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CFD-assisted thin channel membrane characterization module design¹

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Abstract

This project involved the design of a thin channel cross-flow module for the characterization of flat ceramic membranes. A primary objective of this work was to ensure that the flow characteristics over the permeating area were uniform.

To house these membranes, a thin channel module with a long rectangular base was envisioned. The module feed is supplied by a multi-inlet tube-type plenum meant to provide a uniform flow distribution through pressure equilibration attained in its volume.

The design criteria for the module were minimization of both the flow non-uniformity and the pressure drop across the permeating area which was a central rectangular portion of a larger slab-style cell. The flow non-uniformity was taken as the normalized standard deviation of the velocity field above the permeating area. The pressure drops considered were those across the inlet plenum and across the permeating area normalized with respect to the outlet pressure.

The computational fluid dynamics (CFD) scheme which calculated the above module characteristics was a $k-\epsilon$ based turbulent transport model which used the finite difference method.

Design variables considered were: the plenum diameter, module width, height and length, and the diameters, distribution and number of the inlets on the plenum. The distribution of the inlet diameters was determined by two variables: either a linear or parabolic profile of variable slope and model coefficient. The operating variables were the cross-flow velocity and the plenum inlet pressure.

A two-level factorial design was used to screen the design variables. A refined three-level factorial design was used with a reduced set of design variables to optimize the module and study the response surface.

The final module design parameters were chosen such that the design criteria of flow uniformity and low pressure drop were met under a preset range of operating conditions. The local gradients of the response surface were used to verify that the design criteria were not overly sensitive to the selected module design parameters.

Keywords: Cross-flow module; Computational fluid dynamics (CFD); Factorial experimental design; Flow uniformity; Flow distributor

1. Introduction

Membrane characterization can be achieved more readily in a module designed expressly for this purpose. Many present-day test cells are constructed such that they create irregular and confounding hy-

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hydrodynamic conditions. The aim of the present study is to design a membrane module for research purposes with uniform flow characteristics over the permeating area. A well designed and uniform flow will allow a more accurate determination of mass-transfer properties at the membrane surface in cross-flow filtration. Cross-flow filtration is increasingly employed in many industrial separation processes, such as the harvesting of bioreaction products from fermentation broths, the recovery of metal precipitates from waste water, and the removal of fine particles from liquids in electronics industries.

The need for this new module arose in the context of a project to develop and assess ceramic membranes. These ceramic membranes are to be fabricated by a colloidal destabilization technique so that a flat membrane will result. Presently, in our laboratories there are disc-type cells for testing small circular polymeric membranes. These existing cells would not be easily adapted to hold ceramic membranes, so it was decided to design a next-generation test module to supersede the present test cells in their ability to provide data for membrane characterization.

Cross-flow filtration is a process for filtering and concentrating a multi-component feed stock, in which the feed flows parallel to the filtration surface. With this operating configuration any solid or liquid retentate which might be formed on the membrane surface is swept away. Typically, a liquid feed at elevated pressure flows through a channel whose walls are permeable.

Previous work on cross-flow filtration has shown that cross-flow velocity is the key control operating factor for polarized cake or gel formation, where a faster cross-flow velocity usually gives a higher filtration rate [1].

Examples can be found of cross-flow modules designed without inlet pressure equilibration. Such resultant flow characteristics with significant recirculation zones are illustrated in Fig. 1.

For research purposes, it is desirable to have uniform flow and fluid properties over the permeation area. This facilitates decoupling the intrinsic membrane-feed properties from the overall observed properties which are system dependent. Analyses such as the boundary layer model or the osmotic pressure model assume resolved hydrodynamics [2]. The present module design approach would allow

the use of models such as these with less inherent error. Thus, a superior flow diffuser is required for the present module. In this regard, it was decided to position a perforated-pipe type cylindrical header chamber for pressure equilibration with a central inlet in front of the cross-flow channel (Fig. 2).

With the various physical dimensions of the proposed module defined as design variables, it was decided to apply a turbulent CFD simulation for each permutation of them. Accordingly, a statistical approach was adopted to determine the most suitable module parameters for the final concrete module design. Practical mechanical and construction constraints were also considered in this design process.

Concentration polarization, solute retention and hydrodynamic behavior are all important and often interdependent factors. In a module such as is presently being designed, where the hydrodynamics are well defined, other simultaneous effects can be decoupled in a more straightforward manner, thus allowing a more direct means to assessing the intrinsic membrane mass transfer properties.

Very little is available in the literature on work to design such a module, especially in a plate and frame configuration. To study the dependence of permeate flux on operating temperature and to assure a uniform temperature distribution, Robert et al. [3] attempted to produce uniform hydrodynamic conditions. Patents for module design have been granted for apparently minor improvements such as changing the location of the outlet on a tube-bundle module [4].

Even recently, characterization theory is being applied to systems which do not lend themselves well to these analyses which make assumptions about the nature of the hydrodynamics in the module. In an

Fig. 1. Example of flow patterns in a conical type diffuser.

Fig. 2. Schematic diagram of module design.

attempt to characterize membrane performance in a hollow-fiber module, Yoshikawa et al. [5] developed a very complex and parameter-dependent model. The intractability of the whole procedure allowed only the formulation of a model with the conclusion that they would next have to resolve the hydrodynamics in the system.

1.1. Objectives

In view of the above discussion, the objectives to be achieved in this project are summarized as follows:

- to review literature on the subject of module design,

Table 1

Module design variables

Operating variables		Module design parameters	
P_2	pressure	D	largest inlet diameter
u	cross-flow velocity	d_s	smallest inlet diameter ratio
		ϕ	plenum diameter
		l	channel length
		w	channel width
		h	channel height
		n	number of inlets
		L, P	linear or parabolic inlet diameter profile

- to design a thin channel membrane module with very uniform flow properties,
- to employ a statistical experimental design approach to determine the module parameters,
- to select module parameters which give uniform flow conditions over a broad range of practical operating conditions.

1.2. Proposed design

A module was planned to house a small rectangular-shaped slab membrane. The size of the ceramic membrane slab is 35 mm by 50 mm. The membrane would sit in a cut-out area on a porous metal support such that it would be flush with the bottom of the channel and sealed in place with a glass hardener. Fig. 2 is a schematic diagram of the system under design.

The parameters indicated in Fig. 2 are explained in Table 1. The diameters of the inlets to the channel were allowed to vary to compensate for the pressure and flux changes along the plenum. Linear and parabolic profiles were considered, as are shown in Fig. 3.

Fig. 3. Schematic of channel inlet diameter distributions. The view shows the vertical face of the inside front channel wall.

1.3. Design criteria

The design criteria for the module were minimization of both the flow non-uniformity and the pressure drop across the permeating area. This permeating area was a short central rectangular portion of a larger slab-style cell. The flow non-uniformity was defined as the normalized standard deviation of the velocity field above the permeating area. The pressure drops considered were those across the inlet plenum and across the permeating area normalized with respect to the outlet pressure. The plenum feed pressure, P_1 , was the pressure the feed pump should deliver to obtain the expected module entrance pressure, P_2 . For practical purposes, cases where P_1 was in excess of 1000 psi were not considered. The channel inlet diameters were naturally limited by the height of the channel.

1.4. Design methodology

At this point, the procedure adopted to advance towards the objectives can be itemized as follows.

- establish pressure drop algorithm for feed flowing into and out of the plenum,
- conduct preliminary screening tests to set practical ranges for the variables to be used in subsequent systematic analysis,
- use a two-level factorial design to:
 1. evaluate module performance under full range of operating conditions,
 2. determine relationship between design parameters and module performance,
- analyze resulting linear multivariate regression model to point towards eventual module design,
- refine the module configuration with a three-level factorial design and model function analysis to finalize parameter values.

2. Design simulation

The effect of the ensemble of variables was investigated by a numerical simulation which first calculated the trans-plenum pressure drop to achieve a prescribed module inlet pressure, P_2 , and then the channel inlet velocities for the set of parameters under consideration. The channel inlet velocities

along with the dimensional parameters defining the module were then used as boundary conditions for a numerical simulation of the flow through the thin channel.

2.1. Trans-plenum pressure drop calculation

A conventional Bernoulli equation approach was taken to calculate the pressure drop across a prescribed plenum configuration [6]. The fluid from the feed tube enters the plenum chamber, and proceeds along it discharging some of its flux at each inlet to the channel. Fig. 4 schematically illustrates the problem under consideration.

The algorithm for performing this calculation is as follows:

For a required P_2 , the P_1 to deliver the required flow across the plenum is calculated iteratively

1. P_1 is assumed
2. $\Delta P_i = [(2f_i \rho v_i^2 L_i)/D] + \rho[(\Delta v_i^2)/2]$ where $f_i = (0.046/\text{Re}^{0.2})$
3. module inlet velocities were determined with the Bernoulli relation: $u_i = \sqrt{(P_i - P_2)/\rho}$
Flow into the module diminishes Q_{i+1} inside the plenum.
4. $v_i = [Q_i/(D/2)^2]$
5. at the last outlet the residual flux, Q_{RES} is determined
6. if $Q_{\text{RES}} < \text{TOL}$ then the system is converged, otherwise P_1 is adjusted accordingly and the sequence from step 1 is repeated. In this case, TOL was set to $10^{-3} \times Q_0$.

2.2. Numerical simulation

The flow through the module channel was simulated using a finite difference code. This code is based on the TURCOM package developed by Lai [7]. The version employed here is the same as detailed by Pellerin et al. [8] in a previous paper where it was tested against various analytical results, and was able to obtain matching results under specified benchmark conditions. Briefly, the code employs the $k-\epsilon$ turbulence model, which is incorporated into the following governing equations:

Mass:

$$\frac{\partial \rho}{\partial t} + (\rho u)_{j,j} = 0$$

Fig. 4. Specification of the problem for solving the trans-plenum pressure drop.

Momentum:

$$\frac{\partial(\rho u)_i}{\partial t} + ((\rho u)_j u_i)_{,j} = -p_{,i} + \tau_{ij,j}$$

Above, $\tau_{ij,j}$ is obtained from:

$$\tau_{ij} = \mu(u_{i,j} + u_{j,i}) - \frac{2}{3}\mu u_{k,k}\delta_{ij}$$

The fluid was considered to be water at 25°C. The boundary conditions for each case were calculated by the trans-plenum pressure drop routine.

2.2.1. Grid characteristics

In order to produce stable results for all values of the module parameters, a routine was implemented to automatically create the grid. This ensured that the same sufficient numerical resolution was kept for each case. That is, a uniform density of grid cells per unit area was maintained.

One y-direction cell spanning each inlet was used to minimize the x-axis node requirements. This was because the inlets were of a very small dimension relative to the entire module. In order to keep the cell aspect ratios within the required bound of four, it was necessary to have only one cell per inlet for computational economy [9].

A y-axis profile was considered with a higher grid density near both bordering inlets. The following formulae prescribe identical areas under a

parabolic curve, which define the nodes. The local coordinate system (y_G , x_G) was introduced for determining the nodal positions, as shown in Fig. 5. The y-axis nodes were calculated from the profile function, $x_G = Ay_G^2 + By_G + C$. The parameters α and β were resolved in view of the x-axis node spacing to maintain the cell aspect ratios under four. Through trial, β was determined to give sound results when set to 25.7 ℓ . The parabolic coefficients were determined as follows:

$$\text{if: } D_1 \geq D_2 \Rightarrow \begin{cases} A = \frac{B^2}{4D_2} \\ B = -2\frac{\sqrt{D_1 D_2} + D_2}{\ell} \\ C = D_1(1 + \beta) \end{cases}$$

$$\text{if: } D_1 < D_2 \Rightarrow \begin{cases} A = \frac{B^2}{4D_2} \\ B = -2\frac{\sqrt{D_1 D_2} - D_1}{\ell} \\ C = D_2 + \beta D_1 \end{cases}$$

where A , B , C : coefficients of the parabola; D_1 , D_2 : diameters of the adjacent inlets; ℓ : distance between two inlets; α : proportional coefficient (> 0); β :

Fig. 5. Schematic of the parabolic node position profile.

elevation coefficient (> 0); y_G , x_G : orthonormal unit vectors.

3. Simulation results

3.1. Trans-plenum pressure drops

The algorithm outlined in Section 2.1 was employed to calculate the pressure drop across the plenum, and to determine module channel inlet velocities. A quantitative check on the suitability of the design was made through the parameter ζ_{SC} defined by Idelchik [10]. Data for various flow distributors has shown that $\zeta_{SC} \approx 2$ is optimal for thin screens. Its definition is:

$$\zeta_{SC} = \frac{2(P_1 - P_2)}{\rho \bar{u}_i^2}$$

Above, $\bar{u}_i$ is the average velocity on the module side of the screen. It is known that $2 < \zeta_{SC} < 8$ is optimal for screens of finite thickness, as is the case here for the wall between the plenum and the channel [10].

For all the cases considered in the two-level factorial design that is detailed below (Section 3.4), ζ_{SC} was calculated. Table 2 summarizes these calculations.

The values of ζ_{SC} roughly follow a log-normal distribution centered over the optimal range, which indicates the soundness of the general design of the flow distributor under consideration.

3.2. Numerical flow simulation

A typical result of the CFD simulation is given below in Fig. 6, where the boundary conditions arising from the trans-plenum pressure drop calculation are input. The domain is cut along the y -axis of symmetry.

The main feature of note in Fig. 6 is how the jetting at the channel inlets produces some recirculation and irregularities in the pressure field, which are smoothed out relatively quickly as the flow proceeds through the module.

The computational grid generation system was developed to provide common resolution of detail and sufficient numerical precision for all cases tested. Simulations in the z - y -plane were done to verify that there were no unexpected entrance effects and to ensure that a two-dimensional representation was a reasonable approximation. Over the permeation area, z - y -plane flow had a fully developed turbulent profile. The simulation developed by Pellerin et al. [8] can be applied in the z - y -plane for subsequent membrane performance modelling. Further, for the envisioned cross-flow velocities and given the size and

Table 2
Screen drag coefficients in design study

Minimum ζ_{SC}	Maximum ζ_{SC}	Average ζ_{SC}	Standard deviation of ζ_{SC}
2.00	18.96	4.05	3.66

Fig. 6. Boundary conditions and results for an example case of the flow simulation. The shaded area represents the membrane permeation region.

position of the permeation area, z - y -plane velocity components resulting from trans-membrane flux will have a negligible effect on the flow profile.

3.2.1. Simulation responses

The flow uniformity was determined quantitatively by two responses: a velocity and pressure value.

The velocity response was:

$$y_{\text{VEL}} = \frac{1}{\bar{U}} \frac{\sqrt{\sum_{i=1}^{n_v} (U_i - \bar{U})^2}}{n_v - 1}$$

which was the normalized standard deviation of the velocity over the permeation region. The variable n_v was the number of velocity nodes which fall over the permeation region of the membrane.

The pressure drop response was:

$$y_{\text{PRESS}} = \frac{\Delta P (l_{\text{MEMBRANE}}/l_{\text{MODULE}})}{P_2 - (\Delta P/2)}$$

which was the module pressure drop normalized to the permeation region length, divided by the average module pressure.

3.3. Preliminary data

Once the simulation was operational, preliminary runs were conducted to determine trends and to establish variable bounds. As a practical constraint, values of P_1 above 1000 psi were to be avoided. A limit of channel height of 5 mm was imposed to minimize the required volumetric flow rates for the desired cross-flow velocities. For space and materials

considerations, the channel length was to be minimized, but also made sufficiently long so that entrance effects would be smoothed out before the flow reached the permeation area.

In view of the P_1 constraint, the highest cross-flow velocity allowed was 5 m/s and the smallest inlet diameter allowed was 1.4 mm. The largest inlet diameter was selected so as to be no larger than the smallest channel height considered, so as not to confound the subsequent statistical modelling. The channel width was selected to ensure that edge effects were not significant over the permeation area. Table 3 shows the list of variables and their ranges used in the factorial design.

3.4. Experimental design

A full two-level factorial design with 10 parameters requiring 2^{10} (1024) simulations was used for an initial screening. This is an effective way to minimize the number of simulations required to check on all possible variable interdependencies. Fractional two-level designs were not considered as the assumption of insignificant high order interactions was suspect. Three-level or central composite designs were excluded at this stage as the profile type had two levels and the number of inlets was a discrete variable.

An initial examination of the responses from the 10 variable screening study yielded some steps towards an ultimate design. Using coded variables, the operational parameters were always found to give a worst case result at $(u, P_2) = (+1, -1)$ for both

Table 3
Factorial design variable ranges and associated physical precision

No.	Variable	Min.	Max.	Physical precision
1	u – cross-flow velocity	1	5	0.01 [m/s]
2	P_2 – module pressure	10	200	0.25 [psi]
3	D – largest inlet diameter	1.4	3.0	0.025 [mm]
4	d_s – smallest inlet diameter ratio	0.875	1.0	N/A
5	ϕ – plenum diameter	7.5	30.0	0.025 [mm]
6	l – channel length	300	600	0.025 [mm]
7	w – channel width	25.0	45.0	0.025 [mm]
8	h – channel height	3.0	5.0	0.025 [mm]
9	n – number of inlets	4	6	N/A
10	L, P – inlet diameter profile	N/A	N/A	N/A

design criteria. Further, best results were uniformly obtained at $l = +1$. By inspection, stable local minima of the data occurred when $n = -1$, that is, with four channel inlets. The linear and parabolic inlet diameter profiles yielded near identical results, so that the linear profile was chosen for simplicity. Thus, the number of design variables was reduced to only five, as it was only necessary to evaluate each design for the operating conditions $(u, P_2) = (+1, -1)$.

At this stage, further consideration was given to the pumping requirements for the envisioned module design. Based on the initial screening study, the velocity response was qualitatively expected to be minimized with a large channel width. Coupled with a large channel height, this resulted in an excessively large pumping requirement at the highest cross-flow velocity. The channel height was fixed at the center point, which yielded an acceptable pressure drop response of 3.9%. This reduced the module design variables to four.

The two responses were modeled with a multivariate regression routine (IMSL: DRGLM). The general linear functional form for a response y_i (either velocity standard deviation y_{VEL} or pressure drop y_{PRESS}) would be: (below, $j, k, l, m = 1, 2, 3, 4$)

$$y_i = \beta_0 + \beta_j x_j + \beta_{jk} x_j x_k + \beta_{jkl} x_j x_k x_l + \beta_{jklm} x_j x_k x_l x_m + \epsilon \quad (1)$$

Since the data was generated by numerical simulation instead of physical experimentation, the variance was zero. The absence of a stochastic term, ϵ , in Eq. (1), or a reasonable external estimate of the response variances, necessitated simulating this variance. A random number with a normal distribution $N(0, 1)$ was multiplied by a variance estimate based on the physical precision expected during fabrication of the module, as given in Table 3. The three different sample variances of the two responses were estimated from 50 simulations with the design variables varying randomly within their error ranges. Estimates were obtained with all the design parameters set at either their low, mid or high levels. The sample variances of the responses at a given level were calculated by:

$$s_i = \sum_{j=1}^N \frac{(y_{ij} - \bar{y}_i)^2}{N-1} \quad (2)$$

The variance estimates obtained at the three levels were compared to see if they came from the same population. The ratio of the sample variances s_1^2/s_2^2 has an F probability distribution function with a $100(1 - \bar{\alpha})\%$ confidence interval:

$$\left(\frac{s_1^2}{s_2^2} F_{\nu_1, \nu_2, \bar{\alpha}/2}, \frac{s_1^2}{s_2^2} F_{\nu_1, \nu_2, 1 - \bar{\alpha}/2} \right) \quad (3)$$

The value of the F statistic is 1.62 for these cases where ν_1 and $\nu_2 = 49$ degrees of freedom associated with each sample variance. The significance of the parameter estimates in Eq. (1) can be determined by whether a 95% confidence interval contains 0:

$$\beta_i \pm t_{0.05, n} (\text{Var } \beta_i)^{1/2} \quad (4)$$

where $t_{0.05, 49}$ is the statistic taken from the t distribution with n degrees of freedom associated with $\text{Var } \beta_i$, the variance of the i -th parameter and is estimated from the sample variance, Eq. (2) and the diagonal elements of the variance-covariance matrix for Eq. (1).

Determining the significance of terms with the pressure drop response model was straightforward, only 4 terms were significant regardless of which variance estimate was used. These terms were all associated with the channel height. The remaining parameter estimates were essentially zero. The correlation coefficient (r -factor) for this model was > 0.99 . This was expected as the pressure drop across the permeation area should not have been a strong function of the plenum design itself.

The significance of terms in the velocity response model was evaluated using the largest variance estimate which came from the case with all the coded variables at the -1 level. The correlation coefficient, r , for the velocity response model was 0.98, but with only 5 degrees of freedom. An adequate model could not be obtained without consideration of all the higher order interactions. Eliminating all parameter estimates for third and higher order interactions resulted in a very poor fit, $r < 0.6$.

The existing two-level factorial design was expanded to a full three-level design with the remaining four variables. Similar results were obtained with respect to the model adequacy and significance of parameter estimates. The typical quadratic model with second order interactions and quadratic terms

could not adequately describe the results, $r = 0.68$. If all the significant terms were included based on t -tests, the correlation coefficient was high ($r = 0.98$) but only because all but 3 out of 81 possible terms were retained. This suggests that the interactions between the remaining variables are complex.

At this point, the module design was further refined by inspecting the simulation results. The module width was set to the upper level (45 mm) as the velocity response was always lower at this level. This level was always obtained using a bounded function minimization routine (IMSL: DBCOAH) with the quadratic model. The maximum inlet diameter was also fixed at the upper level (3 mm) to minimize the pressure in the plenum. Smaller inlet diameters required a plenum pressure > 1000 psi for cases with the maximum cross-flow velocity. This restriction did not penalize the velocity response.

The plenum diameter, ϕ , and the smallest inlet diameter ratio, d_s , were the two remaining design

variables. The response surface for these variables could only be adequately described by a full quadratic model, again suggesting strong interactions between these two design variables.

The predicted response ($\hat{y}_{VEL}$, Fig. 7) surface was verified by running the flow simulation for an 11×26 grid covering the region of the two final coded variables, Fig. 8. The velocity response is a weak function of d_s over a large part of the region studied. A maximum in y_{VEL} occurs across the range of ϕ . For large plenum diameters ($\phi = +1$), the internal pressure equilibration is very even, providing near constant module inlet velocities contributing to flow uniformity. As ϕ decreases, the pressure equilibration worsens, as higher feed pressures, P_1 , are required. This causes an increase in y_{VEL} . However, as ϕ further decreases, from -0.5 to -1 , P_1 increases to the point where the pressure acting at both ends of the plenum is sufficiently high relative to the overall plenum pressure drop ($P_1 - P_2$).

Fig. 7. $\hat{y}_{VEL}$ response surface for the coded range of ϕ and d_s based on nine data points from the three-level factorial model.

y_{VEL} response surface – 11 x 26 simulation data

Fig. 8. $\hat{y}_{VEL}$ response surface for the coded range of ϕ and d_s generated from the numerical flow simulation run over an 11×26 grid of points.

The true and predicted response surfaces are similar for values of $\phi > 0$. There is poor agreement when $-1 < \phi < 0$. The maximal ridge near $\phi \sim 0.4$ was not predicted by the model since the 2^3 design did not provide any data in this region. This suggests that a third or higher order model is required to adequately represent the true response surface. It has not been uncommon for other multi-variable experimental design models to be highly complex [11,12]. Hsu [13] conducted a 9-variable study of surimi processing using a central composite design. The significant lack of fit can be attributed to the omis-

sion of cubic and/or higher order interaction terms. In view of these previous findings, it was more reliable to use the y_{VEL} contours (Fig. 8) for any further design considerations.

3.5. Final parameter selection

Depending on the starting point, the function minimization routine (IMSL: DBCOAH) arrived at the points $(d_s, \phi) = (+1, -1)$ or $(+0.96, +1)$. Examination of the true response surface (Fig. 8) at these points led to the selection of $(d_s, \phi) = (+0.96, +1)$

Table 4

Finalized values for each design parameter

for the final module design parameters. The velocity distribution predicted for the $\phi = -1$ case for the outlets, (i.e., the u_i values) was a sharp decreasing profile in this case. For the larger plenum case, a very uniform u_i distribution was obtained, owing to the better internal pressure equilibration. Further, the y_{VEL} response surface has contours more widely spaced at the $(+0.96, +1)$ minima than at the $(+1, -1)$ minima, indicating better stability and less sensitivity to system or physical perturbations. The difference between $d_s = +0.96$ and $d_s = +1.0$ in physical terms amounts to inlet diameter increments of 0.03 mm per hole. This length is roughly the machining precision, so for practical considerations, the final value of $d_s = +1.0$ was chosen.

The final design values for each parameter are given in Table 4. Note that the simulations were done using a half-width model assuming symmetry, and the values given below are for the entire module. Corresponding to the parameters in Table 4, the worst case responses were: $y_{VEL} = 0.00100$, $y_{PRESS} = 0.0374$, obtained under the operating conditions of $P_2 = 10$ psi and $u = 5$ m/s. The largest value of P_1 (347.7 psi) occurred at $P_2 = 200$ psi and $u = 5$ m/s.

3.6. Response stability

With the chosen design parameters, the stability of the responses at the extremes of the operating variables was investigated. The CFD simulation was run 25 times at these points with fluctuations corresponding to the expected physical precision, as were given in Table 3.

At the outset, general notional criteria were to keep the response variables below 5%. In view of this, even in the worst case, both responses y_{VEL} and y_{PRESS} fall within this bound, as shown in Table 5.

Table 5
Response error ranges for the extreme points of the operating variables

u	P_2	y_{VEL}	y_{PRESS}	P_1 (psi)
-1	-1	0.00097 ± 0.00007	0.0015 ± 0.0002	15.9 ± 1.1
-1	+1	0.00096 ± 0.00007	0.0001 ± 0.0000	205.9 ± 1.1
+1	-1	0.00100 ± 0.00006	0.0374 ± 0.0015	158.0 ± 5.0
+1	+1	0.00099 ± 0.00006	0.0013 ± 0.0000	347.7 ± 5.0

Using the finalized physical dimensions, work will begin to construct this module.

4. Conclusions

A thin channel cross-flow module with a uniform flow field was designed for the characterization of flat ceramic membranes. A total of ten variables were considered for the module design. A 2^{10} factorial design was used to evaluate the predicted module performance for each combination of the design variables.

It was found that running the simulation at the highest cross-flow velocity and the lowest module pressure always resulted in the worst-case performance for a given module design. Based on the screening study, it was possible to fix $l = +1$, and $n = -1$ with a linear inlet diameter profile. At this point, consideration of pumping requirements led to setting $h = 0$.

Inspection of the subsequent results from a more focused three-level factorial design in the remaining four module parameter variables was able to fix $w = +1$, and $D = +1$. A response surface analysis in the final two module parameters determined optimum values of $d_s = +1$ and $\phi = +1$.

Throughout the design process, a suitable regression model for y_{VEL} was not obtained because of the highly irregular response surface. However, the experimental design approach did provide the data necessary for decisions to fix the design parameters.

A sensitivity analysis of the proposed design showed that it is stable for the full intended range of operating conditions. Thus, its desired performance can be achieved with some margin of safety.

5. List of symbols

- A, B, C coefficients of the parabola for grid generation
- d_s smallest inlet diameter
- D largest inlet diameter
- D_1, D_2 diameters of the adjacent inlets for grid generation
- F statistic from probability distribution for ratio of two independent chi-square distributions

f_f	fanning friction factor
h	channel height
i	subscript: relates to inlet number
i, j, k	subscripts: tensor spatial indices
l	channel length
ℓ	distance between two inlets
L	length along plenum
n	number of inlets
n_v	number of velocity nodes which fall over the permeation region of the membrane
N	variance calculation sample size
p	pressure, in transport equation
P_1	pressure to be delivered by pump for plenum inlet
P_2	pressure at front end of module channel
P_3	pressure at exit of module channel
ΔP	pressure drop across module
Q	volumetric flow rate
r	correlation coefficient, measure of systematic association between two variables
Re	Reynolds number
s_i	sample variance
t	statistic from probability distribution function for a normal distribution with unknown variance
u	cross-flow velocity
u_i	module inlet velocity
$\bar{u}_i$	average velocity at module front end
U	velocity used in determining uniformity response
v	velocity in plenum
w	channel width
y_G, x_G	orthonormal unit vectors for grid generation
y_i	average y_i
y_{ij}	j -th response in variance calculation for y_i
y_{PRESS}	simulation pressure response
y_{VEL}	simulation velocity response
$\hat{y}_{VEL}$	model estimate of velocity response

5.1. Greek letters

$\bar{\alpha}$	significance level
α	proportional coefficient for grid generation
β	elevation coefficient for grid generation
$\beta_{ijk\dots r}$	multivariate regression coefficients
δ_{ij}	Kronecker delta function
ζ_{SC}	screen drag coefficient
μ	fluid viscosity

ν	degrees of freedom
ρ	fluid density
$\tau_{ij,j}$	shear stress tensor
ϕ	plenum diameter

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