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MacKinnon, David; Beraldin, Jean-Angelo; Cournoyer, Luc; Picard, Michel; Blais, Francois

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Lateral Resolution Challenges for Triangulation-based 3D Imaging Systems

David MacKinnon, Jean-Angelo Beraldin, Luc Cournoyer, Michel Picard, François Blais

National Research Council of Canada, Ottawa, Ontario, Canada

ABSTRACT

Lateral spatial resolution is a particularly challenging concept to quantify in triangulation-based 3D imaging systems. In this paper, we present these challenges, then describe an artifact-based methodology for evaluating the lateral resolution of a triangulation-based 3D imaging system that uses laser spots or laser lines. In particular, the response of a 3D imaging system to a spatial discontinuity (step edge) has traditionally been modelled as a first-order linear system. We model the response of a triangulation-based laser imaging system to a spatial step edge from first principles and demonstrate that the response should be modelled as a non-linear system. This model is then used as a basis for evaluating the lateral (structural) resolution of a triangulation-based laser imaging system.

Keywords: lateral resolution, modelling, system response, 3D imaging system, triangulation-based 3D imaging system, laser imaging system

1. INTRODUCTION

The term resolution can be defined as the smallest part of a signal that can be observed¹ or meaningfully distinguished,² the ability of a system to distinguish meaningfully between closely adjacent values,³ the size of the smallest feature discernible by a range scanner,^{4,5} or the smallest spatial interval that can be reproduced.^{6,7} Lateral resolution is a characteristic of some interest to those who use 3D imaging systems; however, quantifying lateral resolution, particularly for triangulation-based systems, has proved to be particularly challenging.

1.1 Importance of Quantifying Resolution

The annex to the VDI 2617 Part 6.2⁸ divides resolution for 3D imaging systems into spatial resolution and structural resolution. Spatial resolution is defined as characterizing the smallest measurable displacement along each of the 3 axes of measurement and is already included in the probing error⁹ so does not require a separate

Further author information: (Send correspondence to D.K.M.)

measurement procedure. Structural resolution is defined as the smallest structure measurable within some defined set of maximum permissible errors^{10, 11} and represents the fundamental limit of resolution of the system. When the term lateral resolution is used in this document, it should be assumed that this refers to structural lateral resolution unless otherwise stated. Resolution, accuracy and precision are interrelated so quantifying spatial resolution requires taking all three parameters into account.

For example, Lira and Woger¹² noted that Annex F of the Guide to Uncertainty in Measurement (GUM)¹³ advocates combining standard uncertainty $u(y)$ (Type A uncertainty) with measurement resolution δ (Type B uncertainty) to generate the combined uncertainty in situations where δ and $u(x)$ are of a similar scale. Specifically, combined uncertainty $u(y)$ should be initially calculated as

$$u^2(y) = u^2(x) + \frac{\delta^2}{12} \quad (1)$$

where δ represents the resolution of the measuring instrument such that the true quantity value would be in the range $(x - \delta/2, x + \delta/2)$. Only if the second term in (1) turned out to be sufficiently smaller than $u^2(x)$ would the effect of resolution be ignored. Meanwhile, den Dekker and van den Bos,¹⁴ stated that resolution is ultimately limited by the imprecision of the measuring system, highlighting the duality between measurement uncertainty and measurement resolution. Without a measure of lateral resolution, uncertainty corrections of the type shown in (1) are impossible.

1.2 Previous Research

Many researchers have examined the problem of quantifying lateral resolution. MacKinnon et al.¹⁵ proposed a test method that involved determining the minimum width of a wedge such that the scanner is able to generate at least one measurement from the wedge surface that is unaffected by either edge. Lichti and Jamtsho⁵ define the effective instantaneous field of view (EIFOV) as a way to measure the angular (lateral in spherical coordinates) resolution of a time-of-flight laser range scanner. Some researchers have designed artifacts specifically to evaluate the resolution of a laser range scanning system.^{4, 16–18}

Some researchers have proposed quantifying resolution using the 3D imaging system's modulation transfer function (MTF).¹⁹ Reichenbach et al.²⁰ proposed the slant-edge test, also referred to as the knife-edge technique, to evaluate the lateral resolution of photographic systems²¹ but was later adapted for 3D imaging systems.²² A range image is generated using a spatial discontinuity with the edge direction slanted with respect to the scan direction, then the scan lines are registered to the edge, differentiated, merged, and binned. The Spatial Frequency Response (SFR) is then produced, consisting of a graph of the normalized modulus of the Fourier transform of the super-resolution derivative of the edge profile versus the spatial frequency.^{21, 23} The standard

ISO 12233 does not require that an SFR cut-off value be generated but it can be extracted from the SFR profile by identifying the spatial frequency of the 3 dB drop of the SFR profile. A limitation of using frequency response terminology is that it is not intuitively clear to technicians who frequently work with 3D imaging systems.

The VDI 2617 Part 6.2⁸ states that the response of any 3D imaging system to a spatial step edge can be modelled as a first-order linear system; however, no studies currently exist to validate this assertion, particularly for triangulation-based systems that typically respond in non-linear ways. In this paper, we develop a response model for a triangulation-based 3D imaging system to determine the goodness-of-fit between the assumed model and one derived from first principals. We begin by considering only laser spot scanners before generalizing into both laser spot and laser line scanners.

2. BACKGROUND

It is important to clearly understand the sources of blur before a broadly-applicable method for measuring blur and, by extension, resolution, can be developed. We examine the sources of blur of a typical laser spot 3D imaging system as an example, then extrapolate to laser line scanners. Two key issues that affect triangulation-based systems, occlusion and shadowing, are then examined. We then assess contemporary and proposed methods for quantifying lateral resolution to highlight their shortcomings when applied to triangulation-based 3D imaging systems. Finally, we define the requirements for a general-purpose measure of lateral resolution.

2.1 Blur

The operation of a triangulation-based 3D imaging system using a laser spot is relatively simple. The light is projected by the emitter into the environment through an optical system and onto a surface. The light is then reflected from the surface and is gathered by the sensor's optical system to be focused onto a digital or analog position-sensitive detector (PSD).²⁴ Ideally the spot should be infinitely sharp but imperfections in, and fundamental limitations of, the 3D imaging system result in blur. Sources of blur include:

- emitter source
- emitter and sensor optics (aberrations and diffraction limits)
- atmospheric effects
- surface diffusion and penetration
- measurement integration
- detector noise and resolution

- lag in the PSD

2.1.1 Emitter Blur

Consider, as an example, the laser spot 3D imaging system illustrated in Figure 1. For systems that use a Gaussian beam, the laser spot can be modelled as an impulse function δ_D convolved with a blurring function $h_e(x, y)$. The resulting beam is generally modelled using the standard Gaussian beam function

$$W(\xi)^2 = W_0^2 + \left(\frac{W_0 \xi}{\xi_0} \right)^2 \quad (2)$$

where W_0 is the radius of the beam waist, ξ_0 is the depth-of-focus of the system, and $\xi = f - z$ is the distance along the z -axis to the beam waist, where f is the lens focal length.²⁵ Figure 1 shows ξ , f , and z . Emitter source and optics generally dominate as sources of blur except when atmospheric conditions are particularly adverse.

In the simple case of an infinite planar surface of arbitrary orientation $\vec{n}$ with respect to the z -axis that intersects the beam, shown in Figure 1, the shape of the beam footprint is an ovoid with the long edge along the line of maximum planar slope as illustrated in Figure 2. The length $a_m(\xi, \vec{n})$ of the longest axis of the ovoid relative to the width $a_w(\xi)$ has a range $a_w(\xi) \leq a_m(z, \vec{n}) < \infty$ depending on the orientation of the surface with respect to the z -axis.²⁶

To visualize the intensity pattern on a simple Lambertian planar surface, consider that if a Gaussian beam is used with distribution

$$I(r, \xi) = I_0 \exp \left(\frac{-2r^2}{W(\xi)^2} \right) \quad (3)$$

where r is the distance from, and orthogonal to, the beam axis, then

$$I(r, \xi) = I_0 \exp \left(\frac{-2\xi_0^2 r^2}{W_0^2 (\xi_0^2 + \xi^2)} \right) \quad (4)$$

represents the beam intensity at any point a distance r from the beam waist. If we place the origin at the point of intersection of the beam axis with a surface such that ξ_z is the distance from the beam waist to the point of intersection and the z -axis is oriented toward the emitter, then (4) becomes

$$I(x, y) = I_0 \exp \left(\frac{-2\xi_0^2 (x^2 + y^2)}{W_0^2 (\xi_0^2 + (z(x, y) - \xi_z)^2)} \right) \quad (5)$$

where $r^2 = x^2 + y^2$ and $z(x, y)$ is a function describing the a surface height as a function of x and y . The farther ξ_z is from the beam waist, the greater $W(\xi_z)$ and, hence, the greater the blur. Translucent surface materials may further blur the beam footprint.²⁷

2.1.2 Detector Blur

On the detection side, the sensor system, shown in Figure 1 with an origin at the centre of the lens, projects the beam footprint onto a PSD. The optics of the sensor system introduce additional blur, and the shape of the projected beam footprint is further distorted by factors such as the baseline distance d between the emitter and sensor, the orientation of the surface with respect to the w -axis, and the orientation of the photo-detector with respect to the w -axis. A combination of factors including the optical roughness of the surface, temporal and spatial coherence, and the apertures of both the emitter and sensor introduces speckle noise σ_p that limits the depth uncertainty of a triangulation-based laser imaging system.^{24,28} For a triangulation-based imaging system using a coherent light source, speckle noise can be reduced in several ways: using larger apertures, using relatively large triangulation baselines, and reducing speckle contrast.^{24,29,30} The aperture diameter has a practical upper limit,^{29,30} and increasing the baseline increases occlusion and shadowing effects.^{24,29,30} The remaining practical approach to reducing speckle noise is to reduce spatial coherence by averaging several measurements in sequence as the laser spot moves over the surface;^{24,29} however, this reduces speckle noise at the cost of introducing motion blur when the measurements are integrated into a single composite measurement.^{24,29,31}

Optical filtering, present on many imaging systems, can introduce additional blurring,³² which is further compounded by quantization effects contributed by the PSD. The sensor system attempts to locate the point $\hat{p}$ on the PSD corresponding to the maximum intensity of the incoming signal, referred to as the signal peak and illustrated in Figure 3. Peak detection can be based on the location of either the mean or the median of the detected signal. According to Baribeau and Rioux,³³ assuming detection is along the p direction, the mean can be defined as

$$p_{mean} = \frac{\int_{-\infty}^{\infty} \int_{q_{min}}^{q_{max}} pI(p, q) dq dp}{\int_{-\infty}^{\infty} \int_{q_{min}}^{q_{max}} I(p, q) dq dp} \quad (6)$$

while the median can be represented by

$$\int_{p_{median}}^{\infty} \int_{q_{min}}^{q_{max}} I(p, q) dq dp = \int_{-\infty}^{p_{median}} \int_{q_{min}}^{q_{max}} I(p, q) dq dp \quad (7)$$

where $q_{min} = -\infty$ and $q_{max} = \infty$ represent the bounds on the PSD; however, as illustrated in Figure 3, q_{min} and q_{max} are, in reality, non-infinite and p is quantized so that

$$\hat{p}_{mean} = \frac{\sum_{p_{min}}^{p_{max}} \int_{q_{min}}^{q_{max}} pI(p, q) dq \Delta_p}{\sum_{p_{min}}^{p_{max}} \int_{q_{min}}^{q_{max}} I(p, q) dq \Delta_p} \quad (8)$$

approximates the mean $\hat{p}_{mean}$ and

$$\sum_{\hat{p}_{median}}^{p_{max}} \int_{q_{min}}^{q_{max}} I(p, q) dq \Delta_p = \sum_{p_{min}}^{\hat{p}_{median}} \int_{q_{min}}^{q_{max}} I(p, q) dq \Delta_p \quad (9)$$

approximates the median $\hat{p}_{median}$. In this case, Δ_p is the width of each pixel element and $\Delta_q = q_{max} - q_{min}$ is the height of each pixel element, illustrated in Figure 3.

The quantization of p represents a significant source of noise in the detection system so that p_{mean} and $\hat{p}_{mean}$, or p_{median} and $\hat{p}_{median}$, can differ significantly. According to Baribeau et al.,²⁸ the uncertainty associated with locating the peak can be modelled as

$$\sigma_p = \frac{\lambda f}{D \cos(\beta) \sqrt{2\pi}} \quad (10)$$

where β is the Scheimpflug angle of the photo-detector. The benefit of Scheimpflug angulation of the PSD is to extend the range over which the projected spot on the PSD remains in focus;³⁴ however, this angulation of the PSD affects the shape of the projected spot and, by extension, the relationship between $\hat{p}$ and quantization noise.

The detection process is further complicated by electrical noise that affects the quality of the detected optical signal.³² Peak detection can then involve, for example, low-pass filtering to eliminate high-frequency noise, linear interpolation to isolate the peak to sub-pixel accuracy, and integration to isolate the location of the peak on the PSD.³⁵ Sub-pixel interpolation is, in particular, strongly affected by the aforementioned quantization noise

Once $\hat{p}$ has been isolated on the PSD, it is combined with information about the orientation of the laser beam and/or position of the scanning system to generate a measured spatial value $\hat{\mathbf{t}} = [\hat{t}_1, \hat{t}_2, \hat{t}_3]$ in the camera coordinate system. Post-processing procedures such as smoothing algorithms may be used to further reduce measurement noise but introduce additional blurring. In some systems, multiple depth maps are combined to generate a composite surface map; however, the merge process may also introduce blurring as spatial measurements that are close to each other are combined into a single measurement to reduce the size of the composite image.

2.2 Occlusion and Shadowing

The greatest challenge to quantifying lateral resolution in 3D imaging systems is that the sensor is separated from the emitter. This results in two complicating factors: occlusion and shadowing. Occlusion occurs when the emitter illuminates a portion of the surface but there exists a spatial structure between the illuminated region and the sensor. shadowing occurs when a portion of the surface cannot be illuminated by the emitter. Both factors result in regions of the surface being effectively invisible to the 3D imaging system.

2.2.1 Effect of Edges

Lateral resolution can, in theory, be quantified either with occlusion and shadowing or without. The former approach is simpler but can generate much larger resolution values. The latter approach requires minimizing the effects of occlusion and shadowing. If occlusion and shadowing affects can be minimized then the remainder of the resolution value represents the ability of the 3D imaging system to digitally represent small structures. Typical structures include:

- step edges
- roof and valley edges
- ridges and gaps
- curves and waves

Consider once again the laser spot scanner described in the previous section, but this time the spatial location of the emitter lens is defined by $\mathbf{p}_e$ and the axis of the laser passes through point $\mathbf{p}_\theta$, and the spatial location of the sensor lens is defined by $\mathbf{p}_s$, as illustrated in Figure 4. Assuming a Gaussian beam, the beam axis corresponds to the point of maximum energy emitted by the laser. If the system has been properly calibrated and the beam axis intersects the surface at a point visible to the sensor then the location of the point of intersection is approximated by $\hat{\mathbf{p}}$ based on where the peak of the detected signal is located on the PSD.

If the detected peak does not coincide with the intersection of the beam axis with the surface then the intersection is located at a point corresponding to the projection of the detected peak onto the beam axis, so is located at either $\hat{\mathbf{p}}_n$ or $\hat{\mathbf{p}}_f$. The effect can be seen in Figure 5, which shows the measured spatial profile (thin line with dots) to a post structure (thick solid line). Note the downward slope in response to the edge on the right side and the slight rise in response to the edge on the left side. Any lateral resolution metric must be able to take into account both system responses.

It should be clear that both spatial and reflectivity discontinuities can result in erroneous spatial measurements results. Any situation in which the peak detected on the PSD does not correspond to the intersection of the beam axis with a surface the spatial measurement result is inaccurate. For example, Figure 4 could also represent a transition between surfaces of different reflectivity. Further complicating the situation is the presence of internal reflection that can result in spikes such can be seen on the left side of the post structure in Figure 5. In this case, reflections from the horizontal surface onto the vertical surface resulted in a detected peak corresponding to the vertical surface but because the laser axis was nearly vertical, the spatial location of the peak was assigned

incorrectly. Any lateral resolution metric should be robust to test artifacts, such as the vertical spikes just discussed, and should be adaptable to measuring the effect of reflectivity transitions.

2.2.2 Spatial Response

To model the behaviour observed in Figure 5 for a laser range scanner, if we assume a planar surface $z(x, y) = n_x x + n_y y$ then we can approximate the intensity cross-section of the light reflected from the surface as a 2D Gaussian equation

$$I(x) = I_0 \exp\left(\frac{-2\xi_0^2 x^2}{W_0^2 (\xi_0^2 + (n_x x - \xi_z)^2)}\right) \quad (11)$$

where n_x is the x-component per unit z of the normal of the plane. If we let $-\infty \leq x_l < x_r \leq \infty$ be the left and right bounds on the planar surface then the beam peak is generally found using one of two possible methods. The first methods is the centroid mean method, which seeks the average point within the beam peak such that

$$x_{avg} = \frac{\int_{x_l}^{x_r} x I(x) dx}{\int_{x_l}^{x_r} I(x) dx} \quad (12)$$

and arises from (6). The second method is the centroid median method that can be approximated as

$$x_{med} = \min_u \left\{ \left| \int_{x_l}^u I(x) dx - \int_u^{x_r} I(x) dx \right| \right\} \quad (13)$$

and arises from (7). In all cases, the depth error

$$e_z = \begin{cases} \frac{e_x}{\tan(\theta)} & \text{where } \theta \neq 0 \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

is a function of $e_x \in \{x_{avg}, x_{med}\}$, the estimated peak location, and θ , the orientation of the line-of-sight between the detected peak and the lens.

If $n_x x \ll \xi_z \forall \{x_l < x < x_r\}$ so that $\psi \ll \theta$ then (11) can be approximated using a simple 2D Gaussian function

$$I(x) \approx I_0 \exp\left(\frac{-2x^2}{W(\xi)^2}\right) \quad (15)$$

where 2 was used to simplify the approximation. One benefit of the 2D, or cross-sectional, approximation, is that it can be applied to both spot and line scanners. Figure 6 shows a simulated depth profile generated using (15) in which the beam was assumed to be directed vertically downward and the angle between the surface normal and the viewing angle was $\theta = \pi/4$. The results approximate the results obtained from the line scanner in Figure 5, after taking into account that the sensor and emitter locations have been swapped. The mean and median peak

methods generate similar results so, for purposes of model fitting, we can use (12) to approximate the depth error as

$$e_z \approx G_z \frac{\exp(-2x_l^2/W^2) - \exp(-2x_r^2/W^2)}{\operatorname{erf}(x_r\sqrt{2}/W) - \operatorname{erf}(x_l\sqrt{2}/W)} \quad (16)$$

where

$$G_z = \left(\frac{W}{\tan(\theta)\sqrt{2\pi}} \right) \quad (17)$$

for $\theta \neq 0$ and the term W is being used to represent the spot radius a distance z from the emitter. We are only concerned with modelling triangulation scanners so $\theta > 0$ should be assumed unless otherwise stated. The depth measurement is shown extending beyond the bounds of the post (solid pale line in Figure 6) to illustrate the effect of a signal return below the detection limit from the more-distant surface (such as due to occlusion, distance, or surface properties) on the measured depth.

For step edges, we are concerned with two cases, illustrated in Figure 7: $x_l \leq x_r$ where $x_r \rightarrow \infty$ and $x_r \geq x_l$ where $x_l \rightarrow -\infty$. In both case we assume that ψ is small enough that the shape of the beam footprint is not significantly distorted. The resulting depth profile is illustrated in Figure 8. Let us assume that the sensor is to the right (positive x) of the emitter and recognize that $W > 0$. For the first case, $x_r \rightarrow \infty$ so

$$e_z(x) \approx G_z \frac{\exp(-2x^2/W^2)}{1 - \operatorname{erf}(x\sqrt{2}/W)} \quad (18)$$

approximates $e_z(x)$ as a function of distance x from the edge. As $x \rightarrow -\infty$, $e_z(x) \rightarrow 0$, and as $x \rightarrow 0$, $e_z(x) \rightarrow G_z$. This case is illustrated in Figure 9 in which the maximum value of each curve increases with decreasing θ . This can be visualized as leverage effect on the depth error; large values of depth error correspond to smaller values of θ for a given non-zero peak location error. Meanwhile, if $x_l \rightarrow -\infty$ then

$$e_z(x) \approx G_z \frac{-\exp(-2x^2/W^2)}{\operatorname{erf}(x\sqrt{2}/W) + 1} \quad (19)$$

approximates $e_z(x)$ as a function of distance x from the edge.

2.2.3 Model Verification

A laser line scanner, referred to here as the System under test (SUT), was used with an artifact created by Physikalisch-Technische Bundesanstalt (PTB), shown in Figure 10, to verify the fit of the response model. The measured sensor-to-emitter distance and the measured distance from the emitter to the surface were used to approximate θ . Beam radius W was estimated using information provided by the manufacturer regarding the depth-of-field and approximate distance between the emitter and the beam waist. The 1.00 mm and 0.50 mm wide post segments were used for the experiment. As can be seen in Figure 11, the model provides a close fit the

measured values. Measured values are shown as a dotted line while the model is represented by a thick line. The vertical line represents the location of the edge used to generate the model so is a predictor of the true location of the edge in the measured data. Measurements were performed at 240 mm, 245 mm, 250 mm, and 255 mm from the emitter resulting in a mean-squared error (MSE) of between 0.050 μm and 0.585 μm .

2.2.4 Frequency Response

If we use $x = -x_l$ so that x always represents the positive distance from the edge then (19) and (18) are the same magnitude but opposite in sign. This means that the same model can be used for both edges of a post with only a change in sign separating them, making it easier to examine the frequency response of the system. In practical terms, this means that we fit to the absolute value of $e_z(x)$. Equation 19 can be further simplified using the first-order approximation of the Maclaurin series for the error function, resulting in

$$|e_z(x)| \approx \left(\frac{W^2}{\tan(\theta)} \right) \frac{\exp(-2x^2/W^2)}{W\sqrt{2\pi} + 4x} \quad (20)$$

as the model to be fit to the data.

If we let $y(x) = 1 - |e_z(x)/e_z(0)|$ where $x \geq 0$ then

$$y(x) = 1 - \frac{W\sqrt{2\pi}}{W\sqrt{2\pi} + 4x} \exp(-2x^2/W^2) \quad (21)$$

is the normalized unit step response of the system. Typically, the response of a 3D imaging system to a step edge has been modelled as a simple first-order system $c(x) = 1 - \exp(-x/X)$ where X is the spatial constant of the system;⁸ however, (21) is not well-modelled by a simple first-order system. There is no simple spatial-frequency domain representation of (21) so the response must be examined in the spatial domain.

The slope of the normalized response at $x = 0$ is

$$\left. \frac{dy}{dx} \right|_{x=0} = \frac{1}{W} \sqrt{\frac{8}{\pi}} \quad (22)$$

so is steeper than a typical first-order linear system. If we define X as the inverse of (22) then the response of the system can be plotted as $y(x = X) \approx 0.772$ and $y(x = 2X) \approx 1.000$. The response is based on the square of x so is much faster than for a typical first-order system. In fact, steady-state is achieved at approximately the width of the beam footprint $2W$ such that $y(x = 2W) \approx 0.999$ as expected because W is the distance to the point at which signal intensity has dropped to approximately e^{-2} of the maximum intensity.

Based on an analysis of the spatial and spatial-frequency response of the system it can be concluded that the response can be only roughly approximated using a simple first-order linear response model. A triangulation-based laser imaging system has a spatial response that is steeper than a simple first-order linear system so the first-order linear system is a poor approximation.

3. LATERAL RESOLUTION

An important question for any 3D imaging system is the size of the smallest structure that can be laterally resolved to some degree of accuracy, what is referred to in the VDI 2617 Part 6.2⁸ as the structural spatial resolution of the SUT. The problem has been studied extensively, as discussed in Section 1.2, but no method has gained widespread acceptance. Part of the problem is the variety of technologies used to generate 3D images. In this paper we have reduced the problem by focusing strictly on triangulation-based 3D imaging laser systems, but the metric used to quantify lateral resolution should be potentially comparable to lateral resolution metrics for other technologies. Figure 12 shows a classification of 3D imaging system technologies as a function of both the depth-of-field and depth noise. Triangulation-based 3D imaging laser systems are generally only functionally comparable to white light pattern projection systems so the resolution metric need only be comparable to resolution metrics for these systems. Lateral resolution metrics for structured-light 3D imaging systems are not yet well-defined so represent an area of future research, the direction of which can be based on the approach presented in this paper.

3.1 Theory

We seek to represent the lateral (structural) resolution δ_x of a triangulation-based 3D imaging laser system using a physical property: the size of the region in the 3D image affected by the presence of a spatial discontinuity. The width of the affected region depends on what happens when the beam axis no longer intersects the top of the post. On the occluded (falling) edge, illustrated in Figure 5, as the beam axis moves off the post top it is occluded so the "peak" continues to be detected on the post top. Once the beam axis has moved far enough that the return signal intensity from the far surface is greater than that of the near surface, the "peak" begins to be attributed to the far surface. The affected region can extend as far as approximately $2\hat{W}$ past the edge. The situation on the visible (rising) edge, illustrated in Figure 5, is even worse. Internal reflections and sensor angle can result in a "peak" being detected on the far surface, or even on the vertical portion of the edge, when the beam axis is still intersecting the post top. In this case, the transition point on the $\hat{\mathbf{e}}_z$ curve may be truncated before reaching the true edge. We, therefore, examine only the occluded edge to maximize the amount of edge-related data available to perform the fit.

The procedure for obtaining a measure of δ_x of a triangulation-based 3D imaging laser system begins with using only the response to the occluded edge so we are only concerned with fitting (18) to the data. The variable x is with respect to the edge; however, the location of the edge is not known, which is why (20) is not a viable fit model. This means that a best fit solution involves three unknowns: the x-correction $\hat{x}_c$ to move the data to the edge in the model, the spot width $\hat{W}$, and the viewing angle $\hat{\theta}$. We can restate (18) as

$$e_i = \left(\frac{\hat{W}}{\tan(\hat{\theta})\sqrt{2\pi}} \right) \frac{\exp\left(-2(\hat{x}_c - x_i)^2/\hat{W}^2\right)}{\operatorname{erfc}\left((\hat{x}_c - x_i)\sqrt{2}/\hat{W}\right)} \quad (23)$$

where $x_i \in \mathbf{x}$ is a member of the range of measured x -values and $e_i \in \mathbf{e}_z$ is the associated measured depth error. The complementary error function was used in this equation to simplify the denominator.

Given that we know $[x_i \ e_i]$, the height of the $\mathbf{e}_z$ curve at the edge is

$$e_i|_{x_i=\hat{x}_c} = \frac{\hat{W}}{\tan(\hat{\theta})\sqrt{2\pi}} \quad (24)$$

so if had an estimate for $\hat{x}_c$ then we could remove $\hat{\theta}$, at least temporarily, from the problem. Given estimates for both $\hat{x}_c$ and $\hat{W}$, (24) could be used to obtain $\hat{\theta}$. A brute-force approach is to let $x_i = \hat{x}_c$ for all $[\mathbf{x} \ \mathbf{e}_z]$ and find the one with the best fit.

What is now required is to solve

$$\hat{e}_i = (e_i|_{x_i=\hat{x}_c}) \frac{\exp\left(-2(\hat{x}_c - x_i)^2/\hat{W}^2\right)}{\operatorname{erfc}\left((\hat{x}_c - x_i)\sqrt{2}/\hat{W}\right)} \quad (25)$$

for the value of $\hat{W}$ that minimizes the l^2 -norm $|\mathbf{e}_z - \hat{\mathbf{e}}_z|$ for each $x_i = \hat{x}_c$ where $\hat{e}_i \in \hat{\mathbf{e}}_z$ is estimated response curve. The $x_i = \hat{x}_c$ that generates the smallest l^2 -norm represents the combination of $\hat{x}_c$ and $\hat{W}$ with the best fit. As illustrated in Figure 13, any point on the curve can be generated by a limited combination of $\hat{W}$ and $\hat{\theta}$ values, represented by each of the two surfaces, one of which is the correct combination, represented by the vertical line. Only a unique combination of $\hat{W}$ and $\hat{\theta}$, illustrated by the intersection of the two surfaces in Figure 13, will generate the same fit at all points on the line.

The estimate of the $\hat{\delta}_x$ of the system is quantified as the distance between the edge and point at which the effect of the edge on the depth measurement becomes negligible so $\hat{\delta}_x = 4\hat{W}$. This means that $\hat{\delta}_x$ represents the width of the smallest structure bounded by two spatial discontinuities in which all measurements are affected by at least one of the edges. As a benefit we also obtain $\hat{x}_c$, which represents the lateral shift required to ensure that the data is correctly aligned to the surface.

3.2 Simulated Results

Figure 14 shows the simulated results of the search for the optimal $\hat{W}_i$ for a given $x_i = \hat{x}_c$. The top graph shows the original simulated curve with added depth noise. A circle with a vertical line through it indicates the true edge location while the circle without the vertical line indicates the $x_i = \hat{x}_c$ being tested. The bottom curve shows the $\hat{W}_i$ values tested and their resulting l^2 -norm. In this simulation the true beam radius was a uniform randomly-generated value between 20 μm and 100 μm so the search step size started at 100 μm and was decreased by decades to 1 μm . The $\hat{W}_i$ that generates the minimum l^2 -norm is clearly visible. The circles on the bottom axis of the bottom graph represent $\hat{W}_i$ values in which the curve does not completely cover the data.

The bottom graph of Figure 15 shows the simulated results of plotting the optimal $\hat{W}_i$ value for each $x_i = \hat{x}_c$ value. The true x_c value ideally corresponds to the point of minimum fit error; however, measurement noise can make isolating this point difficult. As can be seen in the bottom graph of Figure 15, the true x_c value can be approximated by finding the point of locally-minimum fit error that is closest to the point at which the fit-error response increases suddenly. A circle marks the half-way point of the transition between pre- x_c and post- x_c error regions. This observation allows us to further reduce the search space for the best estimate of x_c . Each point on the curve represents a single $\hat{W}_i$ and associated depth error $e_i|_{x_i=\hat{x}_c}$ so if $\hat{x}_c$ can be approximated then $\hat{\theta}$ can be obtained from (24).

The sudden jump in fit error is caused by the rapid change in slope as $\hat{x}_c$ increases through $\hat{x}_c = x_c$, as illustrated in Figure 16. For $\hat{x}_c < x_c$, where the slope is relatively shallow, the slope of the portion of the best-fit curve below $\hat{x}_c$ for any $\hat{x}_c < x_c$ doesn't change significantly, nor does the slope of curve to which it is being fit. As a result, errors tend to accumulate slowly as $\hat{x}_c$ decreases beyond $\hat{x}_c = x_c$. For $\hat{x}_c > x_c$, the slope of the portion of the best-fit curve below $\hat{x}_c$ for any $\hat{x}_c > x_c$ still doesn't change significantly, but the slope of curve to which it is being fit changes rapidly. As a result, large errors tend to accumulate quickly as $\hat{x}_c$ increases beyond $\hat{x}_c = x_c$. Figure 17 shows the best-fit errors at different $\hat{x}_c$ estimates, as well as the best-fit models associated with each estimate. The best-fit lines are only visibly different for $\hat{x}_c > x_c$, while for $\hat{x}_c < x_c$ the differences are small but measurable. The sudden rapid change in fit error provides a marker for estimating the location of the $\hat{x}_c = x_c$ where x_c is not known.

Recall from Figure 9 that the height of the curve increases with decreasing viewing angle. This means that for more distant surfaces, for which the viewing angle is generally smaller, the curve is more pronounced. For closer surfaces, for which the viewing angle is typically larger, the curve is more flattened. This means that the estimation of $\hat{x}_c$ and $\hat{W}$, from the perspective of viewing angle, is easier for more distant surfaces because the

smaller viewing angle increases the effective contrast, resulting in a more abrupt transition. Figure 18 illustrates this effect. The graph with the largest range of fit error values and the most distinct transition has the smallest viewing angle (top graph), while the line with the smallest range of fit error values and the least distinct transition has the largest viewing angle (bottom graph).

Figure 19 shows the effect of varying beam footprint size on the fit error profile. The smallest beam footprint size, shown in the top graph, resulted in the sharpest transition because the fit model is relatively small and changes quickly with respect to the sampling interval. On the other hand, the largest beam footprint, shown in the bottom graph, has the least distinct transition. This means that the estimation of $\hat{x}_c$ and $\hat{W}$, from the perspective of beam footprint size, is easiest when the beam footprint size is smallest relative to the sampling interval.

Figure 20 shows the effect of varying depth noise on the fit error profile. Bin-to-bin separation is fixed so contour uncertainty only exists along the z-axis. Viewing angle and spot size were selected to provide a sharp transition and the edge location, indicated by the circle-with-cross, was fixed at $x_c = 0.1$ mm. Three noise levels are shown: low ($\sigma_z = 1$), medium ($\sigma_z = 7$), and high ($\sigma_z = 49$). The top figure shows a near-ideal response in which $\hat{x}_c$ is easily located. The moderate-noise model shows more variability but $\hat{x}_c$ can still be easily located. The high-noise model; however, has become more smooth so it $\hat{x}_c$ is not as easily located. Recall that each the best-fit $\hat{W}_i$ was obtained with each $x_i \in \mathbf{x}$ as the $\hat{x}_c$ estimate and functions effectively as a leverage point for fitting the model to the data. Greater variability in x_i values results in greater variability in best-fit $\hat{W}_i$ values and, by extension, the best-fit values used to form the graphs in Figure 20. Measurement noise typically increases with distance between the surface and the system so the transition should be more clearly defined for surfaces closer to the SUT than for those farther from the SUT. The means that, from the perspective of measurement noise, the estimation of $\hat{x}_c$ and $\hat{W}$ is easiest when the surface is closest to the SUT.

Monte Carlo simulations were performed in which random Gaussian noise with $\sigma_z = 10$ μm was added to the depth measurement. The edge offset, beam radius, and viewing angle were randomly-generated from a uniform distribution. Results indicate that x_c can be estimated with average uncertainty of 3.50 ± 1.35 μm and W can be estimated with average uncertainty of 3.51 ± 0.87 μm . The noise level of $\sigma_z = 10$ μm was chosen to be representative of a moderately noisy data set. Simulations performed with smaller noise levels, as expected, produced a closer agreement.

3.3 Test Procedure

The PTB artifact shown in Figure 10 was used to generate 10 measurements at 5 mm intervals between 220 mm and 255 mm from the SUT. The artifact is positioned so that the beam spread would be as small as possible

where it crossed the post tops. The edges of the post tops were angled with respect to the laser line so that measurements could be obtained at as many points through the transition between near (post) and far (pit) surfaces as possible. The resulting data was then collapsed along the edge to generate a super-resolution profile that reveals the structural response of the SUT to an edge. Measurements near the sides of the PTB artifact were removed before the profile was generated so that the only effects that would be observed would be those associated with the transition between near and far surfaces. Measurements in the collapsed profile were binned at $10\text{ }\mu\text{m}$ to ensure a sufficient number of samples in each bin to provide noise reduction through within-bin averaging. In this case, the average number of samples per bin was approximately 16. Binning at a finer resolution resulted an average of less than one measurement per bin: too few measurements to provide sufficient noise reduction. As a result, $10\text{ }\mu\text{m}$ was selected as the smallest feasible binning frequency. The narrowness of the PTB artifact ensure that beam spread would be roughly constant within the sampled region, although a wider sampled region would have made it possible to bin at a finer resolution. The SUT used a fixed-position beam and linear translation stage so only lateral spread needed to be considered as an issue in determining the maximum region size. The PTB artifact was oriented so that the surface normal, as much as possible, was oriented along the beam axis.

The analytical portion of the test procedure involved assigning $x_i = \hat{x}_c$ for each $x_i \in \mathbf{x}$, finding the $\hat{W}_i$ that minimizes $|\mathbf{e}_z - \hat{\mathbf{e}}_z|$ for each $x_i = \hat{x}_c$, then finding the $\{\hat{W}_i, x_i = \hat{x}_c\}$ combination that minimizes $|\mathbf{e}_z - \hat{\mathbf{e}}_z|$. Equation 24 was then used to obtain $\hat{\theta}$. This procedure involves two iterative routines so can be time-consuming; therefore, we implement several procedures to reduce the total search time:

- We began with $\hat{W}_i = \hat{W}_0$, the estimated radius of the beam at the waist, as the initial, and smallest possible, $\hat{W}_i$ value.
- The $\hat{\mathbf{e}}_z$ curve can only extend $2\hat{W}_i$ past $x_i = \hat{x}_c$ so we only evaluated $\hat{\mathbf{e}}_z$ curves that were at least as long as the $\mathbf{e}_z$ segment $\mathbf{x} \leq \hat{x}_c$.
- The step size between $\hat{W}_i$ values was initially made large, then reduced until the step size was one order of magnitude less than the expected scale of the lateral resolution. For example, if a system is expected to have a $7\text{ }\mu\text{m}$ lateral resolution then the minimum step size is $0.1\text{ }\mu\text{m}$, but if it is $12\text{ }\mu\text{m}$ then the minimum step size is $1\text{ }\mu\text{m}$.
- We tested only $x_i = \hat{x}_c$ values rather than interpolating between consecutive x_i values to attempt to improve the $\hat{x}_c$ estimate.
- If $e_i \rightarrow 0$ then no curve can be generated. This sets an upper bound on the choice of $x_i = \hat{x}_c$ values so that only x_i from the response curve, and not from the unaffected region, are evaluated.

Table 1. Estimated values for $\hat{W}$ and $\hat{x}_c$ for 1.00 mm and 0.50mm post widths. All entries are mean $\pm$ uncertainty $u = s/\sqrt{N}$ where s is the sample standard deviation and N is the number of samples. All results are based on 10 scans ($N = 10$) of the PTB artifact. Shaded paired elements in each row are significantly different at a probability greater than 0.95.

Distance (mm)	$\hat{W} \pm u_{\hat{W}}$ (μm)		$\hat{x}_c \pm u_{\hat{x}_c}$ (μm)		$\hat{\delta}_x \pm u_{\hat{\delta}_x}$ (μm)	
	1.00 mm	0.50 mm	1.00 mm	0.50 mm	1.00 mm	0.50 mm
220	110 $\pm$ 2.48	109 $\pm$ 1.13	203 $\pm$ 1.53	197 $\pm$ 1.53	441 $\pm$ 9.94	437 $\pm$ 4.50
225	78 $\pm$ 1.07	80 $\pm$ 1.01	150 $\pm$ 1.49	159 $\pm$ 1.80	312 $\pm$ 4.28	321 $\pm$ 4.05
230	60 $\pm$ 0.69	61 $\pm$ 1.35	117 $\pm$ 1.53	108 $\pm$ 2.00	240 $\pm$ 2.76	242 $\pm$ 5.41
235	44 $\pm$ 0.92	42 $\pm$ 0.68	84 $\pm$ 1.63	80 $\pm$ 1.49	178 $\pm$ 3.69	167 $\pm$ 2.74
240	36 $\pm$ 0.34	31 $\pm$ 0.33	66 $\pm$ 1.63	60 $\pm$ 0.00	142 $\pm$ 1.36	124 $\pm$ 1.33
245	36 $\pm$ 0.22	35 $\pm$ 0.10	70 $\pm$ 0.00	70 $\pm$ 0.00	142 $\pm$ 0.89	140 $\pm$ 0.40
250	49 $\pm$ 0.52	49 $\pm$ 0.65	93 $\pm$ 1.53	93 $\pm$ 1.53	197 $\pm$ 2.07	196 $\pm$ 2.60
255	61 $\pm$ 0.65	64 $\pm$ 1.11	121 $\pm$ 1.80	120 $\pm$ 1.49	246 $\pm$ 2.61	257 $\pm$ 4.42

The $\hat{x}_c$ estimate was selected by finding the $\hat{x}_c$ value associated with the locally-minimal fit error immediately before the jump in fit error values. Each $\hat{x}_c$ estimate has associated with it a single $\hat{W}_i$ value representing the model that best fits the measured results when $\hat{x}_c = x_i$.

3.4 Measured Results

Table 1 shows the average $\hat{W}$ and $\hat{x}_c$ estimates obtained for 10 measurements of 1.00 mm and 0.50 mm wide posts on the PTB artifact, shown in Figure 10, at varying distance between the surface and the system. There was no significant difference between results generated by the 1.00 mm and 0.50 mm posts at most of the 8 distances used in the test so the results of both posts can be combined for purposes of discussion. Where significant differences exist the practical difference is negligible except for the $\hat{\delta}_x \pm u_{\hat{\delta}_x}$ values at 240 mm where the 1.00 mm and 0.50 mm results differ by 18 mm. The optimal distance, which is the distance at which lateral resolution is significantly smaller than for all other distances, was at 240 mm from the SUT. The effective $\hat{\delta}_x$ at that distance, after combining the results for both posts, was $133 \pm 2.31 \mu\text{m}$. This value represents the width of the smallest post structure than can be accurately resolved by the SUT so is referred to as the resolution limit of the SUT.

Based on the results presented in Table 1, at 220 mm the resolution is approaching the width of the 0.50 mm post. Figure 23 shows measurements obtained from the top of the 0.50 mm post at 220 mm, showing that the

portion of the measured data that forms a planar region unaffected by either edge is only a fraction of the post width. This observation supports the prediction that at 220 mm post tops smaller than 0.50 mm would be too small for any portion of the top to be resolved without being affected by the presence of edges. Data density, however, is sufficiently large for the 0.50 mm post tops that useful results were able to be obtained. Figure 21 shows the change in $\hat{\delta}_x$ as a function of distance between the PTB artifact and the SUT. Figure 22 shows the $\hat{x}_c$ that correction factor would need to be applied to the lateral position of the data for the predicted and actual edge locations to match. As with the $\hat{\delta}_x$ results, there was no significant difference between the results for the 1.00 mm and 0.50 mm posts at any distance so the results for both posts were able to be combined.

Figures 23 and 24 show the measurements obtained from the top of the 0.25 mm post at between 220 mm and 255 mm respectively. The combination of low sampling density and significant lateral fit error $\hat{x}_c$ at that scale meant that $\hat{W}$ and $\hat{x}_c$ could not be estimated from the measurements of the 0.25 mm post so have not been included in Table 1. Moreover, low sampling density associated with small structures illustrates that estimating lateral resolution from small post tops can be problematic because the scale of the sampling density becomes small with respect to the system resolution. This means that methods depending on the use of small posts will be viable only for producing either qualitative results or quantitative results with a large margin of error. For this reason, only large post tops, relative to the expected resolution, or surfaces with only a single spatial discontinuity within the scan region should be used to ensure a sufficiently large sample size from which to estimate $\hat{\delta}_x$. Using a single spatial discontinuity would avoid the problem of edge-affected regions interacting and allow for a sufficiently large planar segment to ensure good model fit. The problem of low sampling density is of greater concern. Bin-to-bin separation can be reduced but at the expense of reducing the number of measurement results within each bin. A practical limit for bin occupancy is a minimum of 3 measurement results per bin to ensure that at least a minimum of within-bin averaging is performed. Higher bin occupancy levels provided better response to measurement noise at the cost of reducing profile resolution.

The results for the 1.00 mm and 0.50 mm posts in Table 1 indicate that the width of the 0.25 mm post should be larger than the $\hat{\delta}_x$ at $D < 235$ mm and $D > 250$ mm. Qualitative analysis of the graphs in Figure 25 confirms that the tabulated results predict the post widths for which the two edge regions begin to overlap. Figures 25(a)-(b) illustrate post tops that cannot be resolved well. Figure 25(h) shows the point at which both transition regions are beginning to merge, as would be expected for a $\hat{\delta}_x$ value close to the width of the post. Figures 25(c)-(g) show post tops for which the edge regions are not merging so at least a portion of the post top can be resolved.

Figure 26 shows the results of acquisitions by the SUT where the test surface was placed at different distances

from the system. As the distance between the surface and the SUT increased, the transition became more gradual so the estimate of $\hat{x}_c$ becomes less certain. Based on the simulated results presented in Section 3.2, the clearest transition should be observed where the beam footprint and viewing angle are smallest, but also where the measurement noise is lowest, typically the point where the surface is closest to the SUT. The baseline for this system is 100 mm so the change in viewing angle between the 220 mm and 255 mm surface was minimal and can be ignored as having a significant effect. Beam footprint size and measurement noise are the remaining factors so the best results should be observed when the surface is closer than 240 mm which corresponds to the point distance at which the beam footprint is predicted to be smallest. In Figure 26, the sharpest transitions occur at 220 mm to 230 mm, slightly closer than predicted based on the distance corresponding to the minimum $\hat{W}$.

4. CONCLUSION

We have derived a model of the response of a triangulation-based 3D laser imaging system to a sudden spatial discontinuity under the assumption that all surfaces are planar within the resolution of the system. We show that this response model fits experimental results and that it does not follow the typical first-order response for a linear system; rather, the response model is distinctly non-linear. We use the model as the basis for a test procedure to generate a measure of the lateral resolution of the system. For this purpose, we define the lateral resolution to be the total width of the area affected by the presence of a spatial discontinuity. We demonstrate through experimental results that the predicted lateral resolution matches qualitative observations of the response of the system to narrow post tops. As a side benefit, the method also provides the lateral shift required to improve the measured-to-reference fit.

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Figure 1 : Illustration of the sequence of events leading to the generation of a spatial measurement $\hat{\mathbf{t}} = [\hat{t}_1, \hat{t}_2, \hat{t}_3]$.

Figure 2: Contour plot highlighting the ovoid shape of the intensity of emitted light at the point of intersection with a Lambertian planar surface angled with respect to the beam axis. As the surface angulation increases the beam footprint increasingly deviates from the circular shape that is typically assumed.

Figure 3: Position-sensitive detector.

Figure 4: Whether the beam peak arises from $\hat{\mathbf{p}}_n$ or $\hat{\mathbf{p}}_f$, the system erroneously interprets the intersection of the beam axis with the surface as being at $\hat{\mathbf{p}}$.

Figure 5: Response of a laser scanner (solid line with dots) to a post structure (solid line), in which the sensor is to the left of the emitter, illustrating the resulting effect on spatial profile.

Figure 6: Simulated effect of vertical edges on measured depth in response to a post (solid pale line). Mean (solid dark line) and median (dotted line) generated approximately the same results. The position of the emitter and sensor have been swapped from the positions used to generate Figure 4.

Figure 7: Effect of nearby edge on surface measurement. The solid dot indicates the true peak, the cross indicates the mean or median centroid x_m , and the square indicates the measured intersection point.

Figure 8: Simulated depth noise (top) and result after noise filtering (bottom). Beam intensity profile is show centred on the edges.

Figure 9: Effect of viewing angle on the profile near an edge. The arrow indicates the direction of decreasing viewing angle.

Figure 10: Close-up of PTB artifact showing the 1.00 mm, 0.50 mm, and 0.25 mm posts.

Figure 11: Measured (dotted line) versus simulated (thick solid line) response to a 1 mm step edge. The vertical line indicates the expected location of the edge based on model fit.

Figure 12: 3D imaging system technologies as a function of depth-of-field and depth noise.

Figure 13: Convergence behaviour of the $\hat{\mathbf{e}}_z$ curve at two points on the curve. The vertical line represents the target combination of W and θ .

Figure 14: Simulated results for $\hat{W}_i$ estimate for a given $x_i = \hat{x}_c$.

Figure 15: Simulated plot of l^2 -norm versus $x_i = \hat{x}_c$. The true edge is marked in the top graph with a circle and vertical line and in the bottom graph with a vertical line. The circle in the bottom graph represents the upper bound of the search region for $\hat{x}_c$.

Figure 16: Simulation of slope of (19) versus distance from x_c . For $x < x_c$, the slope changes slowly while for $x > x_c$ the slope changes rapidly.

Figure 17: Simulated fit results at different $\hat{x}_c$ estimates. The edge location was fixed at $x_c = 0.1$ mm and is indicated by the vertical line.

Figure 18: Simulated effect of changing viewing angle on fit results. Viewing angles were, from top to bottom, 15° , 40° and 65° . As viewing angle is increased, the fit error becomes more flattened. The edge location was fixed at $x_c = 0.1$ mm and is indicated by the circle-with-cross.

Figure 19: Simulated effect of changing beam footprint size on fit results. As beam footprint size increased, the fit error becomes more flattened. The edge location is indicated by the circle-with-cross.

Figure 20: Simulated effect of depth noise level on fit results. As depth noise increases, the fit error becomes more flattened. The edge location is indicated by the circle-with-cross.

Figure 21: Lateral resolution $\hat{\delta}_x$ versus PTB distance from the SUT.

Figure 22: Lateral correction factor $\hat{x}_c$ versus PTB distance from the SUT.

Figure 23: Measured versus reference surface for the 0.50 mm post with $\hat{\delta}_x$ estimates from the 1.00 mm post. At 220 mm the width of the 0.50 mm post is predicted to be approaching the limit of resolvability but should still be completely resolvable.

Figure 24: Measured versus reference surface for the 0.50 mm post with $\hat{\delta}_x$ estimates from the 1.00 mm post. At 225 mm the width of the 0.50 mm post is predicted to be completely resolvable.

Figure 25: Measured versus reference surface for the 0.25 mm post with $\hat{\delta}_x$ estimates combined from the 1.00 mm and 0.50 mm posts. Graphs with $\hat{\delta}_x > 250$ μm , specifically (a) and (b), correspond to distances at which the 0.25 mm post is predicted to be unresolvable, while those with $\hat{\delta}_x \approx 250$ μm , specifically (h), correspond to distances at which the 0.25 mm post is predicted to be marginally resolvable. All other graphs correspond to distances at which the 0.25 mm post is predicted to be completely resolvable.

Figure 26: Fit error profiles (l^2 -norm versus $x_i = \hat{x}_c$) for the 1.00 mm post with $\hat{x}_c$ estimates. The final $\hat{x}_c$ marked on both graphs with a circle and vertical line. The mean $\pm$ uncertainty for $N = 10$ measurement results is also shown.