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EXPERIMENTAL DETERMINATION OF STRUCTURE AND FOUNDATION PARAMETERS USING WIND-INDUCED VIBRATIONS

BY

H. S. WARD AND J. H. RAINER

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NRCC 13026
DETERMINATION EXPERIMENTALE DES PARAMETRES DE CHARPENTE ET DE FONDATION AU MOYEN DE VIBRATIONS PROVOQUEES PAR LE VENT

SOMMAIRE

On utilise les vibrations provoquées par le vent dans les édifices à plusieurs étages afin de déterminer la rigidité de la fondation et les propriétés modales de la charpente. L'auteur présente deux méthodes permettant de déterminer la rigidité de la fondation, tant pour le mouvement de ballant que pour la direction horizontale, grâce à une combinaison de données expérimentales et de calculs théoriques. Une méthode utilise les rapports de fréquence modale pour tout le système; l'autre utilise les rapports d'amplitude modale. On donne une évaluation quantitative de l'encastrement, sur une grande dalle flottante de fondation, d'un édifice à 13 étages en béton armé.
Experimental determination of structure and foundation parameters using wind-induced vibrations

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J. H. RAINER, PhD


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Experimental determination of structure and foundation parameters using wind-induced vibrations

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J. H. RAINER, PhD†

Wind-induced vibrations in multi-storey buildings are utilized to determine foundation stiffnesses as well as structural modal properties. Two methods are presented which permit the determination of foundation stiffnesses, both rocking and in the horizontal direction, by combining experimental data with theoretical calculations. One method employs modal frequency ratios for the entire system; the other employs modal amplitude ratios. A quantitative assessment of the restraint on a large raft foundation of a 12 storey reinforced concrete building is given.

Introduction

Whenever the theoretical prediction of the behaviour of a large civil engineering system such as a multi-storey building or dam is compared with actual performance, it is not unusual to find discrepancies between prediction and reality. Frequently the causes of the discrepancies are known, but because of a lack of quantitative data on, for example, the characteristics of certain structural loads or the stiffening action of partitions and curtain walls, they have generally been ignored in the design or accounted for by gross approximations. Some of these uncertainties could be resolved by full-scale structural testing, but there has been reluctance to undertake this because until recently it represented a tremendous task. However, in the past few years, methods have been developed which provide quick and simple means of testing a structure. Generally speaking, they involve measuring vibrations generated by mechanical means or by a natural cause such as wind or earthquake.

2. The main object of this Paper is to show that such test methods are effective in investigating the elastic behaviour of large structures. The example used in the study is a 12 storey structure built on a raft foundation, in turn supported by a deep layer of clay. Experimental and theoretical results are interpreted in a way that identifies the stiffness characteristics of the soil and the structure.

The problem

3. As information has been accumulated about the response of structures to earthquakes it has been shown that ground–structure interaction may play an important role. In simple terms the interaction phenomenon represents the boundary condition imposed by the soil on structural movements; this may vary from an almost fully fixed base condition for a rock or till to a fairly flexible support for clay.

4. Numerous theoretical studies have been made of the dynamic character-
istics of foundation elements. Experimental work has been carried out, in some instances to substantiate the theoretical work and in others to investigate the properties of the soil. Mainly in response to the impetus provided by earthquake engineering requirements, there has been a logical extension from the theoretical study of a foundation element on a soil to the coupling of a foundation and superstructure to the soil. If the soil impedance, or the resistance of the soil to the movement of the foundation element as a function of frequency, were known there would be no difficulty associated with this coupling process. However, with current knowledge it is difficult to predict soil impedance and experimental data are required to develop this capability. In Japan earthquakes have been used to obtain information on foundation behaviour at the associated high stress levels and related work at much lower stress levels has been reported from a building excited by a mechanical vibrator. The aim of the work reported in this Paper was to discover whether wind excitation could be used in investigating ground–structure interaction, and thereby to arrive at quantitative structural and foundation properties.

5. Measurements of wind pressure on buildings have shown that for fre-
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Shear walls with 12" elements
Perimeter wall, glazing and 4" brick
Outline of raft foundation

Door openings
14" walls

Interior columns,
24" x 24",
20" x 20" above 5th floor

Perimeter columns,
18" x 18"

Fig. 2. Typical floor plan details of Public Service Alliance of Canada building

For this study the Public Service Alliance of Canada (PSAC) building in Ottawa was chosen. It is a graceful structure with an elliptical floor plan (Fig. 1); the major structural elements consist of three shear wall systems, with 12 in. and 14 in. thick elements, connected by 7-8 in. floor slabs (Fig. 2). In the framed part of the structure the interior columns are 24 in. square below the fifth floor, and 20 in. square above; those on the periphery are 18 in. square. The perimeter of the building is formed by glazing and 4 in. thick brick facing. Most floors do not contain full height interior partitions, thus reducing this source of structural stiffness. The main reason for choosing the building was that the foundation consisted of a thick raft sitting on relatively soft soil. The raft configuration represents the simplest interface reaction between a foundation and the soil and so it is convenient to study it first. The soil below the raft consists of 3 ft of stiff grey clay with a few fissures, 10 ft of medium soft grey clay, followed by soft grey clay with increasing amounts of silt, and dense till at 50 ft. The water content of the clay was generally 70%, and its undisturbed vane test shear strength was typically 1.5 kip/sq. ft.

Measurement and analysis of vibrations

7. The building vibrations were recorded on four occasions when there was a minimum of vibration from street traffic and internal sources such as lifts. Willmore Mk II seismometers, adjusted to flat velocity response for frequencies above 0.5 Hz were used to detect the vibrations.

8. Generally, six seismometers were strung out in the building and their outputs recorded on a seven track FM tape recorder; at least one measuring point was common to successive sets of recording sessions. During the recording sessions the wind speed rarely exceeded 15 mile/h, except when a brief storm affected the end of one set of records.

9. Details of methods for recording and analysing the vibration data have
already been presented.³⁶⁸ Rocking data for the raft foundation were obtained from simultaneous vertical vibration measurements taken near the centre and at the outer wall of the basement. The fundamental objective of the analysis is to determine the amplitude of the vibrations on each floor level as a function of frequency, and the phase relations between these amplitudes on different floor levels.

10. In the past these objectives have been carried out using analogue frequency analysis equipment⁶ or Fourier analysis techniques on digital computers.⁸ In this study a real-time frequency analyser was used. The time required for analysis is thereby drastically reduced and the use of ambient vibration measurements for evaluating structural characteristics becomes more attractive.

Results

Experimental results

11. It was the objective of this work to investigate the feasibility of using wind-induced vibrations to study the soil-structure interaction. The most significant results were obtained from lateral motions in the direction of the short axis, and so attention is focused on these measurements. Results for

![Fig. 3. Fourier spectrum of lateral motion of the base and third floor](image)

![Fig. 4. Experimental first and second mode shapes in the short direction](image)
lateral motion along the long axis and torsional modes of vibration are also tabulated to provide further confidence in the theoretical formulation for the superstructure.

12. The Fourier analyses of simultaneous records of lateral motion along the short axis, on the third floor and in the basement are shown in Fig. 3. The basement record shows one of the difficulties with measurements of this kind, namely the presence of 'noise' from other sources of excitation such as traffic or mechanical equipment.

13. The first four modes of vibration for lateral motion along the short axis of the building were found and the first two mode shapes are reproduced in Fig. 4. Table 1 shows the experimental values of the four lowest resonant frequencies. The first two mode shapes and frequencies of vibration for lateral motion along the other axis of the building and for torsional movements were also derived. Frequency values are shown in Table 2.

14. For lateral movements in the short direction of the building there was a significant rocking motion in the fundamental mode, but this component was less noticeable in the higher modes. Expressed as a percentage of the maximum displacement in a given mode, the horizontal movement of the base was most noticeable in the second mode. A similar situation was found for lateral

<table>
<thead>
<tr>
<th>Mode</th>
<th>Experimental</th>
<th>Method of modal frequency ratios</th>
<th>Method of modal amplitude ratios</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>Model S1</td>
<td>Model S1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Fixed base</td>
<td>Impedance $I_a$</td>
</tr>
<tr>
<td>1</td>
<td>1:7</td>
<td>1:81</td>
<td>1:63</td>
</tr>
<tr>
<td>2</td>
<td>6:3</td>
<td>8:82</td>
<td>6:84</td>
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Table 2. Resonant frequencies (in Hz) for vibrations in long direction and in torsion

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<thead>
<tr>
<th>Mode</th>
<th>Experimental</th>
<th>Vibrations in long direction</th>
<th>Torsion</th>
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<tr>
<td></td>
<td></td>
<td>Model L1</td>
<td>Model L2</td>
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<tr>
<td></td>
<td></td>
<td>Fixed base</td>
<td>Impedance $I_a$, amplitude ratio method</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Impedance $I_a$, amplitude ratio method</td>
<td>Fixed base</td>
</tr>
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<td>1</td>
<td>2:0</td>
<td>1:99</td>
<td>1:90</td>
</tr>
<tr>
<td>2</td>
<td>7:3</td>
<td>8:21</td>
<td>7:70</td>
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Table 3. Experimental damping values, per cent of critical

<table>
<thead>
<tr>
<th>Mode</th>
<th>Short direction</th>
<th>Long direction</th>
<th>Torsion</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>3</td>
<td>0.3</td>
<td></td>
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movements in the long direction. Although rocking movements in this direction were not measured, they tend to be small because of the great length of the raft foundation. In the torsional modes it was possible to detect a small amount of rotational movement in the base at the fundamental frequency, but there was no rotational movement at the second mode frequency.

15. An estimate of the equivalent viscous damping characteristics of the modes of vibration can be obtained by measuring the half-power bandwidth of the power spectrum. The results obtained from the experimental measurements by this method are given in Table 3. The resolution of the damping measurements is determined by the analysis bandwidth, and in this study the resolution was of the order of 2% for fundamental modes, 0.5% for second modes and 0.2% for third modes.

16. Many previous measurements of damping obtained in this manner have been much smaller, but the structures studied were founded on fairly rigid ground. It is probably reasonable to attribute at least part of these higher damping values to the geometric or radiation damping of the soil. There was no significant variation of half-power bandwidth through the height of the structure, for a given mode analysis procedure produces an average value of damping for the whole system.

Theoretical results

17. The interpretation of the experimental data was supported by theoretical analysis of the structure-foundation system. The procedure was to calculate the modal characteristics of a superstructure model in combination with a range of foundation impedances and thus to identify the combination that gave the best match between calculations and observations. Two methods of analysis are considered. The first is based on a comparison of measured and computed ratios of the modal frequencies; this is called the method of modal frequency ratios. The second method compares measured and computed mode shapes, or more specifically the ratios of base to top storey deflexions; this is called the method of modal amplitude ratios.

18. There is little variation in the resonant frequency of a system for low values of damping. For simplicity, zero damping was assumed in the theoretical calculations although it would be possible to incorporate any value of proportional damping. It was also assumed that the soil properties were amplitude-independent.

Modal frequency ratios

19. The first method of analysis is based on the supposition that the structure can be represented adequately by a particular mathematical model, i.e.
that geometric properties and density and stiffness distributions are well established. Although the absolute values of the resonant frequencies of such a structure will depend on actual material properties, such as density and Young's modulus, the ratios of higher order frequencies to the fundamental frequency are invariant relative to these two properties.

20. This fact was used by assuming an appropriate superstructure model and considering the effect of allowing the foundation support condition to vary. A plot could thus be drawn of the frequency ratios as a function of foundation stiffness. If a match could be obtained for the experimental and theoretical ratios then a true definition of the combined system would have been achieved and the value of the foundation impedance determined. The upper and lower bound could be established for the absolute values of the frequency by specifying reasonable values for the modulus of concrete and the weight of the building.

21. The end shear walls and central core were assumed to be cantilevers with flexural and shear flexibilities taken into account. The columns and part of the slabs were treated as frames and were assumed to be connected to the cantilevers by rigid diaphragm action of the floor slabs. Along one side of the building there were two levels of underground parking, but this portion was separated by a construction joint from the main structure and thus it was assumed that there was no interaction between the two structures.

22. To differentiate between the various structural models S is used to designate the model assumed for lateral vibrations in the short direction of the building, L that for lateral vibration in the direction of the long axis and T that for torsional vibrations. The full flexural stiffness of the cantilevers and column bents was used for models S1 and L1. The shear resistance of the walls was based on a uniform shear stress distribution over the walls. Another pair of models, S2 and L2, representing second approximations, was developed at a later stage of the study. Model T1 represents the torsional resistance due to the end shear walls, column bents and core.

23. As far as the superstructure is concerned the action of the soil impedance is to introduce an extra six degrees of freedom at its base. As the structure under consideration is nominally symmetrical about both orthogonal axes, it was assumed that there was no coupling between the lateral displacements of the two principal directions. Thus each degree of freedom of base motion could be considered independently and modelled by a simple spring. Only the horizontal base movements in the long and short direction and rocking in the short direction were considered in detail.

24. The design drawings specified that concrete strength in the columns and shear walls should be 5000 lb/sq. in. below the second storey level and 4000 lb/sq. in. above the second floor. In the calculations, Young's modulus for the concrete in these areas was taken to be $3.9 \times 10^6$ lb/sq. in. and $3.3 \times 10^6$ lb/sq. in., respectively. The corresponding shear moduli were taken as $1.5 \times 10^6$ lb/sq. in. and $1.3 \times 10^6$ lb/sq. in. It was assumed that the density of the concrete was 150 lb/cu. ft and all floor weights above the ground floor were taken as equal. The rotational moment of inertia of the raft and the floor slabs was also considered in the equation of motion involving the rocking of the building. The nominal values for the physical properties of the superstructure were used in the theoretical calculations, but they influence mainly the absolute values of the frequencies rather than the frequency ratios.
25. Some of the theoretical calculations for the model S1 are shown in Fig. 5; here for each mode $i$ the variation of frequency is shown for a range of foundation flexibilities. For comparison the data are plotted as ratios of modal frequency $f_i$ to the fixed base frequency $f_{iF}$. The values of $f_{iF}$ are shown in Table 1 and with these data $f_i$ can be evaluated from Fig. 5. In Fig. 5 the base flexibility has an earlier influence on the higher modes, e.g. the base flexibility $F_H$ of $10^{-9}$ in./lb has scarcely influenced the fundamental mode, but there has been a significant change in $f_4$ when $F_H$ is only $10^{-9}$ in./lb. Another feature, which is general for the sort of structure–foundation model under consideration, is the continuous decrease in the fundamental frequency as ground flexibility increases, whereas the higher mode frequencies tend to approach an asymptotic value.

26. When the data are expressed in terms of modal frequency ratios, as in Fig. 6, the influence of base flexibility is again evident. Here it is possible to plot the experimentally observed modal frequency ratios, and thus to infer the foundation impedance from the intersection between the calculated and experimental curves. If it is postulated that the foundation impedance is independent of frequency, then the vertical dotted line in Fig. 6 probably represents the best choice for constant foundation parameters. Thus if $F_H$ is taken as $8\times10^{-9}$ in./lb and the rotational spring flexibility $F_R$ is $1 \times 10^{-14}$ rad/in. lb, the errors between the theoretical and experimental ratios of $f_2/f_1$, $f_3/f_1$ and $f_4/f_1$ are approximately $+13\%$, $-10\%$ and $-6\%$ respectively. These values of $F_H$ and $F_R$ comprise the foundation impedance designated $I_4$.

27. A plot similar to that in Fig. 6 but with the rotational flexibility $F_R$
plotted on the abscissa, and for various values of horizontal flexibilities $F_H$, enables one to estimate the value of $F_R$ as $1 \times 10^{-14}$ rad/in. lb. However, since the frequency ratios are less sensitive to $F_R$ than to $F_H$, the intersections of the theoretical curves with the experimental frequency ratio are less well defined, and therefore the value of $F_R$ obtained by this method is less precise than that for $F_H$.

28. It is apparent that even better agreement could be achieved if the foundation impedance were assumed to vary with frequency. Thus if it is assumed that $F_R$ remains constant at $1 \times 10^{-14}$ rad/in. lb, the experimental results cross the theoretical curves for the ratios $f_2/f_1$, $f_3/f_1$ and $f_4/f_1$ at values of $F_H$ which...
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are $1.0 \times 10^{-9}$, $8.0 \times 10^{-9}$, and $5.0 \times 10^{-9}$ in./lb, respectively. This suggests a foundation stiffness that increases with frequency. If the values of $F_H$ and $F_R$ are assumed to vary, then the possible range of foundation values becomes broader, although the choice for $F_H$ would still be confined to a relatively small area.

29. The nature of the curves in Fig. 6 means that there is a minimum point in the frequency ratio curves beyond which the ratios increase as the foundation flexibility increases. This means that the experimental results cut the theoretical curves a second time. In this instance this takes place for values of $F_H$ between $10^{-7}$ and $10^{-6}$ in./lb. Although this region leads to correct values for the frequency ratios it can be dismissed from further consideration because the absolute values of the frequencies are much lower than the measured ones.

30. In the method described so far the use of frequency ratios makes it possible to minimize the influence of the structural properties in determining foundation characteristics. It is nevertheless instructive at this point to calculate the absolute frequencies corresponding to the foundation parameters just deduced (impedance $I$) and to compare them with the observed frequencies. Comparison of the second and fourth columns of Table 1 shows that the frequencies agree within 10–15%, which seems adequate in view of the approximations in the initial assumptions about the superstructure.

Modal amplitude ratios

31. In the previous calculations it has been assumed that the only unknown factor is soil impedance. However, if the mathematical model of the structure is not precisely established, or for reasons of economy it is approximated, the method of modal amplitude ratios enables one to obtain foundation stiffnesses as well as progressive improvements in the structural idealization. The method uses comparisons of measured and calculated mode shapes of the structure as well as the modal frequencies.

32. As the mode shapes of a structure serve the same purpose as the frequency ratios in defining the behaviour of the system, characteristics of the modal deformations can also be used to find foundation stiffnesses. The procedure consisted of taking a superstructure model and calculating the mode shapes for different values of the lateral and rotational base flexibilities. The ratio of roof to base deflexion is used as an index of mode shape and is called the modal amplitude ratio. For each particular mode, this ratio was plotted as a function of the foundation properties. From corresponding experimental data it is possible to choose foundation values that provide agreement between the experimental and theoretical modal amplitude ratios. With these foundation values it is possible to calculate the frequencies of vibration and compare them with the experimental values.

33. Calculations using model S1 showed an improvement in the agreement between theory and experiment when the shear resistance of the superstructure was decreased. This form of modification appears justified on account of the door openings in the shear walls, originally not considered. Furthermore, shear stress distribution across a member is in general not uniform, as was originally assumed in model S1. An increase of shear flexibility of 50% in the cantilever walls led to model S2, for which the results of the amplitude ratio plots for the first three modes are shown in Fig. 7. The foundation movement
in the fourth mode could not be measured accurately and so it is not plotted here; its modal frequency was calculated using the foundation stiffnesses found for the third mode. Again the indications are that a frequency-dependent soil impedance is required to obtain agreement between theory and experiment, and this dependency appears to be an increase in foundation stiffness with frequency.

34. The results in Figs 5–7 were calculated on the basis of a given constant value of foundation flexibility for all modes of vibration. In order to continue the procedure described it is necessary to consider the implications of allowing the foundation parameters to take on the values suggested by the experimental results.

35. Although it is possible to handle a frequency-dependent foundation condition by evaluating the transfer function of the structure and soil combined, a simpler solution is to consider that for each mode the superstructure is connected to the corresponding foundation impedance found in Fig. 7. Thus the foundation values that give the best agreement for the fundamental mode were used to calculate only the fundamental mode and frequency of the combined system. Similarly, the foundation values for the second and third modes
found from Fig. 7 were used to compute the modal properties for the second and third modes, respectively. A rigorous proof for the validity of this approach was not sought, but some calculations comparing the transfer function approach with the simpler method show that there is little error near the resonant frequency under consideration.

36. The calculated shapes of the first two modes for model S2, based on agreement between mode shape ratios, are shown in Fig. 8; the mode shapes for the S2 model attached to a rigid base are also given. If these theoretical mode shapes are compared with the corresponding experimental results there is good agreement in the major details. The same procedure was followed in relating the theoretical and experimental amplitude ratios for the L and T models, except that for the L models only lateral flexibility of the foundation was considered, and for the T model calculations were made only for a fixed

Fig. 8. Computed mode shapes for model S2
base condition. For similar reasons (as for the short direction) the shear flexibility in model L2 was increased by 30% over that assumed in model L1. Detailed results are not presented for this case, although the lateral foundation stiffness derived is denoted by impedance $I_4$ in Fig. 9.

Foundation parameters and resonant frequencies

37. An overall impression of the main features of the results may be obtained by showing the nature of the foundation impedances that were suggested by the theoretical work, together with the agreement between the experimental and theoretical values for the resonant frequencies of the structure–ground interaction system.

38. The various values of foundation flexibility derived from matching experimental and theoretical results are given in Fig. 9. The foundation impedance $I_1$ is the constant base flexibility model derived from the modal frequency ratio method for model S1 (Fig. 6). $I_2$ is an arbitrarily selected constant foundation impedance for model S2, intended to give close agreement between measured and computed values of the fundamental frequency. $I_3$ is the foundation impedance for model S2, obtained from the modal amplitude ratio in Fig. 7, and $I_4$ is the impedance obtained for the model L2 in the long

![Fig. 9. Foundation impedances based on measurements and calculations](image-url)
direction of the building. Impedances $I_3$ and $I_4$ show a decrease in foundation flexibility or, conversely, an increase in foundation stiffness, as a function of frequency. In addition to the frequency values on the abscissa scale, the values of the characteristic non-dimensional frequency $a$, used in previous experimental and theoretical studies, are shown for an assumed shear wave velocity of 800 ft/s in the ground. This might be the basis for comparing the results with those obtained from small-scale footing tests.

39. Table 1 summarizes all the information on the resonant frequencies of vibration for the S models. The frequencies in the fourth, seventh and eighth columns agree reasonably well with the experimental values, but it is apparent that the S2 model with a soil impedance between $I_2$ and $I_3$ could lead to agreements for the first four frequencies of vibration that are better than $\pm 15\%$. This is considered to be better agreement than that obtained in the modal frequency ratios calculations because with the $\pm 15\%$ agreement in frequency there is also good agreement between mode shapes.

40. Table 2 compares experimental and computed frequencies for lateral vibrations in the long direction of the building and for torsional vibrations. Only the first two modes of vibration could be determined in these instances and the correlation between theory and experiment is about $\pm 10\%$.

Discussion of results

41. The two sets of theoretical calculations performed in this study represent two different means of evaluating ground-structure interaction parameters.

42. For both methods the basic concept is to use the superstructure as a resonant vibrator to excite the full-size foundation system. The most reliable data on foundation conditions using this technique will be obtained when the superstructure is either simple or reasonably well understood, as for tall chimneys, water towers and bridges. Although a raft foundation was used in this study, it would be possible to obtain similar data for pile foundations. The interpretation of the results would be the same whether the superstructure were excited by wind or by any other form of dynamic force input.

43. In the method of modal frequency ratios it has been shown that if the characteristics of the superstructure are known it is possible to use the results to evaluate foundation properties at the resonant frequencies and amplitude levels associated with the vibrations. The advantage of this approach is that the only experimental data required are the few lowest natural frequencies of the structure. The arduous task of determining measured mode shapes can be avoided.

44. The method of modal amplitude shows the value of experimental results as a source of information for improving the theoretical prediction of structural analysis. In this instance both superstructure and foundation model were chosen to give the best fit to experimental results. There is a potential for investigating trends in the behaviour of structural systems and using this knowledge to improve theoretical formulations. The method of modal amplitude ratios makes possible a more definitive determination of foundation flexibilities than the method of modal frequency ratios. However, it requires information on mode shapes and an accurate determination of foundation movements.
45. The results shown in Fig. 6 for the method of modal frequency ratios, those in Fig. 7 for the method of modal amplitude ratios and the results in the long direction presented in Fig. 9 indicate an increase in foundation stiffness with frequency. A plausible explanation to support this phenomenon may be found in experimental and theoretical results on vibrations of circular as well as rectangular footings. Work by Bycroft, Sung, Kobori et al. shows that the dynamic impedance of footings, both in the rocking and the horizontal modes, increases as the frequency of excitation increases. A detailed quantitative comparison is not attempted in this Paper.

46. It is conceivable that the foundation stiffness is also amplitude dependent. This property could be investigated in an approximate manner by means of wind-induced vibration if attention were restricted to the fundamental mode, since most of the building movement occurs in this mode with nearly sinusoidal variations. Results obtained under different wind speeds would then indicate possible amplitude-dependent stiffnesses. Amplitude dependence can be measured accurately by applying increasing levels of sinusoidal forces to the building with a vibrator.

47. In view of uncertainties involved in wind-induced dynamic measurements, the results obtained in this study for the foundation flexibility conditions must not be regarded at this stage as precisely determined values; more research must be done. In particular, the experimental techniques need to be refined. Base movements of the structure must be carefully monitored, and improved methods of differentiating between wind-induced movements and traffic vibrations of the foundation are required. More data covering a range of structure and soil conditions are needed.

48. Some of these problems can be minimized by using a controlled vibration generator rather than wind. This has been done by Reay and Shepherd. Their results indicate a slight increase in horizontal foundation flexibility from the first to the second mode, whereas the trend in the results reported in this Paper is in the opposite direction. At present there is no explanation for this apparent conflict; more work is necessary on this topic.

Conclusion

49. It has been shown that measurements of the wind-induced vibrations of a structure can be used to obtain quantitative information about the stiffness characteristics of a structure and its foundation. Two methods have been presented that permit the determination of foundation stiffnesses by combining experimental data with theoretical calculations. The advantages and disadvantages of both methods have been outlined. In the building under investigation it was possible to assess the restraint imposed on a large raft foundation by the surrounding soil.

50. The reasonable agreement between theoretical and experimental results indicates that relatively simple structural analysis techniques can be used to predict quite well the dynamic characteristics of cantilever-type buildings.

Acknowledgements

51. The Authors wish to express their appreciation to the owners of the building, the Public Service Alliance of Canada, to the architects for the build-
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52. This Paper is published with the approval of the Director of the Division of Building Research, National Research Council of Canada.

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**Conversion factors**

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<th>Imperial</th>
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</thead>
<tbody>
<tr>
<td>1 in</td>
<td>25.4 mm</td>
</tr>
<tr>
<td>1 ft</td>
<td>0.305 m</td>
</tr>
<tr>
<td>1 lb force</td>
<td>4.448 N</td>
</tr>
<tr>
<td>1 lb/sq. in.</td>
<td>0.007 N/mm²</td>
</tr>
<tr>
<td>1 lb/cu. ft</td>
<td>0.216 kg/m³</td>
</tr>
<tr>
<td>1 mile/h</td>
<td>1.609 km/h</td>
</tr>
<tr>
<td>1 in./lb</td>
<td>5.71 mm/N</td>
</tr>
<tr>
<td>1 rad/in. lb</td>
<td>0.00885 rad/mm N</td>
</tr>
</tbody>
</table>
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