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Snow Creep Pressures: Effects of Structure Boundary Conditions and Snowpack Properties Compared with Field Data

by D.M. McClung and J.O. Larsen

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RÉSUMÉ

Les ouvrages se trouvant dans les manteaux nivaux épais sont soumis à des forces dues à l'interruption des éléments de déformation du manteau nival dans le sens de la pente. Les pressions de fluage qui en résultent constituent souvent le principal facteur à prendre en compte dans la conception. Les auteurs de ce document fournissent des données de terrain (pressions) précises et font une analyse théorique du problème à l'aide d'une loi du fluage linéaire visant à définir la déformation de la neige. Ils formulent des expressions analytiques représentant les pressions et montrent que la théorie linéaire résultante sous-estime d'environ 20 % les pressions moyennes. Pour en arriver à une plus grande précision, il faudra établir une loi de la déformation non linéaire.

SNOW CREEP PRESSURES: EFFECTS OF STRUCTURE BOUNDARY CONDITIONS AND SNOWPACK PROPERTIES COMPARED WITH FIELD DATA

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ABSTRACT

Structures placed in deep snowcovers are subject to forces caused by the interruption of the downslope snowpack deformation components. The resulting creep pressures are often the primary design consideration. In this paper, accurate field data (pressures) and theoretical analysis of the problem using a linear creep law to define snow deformation are presented. Analytical expressions for the pressures are presented and it is demonstrated that the resulting linear theory underestimates the mean pressures by about 20%. Higher accuracy will require a non-linear deformation law.

INTRODUCTION

When structures are erected in deep snowcovers, snow creep pressures are often the primary design consideration. Important examples include: avalanche defences in starting zones, ski lift and powerline towers. Although it is relatively easy to build structures which can withstand creep pressures, the cost penalty for structures which are much stronger than necessary is often prohibitive. High accuracy is also required because anchoring conditions in mountainous terrain are often very poor; even if a structure is built to withstand creep loads, expected creep pressures can dictate the position of structures. Conversely, it can be very expensive and potentially dangerous if structures standing in deep snowcovers fail. These considerations underline the

importance of accuracy in specification of creep loads for both cost and safety.

For a given snowpack, two elements control the distribution and magnitude of forces on structures: (a) the boundary conditions on the face of the structure and at positions where the snowpack contacts the ground; and (b) the rheology of the material. In this paper, the effects of boundary conditions are quantified and compared with two sets of field data from a plane-strain configuration. Calculations are given over the range of expected boundary conditions appropriate to available data. In addition, simple depth variations in snowpack density and stiffness are explored using linear rheology. Taken together, the data and calculations show that a complete definition of the design loads will require a non-linear creep deformation law to be formulated for alpine snow.

MODELLING CONCEPTS

Alpine snow has a unique combination of physical properties which have not yet been formalized in a non-linear deformation law suitable for engineering applications. The properties include: high porosity, high temperature (relative to its melting point), and very low strength (it is the weakest natural geotechnical material). These properties combine to produce slow deformation which can proceed even without an applied load. The high porosity results in continuous densification throughout the winter through irreversible deformation (chiefly

from grain rearrangement). This viscous (or plastic) deformation may be described as non-steady creep.

Since simple non-linear deformation laws are not available, a linear deformation law is applied in this paper. The linear formulation yields analytical expressions and it provides information on the importance and character of non-linear effects for the plane-strain configuration.

Our approach describes the deformation by generalizing linear visco-elastic behaviour from a one-dimensional four parameter Burger Fluid (Fig. 1). For a creep test (the load diagram is a Heaviside function; see Fig. 1), the creep compliance relation may be represented by a proportionality between strain, e , and load stress, σ_0 :

$$e(t) = \sigma_0 \left[\frac{t}{\eta_1} + \frac{1}{E_1} + \frac{1}{E_2} \left(1 - \exp(-E_2 t / \eta_2) \right) \right] \quad (1)$$

In Eq. 1, t is time, η_1 , η_2 are viscous moduli and E_1 , E_2 are elastic moduli. Expression 1 is considered flexible enough to account for deformation of alpine snow to a first approximation (Bader, 1962; Mellor, 1975).

For the model, $t \rightarrow 0$ describes initial elastic response and as t becomes large relative to $t_0 \equiv \eta_2 / E_2$ the loads are approximated in terms of viscous deformation. For the long time limit, the appropriate

state variable, to describe the deformation, is the strain-rate, $\dot{e}$, and Eq. 1 reduces to:

$$\sigma_0 = \eta_1 \dot{e} \quad (2)$$

when $t \gg t_0$.

Our model is an engineering formulation for describing the effects of interruption of slow, viscous creep by a rigid structure on a slope with a deep snowcover. Field data (McClung, 1975) show that transient creep rates in new snow layers persist for several days following a storm but these visco-elastic effects are ignored here.

Even if rapid, transient creep rates are not present, the creep loads on a structure will still be time dependent because alpine snow is continuously densifying (making steady state creep impossible). However, the effect is very slow for a deep snowcover which has been present for several months on a slope. For design purposes, the time of interest is late winter or early spring when snow depth is maximum and densification is slow. Therefore, with a linear model it is possible to treat time dependence by exploring variations in the moduli as time proceeds but with constant values at a given instant of time.

Generalization of the one-dimensional model to three dimensions (including both deviatoric and hydrostatic components) is well known. Lang and Nakamura (1984) provide a rigorous treatment. For long term response, Eq. 2 becomes:

$$\sigma_{ij} = 2\mu \dot{e}_{ij} + (\eta - \frac{2}{3}\mu) \dot{e}_{kk} \delta_{ij} \quad (3)$$

where μ , η are shear and bulk viscosity and σ_{ij} , $\dot{e}_{ij}$ are stress and strain-rate tensor components in rectangular Cartesian coordinates, $\dot{e}_{kk}$ is the sum of the diagonal components of the strain-rate tensor, and δ_{ij} is the Kronecker delta. Equation 3 is equivalent to a linear, compressible Newtonian viscous fluid with neglect of the static pressure term. The static term is unnecessary for describing alpine snow (Salm, 1967) because a state of rest is not possible.

Information about values of bulk viscosity is scarce, so it is natural to relate the shear and bulk viscosity by the viscous Poisson's ratio introduced by Reiner (1946, 1949):

$$\nu = \frac{3\eta - 2\mu}{2(3\eta + \mu)} \quad (4)$$

Fig. 1. Schematic of four parameter Burger fluid and Heaviside load diagram to represent snow deformation.

Values for ν are summarized by Salm (1977) and estimates of μ are given by Haefeli (1967). This pair of parameters can be used to describe linear creep deformation in general but for plane strain solutions ν will be the only parameter to appear.

Both μ and η may depend on snow density, temperature, structure and time for a linear theory. For a non-linear formulation they may also be taken to depend on invariants of the stress and strain rate tensor (a case excluded in this paper).

SNOW GLIDING

The boundary conditions at positions where the snowpack is in contact with the ground are crucial for determining structure forces when the ground is smooth and wet. Snow glide (slip of the entire snowpack over sloping ground) can initiate when the interface between the snowpack and the ground is at 0°C and for slope angles in excess of 15° . When vigorous gliding takes place, the highest forces on structures are produced and, therefore, no serious model would exclude this force component.

The fundamental problem of snow gliding is to relate the snowpack drag to the glide velocity. The present theory contains the assumption that glide occurs by creep over the ground roughness elements. When the interface is at 0°C , the presence of free water is guaranteed there. This condition implies that the internal velocity field is tangential to the interface when the snowpack contacts roughness elements and that there is no shear stress at those positions. At positions for which the snowpack is *not* in contact with the interface (e.g. small roughness elements drowned by water), the drag is negligible (McClung and Clarke, 1987). Assuming the deformation field is governed by Eq. 3, the tangential snowpack drag, τ_0 , is related to the glide velocity, U , by:

$$\tau_0 = \frac{\mu U}{2(1-\nu)} \frac{1}{D^A(x,y)} \equiv \frac{\mu U}{D^*} \quad (5)$$

where $D^A(x,y)$, the stagnation depth, is a function of the geometry of the interface (x and y are up and cross-slope directions) and A , the area for which the snowpack is *not* in contact with the bed. In practice, the stagnation depth must be measured (e.g. Mc-

Clung, 1975) or estimated theoretically. If $A=0$, Eq. 5 reduces to the theory of McClung (1981) for which a continuous, infinitesimal thin water film was assumed all along the interface. McClung and Clarke (1987) provide estimates of D^A for $A \neq 0$. If $A \rightarrow \infty$, $D^* \rightarrow \infty$ and $\tau_0 \rightarrow 0$ (an unstable condition).

If the interface is sufficiently rough or if it is below 0°C , $D^A=0$ and $U=0$. If $\nu \rightarrow \frac{1}{2}$, corresponding to incompressibility, Eq. 5 reduces to an analogous expression derived from the same assumptions, but using a linear, incompressible material initially instead of Eq. 3 (e.g. Kamb, 1970).

Pore pressure effects are expected to be small for alpine snow and we will ignore them. Transient (time dependent) effects implied by variations of supply and drainage of basal water are neglected in the theory but their effect on drag reduction is retained when $A \neq 0$.

PLANE STRAIN EQUATIONS

In this paper, we wish to compare field data with predictions at the centre of a long retaining wall (avalanche defence structure) perpendicular to the snow-earth interface on a long slope without curvature. This configuration is simple enough that one-dimensional solutions are available (McClung, 1982; McClung et al., 1984) to describe the average pressure on the face of the structure using Eqs. 3–5. For a given snowpack, the solutions depend only on ν and D^* (or D^A) (a mean value all along the interface is defined) and the slope angle ψ . The boundary conditions on the face of the structure also influence the solutions significantly.

One-dimensional equations (geometry in Fig. 2) are defined by depth averaged quantities (denoted by a bar):

$$\bar{\sigma}_x = \frac{1}{H} \int_0^H \sigma_{xx} dz \quad (6)$$

where H is snowpack depth measured perpendicular to the snow-earth interface.

For one-dimensional deformation parallel to the slope, Eq. 3 becomes:

$$\bar{\sigma}_x = \frac{2\mu}{1-\nu} \dot{\epsilon} + \frac{\nu}{1-\nu} \bar{\sigma}_z \quad (7)$$

Fig. 2. Schematic of plane strain configuration for measuring creep pressures.

where $\dot{\epsilon} \approx du/dx$ is the compressional strain rate, and u is the slope parallel creep velocity component.

Similarly, the equilibrium equation is:

$$H \frac{d\bar{\sigma}_x}{dx} = \tau_0(x) - \tau_g \quad (8)$$

with $\tau_g = \bar{\rho}gH \sin \psi$, ρ is density, g is acceleration due to gravity, τ_0 is snowpack drag at the interface (due to creep and glide).

With D^* and ν taken constant throughout the zone of influence of the structure, the solution of Eqs. 5, 7 and 8 at $x=0$ (the structure location; see McClung, 1982) is of the form:

$$\bar{\sigma}_x = \left[\frac{2}{1-\nu} \left(\frac{D^*}{H} + \frac{L}{H} \right) \right]^{\frac{1}{2}} \tau_g + \frac{\nu}{1-\nu} \bar{\sigma}_z(0) \quad (9)$$

In Eq. 9, $\bar{\sigma}_z(0)$ and the dimensionless parameter L/H depend on ν , ψ and the boundary conditions on the structure.

BOUNDARY CONDITIONS ON THE STRUCTURE

The boundary conditions on the face of structures buried in snowcovers are unknown. However, it is possible to place bounds on the possible boundary conditions. Regardless of the conditions on traction or displacement parallel to the structure, the creep velocity perpendicular to the structure may be taken as $u=0$ along the face.

For a rough structure in a cold snowpack, the vertical creep velocity may be approximated as $v=0$. The pair of boundary creep velocities ($u=v=0$) will be referred to as the no-slip condition. This condi-

tion is expected from results on snow gliding: glide is not observed on a rough surface unless free water (wet snow) is available. The no-slip condition implies a shear stress along the structure face and causes the maximum force to occur at an angle other than perpendicular to the structure.

At the other extreme, for a smooth structure lubricated by free water, a traction free condition, $\tau_{xz}=0$, along the structure is expected. This pair of conditions ($u=\tau_{xz}=0$) will be termed the traction-free condition. For the intermediate case (both slip and traction occur parallel to the structure), a relation analogous to Eq. 5 may be appropriate. This situation is not explored here explicitly; it should produce pressures intermediate between the no-slip and traction-free conditions which bound the problem.

PLANE STRAIN SOLUTIONS

To compare with field data, it is of interest to predict the forces perpendicular and parallel to the structure (shear forces). These are both represented in the maximum principal stress which is the highest force component on the structure.

Traction-free boundary condition

The traction-free boundary condition is the easier of the two extremes to model in one dimension. In this case $\bar{\tau}(0) = \bar{\tau}_{xz}(0) = 0$ all along the structure and since $u=U=0$ there, the condition is compatible with the glide relation 5. Also, two-dimensional finite-element solutions show that the normal forces are not appreciably changed from their values without the presence of the wall and therefore the depth-averaged normal stress is approximately:

$$\bar{\sigma}_z(0) \approx \frac{1}{2} \bar{\rho}gH \cos \psi \quad (10)$$

To complete the definition of $\bar{\sigma}_x(0)$ in Eq. 9, the dimensionless creep parameter L/H must be specified. It can be shown that $L/H \approx \frac{1}{3}$ if u varies parabolically with depth all along the zone of influence of the structure (termed the backpressure zone). Actually, L/H also depends on ν ; an extensive series of finite-element solutions shows that an em-

Fig. 3. Maximum principal stress as a function of ν and ψ for the traction-free boundary condition, $D^*=0$. (...) model; (O) finite-element calculations.

pirical relation for L/H with the traction-free boundary condition is:

$$\frac{L}{H} = 0.27 + \frac{\nu}{12} \quad (11)$$

With Eqs. 11, 9 and 10 the depth-averaged value of $\bar{\sigma}_x(0)$ (or the maximum principal stress, $\bar{\sigma}_1(0)$) is:

$$\frac{\bar{\sigma}_1}{\bar{\rho}gH} = \frac{\bar{\sigma}_x(0)}{\bar{\rho}gH} = \left[\left(\frac{2}{1-\nu} \right) \left(\frac{L}{H} + \frac{D^*}{H} \right) \right]^{\frac{1}{2}} \sin \psi + \frac{1}{2} \left(\frac{\nu}{1-\nu} \right) \cos \psi \quad (12)$$

The two terms in Eq. 12 result from gravity loads applied parallel and perpendicular to the slope respectively. The terms may be calculated separately by application of gravity loads (body forces) in these directions. Equation 12 was derived from solutions in the ranges $25^\circ \leq \psi \leq 55^\circ$, $0 \leq \nu \leq 0.4$ and $0 \leq D^*/H \leq 3$ (see McClung, 1982, for a discussion of these ranges). The solutions compare with two-dimensional finite-element results within a few per-

cent. Figure 3 gives examples for $D^*=0.0$. For the traction-free condition the resultant force is perpendicular to the face of the structure.

Equation 12 is similar in form to that given by Haefeli (1948). He called the first term the dynamic component of pressure and the second term the static component. The second term of Eq. 12 is identical to Haefeli's formulation but only for the restricted case of the traction free boundary condition (not specified by Haefeli). The first term of Eq. 12 differs substantially from Haefeli's expression.

No-slip boundary condition

Field data show that, in general, the resultant force is not perpendicular to the face of the structure (e.g. Kümmerli, 1958). This result is expected physically: if slip along the structure is inhibited, shear forces will be present there causing the resultant force to have a component perpendicular to the slope. In general, the face of a structure will not be

completely traction-free and we feel the no-slip boundary condition ($u=v=0$) is a closer approximation to conditions encountered in the field except when the snowpack is melting rapidly.

Numerical solutions show that the no-slip condition is more complex to model than the traction-free condition. Not only are shear forces produced on the structure, but the normal forces in the vicinity of the structure are significantly reduced from their values without a wall present; a simple estimate such as Eq. 10 is not available.

The shear stress on the structure may be estimated from Eq. 3:

$$\tau_{xz} = \mu \left(\frac{\partial u}{\partial z} + \frac{\partial v}{\partial x} \right) \quad (13)$$

Since $u=0$ all along the structure, Eq. 13 may be depth averaged to give the average shear stress on the structure:

$$\bar{\tau}(0) = \mu \left(\frac{\partial \bar{v}}{\partial x} \right)_{x=0} = \mu f(\nu, \psi) \quad (14)$$

There is no simple method to evaluate Eq. 14 and it is not compatible with the glide boundary condition.

Similar to Eq. 8, the depth-averaged equilibrium equation in the z -direction yields a value for the average vertical stress (after some algebra):

$$\bar{\sigma}_z(0) = \frac{1}{2} \bar{\rho} g H \cos \psi - \frac{1}{2} H \frac{d\bar{\tau}}{dx} \quad (15)$$

Evaluation of Eq. 15 at the structure gives:

$$\bar{\sigma}_z(0) = \frac{1}{2} \bar{\rho} g H \cos \psi - \frac{1}{2} \mu H \left(\frac{d^2 \bar{v}}{dx^2} \right)_{x=0} \quad (16)$$

In common with Eq. 14, there is no simple method to evaluate Eq. 16 from a one-dimensional model. Similar to Eq. 7, an expression for the vertical stress on the plane perpendicular to the face of the structure can be derived (using $v \equiv 0$ at $x=0$) from Eq. 3:

$$\bar{\sigma}_z(0) = \frac{\nu}{1-\nu} \bar{\sigma}_x(0) = \frac{2\mu\nu}{1-2\nu} \dot{e}(0) \quad (17)$$

Substituting Eq. 16 into Eq. 7 gives a form for $\bar{\sigma}_x(0)$:

$$\bar{\sigma}_x(0) = \frac{2\mu}{1-\nu} \dot{e}(0) - \frac{1}{2} \left(\frac{\nu}{1-\nu} \right) f_1(\nu, \psi) + \frac{1}{2} \left(\frac{\nu}{1-\nu} \right) \bar{\rho} g H \cos \psi \quad (18)$$

We have evaluated Eq. 18 numerically using finite-element solutions to get an approximate empirical expression for $\bar{\sigma}_x(0)$. If the form of Eq. 12 is retained, L/H is given by:

$$\frac{L}{H} \simeq \frac{1}{4} (\sin \psi)^{\frac{1}{2}} + \frac{1}{4} \left(\frac{\nu^2}{1-\nu} \right) \quad (19)$$

to complete the definition of $\bar{\sigma}_x(0)$ in Eq. 12.

An approximate empirical expression for $\bar{\tau}(0)$ from numerical evaluations derived similar to Eq. 19 is:

$$\frac{\bar{\tau}(0)}{\bar{\rho} g H} \simeq \frac{1}{4} (\cos \psi)^{\frac{1}{2}} \left[1 - \frac{3}{2} \left(\frac{\nu}{1-\nu} \right) (\sin \psi)^{\frac{1}{2}} \right] \quad (20)$$

Equations 12, 19, 17 and 20 complete the definitions of stresses at the structure for no-slip boundary condition. The maximum compressive (principal) stress can be estimated by standard methods:

$$\bar{\sigma}_1(0) = \frac{\bar{\sigma}_x(0) + \bar{\sigma}_z(0)}{2} + \left[\left(\frac{\bar{\sigma}_x(0) - \bar{\sigma}_z(0)}{2} \right)^2 + \bar{\tau}(0)^2 \right]^{\frac{1}{2}} \quad (21)$$

A related quantity of interest is direction of the maximum compressive stress. From standard theory, the (downward) angle that the maximum principal stress deviates from the x -axis is:

$$\beta = \tan^{-1} \left(\frac{2\bar{\tau}(0)}{\bar{\sigma}_x(0) - \bar{\sigma}_z(0)} \right) \quad (22)$$

Equations 21 and 22 may prove useful in design applications. Assuming $\psi = 37^\circ$, and Bader's data ($\nu \approx 0$ for low density snow; $\nu = 0.25$ for dense snow) we get $\beta = 25.5^\circ$ and 20.5° . Therefore, the maximum compressive force direction is estimated to vary from 0° (perfectly smooth or wetted structure) to about 25° (rough structure) downward from the x -axis (for $\psi = 37^\circ$).

Equations 21 and 22 should be used cautiously: they are not as accurate as Eqs. 11 and 12 for the traction-free boundary condition and therefore fi-

Fig. 4. Maximum principal stress as a function of D^* for $\psi=45^\circ$, $\nu=0.25$. (....) model; (O) finite-element calculations. The no-slip boundary condition is assumed.

nite element or other numerical procedures are preferable. Errors of up to 15% may be expected in calculations of individual estimates of stress components such as $\bar{\tau}(0)$. Errors for $\bar{\sigma}_1(0)$, $\bar{\sigma}_x(0)$, and $\bar{\sigma}_z(0)$ are generally less than 10%. Figures 4 and 5 compare the results of Eq. 21 with two dimensional numerical solutions. Figure 6 compares the predictions of Eqs. 12 and 21 and the estimates of $\bar{\sigma}_x(0)$ for the extremes in boundary conditions. This comparison shows $\bar{\sigma}_x(0)$ is higher for the traction-free boundary condition with a greater deviation at low slope angles and low values (the difference is about 25% for $\psi=25^\circ$).

When a structure is erected perpendicular to the slope (see Fig. 2), the stress components of interest are $\bar{\sigma}_x(0)$ and $\bar{\tau}(0)$. The magnitude of the resultant force (per unit area) is:

$$\sigma_R = [\bar{\sigma}_x(0)^2 + \bar{\tau}(0)^2]^{1/2} \quad (23)$$

and the direction may be defined (e.g. Haefeli, 1948):

$$\tan \epsilon = \bar{\tau}(0)/\bar{\sigma}_x(0) \quad (24)$$

The ratio in Eq. 24 has been estimated in the field (Kümmerli, 1958). Salm (1977) gives a range for $\tan \epsilon$ from field measurements ($\psi=37^\circ$; $D^*=0$): $\tan \epsilon \approx 0.7$ for low density snow and $\tan \epsilon \approx 0.3$ for high density snow. From Bader's data (Salm, 1977), with $\nu=0.0$ (low density snow) and $\nu=0.25$ (high density snow) calculations with Eqs. 12, 19 and 20 give $\tan \epsilon=0.61$ and 0.29 respectively for our model; finite-element calculations give $\tan \epsilon=0.77$ ($\nu=0.0$) and $\tan \epsilon=0.36$ ($\nu=0.25$). The agreement is surprisingly good for the constant viscosity, constant (depth averaged) density case.

PROPERTIES OF FIELD DATA FROM WESTERN NORWAY

The previous section provides simple estimates for pressures with snow deformation taken as a linear viscous material. In this section, field data are introduced to illustrate how pressures depend on snowpack properties. Our data consist of maximum, σ_m , and average pressures, σ_R , estimated at

Fig. 5. Maximum principal stress as a function of ν for $\psi=45^\circ$, the no-slip boundary condition and $D^*=0$. (...) model; (○) finite-element calculations.

Fig. 6. Comparison of $\bar{\sigma}_x(0)$ as a function of ν and ψ for the analytical model. (·○·) traction-free boundary condition; (·□·) no-slip boundary condition. It is assumed $D^*=0$ in all calculations.

the centre of a 15 m long avalanche defence structure erected perpendicular to the slope. The incline at the site is nearly constant (25°) for a long distance upslope from the structure and the ground surface is fairly smooth rock behind the structure. Previously (Larsen et al., 1985) the field measurements were described in detail. Supporting field measurements include a set of indexes for snowpack properties: H (average snowdepth), $\bar{T}$ (average snowpack temperature) and $\bar{S}$ (average snowpack stiffness). The parameter $\bar{S}$ is defined as the total energy (joules) to drive a ram penetrometer vertically through the snowpack. The values of σ_m and σ_R were estimated from strains measured in the steel supporting beams of the structure (McClung et al., 1984).

In the 1985 paper, a regression analysis was presented to describe σ_m and σ_R in terms of measured field parameters. We now present a similar analysis with an expanded data set (four more winters of data).

For σ_R the best relation found is:

$$\begin{aligned} \sigma_R &= 0.71 \bar{\rho} g H - 0.79 \text{ kPa} \\ r^2 &= 0.70, S_e = 1.61, N = 53 \end{aligned} \quad (25)$$

The descriptors are: r is correlation coefficient, S_e is standard error, and N is number of data points.

If $\bar{\rho} g H$ is excluded as a variable, the best relation is:

$$\begin{aligned} \ln \sigma_R &= 0.44 \ln \bar{S} + 0.077 \bar{T} - 19.5 \\ r^2 &= 0.49, S_e = 0.34, N = 47 \end{aligned} \quad (26)$$

In Eq. 26, the t statistic is 6.5 for $\ln \bar{S}$ and 2.5 for $\bar{T}$. With $\bar{\rho} g H$ in the analysis, no significant improvement was gained by addition of $\bar{T}$, $\ln \bar{T}$, $\bar{S}$ or $\ln \bar{S}$ in a multiple-regression analysis; the resulting equations were found to be overfitted.

For σ_m , the best expression is a power law relation with respect to $\bar{\rho} g H$:

$$\begin{aligned} \ln \sigma_m &= 0.88 \ln \bar{\rho} g H + 0.26 \\ r^2 &= 0.62, S_e = 0.21, N = 45 \end{aligned} \quad (27)$$

Without $\bar{\rho} g H$ the best relationship is:

$$\begin{aligned} \ln \sigma_m &= 0.31 \ln \bar{S} + 0.056 \bar{T} - 13.2 \\ r^2 &= 0.43, S_e = 0.26, N = 39 \end{aligned} \quad (28)$$

For Eq. 28, the t statistic is 2.3 for $\bar{T}$ and 5.1 for $\ln \bar{S}$. With $\bar{\rho} g H$ in the analysis, estimates for $\bar{\rho} g H$ are not improved with a multiple-regression analysis using the other variables.

The best relationship among the snowpack indexes was found to be:

$$\begin{aligned} \ln (\bar{\rho} g H) &= 0.32 \ln \bar{S} + 0.074 \bar{T} - 18.0 \\ r^2 &= 0.64, S_e = 0.19, N = 46 \end{aligned} \quad (29)$$

In Eq. 29, $\ln \bar{S}$ has a t statistic 8.6 and $\bar{T}$ has a t statistic 4.2. This relationship indicates that the effects of temperature and stiffness are largely contained in $\bar{\rho} g H$.

The best relation between σ_m and σ_R is a linear one:

$$\begin{aligned} \sigma_m &= 1.5 \sigma_R - 0.60 \text{ kPa} \\ r^2 &= 0.91, S_e = 1.36, N = 55 \end{aligned} \quad (30)$$

Expression 30 shows that $\sigma_m \approx 1.5 \sigma_R$ to a good approximation. We will subsequently show that this important feature of the data cannot be explained with a constant stiffness, depth-averaged model. It may be noted in Eq. 30 that the constant is not very significant: the t statistic for σ_R is 22.5 and for the constant it is -1 . Without the constant a regression analysis gives $\sigma_m = 1.5 \sigma_R$. Similar comments apply to Eqs. 23 and 25.

COMPARISON WITH FIELD DATA

Two sets of field data exist for the plane-strain problem but they differ in snowpack properties and measurement techniques. Our data set is taken from a relatively low altitude, high latitude site in western Norway. The other data set was taken from a 37° slope in Switzerland (Kümmerli, 1958). For both data sets, the glide component is negligible ($U = D^* = 0$).

Due to climate variations, it is possible that the snowpacks for the two data sets differ. We suspect that one principal difference may be with respect to snowpack stiffness. Our data are subject to unusually strong windpacking effects; this can create a stiffer snowpack. The data from Switzerland (lower latitude, higher altitude) may be influenced more by the effect of solar radiation exposure but we are

unsure of this. The lower slope angle for our location (25°) should introduce a greater settlement component into the snow deformation promoting higher densification and increased stiffness.

Both data sets contain forces in addition to those perpendicular to the structure. In particular, the data from Switzerland show clearly that the resultant force is not perpendicular to the structure. This indicates the presence of shear forces and very little slip on the face of the structure. The condition is expected physically and therefore we believe the no-slip boundary condition will be the more realistic of the two extremes. Since shear forces appear in the data it is more realistic to compare the measurements with $\bar{\sigma}_R$ than with $\bar{\sigma}_x$ in general.

The data from Switzerland are taken by measuring spring displacements to infer forces perpendicular and parallel to a structure placed perpendicular to the ground. Therefore, the magnitude of the resultant is the proper quantity to compare with. The data from Norway were determined largely by axial forces and the structure is slightly inclined down-slope from the perpendicular but we feel σ_R in Eq. 23 is the best quantity to compare with. For the traction-free boundary condition, $\bar{\sigma}_I = \bar{\sigma}_x$ from Eq. 12 is the analogous quantity.

Consider first the average pressure, σ_R , on the face of the structure. For the no-slip boundary condition, linear deformation and the other conditions defined in Fig. 2, the ratio $\sigma_R/\bar{\rho}gH$ should only depend on ν and ψ . From data reviewed by Salm (1977), we regard the *extreme* range of ν to be 0–0.4 for the depth-averaged density range of the data sets (200 kg m^{-3} to 600 kg m^{-3}).

Finite-element calculations show that the mean value of our data ($\sigma_R/\bar{\rho}gH=0.64$; $\psi=25^\circ$) implies $\nu \approx 0.4$ and range 0–0.45 (no-slip boundary condition). For the Swiss data (37° slope), the mean value ($\sigma_R/\bar{\rho}gH=0.58$) implies a mean value $\nu \approx 0.2$ for the data set, which is a very reasonable value (according to Bader's data) for the density range of the experiments. However, while the implied mean value for ν seems appropriate, many of the data imply negative values for ν ; this seems unlikely according to the Poisson ratio data reviewed by Salm.

Figures 7 ($\psi=25^\circ$) and 8 ($\psi=37^\circ$) show comparisons of σ_R (in kPa) from the data with finite-element conditions for constant density and viscos-

ity for the extreme (expected) range of ν (0–0.4). From Figs. 7 and 8, it can be seen that the scatterband of the data sets is wider than the band of predictions for this model. As a percentage of the mean value, the standard deviations of $\sigma_R/\bar{\rho}gH$ for the data sets are nearly identical [21% ($\psi=25^\circ$); 23% ($\psi=37^\circ$)]. An important limitation of the model appears: the prediction range is smaller than the data scatterbands.

Another result (Fig. 7) is that the constant density–stiffness model underestimates the mean of our pressure data by at least 20% assuming values for ν near the middle of Bader's range. This result does not change very much even if we consider vertical stresses $\bar{\sigma}_z$ and compare our data with $\bar{\sigma}_I$ rather than $\bar{\sigma}_R$. From Fig. 8, it appears that the constant density–stiffness model overestimates some of the pressure data on the 37° slope (unless negative values for ν are admitted).

Numerical calculations were performed to explore depth-dependent density variations. Assuming a linear increase in density with depth, these results showed almost no effect on average pressure, or the maximum pressure for either the no-slip or traction-free boundary condition. In combination with previous comparisons, these results imply that variations in snowpack stiffness will be required for a complete explanation of the snowpressure data.

Linear depth variations in snowpack stiffness may be analyzed with viscosity data summarized by Haefeli (1967). He showed that the shear viscosity of snow varies by approximately two orders of magnitude for densities from 300 – 500 kg m^{-3} . In this density range, Haefeli gives values for μ from 10^{10} – $10^{12} \text{ kg (m sec)}^{-1}$. Figures 9 and 10 compare numerical calculations with our data using these values and ν constant in the range (0–0.4); constant ν implies shear viscosity and bulk viscosity increase with the same depth dependence.

With depth dependent stiffness, numerical calculations show that snowpressure is nearly independent of the magnitude (mean value) of the viscosity used but it is sensitive to the form of the depth dependence chosen. With linear variation in shear stiffness (ν taken constant) the pressures are nearly independent of the magnitude of the stiffness gradient taken.

Fig. 7. Resultant pressure (kPa) as a function of $\bar{\rho}gH$ (kPa) for data from Norway. (○) experimental data. Finite-element calculations are shown for the extreme range of ν and the extremes in boundary conditions. $\psi=25^\circ$, $D^*=0$. (---) traction free; (—) no slip.

Fig. 8. Similar to Fig. 7 except the data are from Switzerland. $\psi=37^\circ$; $D^*=0$.

Fig. 9. Resultant pressure data (kPa) from Norway as a function of $\bar{\rho}gH$ (kPa). In this example both stiffness and density are taken to increase linearly with depth, otherwise the assumptions are the same as in Fig. 7. (---) traction free; (—) no slip.

Fig. 10. Similar to Fig. 9 except the data are from Switzerland.

Fig. 11. Maximum pressure versus resultant pressure (both in kPa) for data (○) from Norway. Finite-element calculations for no-slip boundary condition are given for: (—) constant density-viscosity model; and (---) density and viscosity linear with depth (both take $D^*=0$ and $\psi=25^\circ$).

For linear increase in stiffness and density with depth, the values $\sigma_m/\bar{\rho}gH$ and $\sigma_R/\bar{\rho}gH$ decrease substantially from their values when stiffness and density are taken constant. For $\psi=25^\circ$, $\sigma_R/\bar{\rho}gH$ increases from 0.29 to 0.53 for linear variations and no slip as ν increases from 0 to 0.4. The corresponding values for μ and ρ constant are 0.40 and 0.68. These results may be compared with a mean value of the ratio (0.64) for our data. For $\psi=37^\circ$, linear depth dependence of stiffness and density gives $\sigma_R/\bar{\rho}gH$ in the range 0.37–0.64 for the no slip boundary condition. For constant density and stiffness the corresponding values are 0.49 to 0.84. The mean value for the measured ratio from this slope angle is 0.58.

From Figs. 9 and 10, it is clear that the depth dependent density-stiffness model underestimates many of the data for either set. Also, similar to the constant density-viscosity model, the spread of predictions is smaller than the data scatterband for either data set. We conclude neither model pro-

vides a satisfactory description of the data.

Our data contain information about the maximum pressure on the structure. Figure 11 shows σ_m versus σ_R (both in kPa) and comparison of predictions of the two models from finite-element calculations. From Eq. 29, the mean of the data implies $\sigma_m/\sigma_R = 1.5$ to a good approximation. For the constant density-viscosity model, the calculated ratio declines from 1.39 to 1.18 as ν increases from 0 to 0.40 (no-slip boundary condition). The ratio decreases from 1.54 to 1.46 for the same range of ν for the linear depth-dependent density-stiffness model thus providing an excellent fit to the mean of data. If maximum compressive stress $\sigma_1(0)$ is used in the ratio $\sigma_m/\sigma_1(0)$, the no-slip boundary condition gives 1.47 nearly independent of ν assuming linear depth-density and stiffness variations as above. For the traction-free boundary condition (constant density and stiffness) the ratio declines from 1.43 to 1.11 as ν increases from 0 to 0.4 (not shown in Fig. 11).

SUMMARY AND DISCUSSION

Our reformulation of the constant viscosity–constant density one-dimensional treatment of the plane strain snowpressure problem is of both historical and practical interest. The solution represents the analytical model first sought by Haefeli (Bader et al., 1939) in his doctoral thesis. Also, our analytical model is of practical interest since it allows the average pressures to be roughly estimated using a hand calculator. It appears that this simple model underestimates our measured field data by about 20%.

The model does not account for initial stresses. Instead, the long term loads on the structure are defined in terms of viscous stresses and strain rates. These assumptions have a long history in snow mechanics (Bader, 1962; Mellor, 1975). We feel that our new model represents a more accurate representation of the linear problem than our previous attempts which included initial stresses.

Comparison of field data with the two density–stiffness models shows that neither provides a comprehensive explanation of the data. It is surprising that the mean value of $\sigma_R/\bar{\rho}gH$ is higher for the 25° slope (0.64) than it is for the 37° slope (0.58). Linear regression of the Swiss data gives $\sigma_R = 0.69 \bar{\rho}gH - 1.05$ (kPa) which is nearly identical to Eq. 25 for the data on the 25° slope. For the same snow properties, we believe that the mean value of pressure should increase with slope angle if snow is modelled as a linear material. Therefore, either the snow properties are significantly different for the two sites or the deformation conditions affect stiffness variations greatly with slope angle or both. The site in Norway has a very stiff mid-winter snowpack due to windpacking and a minimum effect from solar radiation. Also, the lower slope angle at the site should produce a greater settlement component to increase stiffness. These effects could combine to give pressures as high as those in Switzerland even though the slope angle is higher for the latter by 12°. Our analysis of the ratio σ_m/σ_R further highlights the importance of stiffness variations: we were unable to explain the ratio until depth-dependent stiffness variations were introduced.

Since density variations alone do not provide a consistent match to field data, we believe that variations in stiffness (non-linear viscous relations)

may be the key. Our attempt to vary stiffness (linear increase with depth) is the simplest consistent with snow deformation properties in the field. For well settled snow, field-creep experiments data show the shear strain rate is approximately independent of depth (Mellor, 1975; McClung, 1975) and this implies the shear stiffness increases with the same depth dependence as the shear stress (on open slopes). It is of further interest that creep experiments for well settled snow indicate effective values for ν in the range 0.23–0.38 (McClung, 1975) for final densities (500–550 kg m⁻³) and these values are in excess of Bader's estimates from laboratory experiments. If lower values for the effective ratio of shear to bulk viscosity (higher effective ν) are appropriate in the field (as opposed to the laboratory) our conclusions may be altered when stiffness variations are considered. Salm (1977) provides some motivation for the belief that effective values for ν may be higher when non-Newtonian behaviour takes place. Numerical calculations of $\tan \epsilon$ in Eq. 24 show that $\tan \epsilon$ increases when density and stiffness increase linearly with depth. Therefore, higher effective values for ν are also implied to match the field data of Kümmerli (1958) if stiffness varies linearly with depth.

Until a proper non-linear deformation law is formulated (and applied), the importance of stiffness variations will not be defined. Based on our data, we recommend a safety factor of at least 25% over predictions of the analytical model (constant stiffness) when traditional values for ν (e.g. Bader et al., 1951) are applied. For maximum pressure, we believe the values should be 50% higher than predicted average pressures.

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