NRC Publications Archive Archives des publications du CNRC

Annual heat and mass transfer at an Ottawa site

Gold, L. W.; Boyd, D. W.

This publication could be one of several versions: author's original, accepted manuscript or the publisher's version. /
La version de cette publication peut être l'une des suivantes : la version prépublication de l'auteur, la version
acceptée du manuscrit ou la version de l'éditeur.

For the publisher's version, please access the DOI link below. / Pour consulter la version de l'éditeur, utilisez le lien
DOI ci-dessous.

Publisher's version / Version de l'éditeur:

<https://doi.org/10.1139/e65-001>

Canadian Journal of Earth Sciences, 2, pp. 1-10, 1965-02

NRC Publications Record / Notice d'Archives des publications de CNRC:

<https://nrc-publications.canada.ca/eng/view/object/?id=b3930b6b-36c5-4208-9661-69d83c3bfa6a>

<https://publications-cnrc.canada.ca/fra/voir/objet/?id=b3930b6b-36c5-4208-9661-69d83c3bfa6a>

Access and use of this website and the material on it are subject to the Terms and Conditions set forth at

<https://nrc-publications.canada.ca/eng/copyright>

READ THESE TERMS AND CONDITIONS CAREFULLY BEFORE USING THIS WEBSITE.

L'accès à ce site Web et l'utilisation de son contenu sont assujettis aux conditions présentées dans le site

<https://publications-cnrc.canada.ca/fra/droits>

LISEZ CES CONDITIONS ATTENTIVEMENT AVANT D'UTILISER CE SITE WEB.

Questions? Contact the NRC Publications Archive team at

PublicationsArchive-ArchivesPublications@nrc-cnrc.gc.ca. If you wish to email the authors directly, please see the
first page of the publication for their contact information.

Vous avez des questions? Nous pouvons vous aider. Pour communiquer directement avec un auteur, consultez la
première page de la revue dans laquelle son article a été publié afin de trouver ses coordonnées. Si vous n'arrivez
pas à les repérer, communiquez avec nous à PublicationsArchive-ArchivesPublications@nrc-cnrc.gc.ca.

National Research
Council Canada

Conseil national de
recherches Canada

Canada

Ser
TH1
N21r2
no. 232
c. 2

BLDG

17382
ANALYZED

NATIONAL RESEARCH COUNCIL
CANADA

DIVISION OF BUILDING RESEARCH

ANNUAL HEAT AND MASS TRANSFER
AT AN OTTAWA SITE

BY

L. W. GOLD AND D. W. BOYD

26173

Reprinted from
CANADIAN JOURNAL OF EARTH SCIENCES,
VOL. 2, FEBRUARY 1965, P. 1

BUILDING RESEARCH
- LIBRARY -

FEB 18 1966

NATIONAL RESEARCH COUNCIL

RESEARCH PAPER NO. 232

OF THE

DIVISION OF BUILDING RESEARCH

PRICE 25 cents

OTTAWA, FEBRUARY 1965

NRC 8198

SOMMAIRE

Le composé Fourier d'une période d'un an est calculé pour la radiation nette, la radiation solaire nette, la radiation solaire pénétrante et le flux de chaleur du sol à partir d'observations faites en un lieu couvert d'herbe à Ottawa au cours, de la période de deux ans écoulée entre le 1^{er} janvier 1961 et le 31 décembre 1962. L'approximation Fourier de l'évapotranspiration et de la perte par sublimation est déterminée à partir d'estimations faites du transfert de masse moyen au moment du maximum en été et du minimum en hiver. L'approximation du composé annuel du transfert de convection est obtenue au moyen d'une équation d'équilibre thermique.

Durant la période de six mois écoulée entre le 1^{er} avril et le 30 septembre environ 48% de la radiation solaire nette a été dissipée par évapotranspiration, 42% par radiations à onde longue, 7% par convection, et 3% ont été emmagasinés dans le sol.

ANNUAL HEAT AND MASS TRANSFER AT AN OTTAWA SITE¹

L. W. GOLD

Division of Building Research, National Research Council, Ottawa, Canada

AND

D. W. BOYD²*Meteorological Branch, Department of Transport, Toronto, Ontario*

Received April 10, 1964

ABSTRACT

The Fourier component of period 1 year is calculated for the net radiation, net solar radiation, incoming solar radiation, and ground heat flux from observations made at a grass-covered site in Ottawa in the 2-year period 1 January 1961 to 31 December 1962. The Fourier approximation of the evapotranspiration and sublimation loss is determined from estimates of the average mass transfer at the time of maximum in summer and minimum in winter. The approximation of the annual component of the convective transfer is obtained using the heat balance equation.

During the 6-month period 1 April to 30 September about 48% of the net solar radiation was dissipated by evapotranspiration, 42% dissipated by long-wave radiation, 7% dissipated by convection, and 3% was stored in the ground.

Heat and moisture transfers at the ground surface undergo an annual cycle as a consequence of the annual variation in solar radiation. Observations indicated that net radiation and heat flow from the ground at an Ottawa site were closely approximated by a simple harmonic function with period of 1 year, and that the amplitude and phase of this function did not change appreciably from year to year. This suggested that useful approximations of the other components of the heat balance could be obtained for the site with similar functions. Although it was not possible to measure these components continuously, it was considered that the amplitude and phase of the simple harmonic approximation could be deduced for the moisture transfer from estimates of the average maximum and minimum values, and the times of the year at which these extremes occur. The simple harmonic approximation to the convective exchange could then be obtained as the difference in the heat balance equation. It was realized that the approximations might at times be in considerable relative error, particularly during the period of minimum exchange, but it was considered that they would provide a useful picture of the annual exchange, a reference framework for more accurate measurements and probably as accurate an indication of heat and moisture loss over a long period

¹Issued as N.R.C. No. 8198.

²Seconded to the Division of Building Research, National Research Council, Ottawa, Canada.

of time as could be obtained with present-day instruments. It was with this in mind, as well as the intention of obtaining additional insight into the relationships between climate and the ground thermal regime, that the study recorded in this paper was undertaken. The results presented are based on observations made over the 2-year period January 1, 1961, to December 31, 1962.

INSTRUMENTATION

The observation area was some 60 meters square and covered with field grass allowed to grow to a height of between 15 and 20 cm before being cut. There were a number of one- and two-storey buildings in the vicinity, the nearest about 45 meters north and south of the measurement area. The site was open to the west, and buildings were at least 120 meters away to the east. There was a single row of trees about 30 meters to the southwest.

Air temperature about 120 cm above the soil surface was measured in a Stevenson screen with a ventilated thermocouple immersed in oil in a glass tube about 0.75 cm in diameter and 20 cm long. Wind speed was measured 2 meters above the soil surface with a type 45 anemometer made by the Meteorological Service of Canada. Dew point at the 2-meter level was measured continuously on the north side of a small instrument hut with a "Dewcel" manufactured by the Foxboro Co. Ltd. Evaporation observations were made with a class A pan supplied by the Meteorological Service of Canada. The net radiation observations were made with a radiometer (designed by the Commonwealth Scientific and Industrial Research Organization of Australia) (Funk 1959) mounted about 2 meters above the soil surface. In November of 1961 observations were begun on net solar radiation using two Eppley pyrheliometers mounted horizontally about 2 meters above the soil surface. One instrument faced upward, the other downward.

Ground temperatures have been observed at the site since 1957. A description of the installation is given in a paper by Pearce and Gold (1959). The ground heat flow component was estimated from the measured temperatures, the necessary soil thermal properties having been determined in the earlier study (1959).

CALCULATION OF COMPONENTS

Net Radiation

Monthly averages of the measured daily net radiation are presented in Fig. 1. Shown as well is the Fourier approximation given by

$$q_r = R_0 + R_1 \cos (wt + \delta)$$

where q_r is the daily net radiation, positive for heat supplied to the surface,

w is equal to 360 deg per year,

t is the time in years measured from the beginning of January,

δ is the phase angle in degrees,

R_0 is the annual average of the daily net radiation,

R_1 is the amplitude of the Fourier component with period 1 year.

The monthly average values calculated from the Fourier approximation are within ± 20 cal/cm² day of the observed values. This difference is less than 10%

FIG. 1. Dependence of net radiation on time of year. The plotted points are monthly averages of daily net radiation.

of the observed daily average for each month in mid-summer, but almost 100% of the observed average in mid-winter.

These figures indicate the significance of the winter exchange relative to the larger summer exchange. The relative winter error is not considered significant in this present study because it is very difficult to measure net radiation in the field with an accuracy better than about ± 30 cal/cm² day using radiometers available at present (Boyd 1962).

Ground Heat Flow

The Fourier approximation of the heat flow from the ground is given by

$$(1) \quad q_g = \sqrt{(w/\lambda)k}T_1 \exp(-z\sqrt{(w/2\lambda)}) \cos(wt + \delta - z\sqrt{(w/2\lambda)})$$

where q_g = heat flow from ground, positive for heat flow to the surface,

λ = thermal diffusivity of soil,

k = thermal conductivity of soil,

z = depth of observation below surface,

δ = phase angle of the heat flux at the surface,

T_1 = amplitude at the surface of Fourier component of the ground temperature with a period of 1 year.

Using the soil thermal properties given by Pearce and Gold (1959) ($\lambda = 10.8$ cm² h, $k = 8.45$ cal/cm °C h) and calculating T_1 from ground temperature observations, the following approximations were obtained for the ground heat flow at the surface:

Period	Fourier approximation
1 January 1960 – 31 December 1960	$18.9 \cos (wt + 20^\circ) \text{ cal/cm}^2 \text{ day}$
1 January 1961 – 31 December 1961	$17.5 \cos (wt + 24^\circ) \text{ cal/cm}^2 \text{ day}$

For the purpose of the present discussion, the Fourier approximation for the 2-year period was taken as $18 \cos (wt + 22^\circ) \text{ cal/cm}^2 \text{ day}$.

Mass Transfer

The heat flux associated with the average mass transfer, including evaporation, transpiration, and sublimation, often is estimated for periods of 1 day or longer with semiempirical formulae of the form

$$(2) \quad q_e = A(U_a + B)(e_a - e_s),$$

where q_e is the heat flux associated with the average mass transfer, positive for vapor transfer to the surface,

U_a is the average wind speed at the measuring height a ,

e_a is the average water vapor pressure at the same height,

e_s is the saturation vapor pressure at the average surface temperature,

A and B are constants.

Earlier observations indicated that at Ottawa the average daily snow surface temperature is usually between 0 and 1 C deg colder than the average daily air temperature, and that in summer the average daily soil surface temperature is about 1 to 3 C deg warmer than the average daily air temperature (Gold 1963). The average daily air temperature was used, therefore, in calculating the saturation vapor pressure at the surface for periods of snow cover. The saturation vapor pressure at the surface in summer was assumed to be that for a temperature equal to the average daily temperature of the soil surface at the base of the grass stems. Average daily values for e_a were calculated from observed average hourly values. Average monthly values of $e_a - e_s$ as well as Fourier approximations calculated from 5-day averages of $e_a - e_s$, are plotted in Fig. 2.

Penman (1948, 1956) gave, for a saturated surface, the following values for the constants in equation (2):

$$A = 9.75 \times 10^{-7} \text{ cal/cm}^3 \text{ mb}$$

$$B = 32.5 \times 10^4 \text{ cm/h}$$

for U_a measured in cm/h at a height of 2 meters above the surface; e_a and e_s in mb. The average wind speed at the site in July (the time of maximum evapotranspiration) was $53.4 \times 10^4 \text{ cm/h}$ in 1961, and $59.9 \times 10^4 \text{ cm/h}$ in 1962. The average daily value for $e_a - e_s$ at the time of maximum was about -18.6 mb in 1961, and about -13.3 mb in 1962. Assuming $e_a - e_s$ to be -16 mb and the average wind speed $56 \times 10^4 \text{ cm/h}$, equation (2) gives

$$\begin{aligned} q_e &= -13.8 \text{ cal/cm}^2 \text{ h} \\ &= -330 \text{ cal/cm}^2 \text{ day.} \end{aligned}$$

FIG. 2. Time dependence of the monthly averages of the vapor pressure difference between the 2-meter level and the ground surface. The vapor pressure at the surface is assumed to be the saturation value at the average surface temperature.

The measured average evaporation rate from the class A pan was observed to be 0.49 cm water per day in July 1961 and 0.55 cm in July 1962, equivalent to about 290 cal/cm² day and 320 cal/cm² day, respectively. Converting class A pan observations to lake evaporation gives about 0.38 cm in 1961 and 0.42 cm in 1962, equivalent to about 225 and 250 cal/cm² day, respectively. Since the average surface temperature of a lake is probably cooler than the average surface temperature of a freely evapotranspiring grass surface subject to the same weather, the daily moisture loss from such a surface should be between that for a lake and that for the class A pan. The class A pan observations, therefore, indicate a value for evapotranspiration somewhat lower than that obtained from equation (2) using constants established by Penman. The value of e_s used in equation (2), however, was based on average temperatures measured at the base of the grass stems, and these temperatures would be expected to be higher than the average temperature of the transpiring surfaces. This would result in an absolute value for $e_a - e_s$ in equation (2) that is too large and, therefore, an estimate of the evapotranspiration loss that is too high. It was assumed for the purpose of the present calculations, therefore, that the average moisture loss from a freely evapotranspiring surface during July at Ottawa was equivalent to 280 cal/cm² day, and recognized that this estimate may be in error by about $\pm 10\%$.

If the soil moisture content is not at field capacity, evapotranspiration will be less than that from a saturated surface. Penman found that, for the grass surfaces upon which he made his observations, evapotranspiration was between 0.60 and 0.80 of that calculated from equation (2), depending on the

amount of water stored in the soil (Penman 1956). The value of 0.60 was obtained under winter conditions in southeast England.

During July at Ottawa it is normal for evapotranspiration to exceed rainfall by almost 6 cm of water, but conditions vary a great deal from year to year. In July 1961 the soil moisture content remained nearly at field capacity for 3 weeks (according to calculations based on the Thornthwaite method (Thornthwaite and Mather 1957)) but was rapidly reduced during the last week to about 4.5 cm below field capacity. In July 1962 the soil moisture content was about 8 cm below field capacity at the beginning of the month and remained between 8 and 12 cm throughout the month. These calculations are in agreement with observations made by the Soil Mechanics Section of the Division of Building Research, National Research Council, on the soil water pressure measured with a piezometer at the 3-meter depth at a site about 12 meters east of the row of trees. In July 1961 the soil water pressure at that depth varied between 253 and 300 cm of water, and in July of 1962, between 195 and 241 cm of water.

The observations on soil moisture and the Thornthwaite calculations indicated a good supply of soil water. It was assumed, therefore, based on Penman's observations, that average evapotranspiration during July for the 2 years was 0.80 of the assumed evaporation from a saturated surface (280 cal/cm² day), that is, about 225 cal/cm² day.

The ground at the observation site is fairly level, usually with no surface runoff in summer. Because the underlying soil is clay, it was considered that there was no net loss of water from the site due to ground water flow. Measured rainfall, therefore, should give the lower limit to the evapotranspiration loss. In July of 1961 the average rainfall was 0.38 cm/day, equivalent to about 225 cal/cm² day. In July 1962 the average rainfall was 0.33 cm/day, equivalent to about 195 cal/cm² day. These values are in good agreement with the estimate obtained from equation (2) and the class A pan observations.

For sublimation in winter, reasonable values for the constants in equation (2) are

$$\begin{aligned} A &= 1.5 \times 10^{-6} \text{ cal/cm}^3 \text{ mb,} \\ B &= 0 \text{ (Williams 1959).} \end{aligned}$$

The average wind speed at the site in January is about 65×10^4 cm/h (4 m.p.h.). Figure 2 indicates that the monthly average value for $e_a - e_s$ at the time of minimum average sublimation loss in winter is about -0.67 mb. Substituting these values into equation (2) gives

$$\begin{aligned} q_e &= -0.65 \text{ cal/cm}^2 \text{ h,} \\ &= -15.6 \text{ cal/cm}^2 \text{ day.} \end{aligned}$$

On the basis of the preceding discussion, it was assumed that the Fourier approximation of the heat loss by evaporation and sublimation had a maximum of 225 cal/cm² day in summer, a minimum of 15 cal/cm² day in winter, and an annual average value of 120 cal/cm² day. From the observations on $e_a - e_s$, it

was assumed that the phase angle was -2 deg, with time measured from the beginning of January, giving maximum evapotranspiration in early July and minimum sublimation in early January. In choosing the phase angle, greater weight was given to the observations for 1962 than to those for 1961. The equation obtained is given below:

$$q_e = -120 + 105 \cos (wt - 2) \text{ cal/cm}^2 \text{ day.}$$

Figure 2 indicates that for a saturated surface the Fourier component with period of 1 year is only a fair approximation of the observations of $e_a - e_s$. The reason for this, in part, is the non-linear dependence on temperature of the saturation vapor pressure of water. The drying of the soil from evapotranspiration will reduce the actual difference between the average vapor pressure of the air and at the surface during the summer months. This reduction would probably cause the Fourier component of period of 1 year to be a better approximation of the evapotranspiration loss than is indicated in Fig. 2.

Convective Loss

No observations were available from which estimates could be made of the average convective loss at the time of maximum in summer and minimum in winter. The expression for the convective loss was obtained by putting it equal to the sum of the other three components of the heat balance. The phase angle obtained (7 deg) is about 8 deg greater than that given by observations on the difference between the average daily air and surface temperatures. The approximations for all four components of the heat balance are given below and plotted in Fig. 3.

Fourier approximation of components of heat and mass transfer at grass surface at Ottawa in cal/cm² day ($t = 0$, January 1)

$$\begin{aligned} q_r &= 125 - 157 \cos (wt + 3^\circ), \\ q_g &= 18 \cos (wt + 22^\circ), \\ q_e &= -120 + 105 \cos (wt - 2^\circ), \\ q_c &= -5 + 35 \cos (wt + 7^\circ). \end{aligned}$$

$$\begin{aligned}\text{Incoming solar} &= 329 - 208 \cos (wt + 13^\circ). \\ \text{Net solar} &= 252 - 211 \cos (wt + 4^\circ). \\ \text{Net long wave} &= -127 + 54 \cos (wt + 8^\circ).\end{aligned}$$

SOLAR RADIATION AND ALBEDO

Incoming solar radiation on a horizontal surface was measured near the site with an Eppley pyrliometer supplied by the Meteorological Service of Canada. Average monthly values for the daily radiation for the 2-year period are plotted in Fig. 4. Shown also is the Fourier component with a period of 1 year. Average monthly values for the albedo obtained with the two Eppley instruments mounted back to back are as follows.

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Albedo Percentage	65	61	55 10	10	13	16	15	16	17	15	21	20 65

FIG. 3. Fourier component of period 1 year for the net radiation, convection, mass transfer, and ground heat flow at the Ottawa site.

A comparison with other radiometers indicated that these albedos are accurate to within 5 albedo units. The two albedo values for the months of December and March are for the periods with and without snow cover. Of interest is the marked increase in albedo when the ground becomes covered with snow and the gradual increase for the grass cover during the summer. A low value for the albedo in summer and a relatively high value in winter are characteristic of most terrains in Canada.

The albedos above were used to calculate for each month the average daily net solar radiation from the incoming solar radiation observations. Average daily net long-wave radiation for each month was obtained by subtracting net solar radiation from the net radiation. The monthly averages for the daily net solar and long-wave radiation are plotted in Fig. 4. Given also are the Fourier components with period 1 year.

DISCUSSION

One of the more difficult aspects of establishing the heat balance at ground surfaces is to determine the probable accuracy of observations and calculations. The Fourier approximation for the net radiation given in Fig. 3 agrees within ± 20 cal/cm² day with the observed average daily values for each month for the 2-year period. A check of the calibration of the net radiometer in 1963 gave a calibration factor for long-wave radiation within 2% of that used for the period of observations.

FIG. 4. Time dependence of the incoming solar, net solar, net long-wave, and net radiation.

It is considered that the Fourier approximation for the ground heat flow agrees within ± 3 cal/cm² day with the average daily values for each month for the 2-year period. The approximation for evapotranspiration and sublimation is considered to be within ± 25 cal/cm² day of the average daily values for each month during the period from early winter to the middle of summer, but may be in error by more than this amount in the fall and late summer. Little can be said about the accuracy of the approximation for the convective component, except to acknowledge that it probably has the largest relative error.

A check was made on the evapotranspiration approximation by integrating the equation in Fig. 3 over the period 1 April to 30 September. The loss was calculated to be equivalent to 57 cm of water. This is in good agreement with the observed rainfall of between 45 and 50 cm as there was a net loss of soil water during that period for both years. The heat associated with the approximation for each of the other components was obtained by integration for the above period as well. The values obtained are given in Table I, expressed both in cal/cm² and as the height of a column of water 1 cm² in cross-sectional area that could be evaporated at 0 °C by the heat associated with each component. From this table it may be seen that for the Ottawa site about 48% of the net solar heat gain in summer was dissipated by evapotranspiration, 42% by long-wave radiation, and 7% by convection and that only 3% was stored in the ground. Because the components are almost in phase, the above figures indicate also the approximate average partitioning of heat amongst the components throughout the summer. The heat used for plant growth, probably less than 1% of the net solar radiation, was not considered in the study.

The components given in this paper are expressed as average daily values, but it should be kept in mind that each component undergoes a daily fluctuation

TABLE I
Net heat associated with each Fourier component during the period
April 1 to September 30

Component	Heat, cal/cm ²	Equiv. ht. of column of water evaporated, cm	Percentage of net solar radiation
Incoming solar	8.38×10^4	140.8	
Net solar	7.05×10^4	118.5	
Net long wave	-2.95×10^4	49.6	41.8
q_r	4.10×10^4	68.9	
q_e	-3.41×10^4	57.3	48.4
q_c	-0.51×10^4	8.5	7.0
q_g	-0.20×10^4	3.3	2.8

as well. For example, unless there is marked advection, most of the evapotranspiration will occur during the daytime in association with the absorption of solar radiation. During the daytime convection will dissipate heat, but at night, if there is a significant long-wave radiation loss, convection will probably carry heat from the atmosphere to the ground surface. The partitioning of the heat over long periods as determined in the present analysis, therefore, gives no indication of the partitioning for each instant.

The calculated heat flow rates are all about the same absolute size in winter and, except for the ground heat flow, about equal to the probable error in the observations or calculations. This demonstrates one of the difficulties that is encountered in trying to determine the heat balance at the ground surface in winter for periods of 1 day or longer. It is of interest that the calculations indicate a net convective heat gain to the surface in winter.

ACKNOWLEDGMENTS

The authors record their sincere appreciation for the assistance given by R. Armour and P. Montford in taking and analyzing the observations. This paper is a joint contribution from the Division of Building Research, National Research Council, Canada, and the Meteorological Branch of the Department of Transport, and is published with the approval of their Directors.

REFERENCES

- BOYD, D. W. 1962. Comparison of CSIRO and Suomi radiometers. Symposium on the Heat Exchange at Snow and Ice Surfaces, Associate Committee on Soil and Snow Mechanics, Ottawa. Tech. Mem. No. 78.
- FUNK, J. P. 1959. Improved polythene-shielded net radiometer. *J. Sci. Instr.* **36**(6), 267.
- GOLD, L. W. 1963. The influence of snow cover on the average annual ground temperature at Ottawa, Canada. Presented to the Intern. Union Geodesy Geophys., Intern. Assoc. Sci. Hydrol., General Assembly, Berkeley, Calif. August, 1963. Publ. No. 61, pp. 82-91.
- PEARCE, D. C. and GOLD, L. W. 1959. Observations of ground temperature and heat flow at Ottawa, Canada. *J. Geophys. Res.* **64**(9), 1293.
- PENMAN, H. L. 1948. Natural evaporation from open water, bare soil and grass. *Proc. Roy. Soc. London, Ser. A*, **193**, 120.
- . 1956. Evaporation: an introductory survey. *Netherlands J. Agric. Sci.* **4**(1), 9.
- THORNTHWAIT, C. W. and MATHER, J. R. 1957. Instructions and tables for computing potential evapotranspiration and the water balance. *Publ. Climatology*, **10**(3). Drexel Inst. Technology, Laboratory of Climatology, Centerton, N.J.
- WILLIAMS, G. P. 1959. Evaporation from snow covers in Eastern Canada. Div. Bldg. Res. Natl. Res. Council Canada, N.R.C. No. 5003.