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Quantum circuits based on coded qubits encoded in chirality of electron spin complexes in triple quantum dots

Chang-Yu Hsieh and Pawel Hawrylak

*Quantum Theory Group, Institute for Microstructural Sciences, National Research Council, Ottawa, Ontario, Canada K1A 0R6
and Department of Physics, University of Ottawa, Ottawa, Ontario, Canada K1N 6N5*

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We present a theory of quantum circuits based on coded qubits encoded in chirality of electron spin complexes in lateral gated semiconductor triple quantum dot molecules with one-electron spin in each dot. Using microscopic Hamiltonian we show how to initialize, coherently control, and measure the quantum state of a chirality-based coded qubit using static magnetic field and voltage tuning of individual dots. The microscopic model of two interacting coded qubits is established and mapped to an Ising Hamiltonian, resulting in nonlocal double-qubit gates.

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I. INTRODUCTION

There is currently interest in exploiting electron spins for nanospintronic¹ and quantum information processing.^{2–5} This is partly motivated by electron spin long coherence times⁶ and availability of scalable semiconductor technology. In the simplest approach, a physical qubit is identified with the two states of an electron spin, which can be manipulated by applying local magnetic fields. Much progress has been achieved in using micromagnet^{7,8} technology, electron spin resonance⁹ with spin-blockade measurement and oscillating electric field with spin-orbital coupling¹⁰ to drive coherent oscillations of electron spin qubits. An alternative approach is to encode a qubit in two levels of many-electron spin complexes.³ This includes singlet-triplet two-electron coded qubit¹¹ and a qubit encoded in a degenerate ground state of a linear three-spin complex.¹² DiVincenzo *et al.*¹² proposed the encoding of a qubit in the many-body states of a three-spin complex. They showed how to perform universal quantum computation by only tuning the exchange interaction between pairs of spins. The ability to manipulate the state of a coded qubit with voltages is due to the interplay of Coulomb interactions and Pauli exclusion principle. A detailed microscopic model exploring the orbital and spin relation of a three-electron complex in a single semiconductor quantum dot (QD) was studied and compared with experiments by one of us.¹³ A related microscopic model of three electrons localized in a lateral gated triangular triple quantum dot molecule in the plane of a GaAs/GaAlAs heterojunction was also proposed and explored by one of us in Refs. 14 and 15. Our proposed coded qubit in triple QDs (TQD) is identified with the chirality of three-electron spin complex, or equivalently, two possible directions of a minority spin current. Similar proposals to implement coded qubits with spins in TQDs (Ref. 16) as well as atomic traps^{17,18} exist.

The advantages of working with a coded qubit are two-fold. First, every quantum gate can be implemented electrically. In such a scheme, magnetic field is only used for initialization and measurement of a coded qubit. Second, the coded qubit operates in a decoherence-free subspace^{16,19–21} and is immune to channels of collective decoherence. The suppression of decoherence due to random telegraph noise

for a coded qubit was discussed by us in Ref. 22. Any qubit architecture that exploits charge degree of freedom of an electron suffers decoherence due to electrostatic fluctuations such as $1/f$ or random telegraph noise. The fact that such electrostatic noise does not induce significant decoherence in a chirality-based coded qubit suggests that coded qubits and simple spin qubits may have the same dominant decoherence channel coming from hyperfine interactions with nuclear spins.^{23–25} In short, a coded qubit combines the advantages of a charge qubit, such as the possibility of performing coherent quantum operations with only electrostatic means, and that of the spin qubit, such as the long coherence time.

Other interesting phenomena involving TQD molecules include non-Fermi-liquid behavior when coupled to leads,^{26–28} potential for generating maximally entangled three-partite GHZ and W states,^{29–31} flying spin qubit,^{32,33} and manipulation of the total spin³⁴ of the four-electron complex in TQD.

In this work we develop a microscopic theory of quantum circuits based on qubits encoded in chirality of electron spin complexes in TQD molecules. We use the linear combination of harmonic orbitals-configuration interactions (LCHO-CI) (Ref. 15) method, Hubbard and Heisenberg models to determine a set of optimal conditions for single-qubit operations and derive an effective model of two qubit operations. We show that there exists an in-plane magnetic field direction and magnitude optimal for single-qubit operations. However, it is shown that the magnetic field rotates the coded qubit states in an undesirable way; and we show how this rotation can be controlled by tuning voltages on the gates. Once the optimal set of magnetic fields and voltages have been determined, the exact diagonalization techniques are used to compute parameters of effective qubit Hamiltonians. The present work establishes both single- and double-qubit operations necessary for performing quantum computation with chirality-based coded qubits.

The paper is organized as follows. Section I contains introduction. Section II describes the quantum circuit based on chirality of three-electron spin complexes. Section III presents a definition of a chirality-based coded qubit, the optimal preparation of computational space, and the initialization of a coded qubit. Section IV presents effective Hamiltonians

FIG. 1. (Color online) Three possible layouts for a chain of coded qubits. Each solid and dashed line represents an exchange interaction between a pair of QDs. The layouts (a) and (b) differ in that the constituent coded qubits are implemented with triangular TQD in (a) and with linear TQD in (b). The layouts (a) and (c) differ in the way coded qubits are connected in a chain. If the interqubit distance is reduced beyond a threshold, an additional set of exchange interactions will arise and dominate the behavior of the system in layout (c).

of single and double coded qubits and demonstrates the capability of generating single- and double-qubit operations, respectively. Section V discusses the measurement of a chirality coded qubit. Section VI presents a summary.

II. MODEL

Figure 1(a) shows schematic layout of quantum circuits based on qubits encoded in chirality of electron spin complexes in triangular TQDs. The circles denote individual lateral quantum dots formed in the two-dimensional electron gas (2DEG) at the heterojunctions of AlGaAs/GaAs by metallic gates on the AlGaAs surface. The gates are set to confine a single electron in each dot denoted by an arrow. Additional gates (not shown here) are used to control tunneling, shown schematically as solid lines, between dots in the same TQD molecule. Dashed lines indicate tunneling between neighboring TQDs. The tunneling between any pair of QDs is responsible for exchange interaction of electron spins localized on each QD. The brackets indicate two TQDs isolated from the rest of the circuit. It is assumed that any number of TQDs can be isolated from the rest of the circuit by turning off the exchange interactions with gate voltages.

For comparison, Fig. 1(b) shows the circuit composed of linear, instead of triangular, TQDs, similar to the linear chain of spins proposed by DiVincenzo *et al.*,¹² in which the coded qubits in the chain are implemented with only two exchange interactions. Figure 1(c) is another possible architecture studied by Weinstein *et al.*¹⁶ for spin system. However, in this design, bringing TQDs close together induces interactions

between quantum dots beyond the ones indicated by the dashed lines.

In this work we focus mainly on triangular TQDs-based quantum circuit shown in Fig. 1(a). Since electrons are well localized in each QD, a system of two TQDs in a chain can be described by an extended Hubbard model.³⁵ With $c_{i\sigma}^\dagger$ ($c_{i\sigma}$) electron creation (annihilation) operator for the electron with spin $\sigma = \pm 1$ on the i th QD, the Hubbard Hamiltonian reads

$$\begin{aligned} \hat{H}_2 = & \sum_{n=1}^2 \sum_{i,j=3n-2}^{3n} \sum_{\sigma} E_i \hat{n}_{i\sigma} + \sum_{n=1}^2 \sum_{i,j=3n-2}^{3n} \sum_{\sigma} t_{ij} \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} \\ & + \frac{1}{2} \sum_{n=1}^2 \sum_{i,j=3n-2}^{3n} V \hat{\rho}_i \hat{\rho}_j + \sum_{n=1}^2 \sum_{i,j=3n-2}^{3n} U \hat{n}_{i\uparrow} \hat{n}_{j\downarrow} + \sum_{i=1}^6 g^* \mu_B \mathbf{B} \cdot \mathbf{S}_i \\ & + \sum_{\sigma} t' (\hat{c}_{3\sigma}^\dagger \hat{c}_{4\sigma} + \hat{c}_{4\sigma}^\dagger \hat{c}_{3\sigma}) + V' \hat{\rho}_3 \hat{\rho}_4, \end{aligned} \quad (1)$$

where $n=1,2$ labels the two TQD molecules from left to right, indices i,j range from $3n-2$ to $3n$ and label each QD from left to right in the n th TQD molecule. The intra-TQD Hubbard parameters t_{ij} , V , U , and E_i are the tunneling matrix element between the i th and the j th QDs, the off-site Coulomb-interaction parameter between any two QDs in the same TQD molecule, the on-site Coulomb interaction strength for any QD, and the on-site energy for the i th QD, respectively. g^* and μ_B are the g factor and the Bohr magneton. The Hubbard parameters t' and V' represent the intermolecular tunneling matrix element and intermolecular Coulomb interactions. $\hat{n}_{i\sigma} = \hat{c}_{i\sigma}^\dagger \hat{c}_{i\sigma}$ is the number operator for the σ electron on the i th QD. $\hat{\rho}_i = \hat{n}_{i\uparrow} + \hat{n}_{i\downarrow}$ is the electron charge-density operator on the i th QD. When we discuss triangular resonant TQDs, we drop the subscripts on all the parameters. In our model, we consider the parameters corresponding to regime of strong Coulomb interactions: $t'_{ij} < t_{ij} \ll V' < V < U$.

The intra-TQD Hubbard parameters, E_i , t_{ij} , V , and U , are obtained from a microscopic calculation for single TQD based on LCHO-CI method as explained in Ref. 36. The inter-TQD Hubbard parameter V' is taken to be the direct Coulomb interaction between two charges localized on adjacent edge dots of two neighboring TQD molecules.

The energy spectrum and eigenstates of the Hubbard Hamiltonian for one and two TQD molecules are obtained using configuration interaction technique. For a given number of electrons N_e we construct all possible configurations $|k = i_{N_e} \sigma_{N_e}, \dots, i_2 \sigma_2, i_1 \sigma_1\rangle = \hat{c}_{i_{N_e} \sigma_{N_e}}^\dagger \dots \hat{c}_{i_2 \sigma_2}^\dagger \hat{c}_{i_1 \sigma_1}^\dagger |0\rangle$, build the Hamiltonian matrix in the space of configurations and diagonalize it numerically.

At half filling, the low-energy spectrum of the Hubbard model can be approximated by a Heisenberg model^{35,37,38} describing electron spins localized in each dot,

$$\hat{H}_{\text{heis}} = \sum_{i < j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j. \quad (2)$$

The exchange interactions can be expressed in terms of tunneling matrix elements and quantum dot energies

$$J_{ij} = 2|t_{ij}|^2 \left(\frac{1}{U - V + (E_i - E_j)} + \frac{1}{U - V - (E_i - E_j)} \right). \quad (3)$$

The exchange interaction can be controlled by either tuning the tunneling matrix element t_{ij} by, for example, additional gates controlling the height of the tunneling barrier or by biasing the dots and changing their on-site energy $E_{i(j)}$.

We remark that the effective Heisenberg models of an isolated TQD and a pair of coupled TQDs are sufficient to capture all the relevant physics for a quantum circuit in which the only significant inter-TQD Coulomb interactions are the ones represented by the dashed lines in the Fig. 1(a). This is because all quantum gates can be decomposed into a universal set of all single-qubit gates and one nontrivial double-qubit gate, such as CNOT. Furthermore, in Ref. 39, an effective spin model with only nearest-neighbor interaction for a network of N TQDs was derived and verified against the results obtained from the exact diagonalization of the Hubbard model. The validity of such an effective spin model for the low-energy subspaces of N TQDs supports the claim that we only need to study at most two TQDs isolated from the chain.

III. SINGLE CODED QUBIT: DEFINITION, PREPARATION, AND INITIALIZATION

A coded qubit in a TQD must have one spin pointing in opposite direction with respect to the other two spins. An example of such configurations is $|\uparrow\downarrow\downarrow\rangle = \hat{c}_{1\uparrow}^\dagger \hat{c}_{2\downarrow}^\dagger \hat{c}_{3\downarrow}^\dagger |0\rangle$, which is shown schematically in Fig. 2(a). Another two configurations in the same subspace can be generated by permuting the position of the minority spin in the TQD. The subspace of these three configurations has a doubly degenerate ground state with total spin $S=1/2$ and an excited state with total spin $S=3/2$ separated from the ground state by J_{eff} , as shown in Fig. 2(b). In this work, the quantization axis for the spins are defined along the y axis because a static B field is applied in this direction.

The doubly degenerate ground state forms an effective two-level system. We identify the coded qubit levels $|q_+\rangle$ and $|q_-\rangle$ with

$$|q_\pm\rangle = \frac{1}{\sqrt{3}} \sum_{n=1}^3 e^{\pm i2\pi n/3} |n\rangle, \quad (4)$$

where $|n\rangle = S_n^+ |\downarrow\downarrow\downarrow\rangle$. The two coded qubit levels are the eigenstates of chirality operator $\chi_{ijk} = (\mathbf{S}_i \times \mathbf{S}_j) \cdot \mathbf{S}_k$, which measures the degree of collinearity of the three spins. These two states also can be more intuitively characterized by minority spin moving either to the left or to the right as in resonant valence-bond plaquette.^{40–43}

We note that in the absence of the magnetic field the coded qubit states in $S_y=1/2$ subspace are degenerate with another set of coded qubit states in $S_y=-1/2$ subspace. If we are to work in the computational space corresponding to $S_y=-1/2$, any process which flips the electron spin will remove the coded qubit from its computational space. In order to separate the computational space $S_y=-1/2$ from $S_y=+1/2$ subspace, a magnetic field B_y is applied, as shown in Fig.

FIG. 2. (Color online) (a) A schematic of a single TQD initialized in $S_y=-1/2$ subspace under a B_y field. The B_y field breaks the discrete rotational symmetry, and it affects QD 2 differently from QDs 1 and 3. (b). The energy spectrum of a triangular TQD as a function of ω_c/ω_c^* , where ω_c is the cyclotron frequency used in LCHO-CI calculation. ω_c^* corresponds to a magnetic field B^* such that the gap between computational subspace (the lowest energy level in the plot) and the rest of spectrum is maximized as indicated by the black arrow.

2(a). We avoid preparing the system by applying magnetic field in the z direction (perpendicular to the 2DEG) because it activates undesirable, higher order spin-spin interactions,^{37,38} whereas the magnetic field applied in the plane only modifies the on-site energies of QDs: for B_y field, QDs 1 and 3 energy levels as $E_{1(3)}(B) = E_{1(3)}(0) + 1/2\omega_c(R/2)^2 + g\mu_B S_y$, where R is the spatial separation of QDs 1 and 3 and $\omega_c = eB_y/m_e^*c$ is the cyclotron frequency while the QD 2 energy level only acquires the Zeeman term. Hence magnetic field in the y direction effectively lowers the energy of the QD 2 by $\Delta E_2 = -1/2\omega_c(R/2)^2$ with respect to QDs 1 and 3.

Figure 2(b) shows the energy spectrum of the coded qubit as a function of ω_c obtained in LCHO-CI method. In our LCHO-CI calculation, we use the following parameters for a specific resonant TQD. The interdot distance is $10a_B^*$, where $a_B^* = 9.79$ nm is the effective Bohr radius for GaAs. The confining Gaussian potential on the i th QD is of the form $V_i(x, y) = -V_i^0 \exp\{-(x-x_i)^2 + (y-y_i)^2/d_i^2\}$, where $V_i^0 = 4.0Ry^*$ and $d_i = 2.5a_B^*$. $Ry^* = 5.93$ meV is the effective Rydberg for GaAs.

At $B=0$, the fourfold degenerate ground state is separated from the fourfold degenerate spin-polarized excited state by J_{eff} . The two possible computational spaces corresponding to $S_y = \pm 1/2$ separate energetically with increasing magnetic

FIG. 3. (Color online) (a) The energy spectrum of qubit levels [the zoom in of the lowest level in Fig. 2(b)] as a function of ω_c/ω_c^* . The B_y field splits the doubly degenerate ground state and Δ corresponds to the gap between the two levels at ω_c^* or equivalently at B_y^* . (b) The energy spectrum of the qubit levels under B_y as a function of voltage V_2 on QD2 and V_2^0 is the unbiased voltage at resonant condition when there is no external B field. The splitting of levels under B field can be restored via gate-voltage tuning.

field while the spin-polarized states decrease in energy. At magnetic field B^* , such that $J_{\text{eff}} = 2g\mu B^*$, the energy gap is maximized and should correspond to the working point that can maintain the longest coherence time of the coded qubit. The coded qubit should operate at this value of magnetic field. In Fig. 2(b), the optimal cyclotron frequency ω_c^* corresponds to $B^* = 0.69$ T. However, as discussed above, the magnetic field effectively biases the QD 2. This removes the degeneracy of the two qubit levels and rotates them from their zero magnetic field states. The energy splitting of the two coded qubit levels as a function of applied magnetic field is shown in Fig. 3(a). The splitting is a fraction of the large energy scale J_{eff} . For the largest gap, i.e., $B = B^*$, the splitting is Δ . In order to restore the degeneracy of the two coded qubit levels, one can apply voltage to the QD 2. In Fig. 3(b), microscopic calculations done using LCHO-CI method show that a positive voltage bias on the QD 2 can indeed bring the TQD back on resonance in the presence of B field. We have now established the coded qubit and the optimal working conditions.

In order to operate the coded qubit, we need to initialize it. We propose to initialize the coded qubit by turning off both interactions J_{12} and J_{13} . The only remaining interaction is between QDs 2 and 3. The ground state of a TQD, $|L_0\rangle = |\downarrow_1\rangle|S_{23}\rangle$, becomes a product of a spin-down state of an electron on the QD 1 and a singlet state of electrons across QDs 2 and 3. A singlet across QDs 2 and 3 can be generated in real time starting from two electrons in a biased QD 3, then adiabatically transfer one electron to QD 2. Once the state $|L_0\rangle = \frac{-i}{\sqrt{2}}(e^{i2\pi/3}|q_- \rangle - e^{-i2\pi/3}|q_+ \rangle)$ is generated, we may perform single-qubit operations to obtain a specific chirality state.

IV. COHERENT OPERATIONS: SINGLE- AND DOUBLE-QUBIT GATES

We first discuss single-qubit operations. We rewrite the TQD Heisenberg Hamiltonian in the basis of the two coded qubit levels $\{|q_{\pm}\rangle\}$,

$$\hat{H}_{1q} = \frac{1}{2} \left(J_{12} - \frac{1}{2}J_{13} - \frac{1}{2}J_{23} \right) \sigma_x + \frac{\sqrt{3}}{4} (J_{13} - J_{23}) \sigma_y. \quad (5)$$

The rotated Hamiltonian is a spin Hamiltonian with an effective, in-plane magnetic field given by the exchange interactions. If we take $J_{13} = J_{23}$, and let $2J_{12} > J_{13} + J_{23}$, then the Heisenberg Hamiltonian corresponds to σ_x operation. If we take $J_{23} > J_{13}$ and $2J_{12} = J_{13} + J_{23}$, then the coded qubit Hamiltonian (5) corresponds to σ_y operation. The capability to rotate a qubit with respect to two different axes on a Bloch sphere allows us to generate arbitrary single-qubit operations. In particular, a rotation of angle θ around the z axis on the Bloch sphere can be achieved by the following Euler rotations: $R^z(\theta) = R^y(-\pi/2)R^x(\theta)R^y(\pi/2)$. For the parameters (given in the previous section) we have taken for the GaAs QDs in this study, the resonant exchange interaction J has a magnitude around $30 \mu\text{eV}$, comparable with experiments. It will thus take 0.1 ns to complete a 2π rotation.

In practice, the tuning of the exchange interaction is done through biasing QDs and changing the on-site energies. As long as the biasing condition $|\Delta E| \ll U - V$ is satisfied, we expect the quantum state to remain in the qubit subspace during the process of tuning the exchange interactions, as discussed in Ref. 16. Since the two levels of a coded qubit are extracted from a larger Hilbert space, including spin polarized and doubly occupied configurations, the fidelity of the coded qubit state is determined by interaction with environment and by coupling with remaining configurations. The fidelity of the coded qubit, in the absence of external decoherence, is bounded by $1 - (4t^2/U)$. This limiting factor, t^2/U , indicates how accurately we may treat the low-energy states of a half-filled TQD by localized spins of the Heisenberg model. For instance, in a typical experimental setting,⁴⁴ t is at least two to three orders of magnitude smaller than U . Therefore, the ideal coded qubit has high fidelity.

We now turn to discuss two coded qubit operations using both Hubbard and Heisenberg models. Starting with the Hubbard Hamiltonian, Eq. (1), we derive perturbatively the Heisenberg Hamiltonian for a six-electron complex with a set of exchange interactions given by

$$\begin{aligned}
J_{12} &= J_{56} = J_a = \frac{4t^2}{U - V}, \\
J_{13} &= J_{23} = J_{45} = J_{46} = J_b = \frac{2t^2}{U - V + V'} + \frac{2t^2}{U - V - V'}, \\
J_{34} &= J_c = \frac{4t'^2}{U - V'}, \quad (6)
\end{aligned}$$

and all other exchange interactions set to zero. For all the Hubbard parameters considered in this study (i.e., weak inter-TQD tunneling and strong Coulomb interaction), J_c is about two order of magnitudes smaller than J_a and J_b , mainly due to the fact that $t' < t/2$ in our study.

We treat the inter-TQD exchange interaction, scaled by J_c , perturbatively to derive an effective interaction in the coupled coded qubits subspace, $\{|q_+^L\rangle|q_+^R\rangle, |q_+^L\rangle|q_-^R\rangle, |q_-^L\rangle|q_+^R\rangle, |q_-^L\rangle|q_-^R\rangle\}$, where the superscript L and R stand for left and right TQDs. The effective qubit-qubit Hamiltonian reads

$$\hat{H}_{2cq} = 4|\alpha'\rangle(\hat{n} \cdot \vec{\sigma}_L)(\hat{n} \cdot \vec{\sigma}_R) + 2|\beta'\rangle\hat{m} \cdot \vec{\sigma}_L + 2|\beta'\rangle\hat{m} \cdot \vec{\sigma}_R, \quad (7)$$

where $\alpha' = e^{-i2\pi/3}J_c/9$ and $\beta' = e^{-i\pi/3}(-J_a/2 + J_b/2 + J_c/27)$. $\vec{\sigma}_L$ and $\vec{\sigma}_R$ are the effective Pauli matrices for the left and right coded qubit, respectively. The direction vectors, $\hat{n}$ and $\hat{m}$, are determined by the exchange interactions as follows: $\hat{n} = (\sqrt{|\alpha'| + \text{Re}(\alpha')}/\sqrt{2}|\alpha'|, \sqrt{|\alpha'| - \text{Re}(\alpha')}/\sqrt{2}|\alpha'|, 0)$, $\hat{m} = (\text{Re}(\beta')/|\beta'|, \text{Im}(\beta')/|\beta'|, 0)$. Thus, the effective Hamiltonian for two coded qubits is equivalent to a generalized XY Hamiltonian under a uniform in-plane magnetic field for two spins. With the current form of double-qubit Hamiltonian, it is unclear as to what one should choose for the values of $\hat{n}$, $\hat{m}$, $|\alpha'|$, and $|\beta'|$. Such a dilemma can be avoided if one rotates the effective double-qubit Hamiltonian, Eq. (7), from the chirality basis to the Jacobi basis, $\{|L_0^L\rangle|L_0^R\rangle, |L_0^L\rangle|L_1^R\rangle, |L_1^L\rangle|L_0^R\rangle, |L_1^L\rangle|L_1^R\rangle\}$. The rotated Hamiltonian reads

$$\hat{H}_{2Lq} = \alpha\sigma_L^z\sigma_R^z + \beta\sigma_L^z + \beta\sigma_R^z, \quad (8)$$

where $\alpha = J_c/9$ and $\beta = -J_a/2 + J_b/2 + J_c/18$. The Jacobi state $|L_0\rangle$ is defined in Sec. III where we discuss the initialization of the coded qubit. The other Jacobi state $|L_1\rangle = \sqrt{1/3}|\downarrow_1\rangle|T_{23}^0\rangle - \sqrt{2/3}|\uparrow_1\rangle|T_{23}^- \rangle$, where $|T_{23}^0\rangle$ is a $S_y=0$ triplet state across QDs 2 and 3 and $|T_{23}^- \rangle$ is a $S_y=-1$ triplet state across QDs 2 and 3. Similarly, $|L_1\rangle = -1/\sqrt{2}(e^{i2\pi/3}|q_- \rangle + e^{-i2\pi/3}|q_+ \rangle)$ can also be written as a linear combination of the coded qubit levels. Therefore this rotation of basis only requires single-qubit operations applied to each coded qubit; the two Hamiltonians, Eqs. (7) and (8), are locally equivalent.⁴⁵ In this transformed representation, the effective Ising model gives a much more intuitive and well-studied dynamics in NMR literature.⁴⁶ Now, it becomes clear that we have to suppress the effective field β in Eq. (8) in order to obtain an exact Ising Hamiltonian used in NMR experiments. The J_c exchange interaction is responsible for the coupling between two TQDs, and it should ideally be kept about two

order of magnitudes smaller than J_a and J_b in order for the perturbative analysis to provide good approximations. The value of J_a and J_b should thus be chosen very close in order to make $\beta = J_b/2 + J_c/18 - J_a/2 = 0$. In summary, each TQD involved in double-qubit operation should be kept in an almost resonant condition. The small deviation between J_a and J_b is needed to offset the weak coupling due to J_c in order to suppress the effective magnetic field β in the Ising model.

As Ising interactions can be used to generate the controlled-phase gate,⁴⁷ we show how to generate nonlocal double-qubit operations. We present the sequence of gate operations to simulate the Ising interaction between a pair of chirality-based coded qubits

$$\begin{aligned}
e^{-i\hat{H}_{2Lq}t} &= R_L^x(\pi/2)R_L^z(\pi/6)R_R^x(\pi/2)R_R^z(\pi/6)e^{-i\hat{H}_{2Lq}t}R_L^z(\pi/6) \\
&\quad - \pi/6)R_L^x(-\pi/2)R_R^z(-\pi/6)R_R^x(-\pi/2), \quad (9)
\end{aligned}$$

where $R_{L(R)}^j(\theta) = e^{-i(\theta/2)\sigma_{L(R)}^j}$ represents the rotation of the left (L) or right (R) qubit on its Bloch sphere by angle θ with respect to j axis. If we set $t = \pi/\alpha$ in Eq. (9), where α is defined in Eq. (8), then the controlled-phase gate is generated as follows:⁴⁶

$$U_{cphase} = \sqrt{i}R_L^z(-\pi/2)R_R^z(-\pi/2)e^{-i\hat{H}_{2Lq}(\pi/\alpha)}. \quad (10)$$

Similarly, we can also generate CNOT gate by the following elementary operations:⁴⁶

$$U_{\text{CNOT}} = iR_L^z(\pi/2)R_R^z(-\pi/2)R_R^x(\pi/2)e^{-i\hat{H}_{2Lq}(\pi/\alpha)}R_R^y(\pi/2). \quad (11)$$

The important operation building an entanglement between the two coded qubits is $e^{-i\hat{H}_{2Lq}t}$. For both controlled and CNOT gates, the time needed for the entanglement buildup is given by $\pi/\alpha \approx 1$ ps, about two order of magnitude smaller than typical single-qubit operations.

By counting the number of operations involved in Eqs. (9) and (10), it takes 4 and 12 elementary operations to generate controlled-phase gate with coded qubit in the Jacobi and chirality basis, respectively. To generate the CNOT gate, it will take one additional step in both the Jacobi and chirality basis. At this point, we compare the double-qubit operation derived in this section with the original proposal put forth by Divincenzo *et al.*¹² In the original proposal, it takes 19 elementary steps to generate the CNOT gate, and the quantum states will evolve and escapes the qubit subspace for a finite period of time during the operations. In the present proposal, it takes five steps to generate the CNOT gate in the Jacobi basis and the quantum state never escape the qubit subspace. We compare the results in the Jacobi basis because the qubits on a linear chain proposed by Divincenzo *et al.* do not have the chirality. This significant reduction in the complexity of generating nontrivial double-qubit operations is due to the requirement of weak coupling between the two TQDs in our model. In the case of original proposal, the coupling strength between two coded qubits is as strong as the internal coupling strength between the spins of a coded qubit (i.e., $t' = t$); such a strong coupling imposes the view of one molecule with six constituent dots as opposed to two weakly coupled TQDs.

FIG. 4. (Color online) The four lowest energy levels of two coupled triangular QDs as a function of inter-QD tunneling t' . The energy spectrum resembles that of the Ising model with an external field: a doubly degenerate levels corresponding to states $|\downarrow\uparrow\rangle$ and $|\uparrow\downarrow\rangle$ and two unique levels $|\downarrow\downarrow\rangle$ and $|\uparrow\uparrow\rangle$. “X2” denotes a doubly degenerate level. Inset: the entire energy spectrum at $t'/t = 0.2$ calculated with LCHO-CI method.

Figure 4 shows results of exact diagonalization of the Hubbard Hamiltonian for two coupled TQDs as a function of increasing coupling t' between the two molecules. The inset shows the entire energy spectrum at $t'/t = 0.2$. The low-energy spectrum consists of four levels, characterized mostly by the two lowest levels of each TQD. These four low-lying energy levels are well separated from the rest of the spectrum and constitute the two coupled coded qubit subspace. The low-lying spectrum contains a doubly degenerate level, which is a signature of the Ising model in an external field. The large gap that separates the qubit subspace is due to the weak coupling between the two TQDs. The gap decreases as the inter-QD coupling strength increases. The large energy gap is responsible for the suppression of quantum states escaping out of the qubit subspace. In practice, the inter-QD exchange interaction, J_c , should be tuned by modifying the tunneling parameter t' . Figure 4 shows that the Ising energy spectrum is maintained over a wide range of values of t' , and hence the Ising model Hamiltonian describes the coupling of two coded qubits very well.

V. MEASUREMENT OF SINGLE CODED QUBIT

Next, we discuss the measurement of coded qubits. Several proposals^{17,43,48} have already discussed various methods of detecting chirality of a triangular (three-body) antiferromagnetic cluster. For our specific proposal, we would like to draw analogy with single-shot readout of an electron spin in a single QD as presented in Ref. 49. This scheme of detecting an electron spin in a single QD is based on spin-to-charge conversion. In the presence of an applied magnetic field, the Zeeman effect splits the two spin states of an electron. Furthermore, the QD potential is tuned in such a way

that an electron escapes the dot if it is in the high-energy spin state or it remains in the dot if it is in the low-energy spin state. By coupling a quantum point contact (QPC) to the dot and measuring the QPC conductance, one may determine the number of electrons trapped in the dot. Thus, if the electron escapes the dot, we know it was in a high-energy spin state or vice versa. This simple proposal can be generalized by noticing that all we need are two distinct energy levels, such as the two spin states of a single electron or the two chirality states of a three-electron complex in the presence of an external field.

To obtain two chirality states with different energy, we propose to apply an additional field B_z as part of the measurement procedure. Under B_z , our chirality states splits in exact analogy to the Zeeman splitting of a spin 1/2. Detailed analysis of how the chirality states of a TQD respond to applied field B_z may be found in Refs. 38 and 50. In Ref. 38, Scarola and DasSarma compared the magnitudes of the splitting between chirality states with the Zeeman splitting of a single spin in Fig. 2 of their paper. They considered standard GaAs QD parameters (similar to our parameter choice), with g factor $g^* = -0.44$, electron effective mass $m_e^* = 0.067m_e$, dielectric constant $\epsilon = 12.4$, parabolic confinement at 60 meV, and a 40 nm distance between QDs. In their exact diagonalization calculation, the chirality state splitting is larger than the Zeeman splitting up to $B_z \approx 1.2$ T. With a suitable energy spacing between the two chirality states, we may tune the TQD potential in such a way that an electron may escape the system if it is in a high-energy chirality state. Thus, QPC charge measurement on the TQD help to determine which chirality state the system is in.

Experimentally, we would like to refer to extensive works^{44,51,52} performed to characterize the tunability as well as stability of TQD devices. More recently, Laird *et al.*^{53,54} also demonstrated experimentally the initialization, coherent exchange, and readout of a coded qubit based on a linear half-filled TQD by adopting a specific pulsing technique⁵⁵ that was first exploited to demonstrate coherent exchange and readout in a double dot system. For Laird’s experiments, the Jacobi coded qubit basis is used and the measurement of the qubit is done by measuring the total spin of the electrons in QD 2 and QD 3, respectively. As already shown in Secs. III and IV, the Jacobi states, $|L_{0(1)}\rangle$ have either a singlet or a triplet state across QD 2 and QD 3.

VI. CONCLUSION

In summary, we present a theory of quantum circuits based on qubits encoded in chirality of electron spin complexes in lateral gated semiconductor triple quantum dot molecules with one-electron spin in each dot. Using microscopic Hamiltonian (LCHO-CI formalism) and exact diagonalization techniques we show that there exists an optimized, in-plane B^* field that one should use to single out the computational space and show explicitly how to perform initialization of a coded qubit. We have also shown how to perform specific single-qubit operations around the x , y , and z axes as well as nontrivial double-qubits operations: CNOT and controlled phase gates. The understanding of coupling between

two coded qubits is significantly improved with the successful mapping of the microscopic Hamiltonian for six QDs to two effective spins. The measurement of chirality states in a TQD device is proposed using chirality-to-charge conversion technique, which has been proven to be very successful for electron spin readout in the spin-to-charge conversion scheme.

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