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A SIMPLE EFFECTIVE VISCOSITY FORMULATION FOR TURBULENT FLOW AND HEAT TRANSFER IN COMPACT HEAT EXCHANGERS

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ABSTRACT

A simple algebraic methodology for computing turbulent stresses and fluxes in arrays of heat exchanger elements is proposed. An effective Reynolds number is computed from the back-projection of the friction or heat transfer factor onto the low-Reynolds number profile. This is then used to obtain the turbulent viscosity for fluid friction, or turbulent Prandtl number for heat transfer. It is shown that under certain circumstances, the resulting mathematical expression is consistent with the Brinkman-Forchheimer modified form of Darcy's law and also with Reynolds' quadratic form for frictional/heat transfer resistance. The model is critically appraised in comparison to empirical data for compact and tube bank heat exchangers. The circumstances where it renders a good predictive measure are highlighted and discussed critically.

NOMENCLATURE

b	Forchheimer constant, m^{-1}
F''	equivalent shear stress, N/m^2
F'''	force per unit volume, N/m^3
c_p	specific heat, W/K
D	diameter, m
D_h	hydraulic diameter, m
F	distributed resistance, $kg/(m^2s)$
F_0	laminar resistance, $kg/(m^2s)$
F_T	turbulent resistance, $kg/(m^3)$
f	friction coefficient,
j	heat transfer factor
k	thermal conductivity, $W/(m^2K)$
L	mixing length, m
p	pressure, Pa
$\dot{q}'''$	rate of heat transfer per unit volume, W/m^3
r	volume/area fraction, porosity
s	pitch, m
T	temperature, K
u	velocity, m/s
U	superficial velocity, m/s

Greek letters

α	thermal resistance, $W/(m^3s)$
κ	permeability, m^{-2}
ρ	density, kg/m^3
λ	Brinkman constant, $kg/(ms)$
μ	dynamic viscosity, $kg/(ms)$

INTRODUCTION

Computational fluid dynamics (CFD) has, on occasion, been used to generate numerical data on the performance of heat exchangers. Patankar and Spalding (1974) proposed an original methodology for obtaining pressure-drop and heat transfer calculations for engineering-scale heat exchange devices. A porous media analogy was employed. Only the gross motion of the fluid was considered; detailed flow within the individual passages being modelled on a sub-scale of the computational grid. The approach was subsequently adopted by a number of researchers, (for example; Rhodes and Carlucci 1983; Theodossiou and Sousa 1988; Prithiviraj and Andrews 1998a,b; Coelho 1999a,b; Beale and Zhubrin 2005; Gómez et al. 2007). The methodology is particularly appropriate for calculations involving large-scale industrial processes which include heat exchangers as sub-components.

A typical distributed resistance analogy approach involves solution to equations for which a balance of forces on the fluid may be written in the following form, for incompressible flow,

$$\frac{\partial(\rho \mathbf{u})}{\partial t} + \text{div}(\rho \mathbf{u} \mathbf{u}) + \text{div}(\rho \overline{\mathbf{u}' \mathbf{u}'}) \quad (1)$$

$$= -r \text{grad } p - Fr^2 \mathbf{u} + \text{div}(r \mu \text{grad } \mathbf{u})$$

The quantity

$$F = \frac{1}{2} f \rho |\mathbf{u}| / r D_h \quad (2)$$

is known as a distributed resistance, and $\mathbf{u}$ is the mean interstitial velocity. The viscous term on the right-side

of Eq. (1), was not considered in the original article by Patankar and Spalding's (1974). It is a measure of 'spreading' or re-distribution of momentum in the heat exchanger passages. Assuming the Boussinesq assumption $\rho \overline{\mathbf{u}'\mathbf{u}'} = -\mu_T \text{grad } \mathbf{u}$, and defining an 'effective' viscosity $\mu_{eff} = \mu + \mu_T$, one may equivalently select either interstitial velocity, $\mathbf{u}$, or superficial velocity, $\mathbf{U} = r\mathbf{u}$, as state-variable, so that, for constant volume fraction, r ,

$$\frac{\partial(\rho \mathbf{u})}{\partial t} + \text{div}(\rho \mathbf{u} \mathbf{u}) = -\text{grad } p - Fr \mathbf{u} + \text{div}(\mu_{eff} \text{grad } \mathbf{u}) \quad (3)$$

$$\frac{1}{r} \frac{\partial \rho \mathbf{U}}{\partial t} + \text{div}\left(\rho \frac{\mathbf{U} \mathbf{U}}{r^2}\right) = -\text{grad } p - F \mathbf{U} + \text{div}\left(\frac{\mu_{eff}}{r} \text{grad } \mathbf{U}\right) \quad (4)$$

Both forms are found in the literature. Other forms, similar to Eq. (4) but excluding the volume fractions are also employed in certain commercial CFD codes (see for example, Schwarz and Beale 2009). These are however only strictly correct in the two limits of very large and very small F -values (ie. purely Darcy's and Navier-Stokes' laws, respectively).

The distributed analogy for heat transfer may be written,

$$\frac{\partial(\rho r c_p T)}{\partial t} + \text{div}(\rho r c_p \mathbf{u} T) = \alpha r (T - T_\infty) + \text{div} r k_{eff} \text{grad } T \quad (5)$$

where

$$\alpha = \frac{1}{2} \text{St} \rho |\mathbf{u}| / r D_h \quad (6)$$

A Pr-independent heat transfer factor, j , is defined as $j = \text{Pr}^{1-m} \text{St}$, where $m = 1/3$ is often selected, for lack of problem-specific data.

Some authors have performed calculations for turbulent heat exchanger flow using models such as the k - ϵ and other methods. However few heat exchangers operate in the high Reynolds-number turbulent regime, and indeed for most geometries, the fluid layer near the solid elements is laminar. The book by Kays and London (1984) contains correlations for friction and heat transfer factors, f and j , for many heat exchanger types. Plotted on a log-log scale, these often exhibit a 'hockey-stick' profile starting from an initial negative straight-line of slope near unity, for low Reynolds

number laminar flow, to a flat-horizontal, sic, 'turbulent' profile, with a smooth transition in the absence of a clearly-identified critical Reynolds number. A simple physical explanation for this situation is that, with increased mixing, the effective viscosity is greater than the molecular viscosity, thereby reducing the effective Reynolds number.

It was Reynolds (1875) who first suggested that turbulence may be quantified in terms of a quadratic term for the resistance to fluid and heat flow, namely

$$F''' = F_0 |\mathbf{U}| + F_T |\mathbf{U}|^2 \quad (7)$$

$$\dot{q}''' = \alpha_0 |\mathbf{U}| + \alpha_T |\mathbf{U}|^2 \quad (8)$$

Following a suggestion by Spalding (2007), based on dimensional considerations, it is proposed here to set,

$$\mu_T = \rho L |\mathbf{u}| = \rho c D_h |\mathbf{u}| \quad (9)$$

The non-dimensional constant, c , is the ratio of the mixing-length, L , to hydraulic diameter, D_h . ie a measure of the length-scale of the mixing relative to the physical geometry. It is common in the literature, to correlate the low Reynolds-number laminar friction coefficient in the form,

$$f_0 = a / \text{Re}^m \quad (10)$$

For $m = 1$, $F_0 = \frac{1}{2} \rho a / (\rho D_h^2 \mu)$, i.e. F_0 is independent of $\mathbf{u}$ (but not otherwise if $m \neq 1$). At higher Reynolds numbers, the effect of mixing is to reduce the effective Reynolds number so that,

$$f = a / \text{Re}_{eff}^m = a (c + 1/\text{Re})^m \quad (11)$$

where $\text{Re}_{eff} = \rho D_h |\mathbf{u}| / \mu_{eff}$. Equation (11) generates the quadratic form of Eq. (7) with $F_0 = \frac{1}{2} \rho a c / (\rho D_h^2 \mu)$ for the case $m = 1$. Equation (11) may thus be conveniently correlated to experimental data, to obtain values of a , m , and c for any given geometry.

Figure 1. Friction and heat transfer factor for an offset plate-fin surface, showing the basic methodology

RESULTS

Figure 1 shows a correlation of Eq. (11) with smoothed experimental data for an offset-fin strip-fin heat exchanger, namely surface no. 1/9-24.12, core 105 (from Shah and London 1967; Kays and London 1984). It can be seen that Eq. (11), provides an excellent fit to the empirical friction and heat-transfer factor data. The laminar or low Reynolds number factors, f_0 and j_0 , are shown as a straight lines in the figure.

In the ‘turbulent’ regime, the effective viscosity is obtained, e.g. for the case $Re = 5\,000$ shown in Fig. 1, as the back-projection to the laminar f_0 characteristic (with the resulting $Re_{eff} = 690$ for the case shown) using

$$\frac{\mu_{eff}}{\mu} = 1 + c Re \quad (12)$$

where $Re_{eff}/Re = \mu/\mu_{eff}$. This provides the heat exchanger engineer with a simple means to calculate μ_{eff} , obviating the need for ‘two-equation’ procedures.

Inspection of Fig. 1 reveals that a similar treatment may also be successfully applied to heat transfer calculations for the enumeration of the turbulent conductivity or turbulent Prandtl number. For many heat exchangers, empirical correlations of the form, $Nu = a Re^m Pr^n$ are often provided, with values of m and n given over a range of Reynolds numbers. Thus $j = j(Re, Pr)$ is properly expressed in terms of the Péclet number, $j = j(Pe)$, and Eq. (12) may be re-written as $k_{eff}/k = 1 + c Pe$. If n is constant (or reasonably so), the Péclet number may be replaced by Reynolds number as independent variable for heat transfer correlations. Although this practice is commonplace in empirical correlations and is also adopted here, it generates slight confusion in the form

of two different effective Reynolds numbers for momentum and energy¹ (as opposed to employing a Péclet number as dependent variable in the latter). The reader will note that for the case shown in Fig. 1, the ratio k_{eff}/k is approximately double that of μ_{eff}/μ , i.e., i.e., the effect of free stream turbulence upon heat transfer is apparently more pronounced than on momentum transfer.

Figures 2 to 7 show further examples of correlations of Eq. (11) with various smoothed-experimental data. Figure 2 shows both f and j correlations with data for louvred plate-fin surface no. 3/8 6.006 (from Kays and London 1984). Figure 3 is a similar comparison for plain plate-fin surface, no. 4.00 (again from Kays and London 1984). Figures 4 and 5 show data for friction coefficient, f , for inline square and equilateral triangle tube banks. These are fitted to the empirical correlations of Žukauskas and Ulinskas (2008). The latter are conveniently expressed in the form of power series in Reynolds number, and methods for calculating f for non-square and non-equilateral geometries are also provided in the article.

Figures 6 and 7 are correlations of the heat transfer factor, j , for inline and staggered tube bank geometries compared with three correlations; Žukauskas and Ulinskas (1988), ESDU (1973) and Gnielinski (1979) (see also Žukauskas et al. 2008).

Tables 1 and 2 show values of a , c , and m , obtained for the tube bank fluid flow and heat transfer regression analyses, respectively. The reader will note, that these values were obtained by minimizing the least-squares residual of the function $\ln f$, rather than of f , in order to remove low Re bias in these data, which span numerous decades of values for f and j , as can be seen in Figs. 4–7.

¹ In fact $Nu = Nu(Re, Pr, \text{thermal boundary condition})$.

Figure 2. Friction and heat transfer factor for a louvered plate-fin surface.

Figure 3. Friction and heat transfer factor for a plain plate-fin surface.

Figure 4. Friction factor for inline square tube banks, $s/d = 1.25\text{--}2.00$.

Figure 5. Friction factor for staggered tube banks, $s/d = 1.25\text{--}2.00$.

Figure 6. Heat transfer factor for in-line tube banks.

Figure 7. Heat transfer factor for staggered tube banks.

Table 1. F -coefficients for inline-square and equilateral triangle tube banks.

	$s/d=1.25$			$s/d=1.5$			$s/d=2$		
	a	c	m	a	c	m	a	c	m
Inline-square	218.1	0.319	1.014	85.2	0.272	0.996	24.7	0.203	0.860
Equilateral triangle	158.7	0.294	0.803	52.2	0.234	0.703	7.170	0.123	0.393

Table 2. α -coefficients for inline and staggered tube banks.

	a	c	m
Inline	0.552	0.0015	0.475
Staggered	0.705	0.0015	0.498

DISCUSSION

For the offset geometry shown in Fig. 1, values of the coefficients for fluid flow/heat transfer were as follows, $a = 19.14/5.72$, $m = 0.974/1.09$, $c = 1.24 \times 10^{-3}/2.5221 \times 10^{-3}$. A straight-line laminar-zone f_0 and j_0 region is readily apparent from the figure. For the louvred geometry of Fig. 2, the present approach agrees well with the smoothed experimental data. For both these cases values of $m \approx 1$ are observed. At higher Reynolds numbers, as $Re \rightarrow \infty$, $f \rightarrow ac^m$ and the curve approaches a horizontal line. These features are quite typical of porous-media flows, discussed further below.

Figure 3 shows an example of a compact surface where it can be seen that there is a ‘crisis’ or ‘defect’ region above $Re = 3\,000$ marking transition to turbulence. The inline-square tube-bank friction data of Fig. 4 also display a similar defect (though the staggered geometry, Fig. 5 and heat transfer profiles, Figs. 6 and 7, do not). Under these circumstances there are multiple extrema in the profiles, and Eq. (11), can at best only roughly approximate the data, especially at intermediate Reynolds numbers. For the plain-fin geometry, although a straight-line laminar f_0 line is readily apparent for $Re \leq 2\,000$, the best fit of Eq. (11) over the entire Re range does not necessarily correspond to the best fit (slope) to the laminar line.

For compact inline tube banks, Fig. 4 and Table 1, $m \approx 1$, but for widely-spaced inline tube banks m is substantially less than unity, a trend which is even more apparent for the equilateral triangle geometry of Fig. 5. Inspection of Table 1, shows that values of m used to approximate the friction coefficient f , differ decrease as the pitch-to-diameter ratio increases from $m = 0.803$ for $s/d = 1.25$ to $m = 0.393$ for $s/d = 2$.

The tube-bank heat transfer data of Figs. 6 and 7 are characterised by (i) values of $m \approx 0.5$ i.e., much less than unity and (ii) by very small values of the constant c . The constant, c , is a measure of relative change in j at higher Reynolds numbers. thus small c -values implies little change, $Pr/Pr_{eff} \approx 1$, in other words ‘turbulence’ has little influence upon heat transfer for these tube banks. whereas the pressure losses are substantially affected and c is correspondingly much larger. The reader will note that both fluid flow and

heat transfer data are derived from a very large number of different experiments, (see Beale 1993, for a discussion of this). In particular, the heat transfer data are for a wide variety of different geometries (equilateral triangle, rotated square, etc.) pitch-to-diameter ratios, and thermal boundary conditions (Neumann, Dirichlet, etc). These have been correlated by the authors, as single data sets for all inline and all staggered configurations. Thus the ‘straight-line’ correlations may be, at least partially, an artifact due to data-averaging. It might be better to correlate data for individual geometry/boundary condition combinations.

Porous Media Analogy

For isotropic porous media, the Brinkman-Forchheimer modified form of Darcy’s law (Vafai and Tien 1981) is typically written in terms of superficial velocity;

$$\mathbf{0} = -\text{grad } p - \frac{\mu}{\kappa} \mathbf{U} + \rho b |\mathbf{U}| \mathbf{U} + \text{div } \lambda \text{grad } |\mathbf{U}| \quad (13)$$

where κ is permeability, b is Forchheimer’s constant, and λ is Brinkman’s term. In texts on porous media flow (Scheidegger 1974), b and λ are introduced to account for ‘inertial’ and ‘boundary-layer’ effects respectively. The transient and convective terms are considered negligible, and hence excluded from Eq. (13), provided the permeability, κ , is sufficiently small. NB: High permeability porous matrices are sometimes used to induce mixing in the passages of heat and mass exchange devices, and under these circumstances the left-hand-side of Eq. (13) is not zero. The correspondence with Eq. (4) is clear and obvious for the case $m = 1$, namely $F_0 = \mu/\kappa$, $F_T = \rho b$ and $\lambda = \mu_{eff}/r$. For this case, the f vs. Re curve may be ‘compressed’ onto a single characteristic (Ahmed and Sunada 1969).

For compact surfaces, such as those shown in Figs. 1–3 and also compact inline tube banks $m \approx 1$, and the ‘permeability’ is thus constant in the laminar region. However the porous media analogy is this is certainly not true in the literal sense for widely-spaced inline- and all equilateral triangle tube banks, where the permeability must

be considered to be a function of velocity, even at very low Reynolds number. Of course the porous media (distributed resistance) analogy can still be employed in these circumstances, provided the non-linearities are accounted-for.

In texts on porous media, the inertial (Forchheimer) and viscous (Brinkman) terms are generally treated as being independent. In particular there is some controversy about how to enumerate the latter, and whether it is even significant. From the analysis here, it would appear they are related. In other words, the Forchheimer (drag) term arises naturally, as a direct result of the Brinkman (diffusion) term: the increase in drag and heat transfer being associated with increased effective diffusion coefficient.

In heat exchanger texts, the concept of ‘equivalent shear stress’ is sometimes introduced (Kays and London 1984). This is essentially the area analogue of the volumetric body force, viz, $F'' = D_h F'''$, in Eq. (7). The notion of ‘equivalent shear stress’ thus makes no real distinction between shear and normal stresses, nor for that matter whether these arise due to the action of mean or fluctuating forces. In any event shear forces would tend to increase both Forchheimer and Brinkman terms, whereas normal forces would only increase latter. Thus it may well be that there are difference between the Forchheimer and Brinkman terms, but the model in its present form cannot account for these. Moreover, in addition to these kinetic (dynamic) aspects, there may be purely geometric constraints which also influence the shear forces. The matter is further discussed below.

In favour of the simple model presented here, it must be said that, on the other hand, in computational fluid mechanics and heat transfer, local and overall f vs. Re and j vs. Re correlations are the ‘gold standard’ by which turbulence models (and hence enumeration of μ_T and k_T) are judged as suitable for a given class of problem, ie. the inertial term is used to ‘calibrate’ the increase in shear due to turbulence. Thus for a first analysis, at least, the selection of a single value of μ_T , both in the viscous and resistance terms appears quite reasonable. If a single value of μ_T or k_T cannot effect this, then this is a reflection that a higher-order closure scheme must be found for the problem. Also part of the problem in resolving the Brinkman and Forchheimer terms may also be due to the tacit assumption in standard porous media theory that $m = 1$ in the laminar zone.

Further Comments

The simple algebraic formula for turbulent viscosity and diffusivity, Eqs. (10) and (11), yields excellent results for both f and j for the compact heat exchanger geometry of Fig. 1. The present author previously conducted detailed laminar CFD calculations (Beale 1990; 1993; 2007) for the same geometry and flow conditions as shown in Fig. 1. Those calculations exhibited a straight-line profile similar to f_0 and j_0 , agreeing with the experimental f and j data at low Re , but under-predicting as Re increases. Similar observations were also observed for tube bank geometries

(Beale and Spalding 1998). These previous CFD calculations provide some partial empirical evidence in support of the proposed rationale, and for the need to increase the effective viscosity/conductivity at higher Re . In addition to the present results, the book by Kays and London (1984) contained numerous other heat exchanger surfaces which exhibit similar profiles and are equally amenable to the simple regression procedure above. There are, however, numerous configurations for which the f vs. Re characteristic do not conveniently follow the simple form, Eq. (11), such as the cases shown in Figs. 3–5.

Under those circumstances, the definition of the turbulent viscosity (and conductivity) may be modified to admit higher order terms, such as cubic term in Eq. (7), ie.,

$f = a(d Re + c + 1/Re)^m$, and so forth. However it can be shown that cubic and quartic forms offer only limited improvement over Eq. (11), ie substantially higher-order polynomial or transcendental functions (Žukauskas, Skrinka et al. 2008) are required to obtain a very good fit to f and j curves of the form shown in, e.g., Figs. 3–5

Under such circumstance provided f and f_0 are well-defined, μ_{eff} may still be obtained using the technique described in this paper as follows. For example, if f_0 may be correlated in the form $f_0 = a/Re^m$, the task is particularly simple, and it is straightforward to show.

$$\frac{\mu_{eff}}{\mu} = Re \cdot \left(\frac{f}{a} \right)^{1/m} \quad (14)$$

Thus regardless of the form of the turbulent drag correlation, if a straight-line laminar region is defined, then Eq. (14) provides a means by which an estimate of the effective viscosity may be made. The same reasoning may be applied to the eddy diffusivity in the energy equation.

The cause of mixing in a heat exchanger does not need to be due to ‘turbulence’ in the commonly-perceived sense (though it could be). The $\overline{u'u'}$ and $\overline{u'T'}$ terms may be considered to arise because of volume averaging in the steady-laminar regime and/or and time averaging in the unsteady-laminar and truly turbulent regimes. For the former case these are then simply to be considered the volume-averaged products of the difference between local values u' , v' and average-interstitial values. At intermediate Reynolds numbers, it is well-known that there will be increased mixing due to macroscopic phenomena, rather loosely referred to as ‘free-stream turbulence’, and generally associated with vortex shedding, and shear-layer and wake instabilities in the flow, depending on the geometry.

The assumption here is that the fluctuations are isotropic so that the same μ_{eff} may be found from the distributed resistance (Forchheimer) term as occurs in the viscous (Brinkman) term, ie., that the eddy viscosity/diffusivity which results in augmenting the drag coefficient, f , also effects ‘mixing’ between the passages of a heat exchanger, as was discussed above. Whether a single scalar effective viscosity can account for diffusion and

inertial effect can only be ascertained by detailed experiments and/or by direct numerical simulation, whereby both the drag and the diffusion-rate of an obstacle-filled mixing-layer can be obtained. Such data are not available at the time-of-writing of the present work. Of course the diffusion terms may turn out to be negligibly small, for the case shown with $c = 1.24 \times 10^{-3}$, at $Re = 10\,000$, $\mu_{eff}/\mu = 13.4$, ie the length scale is approximately 13 hydraulic diameters.

If a single value of μ_T or k_T cannot effect this, then this is a reflection of the anisotropy of the problem. The anisotropy may be due to kinetic (fluid motion) or purely geometric factors. Butterworth (1977) argued that F_0 should be represented by a second-order tensor, an approach that has been adopted in various forms by others. This would seem to be a reasonable proposition if the heat exchanger elements are aligned in a rectilinear fashion, though less obvious e.g., for equilateral triangle and other non-rectilinear patterns, where the ‘principal directions’ are not orthonormal. Further corollaries would be to consider F_T , μ_{eff} and k_{eff} all as tensorial quantities. Clearly this would add substantial complexity to the problem.

If however, the cause of the anisotropy is actually geometric, in other words whether or not the different fluid streams can ‘see’ each other as it were; an alternative approach might be to consider the geometric volume fractions to be directional: Equation (1) presumes r to be independent of the choice of volume, a corollary of the so-called ergodic hypothesis. Considering, say, a tube-bank heat exchanger with different pitch-to-diameter ratio in the transverse and longitudinal directions, the area fractions, r_x and r_y do not equal the volume fraction r . It would therefore appear reasonable to re-write the viscous term in Eq. (1), as say, $\mu_{eff} (r_x \partial^2 \mathbf{u} / \partial x^2 + r_y \partial^2 \mathbf{u} / \partial y^2 + r_z \partial^2 \mathbf{u} / \partial z^2)$. This then would have the desired effect of cutting-off diffusion between ‘unmixed’ fluid streams for example when $r_y \rightarrow 0$ but $r_x \neq 0$. However, the area fractions are themselves open to interpretation in complex geometries, for example as shape or view-factors, ‘seen’ by the components of the flow, or as minimum, or mean aperture values etc. For many widely spaced geometries, the premiss $r_x = r_y = r$ would still appear to be a reasonable engineering assumption.

Whether to treat hydraulic resistance or volume fraction, r , as a tensorial quantity is somewhat subjective; the former may be the effect of the shape of the solid elements on the forces acting upon the fluid, whereas the latter is to be considered purely geometric. It is important to appreciate that there are fundamental differences between ordered arrays such as heat exchangers and random porous structures, and that different closure equations (Nakayama 2008) may be appropriate. The goal of this work is not to conduct an exhaustive study of possible closures but rather to propose a simple physical flow model, and to investigate the circumstances under which it is a good model. The situation is more complex still for heat transfer due to the additional heat conduction is also present within the solid matrix, as well as molecular and gross motion of the fluid. In order to further explore the merits of the proposed

algebraic methodology, detailed further detailed experimental data in addition to direct numerical simulation studies are required over a range of geometries and Reynolds numbers.

CONCLUSIONS

An algebraic methodology is proposed for computing the turbulent viscosity and Prandtl number (thermal conductivity) in heat exchangers, based on a value which approximates the value of f or j for any given geometry. The methodology does not resolve the fluctuations within individual passages of a heat exchanger, but rather the overall spatially-averaged behaviour. It is thus suitable for application to CFD simulations in cluttered spaces in industrial equipment, where detailed meshing is not possible. The method assumes that both f_0 and j_0 may be correlated in the form $f_0 = a/Re^m$ in the laminar regime, at low Reynolds numbers. The effective viscosity or conductivity is thus obtained from the back projection onto the laminar distribution. The method may readily be used, provided a well-defined laminar region is identified. A simple expression based on dimensional analyses is shown to generate good correlation with experimental results for compact heat exchanger geometry.

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