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CALCULATION OF THE RADIATED FIELD FROM THE VIBRATIONAL VELOCITY OF A WALL IN A REVERBERANT FIELD

by

R. J. DONATO

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LE CALCUL DU CHAMP DE RAYONNEMENT AU MOYEN DE LA VITESSE DE VIBRATION D'UN MUR DANS UN CHAMP DE REVERBERATION

SOMMAIRE

L'auteur mesure l'énergie de rayonnement d'un mur qui vibre et la compare avec la valeur obtenue d'une connaissance de la corrélation de la vitesse de vibration sur la surface du mur. On a mesuré avec soin un mur simple, mais il est montré qu'on peut prédire le cas d'un mur double avec une précision passable.

Calculation of the radiated field from the vibrational velocity of a wall in a reverberant field

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The radiated energy from a vibrating wall is measured and compared to the value calculated from a knowledge of the correlation of the vibrational velocity over the wall surface. Detailed measurements were made on a single wall, but it is shown that the case of a double wall may be predicted with fair accuracy.

Subject Classification: 2.9.

INTRODUCTION

Sound-transmission loss measurements are commonly made on walls (or floors) between two rooms. With a sound source in one room, the reduction in average sound-pressure level between rooms is measured; this information, together with the area of the separating partition and the absorption in the receiving room, permits calculation of the transmission loss. There are circumstances, however, in which this conventional method does not work well. For example, if there were too much extraneous noise in the receiving room the sound transmitted through the wall could not be measured; then it would be more convenient to measure the wall-surface velocity and calculate the transmitted pressure. In other situations there may be more than one component to be tested and vibration transducers fixed to each component would enable the effects of each to be computed. A similar case is where flanking problems are present and their effect must be identified and allowed for.

Various attempts have been made in the past to calculate the radiated pressure field from velocity measurements on a vibrating wall. Utley and Mulholland¹ made assumptions about the range of coherence of the sound incident on the panel and calculated expressions for the radiation impedance. Their choice of parameter was arbitrary, and they point out that one really needs to know the correlation function of the velocity over the wall. They still, however, found a large discrepancy between the measured velocity and that calculated from the mass law. This is somewhat surprising, inasmuch as the mass law predicts the transmission loss at the low frequencies much better than their 25-dB discrepancy would lead one to expect.

Price and Crocker,² using the statistical energy approach,³ once again obtained discrepancies at the lower frequencies. Even allowing for the correction made in the analysis, the difference is around 7 dB at the lowest frequency of 100 Hz. In all cases the velocity at low frequencies is higher than one would expect. Again the

measurements of transmission loss are higher than their theory predicts at the low frequencies. Better results were obtained when they used a measured value of radiation resistance rather than that calculated by Maidanik.⁴

It is important to consider how this radiation resistance was measured. Price and Crocker use a shaker attached to the wall and this is effectively a point source. Thus the Hankel k transform of the input force is given by

$$F(k) = 2\pi \int_0^\infty f(r) J_0(kr) r dr \quad (1)$$

and if the range of action of $f(r)$ is very small so that

$$\int_0^\infty f(r) 2\pi r dr = F_0,$$

then

$$F(k) = F_0. \quad (2)$$

Thus the wall is actuated by a wide-band k signal. When, on the other hand, the signal comes from a reverberant field, where the correlation function is $(\sin k_a r)/k_a r$ (where $k_a = \omega/c$, and c the velocity of sound in air), then the Hankel transform is $G(k)$, where

$$G(k) = 2\pi \int_0^\infty (\sin k_a r / k_a r) J_0(kr) r dr. \quad (3)$$

This equals zero for $k > k_a$, and

$$2\pi (k_a)^{-1} (k_a^2 - k^2)^{-\frac{1}{2}}, \text{ for } k \leq k_a. \quad (4)$$

Here the k spectrum is restricted to values less than or equal to k_a . Thus, it seems reasonable to suppose that the velocity obtained using a shaker will be relatively larger than that obtained using a reverberant incident sound field.

Although it is true that the above argument holds only for an infinite wall, the effect of limiting the dimensions will be to multiply the k spectrum by a

comb filter in the k domain so that the same general conclusions apply. On the other hand, the statistical energy analysis is probably correct, and the calculated radiation resistance holds for the resonant modes. We run into difficulties, however, when we consider the non-resonant contribution. The nonresonant effect becomes more important for a large wall because the Maidanik radiation resistance is proportional to the perimeter rather than the area.

The approach adopted in this paper is a quasiclassical one in that we assume the modes to be plentiful enough for the classical equations to be used (probably violated at the lower frequencies unless the bandwidth is large), but some allowance must be made for the finite size of the wall. Thus the incident sound field is modified by the aperture in such a way that a convolution must be made between the incident k spectrum and the k spectrum of the wall aperture. This has the effect of broadening the spectrum to beyond k_α and introducing a discrepancy between the pressure calculated from the velocity and that actually measured.

I. THEORY OF METHOD USED TO OBTAIN SPATIAL CORRELATION

One way to measure the spatial correlation between pick-ups attached to the vibrating wall in a narrow band of frequencies would be to pass the two signals through the filters first and then correlate. If we remember, however, that it is spatial correlation at zero time interval that we wish to measure, a more elegant procedure would be to cross-correlate the wideband signals first, convert the time-domain correlation signal into its Fourier components, and then operate on the resulting spectra with a suitable narrow-band filter.

If we consider the broad-band signals from the pick-ups to be represented by $x(t)$, $y(t)$, the impulse response of the two filters by $h(t)$ and the desired signals to be correlated by $e_1(t)$ and $e_2(t)$ we may write from the convolution theorem:

$$\begin{aligned} e_1(t) &= \int_{-\infty}^{\infty} x(t-\alpha)h(\alpha)d\alpha, \\ e_2(t) &= \int_{-\infty}^{\infty} y(t-\beta)h(\beta)d\beta. \end{aligned} \quad (5)$$

The cross correlation of $e_1(t)$, $e_2(t)$ may be now written as

$$\begin{aligned} E[e_1(t+\tau)e_2(t)] &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E[x(t+\tau-\alpha)y(t-\beta)]h(\alpha)h(\beta)d\alpha d\beta, \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_{xy}(\tau-\alpha+\beta)h(\alpha)h(\beta)d\alpha d\beta. \end{aligned} \quad (6)$$

If we carry out the normal Fourier conversions on this

expression, we finally obtain for the correlation function $R_{e_1e_2}(\tau)$

$$R_{e_1e_2}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} P_{xy}(\omega)[H(\omega)]^2 e^{j\omega\tau} d\omega, \quad (7)$$

where P_{xy} is the cross correlation at ω between the two wide-band signals, and $H(\omega)$ is the transfer function of the filter whose impulse response is $h(\tau)$.

As we are interested in the spatial cross-correlation, we put $\tau=0$ in the above expression and obtain

$$R_{e_1e_2}(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} P_{xy}(\omega)[H(\omega)]^2 d\omega. \quad (8)$$

If R_{xy} is an even function in τ , P_{xy} is an even function in ω and

$$R_{e_1e_2}(0) = \frac{1}{\pi} \int_0^{\infty} P_{xy}(\omega)[H(\omega)]^2 d\omega. \quad (9)$$

If R_{xy} is not even in τ , we simply have to take the even component to obtain an equivalent expression.

II. METHODS OF DETERMINING SPATIAL CROSS CORRELATION

Three methods were used to determine the spatial cross correlation of the vibration of the wall as a function of frequency.

A. Method 1: Band-by-Band Correlation

The arrangement is illustrated in Fig. 1. The signals from the two channels to be correlated were passed through two identical filters, then through two integrators to convert the signals from accelerations to velocities. (This step is optional; we are measuring spatial correlations in the true sense of the word, where the cross correlation is normalized by dividing by the square roots of the autocorrelations of the two channels.) The two signals then enter an analog multiplier whose output is averaged with a long time constant and displayed.

B. Method 2: Analogue Operation on Wide-Band Signal

Both this method and the next method to be described use a hybrid correlation technique. The previous theoretical derivation has shown that we may first correlate in time using wideband signals and derive the zero time spatial correlation by subsequent Fourier analysis of the correlator output. The analog method described in this section is less attractive than the digital method of the next section, owing to the limitations of the correlator. The correlator has available at its output terminal the correlation of the two inputs, but the output is only one-sided in time. To do a Fourier analysis of the correlator output in time we need a

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FIG. 1. Narrow-band analog circuit.

double-sided time function, although it is theoretically possible to calculate the double-sided transform from the single-sided if both amplitude and phase relations are known for the input spectrum. With our equipment, however, we could only get the modulus of the power spectrum.

We therefore adopted the system shown in Fig. 2(a). The B channel input was delayed by the use of a subsidiary tape recorder. The signals were originally recorded on the multitrack FM recorder at 15 in./sec. They were played back at $1\frac{7}{8}$ in./sec and channel B signal passed through the subsidiary tape recorder running at $7\frac{1}{2}$ in./sec. The playback output was taken from the recorder and applied to the correlator. This gave a time delay between the two channels of 0.15 sec. Some prewhitening was included in each channel and 20 dB of amplification applied.

The remainder of the analyzing process is shown in Fig. 2(b). The correlator output is multiplied by a window shaping function in an analogue multiplier. This allows either 0.1 sec of the correlator output to be provided to a superheterodyne spectrum analyzer or a triangularly modulated version (Bartlett Window). The correlator output is made repetitive with a period of 63 msec and the frequency analyzer bandwidth taken to be 1 Hz. The output from the analyzer was frequency swept in time and a smoothed version applied to a level recorder operating in the dc mode.

C. Method 3: Hybrid Operation on Wide-Band Signal

In essentials, this method is similar to that described in the previous section, except that here the correlation output is obtained in digital form and recorded on a printer. The subsequent windows are applied by digital filtering in a computer. The basic arrangement is shown in Fig. 3. When possible, the windows were taken to be either a 1/3 or 1/1 octave.

Most of the data were obtained using Method 3, which was found to be more flexible than the other two. Some of the data from the other two methods were, however, used in the analysis.

III. DETERMINATION OF HANKEL TRANSFORM OF SPATIAL CORRELATION FUNCTION

From the Fourier transforms $P_{xy}(\omega)$ of the correlation function in time of two spaced detectors we have

FIG. 2. Wide-band analog circuits. (a) Delay circuit to correlator. (b) Circuit for producing spectrum from shaped correlation function.

obtained the spatial correlation at a particular frequency. In order to determine the sound power radiated from the wall, we now need to transform from the space domain to the wavenumber domain. As we are dealing with a two-dimensional distribution of correlation function, the normalized Hankel transform $H(k)$ will be used where

$$H(k) = \int_0^{\infty} R_{xy}(r) J_0(kr) r dr \quad (10)$$

and

$$R_{xy}(r) = \int_0^{\infty} H(k) J_0(kr) k dk. \quad (11)$$

Three methods were used which will now be described.

Method A. The first method used the direct computation of the Hankel transform, using a spatial window of either uniform weighting to a maximum correlation distance of 48 in. or a Hanning one, also to maximum distance. Although the method has the advantage of being the most direct, it does tend to weight the more distant points where the experimental accuracy in determining the correlation function is the least.

Method B. In this method we fit a curve to the experimentally derived correlation function, which we know

FIG. 3. Wide-band digital system.

must be convergent as the value of r is increased. The chosen curve is of the form $e^{-\alpha r} \sin k_0 r / k_0 r$. Although this choice is partly intuitive, we do at least know that $\sin k_0 r / k_0 r$ describes the correlation of the incident sound field. The exponential part includes departures from an ideal incident reverberant field to account for the transfer function of the wall. The value of k_0 was chosen by computer to correspond to the first zero of the correlation function, and α subsequently determined by a least squares fit.

Method C. This last method once again assumes that the correlation function is of the form $e^{-\alpha r} \sin k_0 r / k_0 r$ but employs a slightly more devious method to determine α and k_0 . Basically, the method makes use of the fact that the Fourier transform ensures a best fit on a least squares basis to the original function. Proceeding to calculate the Fourier transform, on the assumption that the correlation function may be expressed as above,

$$F(k) = \int_{-\infty}^{\infty} e^{-\alpha|r|} (\sin k_0 r / k_0 r) e^{-jkr} dr \quad (12)$$

$$= 2 \int_0^{\infty} e^{-\alpha r} (\sin k_0 r / k_0 r) \cos kr dr \quad (13)$$

$$= \int_0^{\infty} e^{-\alpha r} \frac{[\sin(k_0 + k)r + \sin(k_0 - k)r]}{k_0 r} dr. \quad (14)$$

The integrals in Eq. 12 are of the form

$$I = \int_0^{\infty} e^{-\alpha x} \frac{\sin mx dx}{x} \\ = \tan^{-1}(m/\alpha).$$

Thus,

$$F(k) = \frac{1}{k_0} \left[\tan^{-1} \left(\frac{k_0 + k}{\alpha} \right) + \tan^{-1} \left(\frac{k_0 - k}{\alpha} \right) \right] \quad (15)$$

$$= \frac{1}{k_0} \tan^{-1} \frac{2k_0/\alpha}{1 - (k_0^2 - k^2)/\alpha^2}. \quad (16)$$

Equation 3 may now be used to determine k_0 and α . If we assume that $\alpha \ll k_0$, then

$$F(k_0) \approx \frac{1}{2} F(0) \quad (17)$$

and

$$\left[\frac{dF(k)}{dk} \right]_{k=k_0} \approx \frac{1}{\alpha k_0}. \quad (18)$$

Also

$$F(0) \approx \frac{\pi}{k_0}. \quad (19)$$

IV. CORRELATION FUNCTIONS AND THEIR ASSOCIATED k SPECTRUM

Correlation measurements were made on a lightweight concrete wall (24 lb/ft²) having a coincidence

frequency in the 400-Hz region. The transducers were ceramic accelerometers mounted on short lengths of rod screwed into the wall. The accelerometers were spaced about the center of the wall, parallel to the length (10 ft) and height (8 ft). Spacings between adjacent transducers varied from 2 to 16 in. in a geometric progression. Five transducers were used so that we were able to make correlation measurements at 2, 4, 6, 8, 12, 16, 24, 32, 48, and 64 in. The wall formed the partition between two reverberant enclosures. The transmitting room was actuated from loudspeakers by a noise signal in the band 70 to 7000 Hz. The signals were processed in a wideband form and filtered in 1/3 or 1/1 octaves. For the wideband case we found it advantageous to split the frequency band into two regions by using a filter before the analog correlator. To analyze signals below 1 kHz we set the upper cut-off just above 1 kHz, and for signals up to 4 kHz we set the upper frequency at 5 kHz. The correlator measured one-sided correlation and displayed the results for 100 increments of τ , the time delay. Applying a window equal in width to the total delay gives a bandwidth of $(100\tau)^{-1}$. For lower frequencies (less than 1 kHz) $100\tau = 10$ msec, and for frequencies greater than 1 kHz, $100\tau = 2$ msec. Thus for the lower frequencies the sampling frequency was 10 kHz and for the higher frequencies 50 kHz. These are considerably higher than the Nyquist sampling frequency and were chosen primarily to improve the signal to noise ratio. In addition to the rectangular bandwidth a Hanning window was applied, increasing the bandwidths by a factor of 1.7.

Typical correlation functions, calculated by computer, are shown in Fig. 4. The associated k spectra

FIG. 4. Correlation coefficients at octave spacing.

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FIG. 5. Normalized $kF(k)$ vs frequency.

calculated from the Hankel transform are given in Fig. 5. One sees immediately that at the lower frequencies the spectra extend beyond $k_a = \omega/c$. Thus the finite size of the wall modulates the input correlation function and results in a convolution of the input k spectrum with that of the wall.

Simple reasoning may be applied to explain the form of the spatial correlation functions and their variation with frequency. Figure 6 shows the distance at which the first zero of the spatial correlation function occurs. We shall simplify the situation by assuming the wall area to be infinite. If the power k spectrum of the incident excitation is $F(k)$ and the impedance of the wall $z(k)$, then the corresponding k spectrum of the velocity is given by

$$[V(k)]^2 = F(k)/[z(k)]^2, \quad (20)$$

and the corresponding spatial correlation of the velocity will be proportional to $R_v(r)$, where

$$R_v(r) = \int_0^{k_a} \frac{F(k)}{[z(k)]^2} J_0(kr) k dk, \quad (21)$$

with

$$[z(k)]^2 = (A^4 m / \omega)^2 (k^4 - k_0^4)^2 + (2\rho c)^2 (1 - k^2/k_a^2)^{-1}, \quad (22)$$

$$k_0 = (\omega)^{1/2} / A,$$

$$A^4 = \frac{Eh^2}{12(1-\sigma^2)\rho_s},$$

E the Young's modulus of the plate, σ Poisson's ratio, h the thickness, ρ_s the density, and $m = \rho_s h$. The characteristic impedance of the air surrounding the wall is ρc .

As we are considering an infinite wall, the upper limit in the integral of Eq. 21 is k_a . For low frequencies $k_a \ll k_0$, so that apart from an extra contribution near $k = k_a$, $R_v(r)$ should have the same form as the correlation function of the applied sound field, i.e., $\sin k_a r / (k_a r)$ for a diffuse field. Zero correlation will occur when $k_a r = \pi$. For higher frequencies where $k_a \gg k_0$, $R_v(r)$ will have the form $J_0(k_0 r)$ and will have a zero when $k_0 r \approx 2.5$. These two points were calculated at $f = 250$ Hz and 4 kHz and are shown to fall on the curve of Fig. 6. For frequencies below 250 Hz the infinite wall situation is grossly violated, the upper

FIG. 6. Variation of first correlation zero with frequency: ● experimental, ○ theoretical.

FIG. 7. Average values of the ratio (first minimum/first maximum).

limit in the integral is appreciably larger than k_a and the corresponding value of r for zero correlation is reduced; this effect may be seen from the curve of Fig. 6.

One other feature of the correlation curve we can look at is the amplitude of the first trough. On the above reasoning, we would expect these to be 0.21 at low frequencies and 0.40 at higher frequencies. The results are not so accurate as those using the zero values owing to scatter in the results at the larger kr values. The trend is clear, however, and may be seen in the curve of Fig. 7.

V. EFFECT OF FINITE WALL

When we come to calculate the power W radiated per unit area from the wall, we take the Hankel transform of the velocity correlation function, i.e., $[v(k)]^2$, multiply by $[1 - k^2/k_a^2]^{-1/2}$ and integrate from 0 to k_a , i.e.,

$$W = \int_0^{k_a} \frac{[v(k)]^2}{[1 - k^2/k_a^2]^{1/2}} 2\pi k dk \rho c. \quad (23)$$

This expression contains $v(k)$, the normalized velocity, so that we must multiply by the rms velocity squared v_0^2 to measure the true power per unit area. The expression holds only for an infinite area wall. To allow for the fact that the wall is finite, we must apply a modulating function so that the higher separations contribute a smaller amount to the correlation function. We shall assume for simplicity that the wall is circular with a radius of 4 ft. Consider now a point distance r_1 from the origin as in Fig. 8. This point is correlated

FIG. 8. Diagram showing range of action of correlation function.

$R_0(r)$ with points distance r from it. As r_1 sweeps out a circle we may consider the annulus of area $2\pi r_1 dr_1$ receiving contributions from a swept area of $2\pi r_1 2\pi r$. For a fixed r this will hold for all annuli up to $r_1 = R - r$ and the total number of points contributing to the radiated power is

$$\int_0^{R-r} 2\pi r_1 2\pi r dr_1 = 2\pi r 2\pi \frac{(R-r)^2}{2}. \quad (24)$$

If the wall had been infinite, the amount of power radiated from a circle radius R would have been proportional to $R_0(r) 2\pi r \pi R^2$. Thus the modulating function is $(1 - r/R)^2$. It should be noted that this function is different from that mentioned in a previous paper⁵ where the form was $1 - (r/2R)^2$. This latter expression was obtained as an extrapolation from the linear problem, but it is now thought to be less accurate than the new expression. The effect on the results of the previous paper will be discussed later.

One further correction is made for the variation of the rms value of velocity over the wall face. We find by experiment that we may express the velocity as

$$V^2(r) = \frac{V^2(0)}{(1 + ar/R)^n}.$$

Thus the correction to apply is

$$\int_0^R \frac{2\pi r V^2(0) dr}{(1 + ar/R)^n} / \int 2\pi r V^2(0) dr.$$

Both a and n may be determined and thence the proper correction.

VI. COMPARISON OF MEASURED AND CALCULATED PRESSURES

Figure 9 shows the total correction to be applied to the wall velocity to be applied to the term $\rho c (V_0)^2 S$ for the power radiated from the wall. S is the area and V_0 the rms value of the velocity at the center. The com-

FIG. 9. Total decibel correction to be applied to velocity.

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FIG. 10. Comparison of measured and calculated receiving room pressures: ● calculated, ○ measured. (a) Single wall—using correlation function. (b) Double wall—using same correction as in (a). (c) Single wall—formula corrected. (d) Double wall—formula corrected.

parison of the measured receiving room pressure and the pressure calculated from the wall velocity is given in Fig. 10(a). The biggest discrepancy occurs at the higher frequencies and is most probably a resonance in the transducer mounting system. Figure 10(b) shows similar results for a double wall (9 lb/ft²). The correlation function was not measured for this wall and we applied the single wall correction. Clearly the results are not so good. This is not surprising, because the velocity k spectrum will be different in the two cases. Once again we found a discrepancy at the high frequencies. The velocity results were corrected in this case by comparing the accelerometer output with those from semiconductor strain gauges and a correction applied.

These results may be compared with those calculated using the method given in Ref. 5. There, considering the transmission through a double wall, we convolved the incident k spectra with the k spectrum of the modulating function of the correlation. Making allowances for the fact that we are now using $(1-\tau/R)^2$ rather than $[1-(\tau/2R)^2]$, we obtain the results of Fig. 10(d). It will be seen that they fit somewhat better. The procedure is repeated for the single wall, only now the analysis is slightly more accurate in that we will apply the modulating function to the velocity itself. For low frequencies, we assume $V(k)$ to be constant from $k=0$ to $k=k_a$, and we obtain the results of Fig. 10(c). The fit is almost as good as that using the measured correlation functions.

TABLE I. Correction for variation of rms velocity over wall face.

Frequency (Hz)	Correction (dB)
125	4
250	3
500	2
1000	1.5

In passing, one should perhaps mention the effects on transmission loss of the new modulating function. The lower frequencies (up to 200 Hz) would make the transmission loss 2 dB higher than those shown, and from 200 to 500 Hz the increase would be 1 dB. On the whole this improves the fit to the measured data.⁵

VII. CONCLUSIONS

The pressure calculated from the velocity k spectrum agrees well with the measured value. The theoretical derivation for the single wall case also predicts the pressure reasonably well. For the double wall the theoretical prediction is not so good. When applying the theoretical correction, we have taken the aperture modulating function to apply to the velocity of the surface of the wall rather than to the incident sound field. Thus in the double wall case we apply the modulating function to $(1-k^2/k_a^2)^{-1/2}$ which, although of the correct general shape, is perhaps not really peaked enough. By applying a more highly peaked function we would obtain the larger discrepancies between velocity and pressure that we need.

One interesting feature in the single wall case was that although the velocity k spectra showed some variation for different windows (and, hence, bandwidth) the correction to the radiated field showed remarkable consistency.

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