and temporal dimensions of distribution systems, where redundancy of information is routinely observed as well as the credibility of available data is varied. In this paper, the DS rule of combination and its modifications including Yager-modified rule, Dubois-Prade disjunctive rule and Dezert-Smarandache rule are described in detail. The inferencing results through different rules of combination are compared using an example of microbial monitoring data to interpret water quality in a distribution network.

Keywords: microbial water quality, data fusion, probabilistic reasoning, and rules of combination

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INTRODUCTION

The microbial quality of water can change as the water travels from the treatment plant to the extremities of the distribution network. Microbial proliferation is influenced by many factors, which include residence time, condition and type of piping materials, water temperature, disinfectant residual, hydraulic conditions and other physico-chemical characteristics of the distributed water. Safe and aesthetically acceptable water quality requires effective management of the operation and maintenance of the distribution network.

Suspended particles, capable of surviving the various phases of water treatment, can transport microorganisms adsorbed on their surface. The microorganisms may be protected from disinfectant if the particles contain reducing compounds, such as iron oxides or organic matter. Therefore, low turbidities (less than 1 NTU) in water entering the distribution network, can significantly reduce the risk of breakthrough of pathogenic microorganisms.

Microorganisms present in the biofilms, sediments and corrosion products of pipes could be released into the bulk water during repair and cleaning operations. Microorganisms present in the distribution network are generally benign, but they are at lower end of food chain for organisms such as fungi, protozoa, worms and crustaceans, whose presence in the distribution network may pose some health risks. Excessive microbiological activity can also lead to aesthetic water quality failures including taste, odor and color.

Microbial water quality

Testing drinking water for all possible pathogens (disease-causing organisms) is complex, time-consuming, and expensive. The burden of excessive time and money spent on daily analysis could be limiting factor for utilities. Regulatory agencies require utilities to monitor their potable water, not for the pathogens, but for a group of specific (indicator) bacteria, which indicate the 'probable' presence of the pathogenic bacteria. The total coliform group, which serves as the indicator organism is relatively easy and inexpensive to test. If a certain level of total coliforms is found in a water sample, steps are taken to find the source of contamination and restore safe drinking water.

Total coliforms are organisms that are present in the environment and in the feces of all warm-blooded animals and humans. Total coliforms will not likely cause illness, however, the presence of coliform bacteria in drinking water indicates that pathogens may be present. Therefore, total coliform provides only a partial and incomplete ‘evidence’ of fecal contamination. While the total coliform group is the best available choice as an indicator, however, some caveats do exist:

- Total coliform group can be found in the environment naturally as well as in the feces of humans or other warm-blooded animals;
- Some bacteria of the total coliform group can live and multiply outside the human body therefore, if samples are analyzed to obtain a direct count of total coliforms, this could lead to an overestimation of their original density;
- Occasionally false positive results are obtained in presumptive test due to noncoliform bacteria. The false positives can lead to regulatory actions even though a true threat to the consumer's health is not present. Therefore, a confirmatory test is performed to ensure that a positive total-coliform test result actually caused by a total coliform; and
- False negative results can be caused by the presence of excessive noncoliform bacteria. False negatives can lead utilities to have false confidence in the quality of the finished product when a true health threat does exist.

Heterotrophic plate counts (HPCs) cover a wider range of organisms and are generally considered better indicators for these conditions than the total coliforms (WHO, 2004). HPCs are indigenous to water (and biofilms) and are always present in distribution networks in greater numbers than total coliform. An increase in HPC numbers indicates treatment breakthrough, post-treatment contamination or growth within the water or the presence of deposits and biofilms in the system. A sudden increase in HPCs above historic baseline values should trigger actions to investigate and, if necessary, remediate the situation. However, there is no evidence that heterotrophic microorganisms in distribution networks are responsible for public health effects in the general population through ingestion of drinking water (WHO, 2004).

HPCs have a long history of use in water microbiology and have been employed as indirect indicators of water safety (WHO, 2004). Commonly, the HPCs measurements are used:

- to specify the effectiveness of water treatment processes, therefore as an indirect indication of pathogen removal;
- as a measure of numbers of regrowth organisms that may or may not have sanitary significance; and
- as a measure of possible interference with coliform measurements in lactose-based culture methods (this application is of declining value, as lactose-based culture media are being replaced by alternative methods that are lactose-free).

There is no evidence, either from epidemiological studies or from correlation with occurrence of waterborne pathogens, that HPC values alone directly relate to health risk. They are therefore unsuitable for public health target setting or as a sole justification for issuing “boil water” advisories. Abrupt increases in HPC levels might sometimes be associated with faecal contamination; tests for *E. Coli* or other faecal-specific indicators and other information are essential for determining whether a health risk exists.

The current *Guidelines for Canadian Drinking Water Quality* do not specify a maximum allowable concentration for HPCs but recommend that HPCs levels in drinking-waters should be less than 500 cfu/ml. If the acceptable HPCs levels are exceeded, an inspection of the system should be undertaken to determine the cause of the increase in heterotrophic bacteria. After analysis of the situation, the guidelines recommend that appropriate actions should be taken to correct the problem and special sampling should continue until consecutive samples comply with the recommended level. Originally, the HPCs guideline was established not to directly protect human health; but rather, it was based upon the knowledge that higher counts of heterotrophic bacteria interfered with the lactose-based detection methods used for total coliform bacteria. New total coliform methods, are not affected by high numbers of heterotrophic bacteria and therefore do not require a set upper limit for HPC. Under these circumstances, utilities are encouraged to use HPC bacteria as a quality control tool (WHO, 2004).

Data fusion

Data fusion refers to the synergistic aggregation of complementary and/or redundant observations and measurements. Different methods (e.g., various microbial water quality sensors) that are used to predict the status of water quality can provide complimentary

information that can increase the accuracy of the prediction. Information collected from various sources can also be redundant rather than complimentary, if it deals with the same aspect of the problem. Redundant data can improve the reliability of an observation / measurement where one measurement / observation is confirmed or rejected by the others. Data fusion is useful for objective aggregation that can be reproducible and interpretable. Many infrastructure engineering problems, e.g., condition assessment of assets, production process quality control, and water quality monitoring require more than one performance indicator to define the overall *condition rating*.

The quantitative aggregation of incomplete, non-specific (ambiguous) and imprecise (vague) information / data warrants soft computing methods, which are tolerant to partial truth and imprecision (Zadeh, 1984). The term *soft computing* comprises an array of heuristic techniques (such as fuzzy logic, probabilistic reasoning, neural networks, and genetic algorithms), which essentially provide a rational and reasoned out framework for solving complex real-world problems (Bonissone, 1997).

DEMPSTER–SHAFFER (DS) THEORY

Two major types of uncertainties are observed, *aleatory* (natural heterogeneity and stochasticity) and *epistemic* (subjectivity, ignorance). The traditional approach to handle aleatory uncertainty is through probabilistic analysis based on historical data (a *frequentist* approach). Traditionally, epistemic uncertainty was addressed through Bayesian approach, however, the approach was limiting, as it required priori assumptions (Sentz and Ferson, 2002).

Consider a case of water quality deterioration in distribution network in which the possible outcomes (*condition states*) of a failure event are *low*, *medium* and *high* denoted by {L}, {M}, and {H}, respectively. The traditional Bayesian approach can treat these outcomes only as *disjoint* bodies of evidence, i.e. probabilities can be assigned to only *singletons* {L}, {M}, and {H}. Further, according to the basic axiom of probability, $p(L) + p(M) + p(H) = 1$. Consequently, $p(\neg L) = 1 - p(L) \Rightarrow p(M) + p(H)$. The inference about the probability of the complement {L}, $p(\neg L)$ is based on a rather strong assumption, i.e., the *Principle of Insufficient Reason* (Sentz and Ferson, 2002) that ignorance has to be distributed uniformly among all remaining singletons {M} and {H}.

The Dempster-Shafer (DS) theory is a relatively new approach, which extends the traditional Bayesian approach. The DS theory is based on a seminal work of Dempster (1967) and Shafer (1976). The DS theory can be interpreted as a generalization of the Bayesian theory where probabilities are assigned to *subsets* and not only to mutually exclusive *singletons* (Sentz and Ferson, 2002). For example, in the above case, in addition to singletons {L}, {M}, {H}, subsets of outcome (with less specificity) such as {L, M} (read: L or M), {M, H}, {L, H} and {L, M, H} are also considered as candidates for a probability mass assignment, which is discussed in detail in the next section. The Bayesian approach could therefore be viewed as a special case of DS theory, where sufficient evidence exists to assign probability to singletons (highly specific situation) only and ignore under-specific subsets. Thus, Bayesian analysis is unable to differentiate both aleatory and epistemic uncertainties efficiently and cannot handle under-specific and ambiguous evidences without making strong assumptions. The DS theory or evidential reasoning (or theory of evidence) addresses these issues effectively.

The applications of DS theory in civil and environmental engineering vary from slope stability (Binaghi *et al.*, 1998), environmental decision-making (Attoh-Okine and Gibbons, 2001; Chang and Wright, 1996), seismic analysis (Alim, 1988), failure detection (Tanaka and Klir, 1999), construction management (Sönmez *et al.*, 2002), water quality and water treatment (Sadiq and Rodriguez, 2005; Demotier *et al.*, 2005; Boyd *et al.*, 1993), pipe deterioration modeling (Najjaran *et al.*, 2005), and remote sensing (Wang and Civco, 1994) to climate change (Luo and Caselton, 1997). Many more engineering applications of DS theory can be seen in detailed bibliography provided by Sentz and Ferson (2002).

The objective of this paper is to demonstrate the potential of DS the theory as a tool for interpreting water quality data. A simple example with two microbial water quality parameters is used. The basic concepts of evidence theory and various rules of combination (modifications) including Yager (Yg) modified rule, Dubois-Prade (DP) disjunctive rule, and Dezert-Smrandache (DSm) rule are described in detail using inferencing results of the example.

Basic concepts of DS theory

In DS theory, the *frame of discernment* Θ is defined as a set of mutually exclusive alternatives, which allows the *power set* “A” to have a total of $2^{|\Theta|}$ subsets in the domain, where

$|\Theta|$ is the cardinality of the *frame of discernment*. For example, if the *frame of discernment* $\Theta = \{L, M, H\}$, its power set comprises 8 subsets (the cardinality is 3), due to *closed world* assumption over “union” (i.e., the possible outcomes are exhaustive and can not be outside the frame of discernment). This power set A contains the 8 subsets A_i ($i = 1, 2, \dots, 8$), i.e., ϕ (a null set), $\{L\}$, $\{M\}$, $\{H\}$, $\{L, M\}$, $\{M, H\}$, $\{L, H\}$, and $\{L, M, H\}$. Thus, depending on the evidence, masses can be assigned to *low*, *medium*, *high*, *low or medium*, *low or high*, *medium or high*, and *low or medium or high* (the last subset denotes a fully ignorant situation). Recall that this concept is different from the Bayesian approach in which possible outcomes on this *frame of discernment* Θ are $\{L\}$, $\{M\}$ and $\{H\}$.

Three important concepts, namely, *basic probability assignment* (m or *bpa*), *belief* (*bel*), and *plausibility* (*pl*) functions are used in DS theory (Alim, 1988). These are explained using an example of microbial water quality in distribution networks.

Example: To maintain an acceptable water quality in the distribution network, a large amount of water quality data is generated continually. Data are gathered on water quality indicators using different sampling techniques (grab sampling or auto-samplers and subsequent laboratory analysis). To describe the microbial quality of water in the distribution network, two indicators, total coliforms (TC) and HPCs are commonly monitored using grab sampling (as described in the introduction), followed by an analysis in the laboratory or using portable kits in the field.

Assume that total coliform and HPCs are two viable indicators for possible microbial activity in the distribution network. Assume further that concentrations of TC and HPCs measured in the laboratory are mapped over a qualitative scale of potential health risk at three “risk” levels — *low* (L), *medium* (M) and *high* (H). Assume further that data collected from total coliform sampling in a distribution network suggest that $\{M\} = 0.5$ (i.e., TC concentration is such that there is a 50% probability that microbial activity may cause a *medium* risk). This is the case of incomplete information!

Basic probability assignment

The *basic probability assignment* (*bpa* or m) expresses the proportion of all available relevant evidence that supports the claim that a particular element of power set A belongs to the (sub)set A_i but to no particular subset of A_i (Klir, 1995). For a given $m(A_i)$, every subset A_i for which $m(A_i) \neq 0$ is called a *focal element*. The mass $m(A_i)$ is defined over the interval $[0, 1]$, but it is different from the classical definition of probability. The *bpa* of the null subset $m(\phi)$ is zero

and the sum of the *basic probability assignments* $m(A_i)$ in a given evidence set “ $\langle m(A_i), A_i \rangle$ ” is “1”. Thus,

$$m(A_i) \rightarrow [0, 1]; \quad m(\phi) = 0; \quad \sum_{A_i \subseteq A} m(A_i) = 1 \quad (1)$$

In our example, the *focal element* of a given evidence “ $\langle m(A_i), A_i \rangle$ ” can be written as $m(M) = 0.5$, therefore $m(\Theta) = m(L, M, H) = 0.5$. This is because $\{L, M, H\}$ represents complete ignorance and the DS theory dictates that all missing evidence is always assigned to ignorance (as opposed to the Bayesian approach that distributes missing evidence in remaining disjoint subsets).

The lower and upper bounds of a probability can be determined from the *basic probability assignment*, which contains the probability set bounded by two non-additive measures *belief* and *plausibility*.

Belief function

The lower bound, belief (*bel*), for a set A_i is defined as the sum of all the *basic probability assignments* of the proper subsets A_k of the set of interest A_i , i.e., $A_k \subseteq A_i$. The general relation between *bpa* and belief can be written as

$$bel(A_i) = \sum_{A_k \subseteq A_i} m(A_k) \quad (2)$$

It can be shown that

$$bel(\phi) = 0; \quad bel(\Theta) = 1 \quad (3)$$

Example (cont’d): The belief functions are given by

$$bel(L) = m(L) = 0; \quad bel(M) = m(M) = 0.5; \quad bel(H) = m(H) = 0$$

$$bel(L, M) = m(L) + m(M) + m(L, M) = 0 + 0.5 + 0 = 0.5$$

$$bel(L, H) = 0; \quad bel(M, H) = 0.5; \quad bel(L, M, H) = m(L) + \dots + m(\Theta) = 1$$

It can be noticed that $bel(L, M) \geq bel(L) + bel(M)$ because DS theory allows some mass to be assigned to under-specific subset $m(L, M)$, which was not allowed in case of Bayesian approach. Therefore DS theory relaxes a strong *additivity* constraint of probability theory to more relaxed constraint of *monotonicity*.

Plausibility function

The upper bound, *plausibility*, is the summation of *basic probability assignment* of the sets A_k that intersect with the set of interest A_i , i.e., $A_k \cap A_i \neq \emptyset$, and therefore it can be written as

$$pl(A_i) = \sum_{A_k \cap A_i \neq \emptyset} m(A_k) \quad (4)$$

The *plausibility function* can be linked to *belief function* through the *doubt function*, which is defined as the compliment of belief

$$pl(A_i) = 1 - bel(\neg A_i) \quad (5)$$

where $\neg A_i$ is the complement of A_i . In addition, the following relationships for belief and plausibility functions hold true in all circumstances

$$pl(A_i) \geq bel(A_i); \quad pl(\emptyset) = 0; \quad pl(\Theta) = 1; \quad pl(\neg A_i) = 1 - bel(A_i) \quad (6)$$

In our example, the *plausibility function* are given by

$$pl(L) = m(L) + m(L, M) + m(L, H) + m(\Theta) = 0.5$$

$$pl(M) = 1; pl(H) = 0.5; pl(L, M) = 1.0; pl(L, H) = 0.5; pl(M, H) = 1; \text{ and } pl(\Theta) = 1$$

Belief interval

The belief interval (I) is an interval between *belief* and *plausibility* representing range in which true probability may lie. A narrow belief interval represents more precise probabilities. It can be shown that the probability is uniquely determined if $bel(A_i) = pl(A_i)$; probability theory is applicable only where all probabilities are unique and disjoint (Yager, 1987). If $I(A_i)$ has an interval $[0, 1]$, it means that no information is available; on the other hand, if the interval is $[1, 1]$, it means that A_i has been completely confirmed by $m(A_i)$.

The belief interval for our example is

$$I(L) = [0, 0.5]; I(M) = [0.5, 1]; I(H) = [0, 0.5]; I(L, M) = [0.5, 1]; I(L, H) = [0, 0.5]; I(M, H) = [0.5, 1]; \text{ and } I(\Theta) = [1, 1]$$

Dempster–Shafer (DS) rule of combination

The purpose of data fusion is to summarize and simplify information in a rational manner. The DS theory assumes that the sources of information are independent. Alim (1988) described that the “combined” belief not only represents the total belief of a set A_i and all of its subsets but also takes into account the contribution of different sources of evidence that focuses on A_i . The DS inference uses combination operators that compromise on precision but require less information than the Bayesian inference (Sentz and Ferson, 2002).

The DS rule of combination strictly emphasizes agreement between multiple sources and ignores all the conflicting evidence through *normalization*. A strict *conjunctive logic* through AND-type operator (product) is employed in combination of evidence. The DS rule of combination determines the joint m_{1-2} from the aggregation of two *basic probability assignments* m_1 and m_2 by the following equation:

$$m_{1-2}(A_i) = \frac{\sum_{A_p \cap A_q = A_i} m_1(A_p) m_2(A_q)}{1 - K} \quad \text{when } A_i \neq \phi; \quad \text{and } m_{1-2}(\phi) = 0 \quad (7)$$

where $K = \sum_{A_p \cap A_q = \phi} m_1(A_p) m_2(A_q)$ is the *degree of conflict* in two sources of evidence and $m_1(A_p)$ and $m_2(A_q)$ are their corresponding masses.

The denominator $(1-K)$ is a normalization factor, which counterbalances the effect of conflicting evidence on aggregation. The above equations can be rewritten as

$$m_{1-2}(A_i) = \frac{\sum_{A_p \cap A_q = A_i} m_1(A_p) m_2(A_q)}{\sum_{A_p \cap A_q \neq \phi} m_1(A_p) m_2(A_q)} \quad (8)$$

Example (contd.): In addition to total coliform, the HPC is used as second body of evidence for evaluation of microbial water quality. Assume that after qualitative evaluation the following body of evidence $\langle m_2(A_q), A_q \rangle$ is obtained

$$m_2(L) = 0.5; \text{ and } m_2(L, M) = 0.5$$

The basic probability assignment shows that there is a 50% probability (mass) that microbial water quality is *low*, and the same probability (mass) that it is *low* or *medium* (under-specific). The earlier evidence $\langle m_1(A_p), A_p \rangle$ obtained from total coliform sampling results implied that,

$$m_1(M) = 0.5 \text{ and } m_1(\Theta) = 0.5$$

It is noted that the above *bpas* are obtained under the assumption that the subset $\{L, H\}$, i.e. “L” or “H” is practically not possible (i.e., “or” condition requires two contiguous states); and hence, no mass can be

attached to this subset. The aggregation of the two sources B and C is obtained using the DS rule of combination (Equation 8). Thus,

Degree of conflict = $K = 0.25$

Normalization factor = $1 - K = 0.75$

The combined evidence masses are

$m_{1-2}(L) = 0.33$; $m_{1-2}(M) = 0.33$; $m_{1-2}(H) = 0$; $m_{1-2}(L, M) = 0.33$; $m_{1-2}(L, H) = 0$; $m_{1-2}(M, H) = 0$; and $m_{1-2}(\Theta) = 0$

Similarly, *belief* and *plausibility* functions are derived using Equations (2) and (4), respectively. Subsequently, the belief intervals can be derived.

Subsets	$m_{1-2}(\cdot)$	$bel_{1-2}(\cdot)$	$pl_{1-2}(\cdot)$	$I_{1-2}(\cdot)$
{L}	0.33	0.33	0.66	[0.33, 0.66]
{M}	0.33	0.33	0.66	[0.33, 0.66]
{H}	0	0	0	[0, 0]
{L, M}	0.33	1	1	[1, 1]
{M, H}	0	0.33	0.66	[0.33, 0.66]
Θ	0	1	1	[1, 1]

Based on the two bodies of evidence in the example that the water quality can be certainly rated as *low* or *medium*.

The DS rule of combination has interesting characteristics. First, the order of fusion (combination) does not affect the final results. Second, the DS rule of combination is both *commutative* (i.e., $B \oplus C = C \oplus B$) and *associative* (i.e., $A \oplus (B \oplus C) = (A \oplus B) \oplus C$), but not *idempotent* (i.e., $A \oplus A \neq A$).

Some drawbacks / issues

Despite the versatility of the DS theory in dealing with uncertain knowledge, serious drawbacks have been identified for the DS rule of combination. Zadeh (1984) presented an intriguing example of a patient who is diagnosed by two physicians A and B . Physician A diagnosed that the patient has disease x with a probability (confidence) of 99% and has disease y with a probability of only 1%. The physician B , on the other hand, believed that the patient has disease z with a probability of 99% but again has disease y with a probability of 1%. The *frame of discernment* for the disease is $\Theta = \{x, y, z\}$. The DS rule of combination implies that

Degree of conflict = $K = 0.9999$ $\therefore$ Normalization factor = $1 - K = 0.0001$

$m_x(disease) = 0$; $m_y(disease) = 1$; and $m_z(disease) = 0$.

These results are counterintuitive. This problem arises because the DS rule of combination relies only on “non-conflicting evidence”. In this example, the non-conflicting evidence has a mass of 0.0001 and the remaining 0.9999 mass was neglected while deriving mass for the fused information obtained from the two doctors. This example raises two questions about the DS rule of combination: i) how to handle the “conflict”, and ii) is “normalization” (i.e., to make the sum of fused masses equal 1) necessary?

MODIFICATIONS OF DS RULE OF COMBINATION

To address the issues of “conflict” and “normalization”, various techniques have been proposed in the literature focusing on the extension of the DS theory. The most common extensions/ modifications on the DS rule of combination have been proposed by Yager (1987), Smets (1990), Inagaki (1991), Dubois and Prade (1992), Zhang (1994), Murphy (2000), and more recently by Dezert and Smarandache (2004). The *theory of hints* proposed by Kohlas and Monney (1995) may also fall in the category of evidential reasoning. Sentz and Ferson (2002) provided a comprehensive but non-exhaustive review of these modifications. In general, the main difference between the modified rules of combination and the traditional one is in regard to handling the “conflict” and their assumption on *closed world* (exhaustive) of power set (e.g., traditional DS is closed over “union”, whereas Smets (1990) uses *open world* assumption in its Transferable Belief Model, i.e., frame of discernment is not assumed exhaustive). A detailed discussion on this topic can be found in Dezert and Smarandache (2004).

In this paper, three modified rules including Yager (Yg), Dubois and Prade (DP), and Dezert and Smarandache (DSm) are discussed in detail and compared using the microbial water quality example.

Yager (Yg) rule of combination

Yager (1987) rule of combination is very similar to DS rule of combination, except that joint evidence is not normalized with non-conflicting evidence. Thus,

$$m_{1-2}(A_i) = \sum_{A_p \cap A_q = A_i} m_1(A_p)m_2(A_q) \quad (9)$$

The total conflicting evidence (K) is shifted to ignorance Θ so that $m_{1-2}(\Theta)$ is given by,

$$m_{1-2}(\Theta) = \sum_{\substack{A_p \cap A_q = \Theta \\ A_p \cap A_q = \phi}} m_1(A_p)m_2(A_q) \quad (10)$$

Sentz and Ferson (2002) describe this rule as an “epistemologically honest” interpretation of a body of evidence, which does not change it through normalization by non-conflicting evidence. The Yager’s rule of combination is *commutative* but not *idempotent* and *associative*.

Example (cont’d): The Yager’s rule of combination gives the following results

Subsets	$m_{1-2}(\cdot)$	$bel_{1-2}(\cdot)$	$pl_{1-2}(\cdot)$	$I_{1-2}(\cdot)$
{L}	0.25	0.25	0.75	[0.25, 0.75]
{M}	0.25	0.25	0.75	[0.25, 0.75]
{H}	0	0	0.25	[0, 0.25]
{L, M}	0.25	0.75	1	[0.75, 1]
{M, H}	0	0.25	0.75	[0.25, 0.75]
Θ	0.25	1	1	[1, 1]

It is noted that the belief intervals are greater than those obtained from the DS rule of combination. Also, there is a certain mass attached to ignorance Θ due to conflict in evidence from the two sources of information.

Dubois and Prade (DP) rule of combination

Dubois and Prade (1992) modified the DS rule by disjunctive consensus. More precisely, when there is a certain mass assigned to a conflict (e.g., one source of information believes in {L} and the other source believes in {M}) instead of transferring that mass to either Θ (i.e., Yg rule) or normalized based on the non-conflicting mass (Equations 7 and 8, i.e., DS rule), the DP rule assigns that mass to subset {L, M} (i.e., {L} or {M}). In this case, the conflict vanishes, and so does the need for normalization. It is noted that this method yields more non-specific results,

but unlike the Yg rule, it does not artificially translate the specificity of a case from $\{L, M\}$ into ignorance Θ (i.e., $\{L, M, H\}$). The DP rule is described as,

$$m_{1-2}(A_i) = \sum_{A_p \cup A_q = A_i} m_1(A_p)m_2(A_q) \quad (11)$$

The DP rule is commutative, associative but not idempotent.

Example (cont'd): Continuing on the same example, the DP rule of combination gives the following results

Subsets	$m_{1-2}(\cdot)$	$bel_{1-2}(\cdot)$	$pl_{1-2}(\cdot)$	$I_{1-2}(\cdot)$
$\{L\}$	0	0	1	$[0, 1]$
$\{M\}$	0	0	1	$[0, 1]$
$\{H\}$	0	0	0.5	$[0, 0.5]$
$\{L, M\}$	0.5	0.5	1	$[0.5, 1]$
$\{M, H\}$	0	0	1	$[0, 1]$
Θ	0.5	1	1	$[1, 1]$

It is noted that the belief intervals are greater than those obtained from the DS and Yg rules of combination. This rule falsely introduces more uncertainty among its subsets due to its disjunctive nature during fusion.

Dezert-Smarandache (DSm) rule of combination

The DS, Yg and DP rules of combination deal with mutually exclusive and exhaustive sets. For example, the exclusivity assumption means that $\{L\}$, $\{M\}$ and $\{H\}$ are three possible outcomes to describe condition ratings of water quality. Though, bodies of evidence can allow less specific situations such as $\{L, M\}$ (i.e., “L” or “M”). In other words, these rules are *closed* over “union” i.e., they allow ambiguous and under-specific situations such as “L” **or** “M”, but not nonexclusive situations such as “L” **and** “M”. The Dezert and Smarandache rule of combination relaxes the constraint of exclusivity, which means that it is “closed” over both “intersection” and “union”. Thus, the frame of discernment $\Theta = \{L, M, H\}$ has a hyper power set called *Dedekind* distributive lattice D^Θ , which is also closed over “intersection” and “union”. Therefore, the hyper power for our example (cardinality = 3) consists of 19 subsets including,

$\phi, L, M, H, L \cup M, M \cup H, L \cup H, L \cup M \cup H, L \cap M, M \cap H, L \cap H, L \cap M \cap H, L \cap (M \cup H), M \cap (L \cup H), H \cap (L \cup M), L \cup (M \cap H), M \cup (L \cap H), H \cup (L \cap M), (L \cap M) \cup (L \cap H) \cup (M \cap H)$

If all the subsets that include combination of L and H are ruled out as not possible, the hyper power set can be reduced to

$\phi, L, M, H, L \cup M, M \cup H, L \cup M \cup H, L \cap M, M \cap H, L \cap M \cap H, L \cap (M \cup H), H \cap (L \cup M), L \cup (M \cap H), H \cup (L \cap M)$

To further simplify the analysis, the masses associated with the subsets containing three elements are assigned to ignorance $\Theta = L \cup M \cup H$. Therefore the hyper power set can be further reduced to the following subsets

$\phi, L, M, H, L \cup M, M \cup H, L \cap M, M \cap H, L \cup M \cup H$

The relaxation of the mutual exclusivity constraint can be interpreted to mean that the focal elements are fuzzy in nature. The intersection represents the situation in which the two bodies of evidence not only conflict but are also equally reliable. The union, on the other hand, represents the situation in which the two bodies of evidence are less specific or ambiguous.

The DSm rule of combination is defined as,

$$m_{1-2}(A_i) = \sum_{\substack{A_p, A_q \in D^\Theta \\ A_p \cap A_q = A_i}} m_1(A_p) m_2(A_q) \quad (12)$$

The major practical issue related to use of DSm theory is the “curse of dimensionality” which increases exponentially with an increase of cardinality of frame of discernment. The DSm rule is commutative and associative but not idempotent.

Example (cont’d): The DSm rule of combination gives the following results

To estimate the belief of any basic focal element, e.g., “L” and “M”, the intersection mass is equally distributed to both elements (i.e., similar to *Principle of Insufficient Reason*). Thus, the belief of “L” is given by

$$bel_{1-2}(L) = m_{1-2}(L) + \frac{1}{2} [m_{1-2}(L \cap M)] = 0.25 + \frac{1}{2} [0.25] = 0.38$$

Similarly, the plausibility of “L” is given by

$$pl_{1-2}(L) = m_{1-2}(L) + m_{1-2}(L \cap M) + m_{1-2}(L \cup M) = 0.25 + 0.25 + 0.25 = 0.75$$

Interestingly, the DSm yields the smallest belief intervals in comparison to all other rules. This effect is due to relaxation of the exclusivity constraint. Theoretically, the *belief* of subset $\{L \cap M\}$ can be

determined independently without distributing the mass to basic elements L, M and H. This mass which is attached to $\{L \cap M\}$ shows the fuzziness of focal elements, i.e., $\{L\}$ and $\{M\}$ are overlapping on the universe of discourse.

Figure 1 illustrates the belief and plausibility functions obtained from the four rules of combination in the microbial water quality example. The area between *bel* and *pl* lines represents the belief interval (*I*) for each condition state. For example the highest level of uncertainties can be observed in the DP rule of combination.

Subsets	$m_{1-2}(\cdot)$	$bel_{1-2}(\cdot)$	$pl_{1-2}(\cdot)$	$I_{1-2}(\cdot)$
L	0.25	0.38	0.75	[0.38, 0.75]
$L \cap M$	0.25			
$L \cup M$	0.25			
M	0.25	0.38	0.75	[0.38, 0.75]
$M \cap H$	0			
$M \cup H$	0			
H	0	0	0	[0, 0]
Θ	0			

Estimating Utilities

The quality ordered weights $q \in [0, 1]$ can be assigned to probabilities of possible outcomes to evaluate the overall impact (utility). This concept is similar to risk analysis where, consequences are associated with failure probabilities to determine the overall risk. In our case, quality ordered weights are assigned to *belief* and *plausibility* functions of possible outcomes $\{L\}$, $\{M\}$, and $\{H\}$ to determine the condition rating of the water quality based on given bodies of evidence.

Yang and Xu (2002) discussed a probabilistic method to determine the utility values in a heuristic way. Liuo and Lo (2005) proposed an approach to determine the utilities given by,

$$\left. \begin{aligned}
 \text{Lower utility} &= U_L = [q_L \times bel(L) + q_M \times bel(M) + q_H \times bel(H)] \times 100 \\
 \text{Upper utility} &= U_U = [q_L \times pl(L) + q_M \times pl(M) + q_H \times pl(H)] \times 100 \\
 \text{Expected utility} &= (U_U + U_L)/2
 \end{aligned} \right\} \quad (13)$$

where $q_L = 0 \quad q_M = 0.5 \quad q_H = 1$

Figure 1. *Belief* and *plausibility* functions obtained from different rules of combination as applied to water quality example

The interval $[U_U - U_L]$ represents the uncertainties associated with bodies of evidence and the type of data fusion technique used, whereas the expected utility U_A is the best point estimate based on averaging lower and upper utilities.

Example (cont'd): Figure 2 compares the upper and lower utility values obtained from the four rules of combination. It is noted that the smallest interval will be obtained from the DS_m rule while the largest interval is obtained from the DP rule due to relaxation of mutual exclusivity constraint in DS_m rule and disjunctive operation in DP rule of combination, respectively. The DP disjunctive rule increases the *plausibility* function (i.e., increase the utilities interval), therefore in turn pushes the expected utility U_A further up, giving more conservative results as compared to other methods.

Figure 2. Comparison of upper and lower utilities for 4 data fusion rules

SOME SPECIAL CASES FOR COMBINING BODIES OF EVIDENCE

Combining bodies of evidence from different sources of information involves two critical points - type of evidence and the way the conflict is handled. Sentz and Ferson (2002) identified 4 different types of evidence as shown in Figure 3. Consonant bodies of evidence are nested evidence, in which each new source of information fully supports the prior belief in a particular proposition (Figure 3A). This type of evidence provides a linkage between the Dempster-Shafer theory and with the fuzzy sets interpretation of *possibility theory* (Dubois and Prade, 1988).

Consistent bodies of evidence (Figure 3B) mean that the belief in a particular proposition is consistently supported by other propositions that are in conflict with each other. Therefore, there is consensus among the bodies of evidence, but the consensus is less than that of consonant evidence. Arbitrary bodies of evidence (Figure 3C) refer to the situation in which no proposition is completely supported by the other bodies of evidence. Thus, the consensus is less than consistent evidence. Disjoint bodies of evidence (Figure 3D) is a typical case of traditional probability theory in which all sources of information provide evidence as mutually exclusive subsets. Evidential reasoning can handle all four types of bodies of evidence.

Figure 3. Four types of evidence from three different sources

To elaborate the difference in combining various types of evidence, 11 cases are summarized in Table 1. In this table, the mass of the first body of evidence (total coliform, m_1) remains constant $\{M\} = 0.5$ and $\Theta = 0.5$, while the mass of the second body of evidence (HPC, m_2) varies to represent these cases. Table 2 shows the estimated utilities obtained from the four rules of combination.

1. m_2 disjoint (& focussed) and non-conflicting with m_1

The sources of information are disjoint and non-conflicting, i.e., both sources confirm that masses are assigned to $\{M\}$, which is disjoint and focussed (i.e., the whole probability mass is assigned to one disjoint alternative). Except for the DP rule, all three methods reduce the ignorance Θ to 0.15, from $\Theta_1 = 0.5$ and $\Theta_2 = 0.3$. The DP rule uses disjunctive logic and rather increased the ignorance to 0.65. The expected utility obtained from the DP rule is also highest (66), which means that it predicts with a highest conservatism than the actual evidence warrants.

Table 1. Some special cases for combining different types of evidences

Case	Type of evidence	m_1	m_2
1	Disjoint (& focussed) and non-conflicting	$\{M\} = 0.5; \Theta = 0.5$	$\{M\} = 0.7; \Theta = 0.3$
2	Disjoint (& distributed) and non-conflicting	$\{M\} = 0.5; \Theta = 0.5$	$\{L\} = 0.5; \{M\} = 0.5$
3	Under-specific and non-conflicting	$\{M\} = 0.5; \Theta = 0.5$	$\{L, M\} = 0.7; \Theta = 0.3$
4	Disjoint (& focussed) and conflicting	$\{M\} = 0.5; \Theta = 0.5$	$\{L\} = 0.7; \Theta = 0.3$
5*	Disjoint (& distributed) and conflicting	$\{M\} = 0.5; \Theta = 0.5$	$\{L\} = 0.5; \{H\} = 0.5$
6	No evidence (or complete ignorance)	$\{M\} = 0.5; \Theta = 0.5$	$\Theta = 1$
7	Disjoint (& uniformly distributed)	$\{M\} = 0.5; \Theta = 0.5$	$\{L\} = \{M\} = \{H\} = 0.33$
8	Under-specific (& uniformly distributed)	$\{M\} = 0.5; \Theta = 0.5$	$\{L, M\} = \{M, H\} = 0.5$
9	Consistent	$\{M\} = 0.5; \Theta = 0.5$	$\{M\} = 0.5; \{M, H\} = 0.5$
10	Contradictory	$\{M\} = 0.5; \Theta = 0.5$	$\{L\} = 1$
11	Mixed or arbitrary	$\{M\} = 0.5; \Theta = 0.5$	$\{L\} = 0.5; \{M, H\} = 0.5$

Table 2. Estimated utilities for different types of evidence

Case	DS				Yg				DP				DSm			
	U_L	U_U	U_A	Θ	U_L	U_U	U_A	Θ	U_L	U_U	U_A	Θ	U_L	U_U	U_A	Θ
1	43	65	54	0.15	43	65	54	0.15	18	115	66	0.65	43	65	54	0.15
2	33	33	33	0	25	63	44	0.25	13	100	56	0.5	31	38	34	0
3	25	65	45	0.15	25	65	45	0.15	0	115	58	0.65	25	65	45	0.15
4	12	46	29	0.23	8	83	45	0.5	0	115	58	0.65	16	48	32	0.15
5*	50	50	50	0	25	100	63	0.5	0	100	50	0.5	44	75	59	0
6	25	100	63	0.5	25	100	63	0.5	0	150	75	1	25	100	63	0.5
7	50	50	50	0	34	84	59	0.34	8	100	54	0.5	46	67	57	0
8	25	50	38	0	25	50	38	0	0	100	50	0.5	25	75	50	0
9	38	50	44	0	38	50	44	0	13	100	56	0.5	38	75	56	0
10	0	0	0	0	0	75	38	0.5	0	100	50	0.5	13	25	19	0
11	17	33	25	0	13	63	38	0.25	0	100	50	0.5	19	63	41	0

2. m_2 disjoint (& uniformly distributed) and non-conflicting with m_1

This case refers to the situation that one body of evidence is distributed (i.e., the whole probability mass is distributed uniformly in disjoint evidence) between two disjoint alternatives, $\{L\}$ and $\{M\}$. Both DS and DSm rules give similar results, but in the DS rule, 25% of evidence is not used and the results are normalized based on the remaining 75% non-conflicting evidence. The average utility values obtained from both Yager and DP rules are significantly higher than that obtained from the DS and DSm rules.

3. m_2 under-specific and non-conflicting with m_1

The second source of information m_2 is under-specific because 70% of total mass is assigned to $\{L, M\}$ and remaining mass is given to ignorance. The average utility value is 45 for the DS, Yager and DSm rules, but the DP rule gives a higher value of 58.

4. m_2 disjoint (& focussed) and conflicting with m_1

In case of conflicting evidence, the ignorance mass is high in all rules except the DSm rule. However, this is not an appropriate comparison since in case of the DS rule the masses are normalized based on non-conflicting evidence, which may decrease the ignorance mass. The largest utility interval $[0, 115]$ is in case of the DP rule, which is incapable of handling conflicting evidence.

5. m_2 disjoint (& distributed) and conflicting with m_1

The asterisk sign marked in the table indicates that the case is practically not possible (according to our assumption) since the mass is attached to both $\{L\}$ and $\{H\}$ simultaneously in the disjoint evidence. But the analysis is carried on to illustrate this particular case of evidence. The DS rule cannot use 50% of the conflicting evidence. The estimated upper and lower utility values are the same due to the nature of the utility Equation 13 in which the quality ordered weight (q_L) assigned to $\{L\}$ is zero. Again, the largest belief interval $[0, 100]$ was obtained through the DP rule of combination.

6. m_2 provides no evidence (or complete ignorance)

The ignorance case maintains the same body of evidence m_1 whereas the second source of information m_2 provides no-evidence.

7. m_2 disjoint (& uniformly distributed)

A 67% conflict of evidence was obtained in case of the DS rule of combination. Again the largest utility interval [8, 100] is obtained using the DP rule of combination.

8. m_2 under-specific (& uniformly distributed)

No conflict is obtained while using the DS rule of combination. Consistently, the largest utility interval [0, 100] is obtained for the DP rule of combination. The utility interval is same for DS and DSm, but higher average utility was obtained in case of the DSm rule due to higher mass attached to plausibility of {H}.

9. m_2 consistent and non-conflicting with m_1

Due to consistent evidence, the DS and Yg rules of combination provide a smaller utility interval because these rules rely on an alternative {M}, which is common in both bodies of evidence. On the contrary, the DSm rule uses the conflicting mass {H}; and hence, the average utility value (56) estimated obtained from this rule is higher than that obtained from the DS and Yg rules.

10. m_2 and m_1 contradictory

The DS rule of combination cannot handle contradictory evidence. The smallest utility interval [13, 25] is obtained in case of the DSm rule of combination.

11. m_2 and m_1 arbitrary or mixed

The highest ignorance mass is obtained in case of the DP rule of combination. In case of the DS and DSm rules, the ignorance mass is zero.

Combining Sources of Varying Credibility

The comparison described above implicitly assumes that all sources of information are equally credible. Sampling locations for monitoring water quality may be representative of a particular part of the water distribution system, e.g., if one sample is collected from main distribution line and the other is collected from a minor line, the influence zones of the two samples are different. Similarly, if the samples are collected at the same point when two different flow conditions prevail, the evidence of water quality also needs to be adjusted based on the flow

conditions. Also, if water utility staff with different levels of expertise collects water samples, the observations need to be adjusted based on their credibility.

Therefore, the bodies of evidence obtained from different sources of information need to be discounted using *credibility factor* (α) depending on the relative strength and/or reliability of each source. The evidence can be discounted as

$$\left. \begin{aligned} m_{1-2}(A_i)_\alpha &= m_{1-2}(A_i) \cdot \alpha \\ m_{1-2}(\Theta)_\alpha &= m_{1-2}(\Theta) \cdot \alpha + (1 - \alpha) \end{aligned} \right\} \quad (14)$$

The credibility factor is constrained by $0 \leq \alpha \leq 1$, where “0” represents “fully incredible evidence”, and “1” represents “fully credible evidence”.

Yager (2004) discussed the credibility issue in detail and suggested a *credibility transformation function*. This approach discounts the evidence with a credibility factor (α) and distributes the remaining evidence ($1-\alpha$) equally among the other elements of *frame of discernment*.

$$m_{1-2}(A_i)_\alpha = m_{1-2}(A_i) \cdot \alpha + \frac{1 - \alpha}{n} \quad (15)$$

where α is the *Credibility factor*, and n is the number of the *focal elements* of the *frame of discernment*.

CONCLUSIONS

In this paper, evidence theory was introduced as an innovative methodology that can be used to simplify and improve the understanding and interpretation of data generated through routine water quality monitoring in distribution systems. Here we would like to refer to a statement by Halpern and Fagin (1992) about data fusion who said that *Data fusion is not a problem of mathematics rather its a problem of judgment!*

The theory of evidence can effectively deal with the difficulties related to the multiplicity of indicators describing water quality, coupled with the spatial and temporal dimensions of distribution systems, where redundant information is routinely collected from sources that may have variable credibility. A hypothetical example of water quality monitoring was used to

demonstrate the concepts. This example included two complimentary sources of information, the total coliforms and HPCs, to compare the *belief* and *plausibility* functions (and the *belief intervals*) obtained from four alternative rules of combination. Each of these rules of combination deals differently with ignorance and conflict and hence predicts different *belief* and *plausibility* values. These values can then be used in conjunction with empirical *quality-ordered* weights to determine overall utilities, which are a precursor to condition assessment and data interpretation in the water quality context. The four rules of combination yield different utilities and there is no general criterion to decide which rule is the most appropriate choice. However, within a specific context, one may be able to distinguish the advantages of one rule over the others. For instance, if a water distribution system is vulnerable and hence prone to higher risk of water quality failure, it may be preferable to use the rule of combination that tends to yield more conservative results (i.e., a larger *belief interval*). Similar arguments can be made to justify other fusion rules in different situations.

Future research should focus on the implementation of decision-making tools using the theory of evidence that can be adapted to specific water utility conditions and managers' needs. The potential combination of theory of evidence with modeling techniques, such as linear and nonlinear time-series analysis, neural networks, and genetic algorithms, to predict the *condition ratings* of water quality should also be evaluated through future research efforts to implement more powerful decision-making tools.

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