NRC Publications Archive Archives des publications du CNRC

Analog simulations of earthquake motions

Ward, H. S.

This publication could be one of several versions: author's original, accepted manuscript or the publisher's version. /
La version de cette publication peut être l'une des suivantes : la version prépublication de l'auteur, la version
acceptée du manuscrit ou la version de l'éditeur.

Publisher's version / Version de l'éditeur:

*Journal of the Engineering Mechanics Division, American Society of Civil
Engineers, 91, EM5, pp. 173-190, 1966-09-01*

NRC Publications Record / Notice d'Archives des publications de CNRC:

<https://nrc-publications.canada.ca/eng/view/object/?id=a76601da-d905-47e7-9d94-04826083c5a2>;
<https://publications-cnrc.canada.ca/fra/voir/objet/?id=a76601da-d905-47e7-9d94-04826083c5a2>

Access and use of this website and the material on it are subject to the Terms and Conditions set forth at

<https://nrc-publications.canada.ca/eng/copyright>

READ THESE TERMS AND CONDITIONS CAREFULLY BEFORE USING THIS WEBSITE.

L'accès à ce site Web et l'utilisation de son contenu sont assujettis aux conditions présentées dans le site

<https://publications-cnrc.canada.ca/fra/droits>

LISEZ CES CONDITIONS ATTENTIVEMENT AVANT D'UTILISER CE SITE WEB.

Questions? Contact the NRC Publications Archive team at

PublicationsArchive-ArchivesPublications@nrc-cnrc.gc.ca. If you wish to email the authors directly, please see the
first page of the publication for their contact information.

Vous avez des questions? Nous pouvons vous aider. Pour communiquer directement avec un auteur, consultez la
première page de la revue dans laquelle son article a été publié afin de trouver ses coordonnées. Si vous n'arrivez
pas à les repérer, communiquez avec nous à PublicationsArchive-ArchivesPublications@nrc-cnrc.gc.ca.

National Research
Council Canada

Conseil national de
recherches Canada

Canada

Ser
TH1
N21r2
no. 291
c. 2

BLDG

17389

ANALYZED

NATIONAL RESEARCH COUNCIL
CANADA

CONSEIL NATIONAL DE RECHERCHES

31655

ANALOG SIMULATIONS OF EARTHQUAKE MOTIONS

(WITH DISCUSSION AND AUTHOR'S CLOSURE)

BY

H. S. WARD

REPRINTED FROM
JOURNAL OF THE ENGINEERING MECHANICS DIVISION
PROCEEDINGS OF THE AMERICAN SOCIETY OF CIVIL ENGINEERS
VOL. 91, NO. EM5, OCTOBER 1965,
P. 173-190

RESEARCH PAPER NO. 291
OF THE
DIVISION OF BUILDING RESEARCH

OTTAWA

PRICE 25 CENTS

SEPTEMBER 1966

NRC 9228

SIMULATIONS A L'AIDE D'ORDINATEURS DES
MOUVEMENTS DU SOL CAUSES PAR LES
TREMBLEMENTS DE TERRE

SOMMAIRE

L'auteur présente deux méthodes de simulation par ordinateurs des mouvements du sol causés par les tremblements de terre. L'une de ces méthodes utilise une partie d'une courbe gaussienne de probabilité avec un diagramme de réponse donné. La seconde méthode emploie un ordinateur hybride permettant la simulation de n'importe quel genre de mouvement du sol. Les déplacements dûs aux tremblements de terre simulés agissaient sur des systèmes linéaires à un seul degré de liberté, dont on mesurait les réponses. L'effort de cisaillement basal a été utilisé comme mesure de la réponse, et l'auteur établit des formules empiriques permettant de calculer l'effort de cisaillement en fonction de la masse et de la période du système, et de l'importance de la force d'accélération transmise au sol.

Journal of the
ENGINEERING MECHANICS DIVISION
Proceedings of the American Society of Civil Engineers

ANALOG SIMULATIONS OF EARTHQUAKE MOTIONS

By Howard S. Ward¹

INTRODUCTION

A major problem facing engineers in earthquake zones is that of determining the response of structures to the dynamic action of earthquakes. The present (1965) state of earthquake engineering is reflected in the building codes of countries in earthquake zones. A survey shows that many of them incorporate a feature that has been influenced by G. W. Housner's² study of earthquake spectra, i.e., a consideration of the response of a structure, considered to be a single-degree-of-freedom system, to an assumed earthquake.

Housner measured the response to actual strong-motion earthquakes of single-degree-of-freedom systems with and without viscous damping. His well-known earthquake response spectra were thus developed, having been determined by two parameters: (1) The frequency content of the earthquake disturbance; and (2) the degree of damping of the system.

Owing to the limited number of actual earthquakes available for structural design studies, methods of simulating earthquakes are of interest to structural engineers. Housner and P. C. Jennings³ recently introduced a method of producing a digital simulation of pseudo-earthquakes. They produced eight simulated earthquakes by using sections of a stationary, gaussian random function with a given power spectrum. These simulated earthquakes provide a source of excitation for structures simulated on a digital computer.

Note.—Discussion open until March 1, 1966. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Engineering Mechanics Division, Proceedings of the American Society of Civil Engineers, Vol. 91, No. EM5, October, 1965.

¹ Research Officer, Bldg. Physics Sect., Div. of Bldg. Research, Natl. Research Council, Ottawa, Canada.

² Housner, G. W., "Limit Design of Structures to Resist Earthquakes," Proceedings, 1st World Conf. on Earthquake Engrg., San Francisco, Calif., June, 1956.

³ Housner, G. W., and Jennings, P. C., "Generation of Artificial Earthquakes," Journal of the Engineering Mechanics Division, ASCE, Vol. 90, No. EM1, Proc. Paper 3806, February, 1964, pp. 113-150.

It is the main purpose herein to consider alternative forcing functions that might be considered equivalent to real earthquakes and thus provide an unlimited assortment of forcing functions for analog studies of structures. Three types of forcing function were studied, each offering a simulation of some of the characteristics of earthquakes. Some previous work has been done along this line by G. N. Bycroft.⁴

One of the forcing functions, filtered white noise, gave velocity response results similar to the average response results obtained by Housner. It is suggested that this method of earthquake simulation can be used to predict in statistical terms the probable response of a structure to an earthquake. The results obtained from two of the forcing functions can be used to calculate the base shear of the structural systems as a function of their fundamental period and damping characteristics. Empirical equations are developed for the calculation of base shear as a function of the peak ground acceleration and the fundamental period of free vibration, T .

Notation.—The symbols adopted for use in this paper are defined where they first appear and they are listed alphabetically in the Appendix.

ANALOG SIMULATION OF A ONE-DEGREE-OF-FREEDOM SYSTEM

The equation of motion for a single-degree-of-freedom system, the base of which is excited by a ground acceleration, $\ddot{x}_g$, is,

$$m \ddot{x} = -k x - m \ddot{x}_g \quad \text{— Damping Force} \quad (1)$$

in which m = the mass of the system, x = the displacement of the mass relative to its foundation, and k = the stiffness of the system.

Eq. 1 can be put in the form,

$$\ddot{x} = \frac{-k x}{m} - \ddot{x}_g \quad \text{— Damping Acceleration} \quad (2)$$

which can be simulated with an electronic analog computer. The mechanical system and the basic electronic circuit of its analog are shown in Fig. 1. Eqs. 1 and 2 are based on the assumption that the structure rests on a rigid foundation, i.e., that the base of the structure moves as a whole with acceleration $\ddot{x}_g$.

The lateral force, which is exactly equal to the base shear for a single-degree-of-freedom system, is

$$F = k x \quad (3)$$

EXPERIMENTAL PROCEDURE

One of the parameters most frequently used in describing random vibrations is power spectral density, which may be defined as the curve representing the power of the random vibration at different frequencies. A power spectral density analyzer was used to determine the power spectra of the

⁴ Bycroft, G. N., "White Noise Representation of Earthquakes," *Journal of the Engineering Mechanics Division*, ASCE, Vol. 86, No. EM2, Proc. Paper 2434, April, 1960, pp. 1-16.

three inputs investigated. The system incorporates a frequency-modulated tape recorder, an analyzer capable of dealing with frequencies from 3 cps to 10,000 cps, and a chart recorder. The analyzer section consists of a system of filters, the center frequency of which automatically changes at a pre-determined rate that depends on the length of the input signal. The bandwidth of the filters can also be set at a value dependent on the nature of the signal (a bandwidth of $2/3$ cps was used during most of the experimental work).

(a) MECHANICAL SYSTEM

(b) BASIC ANALOG CIRCUIT

FIG. 1.—THE ANALOG

The signal on the tape is passed into the analyzer, the filter of which will have a maximum response at a particular frequency, f_n , at a given instant of time. The bandwidth, Δf , of the filter determines the frequencies at which the output is down by 3 decibels (db). The output from the filter is now squared and the average result plotted on the chart recorder. The specifications of the filters in the analyzer limit the frequency range of analysis from 3 cps to 10,000 cps. This limitation was removed by the simple expedient of increasing the speed of playback of the tape over the speed of the original recording. A number of 25-sec bursts from a band-limited (0-35 cps) white noise generator were analyzed. This seems to be the approximate duration of strong-

motion earthquakes. A typical power spectrum of the output is shown in Fig. 2, in which the ordinate is given in arbitrary units.

The response of systems with zero damping to bursts of white noise was investigated; results for the average maximum values of the velocity response are shown in Fig. 3. The three sets of results are related to the three different levels of the input, $\ddot{x}_g$, which are shown in Fig. 3. Although there is a certain quantity of scatter, the most reasonable curve representing the velocity response appears to be a horizontal line. The scatter is explained by the fact that only a finite number of samples were taken.

TABLE 1.—STATISTICAL PROPERTIES FOR THE VELOCITY RESPONSE OF UNDAMPED SINGLE-DEGREE-OF-FREEDOM SYSTEMS SUBJECTED TO WHITE NOISE EXCITATION

Average maximum ground acceleration, in feet per second ²	Average value of maximum velocity response, in feet per second	Standard deviation of velocity response, in feet per second
56.8	8.40	1.245
27.7	4.55	1.104
17.0	2.60	0.956

FIG. 2.—TYPICAL POWER SPECTRUM

If it is assumed that the distribution of the average maximum velocity response for a burst of white noise of 25-sec duration is a random variable with a normal distribution, then certain statistical calculations can be made. The average standard deviation and the mean value of the average maximum velocity, obtained from Fig. 3, are drawn up in Table 1. If the average value is now considered to be the value obtained from an experiment consisting of n trials, it is possible to establish a region within which there is a probability that the true mean will exist. If the errors associated with the true mean, u_0 , (which is greater than or less than the measured mean, $\bar{x}$) are equal, then the acceptance region for a given level of significance is,

$$-K_{\alpha/2} \leq U \leq K_{\alpha/2} \dots \dots \dots (4a)$$

FIG. 3.—VELOCITY RESPONSE OF UNDAMPED SINGLE-DEGREE-OF-FREEDOM SYSTEMS

in which $K_{\alpha/2}$ = the normal deviate, corresponding to $\alpha/2$, of the normal distribution, and

$$U = \frac{(\bar{x} - \mu_0) \sqrt{n}}{\sigma} \dots \dots \dots (4b)$$

is the test statistic.

The values shown in Table 1 have been used with $n = 10$ to calculate the regions where the probability is 0.95 that the true mean will exist. The value of $n = 10$ was chosen because this was generally the number of tests used to calculate an average value. It is also assumed that the sample standard deviation equals that of the population.

If the velocity response, $\dot{x}$, is independent of the natural frequency of the system, and if the system vibrates in its fundamental mode as a result of white noise excitation, then it follows that the displacement response, x , will be inversely proportional to the frequency. This follows, because under the above conditions,

$$\dot{x}_{\max} = 2 \pi f x_{\max} = \text{constant} \dots \dots \dots (5)$$

in which f = the frequency, and the constant depends on the level of the input acceleration. The fact that the vibrations do occur at the fundamental

TABLE 2.—RESPONSE OF UNDAMPED SYSTEMS IN TERMS OF GROUND ACCELERATION PARAMETERS

Average peak ground acceleration in feet per second ²	56.8	27.7	17.0
Velocity response			
peak acceleration, in seconds	0.1478	0.1661	0.1529
Velocity response			
rms of ground acceleration, in seconds	0.4939	0.5382	0.4576

frequency is demonstrated by the results shown in Fig. 4. These represent the displacements of five single-degree-of-freedom systems that were subjected to the same 25-sec burst of white noise. The natural periods of the systems shown range from 3.0 sec to 1.0 sec, and they were subjected to 2% critical viscous damping, with a maximum acceleration of 17.0 ft per sec. The critical viscous damping acceleration for a system with a natural frequency of f is equal to $4 \pi f \dot{x}$. Viscous damping is then defined as a ratio of the critical damping; thus β is the fraction of critical damping.

Although the systems are linear it is not obvious to which property of the ground motion the velocity is directly proportional. The average results shown in Table 1 have been used to calculate the ratios of velocity to the peak ground acceleration and to the rms of ground acceleration. The values in Table 2 are not exactly the same for the different levels of the input, but

the discrepancy is negligible from a statistical viewpoint. It appears that the velocity is proportional to both the peak ground acceleration and the rms of the ground acceleration.

FIG. 4.—RESPONSE OF SINGLE-DEGREE-OF-FREEDOM SYSTEMS TO WHITE NOISE EXCITATION

Eqs. 3 and 5 can be used to evaluate the base shear, V_B , in terms of the velocity of a system. Thus

$$V_B = \frac{k \dot{x}}{2 \pi f} = \frac{W 2 \pi f \dot{x}}{g} \dots \dots \dots (6)$$

in which W = the weight of the system and g = the acceleration resulting from gravity. If average values of the results shown in Table 2 are used, it is

FIG. 5.—BASE SHEAR RESPONSE TO WHITE NOISE EXCITATION

FIG. 6.—VELOCITY RESPONSE OF VISCOUSLY DAMPED SYSTEMS TO WHITE NOISE

possible to formulate equations for the base shear in terms of the peak ground acceleration or rms of the ground acceleration. These are,

$$V_B = \frac{W}{g} \frac{2}{T} (0.1556) (\text{peak ground acceleration}) \dots\dots\dots (7a)$$

and
$$V_B = \frac{W}{g} \frac{2}{T} (0.4965) (\text{ground acceleration rms}) \dots\dots\dots (7b)$$

in which T = the fundamental period of vibration of the system, and the value for the velocity represents the average value. There is, however, a probability associated with this value being larger. The standard deviations of the velocity shown in Table 1 have been used to calculate the upper limit within which the probability of the occurrence of $V_B g/W$ (peak ground acceleration) is 0.990. Both the average value and this upper value of the nondimensional parameter are shown as the top two curves in Fig. 5. The top curve indicates the upper limit of base shear response beyond which the probability is 0.01 that the base shear will occur. In Fig. 5 each curve indicates the average maximum response accompanied by various degrees of viscous damping.

The two top curves in Fig. 5 indicate the maximum level of response that an undamped single-degree-of-freedom system will provide to a ground acceleration excitation whose power spectrum is flat from 0 cps to 35 cps. Perhaps the only type of structure that will approximate an undamped system is an elevated water tank, in which case the appropriate set of results shown in Fig. 5 might be used as a design criterion. If the weight and fundamental period of the structure together with the peak ground acceleration are known, then the base shear can be determined from Fig. 5.

In actual structures, the effect of damping is apparent. The average maximum response resulting from the same white noise source of excitation was investigated with different values of viscous damping. Results were obtained at the three levels of input, but only the results from the level in which the maximum average peak acceleration is 27.7 ft per sec² and the input acceleration rms value is 8.5 ft per sec² are shown in Fig. 6, in which it may be seen that the velocity response is no longer independent of frequency. Exactly the same process as that described for the no-damping case was conducted. For viscous damping the velocity response results for an input acceleration of 27.7 ft per sec² have been used; it is for this set of results that the response down to 20 cps has been obtained.

It was assumed that the relationship between the average maximum velocity response, $\dot{x}_{\max}$, and the fundamental period, T , was

$$\dot{x}_{\max} = A T^n \dots\dots\dots (8)$$

in which A = a constant. The results of plotting $\log_{10} \dot{x}_{\max}$ against $\log_{10} T$ are shown in Fig. 7. As the response appears to fall off rapidly beyond 0.05 sec and levels off at values of T around 3 sec, the approximation in Eq. 8 is only really valid between these values. The exponent n in Eq. 8 is equal to the slope of the lines in Fig. 7. The empirical equations for the velocity response at different levels of damping are

for 2% damping
$$V = 2.855 T^{0.375} \dots\dots\dots (9a)$$

$$\text{for 5\% damping} \quad V = 2.020 T^{0.375} \quad \dots\dots\dots (9b)$$

$$\text{for 10\% damping} \quad V = 1.440 T^{0.375} \quad \dots\dots\dots (9c)$$

in which the exponent $n = 0.375$, in each case.

The standard deviation of the samples obtained are considerably smaller for damping than for no damping. For example, at 5% critical damping the standard deviation for the velocity response results was 0.0413 ft per sec. This means that the average response can be used as an upper limit for design purposes.

FIG. 7.—VELOCITY RESPONSE VERSUS PERIOD
RELATION FOR VISCOUSLY DAMPED SYSTEMS

Values of $V_B g/W$ (peak ground acceleration) can now be obtained by substituting Eq. 8 in Eq. 7. Thus,

$$\frac{V_B g}{W a_p} = \frac{2 \pi A}{27.7 T^{0.625}} \quad \dots\dots\dots (10)$$

in which a_p = the peak ground acceleration.

Eq. 9 is plotted for the different values of critical damping on Fig. 5; the curves indicate that for white noise excitation viscous damping significantly influences the response of systems with small values of T .

It can be argued that white noise is a reasonable representation of the ground motion only in epicentral regions. At distances of a few miles the high frequency content of the ground motion acceleration is largely attenuated. To obtain some idea of the possible effect of the attenuation of higher frequencies a low-pass filter was placed on the output of the white-noise generator. An analysis was made of the signal by recording the response of the filter to white noise excitation. The resultant power spectrum is shown in Fig. 8. The filter gives an almost uniform fall-off of 4.6 db per octave.

FIG. 8.—POWER SPECTRUM FROM FILTERED WHITE NOISE

FIG. 9.—VELOCITY RESPONSE TO FILTERED WHITE NOISE

This form of excitation was now used and the velocity responses of single-degree-of-freedom systems were obtained. The results are shown in Fig. 9 in which the maximum average ground acceleration is 27.5 ft per sec². The points shown on the curve representing the response for no damping are the result of averaging twenty samples. If the results obtained with white noise and filtered white noise are compared it can be seen that the velocity responses with damping are almost identical to each other and that the no-damping cases are different. The action of viscous damping tends to mask the effect of the frequency content of the ground motion on the response of structures with a low natural period.

It is not surprising, therefore, that the results for the damped response to white noise and filtered white noise are similar to those of Housner. It is the response for no damping that shows the real nature of the frequency content

FIG. 10.—UNDAMPED RESPONSE TO FILTERED WHITE NOISE

of the ground motion. The results for the no-damping case when filtered white noise was used resemble the form of Housner's average velocity spectrum curve. It seems reasonable to say, then, that filtered white noise can be used to represent an earthquake, certainly at a point near the epicenter.

A curve of $V_B g / W a_p$ for no damping and an input of filtered white noise is shown in Fig. 10. It follows almost exactly the curve shown in Fig. 5 for the range $T = 3.0$ sec to $T = 0.4$ sec. Its values decrease as T decreases. Because the damped velocity response curves are the same for both types of ground acceleration input, the curves plotted in Fig. 5 also represent $V_B g / W a_p$ for the response to the filtered white noise input.

At this stage a comparison was made of the effect of the duration of the simulated earthquakes. Filtered white noise was used as the input with

systems having 2% critical viscous damping. Comparison was made between bursts of excitation of 25-sec and 35-sec duration, respectively. The results, shown in Fig. 11, indicate that this will probably be of no significance for lightly damped structures.

It is well known that outside the epicentral region an earthquake is generally heralded by P waves, which are followed by lower frequency S waves. It is hoped that in the future a hybrid computer facility might be used to prepare a large number of simulated earthquakes on magnetic tape that will feature this variation of frequency and magnitude of ground acceleration with time.

FIG. 11.—EFFECT OF DURATION OF GROUND MOTION ON LIGHTLY DAMPED SYSTEMS

This technique was used to generate the third type of ground motion considered herein. The accelerograph record of the S 86°W - N 86°E component of the Olympia, Washington, record of April 13, 1949 was used as the model. Digital information was taken from the record and consisted of 862 three-digit numbers. The digital information was not an exact model of the Olympia earthquake, but it provided a close approximation. The digital information was entered into a G.15 digital computer connected to the analog computer by a digital-to-analog converter. The stepwise analog signal generated by the digital input was smoothed by an analog interpolation circuit. The output from this circuit was stored on magnetic tape with the aid of an FM tape recorder.

FIG. 12.—VELOCITY RESPONSE OF SOME SINGLE-DEGREE-OF-FREEDOM SYSTEMS TO THE GROUND MOTION GENERATED BY THE HYBRID COMPUTER

FIG. 13.—VELOCITY RESPONSE SPECTRUM FOR HYBRID SIMULATION

The recorded signal was used to obtain the maximum velocity response of undamped single-degree-of-freedom systems. Some of the resulting responses, together with the simulated earthquake, are shown in Fig. 12. The signal was scaled so that the peak ground acceleration was equal to 11 ft per sec². The complete velocity spectrum results are shown in Fig. 13. A power spectrum

FIG. 14.—POWER SPECTRUM OF EARTHQUAKE SIMULATED THROUGH HYBRID COMPUTER

analysis and an amplitude probability density analysis were made of the recorded ground acceleration. The results are shown in Figs. 14 and 15.

ANALYSIS OF RESULTS

It is impossible to represent an earthquake by a single model. Many factors influence the form of the ground acceleration that will be experienced

in any given location during an earthquake. Perhaps the most important factors are the distance from the epicenter and the nature of the ground through which the earthquake waves are propagated. Although the general effects of these factors are known, there is not a sufficient number of recordings of earthquakes to provide a quantitative understanding.

Because the problem of earthquake-resistant design involves the dynamic analysis of structures, the most important feature of the ground motion is its power spectrum. If an explicit definition could be made for an earthquake spectrum at any location, then one of the major hurdles involved in earthquake design would be overcome. Unfortunately this is not the case, and the most logical method of analysis for the time being seems to be that based on a

FIG. 15.—AMPLITUDE PROBABILITY DENSITY OF EARTHQUAKE SIMULATED BY HYBRID COMPUTER

statistical approach. The three types of ground motion investigated during this work were used with these concepts in mind.

The frequency content of an earthquake in an epicentral region will depend on the mechanism involved in producing the earthquake. The question of earthquake mechanisms is still in debate, but it seems reasonable to assume that the power spectrum of the ground motion in the epicentral region will be essentially flat. If this supposition is accepted, then it is possible to evaluate statistical information concerning structural response in epicentral regions by exciting mathematical models of the structure to a large number of bursts of white noise. The results shown in Figs. 3 and 6 represent this statistical information for single-degree-of-freedom systems with different values of viscous damping. These results show that for certain values of the fundamental

period the inertia load acting on the structure exceeds the load calculated by multiplying the mass of the structure by the peak ground acceleration.

It is known that the higher frequencies in earthquake ground motions are more rapidly attenuated than the lower frequencies. The results obtained by filtering white noise are shown in Fig. 9. These results show that, for the filter used, the undamped velocity response falls off at either side of the response when $T = 0.75$ sec. This general property of peaking has been demonstrated by Housner for actual earthquake records. Only one filter was used during the work, but the results show that the method of filtering white noise is a possible approach to the simulation of earthquakes at locations distant from the epicenter.

The first two types of ground motion are concerned with the effect of location with respect to the epicenter. The second factor that influences the characteristics of the ground motion is the type of foundation material through which the waves pass. Experimental observations of quarry blasts⁵ show that higher frequencies are propagated more readily in rock than in less well consolidated material. K. Kanai⁶ has shown that predominant frequencies of ground vibration can be explained by the multiple reflection theory of elastic waves in ground layers.

In this case, the ground motion will be modified by the characteristics of the material through which it passes; in particular, there is the possibility that a large percentage of the power in an earthquake will occur in a small frequency bandwidth. This means that the power spectrum of an earthquake will have a peak in it similar to that of Fig. 14. The position of this peak will then depend on the path taken by the earthquake waves. The hybrid computer approach outlined herein provides a useful method of producing numerous simulated analog earthquakes with "tailored" properties; these may be frequency or amplitude variations with time.

CONCLUSIONS

The work described herein indicates the importance of the relationship between the dynamic characteristics of a structure and the type of dynamic excitation to which it is subjected. There is not sufficient information available to allow for all the factors that influence the characteristics of earthquakes. Until such information is available the methods described herein provide means of simulating some of the basic characteristics of earthquakes, a statement that is based on the observation that the results obtained during this work bear a close resemblance to the results obtained from actual earthquake records.

The simulation of earthquakes by the hybrid computer has the advantage that it is possible to incorporate any form of frequency or amplitude variation with time. It is not possible to achieve as much with the white noise. Both

⁵ Thoenen, J. R., and Windes, S. L., "Seismic Effects of Quarry Blasting," Bulletin No. 443, U. S. Dept. of the Interior, Washington, D. C., 1942.

⁶ Kanai, K., "On the Spectrum of Strong Earthquake Motions," Bulletin, Earthquake Research Inst., Univ. of Tokyo, Tokyo, Japan, Vol. 40, 1962, pp. 71-90.

types of simulations should provide useful statistical information concerning the structural response to earthquakes of multi-degree-of-freedom systems.

ACKNOWLEDGMENTS

This paper is a contribution from the Division of Building Research, National Research Council, Canada, and is published with the approval of the Director of the Division.

APPENDIX I.—NOTATION

The following symbols have been adopted for use in this paper.

- A = constant;
- a = peak ground acceleration;
- p
- F = lateral force;
- f = frequency of natural vibrations;
- f_n = center frequency of a filter;
- Δf = bandwidth of a filter;
- g = acceleration of gravity;
- K_{α/2} = acceptance limits for a two-sided procedure;
- k = stiffness of a single-degree-of-freedom system;
- m = mass of a single-degree-of-freedom system;
- T = period of natural vibrations;
- U = test statistic;
- u₀ = true mean of a random variable with normal distribution;
- V_B = base shear;
- W = weight of single-degree-of-freedom system;
- x = displacement of mass relative to foundation;
- $\dot{x}$ = velocity response of a structure
- $\bar{x}$ = measured mean of a random variable;
- $\ddot{x}_g$ = ground acceleration;
- β = fraction of critical damping; and
- σ = standard deviation of a random variable.

KEY WORDS: analog; computers; earthquakes; engineering mechanics; shear stress; simulation

ABSTRACT: Two analog methods of simulating earthquake motions are presented. One method uses sections of a gaussian random function with a given power spectrum. The second method uses a hybrid computer that permits any given type of ground motion to be simulated. Linear single-degree-of-freedom systems were excited by the simulated earthquakes and their responses were measured. Base shear was used as a measure of the response and empirical formulas were developed for base shear as a function of the mass and period of the system and the magnitude of the ground acceleration.

REFERENCE: Ward, Howard S., "Analog Simulations of Earthquake Motions," Journal of the Engineering Mechanics Division, ASCE, Vol. 91, No. EM5, Proc. Paper 4522, October, 1965, pp. 173-190.

ANALOG SIMULATIONS OF EARTHQUAKE MOTIONS^a

Discussion by Jack R. Benjamin

JACK R. BENJAMIN,⁷ M. ASCE.—This interesting paper adds to knowledge of the simulation of earthquakes using white noise models. The modeling of a complex statistical phenomena is always subject to approximations that must not be forgotten in the interpretation of results. The random vibration technique dominates current (1965) thought in the estimation of structural response to the earthquake forcing function. However, the model is far from perfect and both its assets and liabilities are important.

The general concept compared surface features of records with the corresponding characteristics of white noise and generally accepted the model if partial correlation was found. The mathematical usefulness of the random vibration technique cannot be denied. However, liabilities are involved in acceptance of model.

There are two primary faults of the white noise model. First, a surface representation is given that does not recognize or allow study of the fundamental processes involved. Secondly, the process is stationary. The first fault leads to the rule-of-thumb limitation and the second fault presents a physical impossibility. The mathematical properties of a given statistical model are fixed. If the model satisfies the engineer's knowledge of nature, it may provide a useful approximation of reality. However, any model that concentrates on gross effect and has major liabilities must always remain suspect.

Other models are available although the mathematical details are unsolved. The earthquake itself can be modeled as a queue. The building of strain energy is a demand and the release of a part of this energy in the form of wave energy is a service. Similarly, the ground motion is a demand and the structural response is a service. Limited studies indicate that this model generates ground motions similar to those of an earthquake without assuming a stationary process. In fact, a Markov type of dependency is involved. Transient characteristics are reproduced and it appears probable that characteristics of occurrence can be studied. These investigations are in their infancy. The only reason for mentioning them is to show that other models can be constructed.

The statement of a confidence interval must be carefully qualified when statistical models are used. Normality cannot be proven because other statistical distributions with vastly different extreme characteristics can be found to fit observed data as well as the normal distribution. The Central Limit Theorem states a trend that can only be verified in the region of the mean.

The statement of a conclusion for design requires even more care with regard to assumptions. This is especially true with the use of random vibration theory in earthquake studies.

---

^a October, 1965, by Howard S. Ward (Proc. Paper 4522).

⁷ Prof. of Structural Engrg., Stanford Univ., Stanford Calif.

ANALOG SIMULATIONS OF EARTHQUAKE MOTIONS^a

Closure

HOWARD S. WARD,⁸—The writer wishes to thank Benjamin for his discussion. Undoubtedly, there are other ways of simulating earthquake motions besides the band-limited white noise model. However, every simulation technique would have similar imperfections unless they were based on a detailed analysis of numerous records of strong-motion earthquakes. Only then would it be possible to realistically approach the simulation of earthquake motions.

Unfortunately, although strong-motion seismograph programs have been in existence for several years in both the United States and Japan, there is only a limited number of records available. The accumulation of sufficient data to improve the simulation of earthquakes will take many years, although UNESCO is encouraging the installation of strong-motion instrumentation in all those countries that experience earthquakes. This may accelerate the process of data collections.

Meanwhile there does not appear to be any justification for complicated methods of simulating earthquakes provided the superposition technique is used for the earthquake-resistant design of multistory structures. This technique involves determining individual maximum response of each of the separate modes of vibration, considered as an equivalent single-degree-of-freedom system. An estimate of these responses can be provided in a simple manner by the random vibration technique. If the earthquake-resistant design were based on the most logical conditions, namely, the response of a single multi-degree-of-freedom system, then there would be justification for complicating the simulation of earthquakes.

---

^a October, 1965, by Howard S. Ward (Proc. Paper 4522).

⁸ Research Officer, Bldg. Physics Sect., Div. of Bldg. Research, Natl. Research Council, Ottawa, Ont., Canada.