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Possibilistic approach for consideration of uncertainties to estimate structural capacity of ageing cast iron water mains

Solomon Tesfamariam, Balvant Rajani, and Rehan Sadiq

Abstract: Drinking water distribution networks form essential components of all urban centres. Water mains buried in the soil–backfill are exposed to different deleterious reactions, with the result being that the design factor of safety may significantly degrade, leading to structural failure. In particular, metallic distribution and trunk mains are subject to corrosion. Proactive pipeline management, which entails timely maintenance, repair, and renovation, can increase the service life of pipes. Several nondestructive evaluation techniques have recently become available to measure the remaining wall thickness of metallic pipes. In this paper, a previously developed analytical model based on Winkler-type pipe–soil interaction (WPSI) is cast in a “possibilistic” framework to translate the remaining pipe wall thickness to current structural factor of safety. The WPSI model takes into consideration external (traffic, frost, etc.) and internal (operating and surge pressures) loads, temperature changes, and loss of bedding support and the reduction of pipe structural capacity in the presence of corrosion pits. Uncertainties associated with the input data–parameters are handled using fuzzy arithmetic operations and interpreted through possibility theory. A Monte Carlo type random sampling method is carried out for performing sensitivity analyses to identify the critical data–parameters that merit further investigation.

Key words: water mains, pipe–soil interaction, uncertainty analysis, fuzzy arithmetic, possibility theory.

Résumé : Les réseaux de distribution d'eau sont parmi les composantes essentielles de tout centre urbain. Les conduites de distribution enfouies dans le sol/remplissage sont exposées à plusieurs réactions délétères et il peut en découler que le coefficient de sécurité se dégrade de manière importante, engendrant une défaillance structurale. Plus particulièrement, les conduites métalliques principales et d'alimentation sont sujettes à la corrosion. La gestion proactive des conduites, qui demande de la maintenance, des réparations et des rénovations en temps opportun, peut accroître la durée de vie des conduites. Plusieurs techniques d'évaluation non destructives pour mesurer l'épaisseur résiduelle des parois des conduites métalliques sont maintenant disponibles. Le présent article met un modèle analytique développé antérieurement et basé sur une interaction conduite-sol de type Winkler (WPSI) dans un cadre « possibiliste » afin de traduire l'épaisseur résiduelle des parois des conduites en un coefficient de sécurité structurale actualisé. Le modèle WPSI tient compte des charges externes (circulation, gel, etc.) et internes (pression de fonctionnement et coups de bélier), les changements de température et la perte de support de remplissage ainsi que la réduction de la capacité structurale des conduites en présence de piqûres de corrosion. Les incertitudes associées aux données/paramètres d'entrée sont traitées par des opérations arithmétiques floues et interprétées par la théorie des possibilités. Une méthode d'échantillonnage aléatoire de Monte-Carlo est effectuée pour les analyses de sensibilité afin d'identifier les données/paramètres critiques qui méritent une attention plus approfondie.

Mots clés : conduites de distribution, interaction conduite-sol, analyse d'incertitude, arithmétique floue, théorie des possibilités.

[Traduit par la Rédaction]

Introduction

The ability to reliably deliver safe potable water without major interruptions is paramount. The deterioration of age-

ing water mains is of major concern to water utilities and their customers. The financial costs to repair and replace these water mains are estimated to exceed Can\$1 billion annually in Canada alone. The implementation of a proactive

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asset-management strategy is essential to maintain a water distribution network that is both reliable and safe, given the limited availability of financial resources.

Figure 1 shows a risk-based approach to decision making for renewal of deteriorating buried pipes. The risk-based decision-making process entails quantifying the initiation of deterioration, condition evaluation, and determination of condition state and failure risk. Pipe inspection typically identifies different distress indicators, i.e., geometry of corrosion pits, frequency of pits along pipe length, number of broken wires and their frequency in prestressed concrete cylinder pipe (PCCP), etc. One of several approaches to translate distress indicators to condition state is to use a mechanistic (physical) model to determine the residual factor(s) of safety.

In the past few years, different nondestructive techniques (NDT) have become available (Jackson et al. 1992; Hartman and Karlson 2002; Lillie et al. 2004; Rajani and Kleiner 2004) to measure remaining wall thickness, corrosion pits (ductile iron), or graphitization depths (cast iron) along pipes. These NDT measurements have to be incorporated within a broad decision support system to assess the pipe condition, determine remaining service life, and make decisions whether to renew–replace the pipe and when to do it.

In most cases, a combination of circumstances leads to the pipe failure. It is very difficult to ascertain the precise cause of failure because all the operational factors, design parameters, or data that define a problem are known imprecisely and (or) recorded inaccurately, e.g., external (traffic, frost, etc.) and internal (operating and surge pressures) loads, temperature changes, loss of bedding support, pipe condition, and corrosion pit geometry. Any attempt to establish or estimate the condition of the pipe has a degree of uncertainty, given that many of the factors that contribute to pipe failure have significant uncertainties, either because of the large spatial variability (e.g., soils) or because of inherent variability (e.g., pipe material properties). Uncertainties in the determination of the remaining service life are further exacerbated by the difficulties associated with obtaining reasonable estimates of past, present, and future corrosion rates. Rajani et al. (1996) and Rajani and Tefamariam (2004) have developed Winkler-type pipe–soil interaction (WPSI) models (based on mechanics and hence termed mechanistic or physical models) that account for most of the predominant factors or data–parameters identified earlier. The WPSI may also involve model-associated uncertainties, which occur as a result of model formulation and model parameters–coefficients.

Most existing mechanistic models for pipe–soil interaction coupled with material failure theories are deterministic and provide point estimates for the factor of safety (FS). Deterministic models do not consider the uncertainties involved in the data and parameters and therefore may result in false precision. A possible approach to overcome this limitation is to estimate “probability” or “possibility” of pipe failure by using various methods such as Monte Carlo simulations, analytical methods (e.g., first- and second-order reliability methods), probability-bound analysis (p-boxes), random set theory, and possibilistic analysis using fuzzy arithmetic (Joslyn and Booker 2004).

This paper builds on our previous work (Rajani et al. 1996; Rajani and Tefamariam 2004) in which WPSI models were developed to determine stresses, strains, and displacements at any point along the length of a jointed pipe. In this paper these models are recast in “fuzzy” terms using a possibilistic approach to describe uncertainties in operational factors and input data as discussed earlier. In the following sections, material failure theories specific to cast iron mains are described and combined with fuzzy stress solutions to determine the fuzzy structural capacity in terms of factor(s) of safety (FS). It should also be noted that the discussion in this paper is limited to the translation of distress indicators in the form of corrosion pits to a condition rating expressed in term of factors of safety (box denoted by the broken line in Fig. 1). Although the focus in this paper is on cast iron mains, the proposed approach should be equally applicable to other pipe materials. Sensitivity analyses (through Monte Carlo type random sampling techniques) using rank-correlation coefficients are conducted to identify and evaluate the contributions of critical factors, parameters, and data that dictate the factor(s) of safety.

Pipe–soil interaction models

As noted previously, Rajani and Tefamariam (2004) recently developed an approach, based on the Winkler-type pipe–soil interaction model, that accounts for the unsupported length (likely to develop as a result of prolonged leakage or washout) and soil elastoplasticity. Further, the axial, flexural, and circumferential stress components were consolidated with the previously reported responses (Ugural and Fenster 1987; Rajani et al. 1996) to provide an overall picture of the response of buried water mains under the influence of earth, frost, and live loads, water pressure, temperature differential, unsupported length, and pipe–soil interaction. It must be noted that these responses consider the loss of support in the longitudinal direction only, and not in the in-plane direction.

Only succinct details of the WPSI models developed by Rajani et al. (1996) and later by Rajani and Tefamariam (2004) are given here to explain the extension of the model to include failure analysis of cast iron mains. In general, the stress components due to external loads σ_x^w , internal pressure σ_x^p , temperature differential σ_x^T , and longitudinal bending σ_x^f that contribute to the total axial stress (σ_x^a) are

$$[1] \quad \sigma_x^a = \sigma_x^w + \sigma_x^p + \sigma_x^T + \sigma_x^f$$

Similarly, the total hoop stress ($\sigma_\theta^{\text{Total}}$) has contributions from external loads (σ_θ^w), internal pressure (σ_θ^p), temperature differential (σ_θ^T), and longitudinal bending (σ_θ^f):

$$[2] \quad \sigma_\theta^{\text{Total}} = \sigma_\theta^w + \sigma_\theta^p + \sigma_\theta^T + \sigma_\theta^f = \sigma_\theta^w + \sigma_\theta^p + \sigma_\theta^T - \nu_p \sigma_x^f$$

where ν_p is the Poisson’s ratio. The thermal axial (σ_x^T) and thermal hoop (σ_θ^T) stresses in eqs. [1] and [2] are a consequence of temperature differential (ΔT) between the inside of the pipe and the surrounding soil. Each of the stress components in eqs. [1] and [2] can be estimated using the solutions provided in the references cited earlier.

Fig. 1. Schematic diagram for water mains from installation to renewal–replacement. The discussion in this paper is limited to those aspects within the box formed by the broken line.

Failure theories

In-service water mains or pipes are subjected to continuous deleterious reactions and internal and external loads that undermine the intended design factor of safety (FS). Consequently, the service life is significantly reduced if existing stresses on structurally deteriorated pipe exceed the expected or admissible design loads or stresses. Pipe failure is defined as an event in which the factor of safety (FS) falls below a critical value, FS_{cr} (usually set to 1), i.e., $FS < FS_{cr}$. Cast iron, a brittle material, typically fails through fracture at strains of 0.5%, rather than through yielding. Thus, the failure of brittle materials such as cast iron is dictated by its ultimate strength. In contrast, yield strength is used to describe failure of ductile materials like that in ductile iron mains. Two specific failure criteria (theories) applicable to cast iron, namely, in-plane (ip) and biaxial distortion energy (de), are discussed in the following. The omission of subscripts “ip” and “de” in reference to FS means that either or both criteria are applicable. The “ip” criterion, as the name implies, envisages in-plane failure modes, and the “de” criterion explains longitudinal and (or) flexural action failure modes. In most circumstances the failure criterion that leads to an FS that is below FS_{cr} dictates the failure mode.

In-plane failure criterion for cast iron

Several theories (e.g., Ugural and Fenster 1987) explain the failure of cast iron materials, but the most common failure theory used for the design of cast iron mains (AWWA 1977) is based on interaction curves developed by Schlick (1940). The AWWA (1977) design guidelines C101-67 treat a cast iron pipe as a rigid structural ring element; as a result,

loads are supported by resistance in the in-plane direction and not in longitudinal bending, since a well-constructed new pipe is not expected to undergo differential lateral movement–displacement (along the length of the pipe). Experimental work carried out by Schlick (1940) showed that the failure of a grey cast iron pipe is governed by parabolic interaction curves (Fig. 2) expressed in terms of normalized internal pressure, p_i/P , and normalized external bearing load, w/W :

$$[3] \quad (w/W)^2 + p_i/P \leq 1$$

where p_i is the internal pressure; w is the external bearing load; $W = \pi t^2 \sigma_r / 3(D + T)$ is the three-edge crushing (ring) load necessary to cause failure in the absence of internal pressure, where σ_r is the rupture modulus, D is the pipe diameter, and t is the pipe wall thickness; and $P = 2\sigma_{bu}t/D$ is the internal bursting pressure necessary to cause failure in the absence of external load, where σ_{bu} is the burst tensile strength. The external load, w , in the AWWA (1977) design guidelines C101-67 considers traffic and overburden loads; however, additional frost loads induced during cold seasons in frost-susceptible soils or backfills are not explicitly considered. Rajani and Makar (2000) suggested that the frost load could be estimated as a percentage, ranging from 0% to 100%, of the earth load in accordance with the frost load theory developed by Rajani and Zhan (1996).

AWWA (1977) advocates the application of a factor of safety of 2.5 to internal pressure (p_i) and external loads (w) for the design of cast iron pipes. Now one would use a “load factor,” since the design is based on limit states. Melchers (1987) noted that it makes little difference which term is used in the design procedure as suggested by AWWA

Fig. 2. In-plane failure criterion interaction diagram for cast iron water mains (modified after Watkin and Anderson 2000).

(1977), since the same (proportional) factor of safety of 2.5 is advocated for both internal pressure (p_i) and external loads (w). An additional pipe wall thickness of 2.03 mm (0.08 in.) is suggested for corrosion (graphitization) allowance, which should increase the FS of new pipes to values beyond that suggested for structural resistance only (Moore 1939). Nevertheless, as corrosion or graphitization pits initiate randomly and subsequently grow over time, the structural FS of a pipe diminishes (Rajani and Makar 2000; Rajani and Tefamariam 2004) until failure occurs ($FS < 1$), even though external and internal loads may not have changed substantially.

The in-plane failure criterion (eq. [3]) can be rewritten in terms of the in-plane factor of safety, FS_{ip} , as follows:

$$[4] \quad FS_{ip}^2 (w'/W)^2 + FS_{ip} (p_i'/P) = 1$$

where w' and p_i' are working loads and pressures, respectively. AWWA (1977) also specifically indicates that two load cases should be examined for design. Load case 1 refers to the consideration of normal operating pressures, earth and traffic loads, and load case 2 refers to the consideration of normal operating pressures, transient pressure and earth loads, which recognizes the low probability of concurrence of transient pressures and traffic loads. Although working loads and working pressures w' (and p_i') may not vary with time, its resistance strength (as determined by σ_r and σ_{bu}) is undermined as a result of corrosion-induced deterioration. Failure is imminent when $FS_{ip} < 1$. As illustrated in Fig. 2, longitudinal split failures are likely when normalized pressures (p_i'/P) dominate, and crushing failures are likely when normalized external loads (w'/W) dominate. A combined failure mode is likely to occur, however, when the normalized external loads and pressures are in mid-range, arbitrarily set at, for example, $0.4 < p_i'/P < 0.8$ and $0.4 < w'/W < 0.8$.

Biaxial distortion energy failure criterion for cast iron

The development of cast iron fracture criteria under biaxial (hoop and axial) stresses dates back to the 1950s (Cof-

fin 1950; Fisher 1952; Mair 1968). These researchers investigated the response of cast iron in the presence of graphite flakes. Typically, the ferrite-graphite matrix transmits the load when the cast iron is in compression; however, the graphite flakes act as stress raisers when subjected to tension. Consequently, the ultimate tensile strength of a cast iron pipe is substantially lower than its ultimate compressive strength. Mair (1968), based on his experimental work and that of Coffin (1950) and Fisher (1952), arrived at the conclusion that the cast iron failure criterion is best represented by the distortion energy theory developed by von Mises. This theory states that failure by yielding or fracture occurs when the distortion energy per unit volume in a state of combined stress at any point in the body becomes equal to that associated with yielding or fracture in a simple tension test (Ugural and Fenster 1987). The biaxial failure criterion based on distortion energy theory is

$$[5] \quad \sigma_1^2 - \sigma_1 \sigma_2 + \sigma_2^2 = \sigma_u^2$$

where σ_1 and σ_2 are biaxial stresses, and σ_u is the ultimate tensile strength. The distortion energy failure criterion for biaxial stresses (eq. [5]) can be rewritten for cast iron material in terms of the distortion energy factor of safety, FS_{de} , and stress concentration factor, K , as follows:

$$[6] \quad K\sigma_i = \sigma_u / FS_{de} \quad \text{for } \sigma_1 > 0 \text{ and } \sigma_2 > 0$$

($K = 3$, tension-tension)

$$(K\sigma_1)^2 - K\sigma_1\sigma_2 + \sigma_2^2 = (\sigma_u / FS_{de})^2 \quad \text{for } \sigma_1 > 0$$

and $\sigma_2 < 0$ ($K = 3$, tension-compression)

$$(K\sigma_1)^2 - K\sigma_1\sigma_2 + \sigma_2^2 = (\sigma_u / FS_{de})^2 \quad \text{for } \sigma_1 < 0$$

and $\sigma_2 < 0$ ($K = 1$, compression-compression)

The stress concentration factor, K , discussed here should not be confused with that typically used to calculate stresses in the presence of defects with specific geometry, although it has the same connotation because the stress concentration factor discussed here represents an aggregate reduction in tensile strength in the presence of carbon flakes that act as stress raisers.

Figure 3 illustrates the interaction diagram for axial and hoop stresses that correspond to a specific pipe design as expressed by eq. [6] for cast iron with an ultimate strength of $\sigma_u = 206$ MPa. The outer and inner strength envelopes in Fig. 3 represent stress conditions for factors of safety (FS_{de}) of 1.00 and 2.50, respectively. An increase in deterioration diminishes the failure envelope shown in Fig. 3, and the deterioration may compound with the increase in biaxial stresses. Consequently, failure is imminent when $FS_{de} < 1$. The longitudinal, combined, and circumferential failure modes are depicted in Fig. 3. As indicated earlier, AWWA (1977) did not consider longitudinal action, and therefore the selection of load cases does not arise for design on this basis. Therefore, it is deemed appropriate to use the same two load cases adopted for longitudinal analyses and use distortion energy failure criterion, as is customary for in-plane analyses.

Fig. 3. Failure envelopes for cast iron in accordance with biaxial distortion energy failure criterion.

Residual ultimate strength

The ultimate strength referred to in eq. [3] (by way of definition of P) and eq. [5] is pertinent to cast iron that has no defects or corrosion (or graphitization) pits, i.e., intrinsic material property. The corrosion pits diminish the intrinsic material strength of cast iron to the so-called residual ultimate strength (σ_{ur}) in accordance with fracture mechanics theory (Rajani et al. 2000) as follows:

$$[7] \quad \sigma_{ur} = \frac{\alpha_{ur} K_q}{\beta (d/t) \sqrt{a_n}}^s$$

where α_{ur} and s are constants; K_q is the provisional fracture toughness; d is the corrosion pit depth; $a_n = L_1 d$ is the lateral dimension of a pit (in the absence of data, the multiplier L_1 is assumed to have a value in the range of 1–5); and $\beta = a_1 (d/t)^{b_1}$ is a geometric factor, where a_1 and b_1 are constants. Rajani et al. (2000) established fracture toughness and other related parameters (K_q , d , a_1 , b_1) based on limited experimental tests conducted on cast iron specimens taken from cast iron pipe samples. Others (ASME 1991) have developed similar relationships for materials such as steel. It must be emphasized that the application of residual strength is most suitable (Rajani and Makar 2000) when stresses are calculated nominally, without the presence of corrosion pits, rather than rigorously, with the presence of corrosion pits.

Uncertainty analyses

Models often involve input data or parameters with uncertainties, which are commonly represented by random variables with known or assumed probability distributions (Ross 2004) or ranges (interval) of values based on experience and engineering judgment. In addition to many other taxonomies of uncertainty, uncertainties can be broadly grouped under two major classes. The first class of uncertainty is model-associated, which includes the model formulation itself and

coefficients–parameters involved in the model. This uncertainty is the result of the abstraction (oversimplification) of natural processes. The second class of uncertainty is associated with the state of input data, which itself can be classified into two main types. Type I (variability, aleatory) uncertainty arises as a result of natural heterogeneity or stochasticity which cannot be reduced. Type II (epistemic) uncertainty is due to partial ignorance that results from measurement and systematic errors or subjective uncertainty. It is postulated in this paper that epistemic uncertainty (incomplete knowledge and (or) lack of data) dominates the decision analysis of the problem at hand. Contrary to aleatory uncertainty, the epistemic uncertainty can be reduced by collecting more information and data. It plays an important role when the evidence base is small, i.e., sparse data situation. High consequences associated with failures of major water supply lines make it vital to analyze these uncertainties (Ferson and Ginzburg 1996; Ferson et al. 2004).

Different uncertainty analysis techniques have been used to study the reliability of cast iron pipes. Ahammed and Melchers (1994) used the first-order reliability method (FORM) to incorporate uncertainties in the input parameters. Sadiq et al. (2004) used Monte Carlo (MC) simulations based on Latin hypercube sampling to develop a hazard function of time to failure of cast iron pipes. Both MC simulations and FORM methods are parametric techniques, which assume that the model inputs are random variables that can be represented by probability density functions. Such probability distributions can best be derived from statistical data that adequately represent the populations of interest. Unfortunately, such statistical samples are not always available for most buried pipe infrastructure.

In this paper, fuzzy set theory is used to represent uncertainties in input parameters. This theory is able to deal effectively with epistemic uncertainties (including subjective judgment) and allows approximate reasoning. It also has an intrinsic ability to propagate uncertainties through various stages that constitute a model (Zadeh 1965). Fuzzy-based techniques are a generalized form of interval analysis used to address uncertain and (or) imprecise information. Fuzzy sets qualify as fuzzy numbers if they are normal, convex, and bounded (Klir and Yuan 1995). The use of fuzzy-based techniques obviates the requirement to make assumptions about the type of probability distribution, as required in MC simulations or FORM. In cases where some input data or parameters are supported by statistical evidence to establish probability density functions, these functions can be combined with fuzzy numbers to form a hybrid approach (Guyonnet et al. 2003).

A fuzzy set theory is an extension of the classical set theory (in which x is either a member of set A_i or not) in that x can be a member of set A_i with a membership μ_x . A fuzzy number describes the relationship between an uncertain quantity x and a membership function μ_x , which ranges between 0 and 1. Triangular fuzzy numbers (TFNs) are used here for simplicity, although any normal and convex shape is possible. The selected shape should be justified by available information (Guyonnet et al. 1999).

Another important concept related to fuzzy sets requires the introduction of the α -cut level, denoted as A_i^α . The A_i^α , a subset of A_i , represents a cut on a fuzzy set A_i and consists

Fig. 4. Possibility distribution for factor of safety and illustration of the α -cut concept.

of an interval for which membership $\mu_{A_i}(U) \geq \alpha$, where U is the universe of discourse. Figure 4 illustrates the α -cut concept applied to the factor of safety. At $\alpha = 0.4$, the subset $FS^{0.4}$ represents the FS interval of [0.18, 1.65]. The α -cut can be used to form a fuzzy confidence band, which can be viewed as a possibilistic confidence interval analogous to the probabilistic confidence interval. The α -cut concept is used later to perform sensitivity analyses.

Possibility theory

A possibility distribution represents a state of knowledge about an issue, distinguishes between what is “plausible” and what is less plausible and between what is “surprising” and what is “expected” (Dubois and Prade 1988). Earlier, the idea of “possibility” was supported by the famous economist Shackle (1961), who stated “... a man cannot, in general, tell what *will* happen, but his conception of nature of things, the nature of the men and their institutions and affairs, and of the non-human would enable him to form a judgment as to whether any suggested thing can happen.” Zadeh (1978) later formulated the contemporary possibility theory as an extension of fuzzy set theory, in which a possibility distribution is numerically equal to the membership function. In this paper, uncertainty analysis using fuzzy arithmetic is performed by TFNs of input parameters. The computed value of factor of safety is also a TFN, which can be interpreted as a possibility distribution.

Epistemic uncertainty in possibility theory is described using “dual measures” of possibility denoted by Π and necessity denoted by N (Dubois and Prade 1988). The dual measures of possibility theory can be interpreted as the Dempster–Shafer plausibility and belief, respectively (Alim 1988). In a possibilistic framework, the “possibility of failure” is defined in terms of the fuzzy factor of safety by the hypothesis, $FS \leq FS_{cr}$. It has been shown (Joslyn and Booker 2004) that the cumulative distribution function is bounded by a possibility distribution, i.e., “true probability” measure of failure lies between necessity N and possibility Π measures.

The possibility and necessity measures in terms of FS can be defined by the following relationships (Dubois and Prade 1988):

$$[8] \quad \Pi(FS \leq FS_{cr}) = \sup_u \min[\mu_{FS}(u), \mu_{FS_{cr}}(u)]$$

$$[9] \quad N(FS \leq FS_{cr}) = \inf_u \max[1 - \mu_{FS}(u), \mu_{FS_{cr}}(u)]$$

where $\Pi(FS \leq FS_{cr})$ is read as the possibility that $FS \leq FS_{cr}$; $\mu_{FS}(u)$ is the membership function of FS for any value u ; $\mu_{FS_{cr}}(u)$ is the membership function of FS_{cr} for any value u ; $\sup_u$ is Supremum, the largest value over the universe of discourse of u ; min is the minimization operator; $\inf_u$ is infimum, the smallest value over the universe of discourse of u ; and max is the maximization operator. How the dual measures of necessity N and possibility Π can be interpreted in the present context is discussed later in the section titled Sensitivity analyses.

Deriving possibility (risk) of failure to make decision

In possibilistic analysis, epistemic uncertainty manifests itself as an interval of dual measures $[N, \Pi]$, which represents ignorance or lack of knowledge. Therefore, the larger the interval, the bigger the epistemic uncertainty, and vice versa. This raises the issue of what value of possibility of failure should be appropriate to make a decision or manage risk. There are three common approaches for decision-making under ignorance, which can be related to the attitude of a decision-maker. The first approach accounts for a pessimistic attitude, where Π is selected as a representative value for the possibility of failure. The second approach accounts for an optimistic attitude, where N is selected as a representative value for the possibility of failure. The third approach is normative or taking a compromising attitude, which uses the Laplace principle of insufficient reason and takes the arithmetic average of Π and N to derive the possibility of failure. These three attitudes have one-to-one correspondence to *plausibility*, *belief*, and *pignistic* probability within the Dempster–Shafer framework (Smets 2000). These three attitudes are special cases of Hurwicz decision strategy, which defines a level of optimism or degree of *orness*, $\lambda_d \in [0, 1]$ using the following function:

$$[10] \quad \text{possibility or risk of failure } (FS \leq FS_{cr}) = \lambda_d N + (1 - \lambda_d) \Pi$$

Table 1. Interpretation of dual measures in possibility theory as deterioration increases and the hypothesis FS $\leq$ FS_{cr}

Scenario	Π	N	Evidence that pipe will fail		Evidence that pipe will not fail	
			No support	Support	No support	Support
1	0	0	True			True
2	<1	0	Partially true	Partially true		True
3	1	0		True		True
4	1	<1		True	Partially true	Partially true
5	1	1		True	True	

Note: For example, scenario 1 can be read as “it is true that the evidence has no support for the hypothesis that pipe will fail” and “it is true that the evidence has support for the hypothesis that pipe will not fail.”

This function reduces to Π when $\lambda_d = 0$ (the lowest possible optimism); to N, i.e., optimistic attitude, when $\lambda_d = 1$ (the highest possible optimism); and to an arithmetic average of Π and N when $\lambda_d = 0.5$ (the compromising optimism). Therefore, the Hurwicz decision strategy can help derive the possibility (risk) of failure at any desired level of optimism using an interval of $\lambda_d \in [0, 1]$. The concept of Hurwicz decision strategy can be further generalized using ordered weighted average (OWA) operators. The OWA operators are beyond the scope of this paper, and interested readers are referred to Yager (1988) for details.

The interpretation of Π and N using eqs. [8] and [9], respectively, is illustrated in the figures with Table 1, which show five scenarios where factors of safety decrease with an increase in deterioration. Scenario 1 shows that the possibility distribution (TFN) of FS is greater than FS_{cr} = 1. Thus, both possibility and necessity measures are equal to 0. Thus, scenario 1 can be read as “it is true that the evidence has no support for the hypothesis that pipe will fail” and “it is true that the evidence has support for the hypothesis that pipe will not fail.” These two statements are complementary in probabilistic framework but they are not in the possibilistic framework due to the existence of ignorance or epistemic uncertainty.

With increasing deterioration, the lower vertex of the FS intercepts with FS_{cr} = 1. Hence, $0 < \Pi < 1$ and N = 0 can be

related to the physical condition of the pipe that “it is true that the evidence has *partial support* for the hypothesis that pipe will fail.” Similar interpretations can be extended to scenarios 3 and 4. Scenario 5, an extreme case, illustrates that “it is true that the evidence has *support* for the hypothesis that pipe will fail” and “it is true that the evidence has *no support* for the hypothesis that pipe will not fail.”

Integration of uncertainty analysis with pipe-soil interaction models and failure theories

The data-parameters required for the pipe-soil interaction models discussed in an earlier section can be grouped into three broad categories: pipe properties (geometry and material); soil properties (unit weight, Poisson’s ratio, internal friction, etc.) and trench characteristics (trench depth, trench width); and traffic type and operational conditions (internal pressure, transient pressure, temperature variation, unsupported length, etc.). The uncertainties in the data-parameters that define a typical buried 150 mm diameter (6 in.) cast iron main are shown in Table 2 in terms of TFNs (minimum, most likely, maximum). The uncertainties (vagueness) expressed for the data-parameters are based on the authors’ best judgment and experience. Crisp (deterministic) values are assigned to properties known with near certainty.

Table 2. Input parameters for pipe, soil, and operational conditions for 150 mm diameter cast iron mains.

	Min.	Most likely	Max.
Pipe			
Nominal diameter, D (mm)		150	
Wall thickness, t (mm)		13.12	
Pipe length, $2L$ (m)		6.0	
Elastic modulus, E_p (MPa)	196 000	206 000	216 000
Tensile strength, σ_u (MPa)	132	207	282
Ring modulus of rupture, σ_r (MPa)	235	310	385
Bursting tensile strength, σ_{bu} (MPa)	94.8	145.0	195.0
Poisson's ratio, ν_p	0.24	0.26	0.28
Fracture toughness, K_{Ic} (MPa·m ^{1/2})	7	10	13
Thermal coefficient, α_t ($\times 10^{-6}/^\circ\text{C}$)		10.5	
Soil			
Elastic modulus, E_s (MPa)	90	100	110
Poisson's ratio, ν_s	0.25	0.30	0.35
Soil unit weight, γ_s (N/m ³)	22 488	22 988	23 488
Trench characteristics and operational conditions			
Trench depth, H (m)	1.47	1.52	1.57
Trench width, B_d (m)	0.71	0.81	0.91
Unsupported length, b (mm)	0	203	406
Earth load and traffic load, q (N/mm)	25.3	34.3	45.8
Water pressure, p_i (kPa)	172	345	689
Transient water pressure, p_t (kPa)	621	827	1034
Remaining wall thickness, $t_{\min}$	$t_{\min} - 10\%$	$t_{\min}$	$t_{\min} + 10\%$
Temperature difference, ΔT ($^\circ\text{C}$)	0	-14	-28

Table 3. Common fuzzy arithmetic operations using two triangular fuzzy numbers.

Operator	Formula	Result
Summation	$A + B$	$[a_1 + b_1, a_2 + b_2, a_3 + b_3]$
Subtraction	$A - B$	$[a_1 - b_3, a_2 - b_2, a_3 - b_1]$
Multiplication	$A \times B$	$[a_1 \times b_1, a_2 \times b_2, a_3 \times b_3]$
Division	A/B	$[a_1/b_3, a_2/b_2, a_3/b_1]$
Scalar product	$Q \cdot B$	$[Q \cdot b_1, Q \cdot b_2, Q \cdot b_3]$
Exponent	B^n	$[b_1^n, b_2^n, b_3^n]$

Note: The values of A and B are positive and, if negative numbers are used, the corresponding minimum and maximum values have to be selected. $A = [a_1, a_2, a_3]$; $B = [b_1, b_2, b_3]$; $a_1 < a_2 < a_3$; $b_1 < b_2 < b_3$; a_i and $b_{i(i=1 \text{ to } 3)} > 0$; $n > 0$; $Q > 0$.

The solutions for each of the stress components in eqs. [1] and [2] and the failure theories criteria (eqs. [4] and [6]) were rewritten in terms of fuzzy numbers to obtain fuzzy factors of safety. As noted previously, fuzzy factors of safety reflect the uncertainties present in the input data-parameters. Commonly used fuzzy arithmetic operations are shown in Table 3. Fuzzy arithmetic operations require extreme care, and specific rules and applicable procedures (Klir and Yuan 1995; Cooper et al. 2000) must be followed to ensure reliable outcomes, e.g., simplification of equations prior to establishing their fuzzy form.

In general, the proposed method can be used for design and (or) condition assessment of buried pipes. Condition assessment is usually exercised for ageing mains after inspec-

tion to determine their current factor of safety. Sensitivity analysis is pursued to identify the influence of different input data-parameters in the safety of a structure. In this paper, only sensitivity analysis and condition assessment are addressed, primarily because they illustrate the versatility of fuzzy sets to assess uncertainties and how pipe condition can be assessed in practical circumstances.

Sensitivity analyses

This paper describes two types of sensitivity analyses. The first considers (Fig. 5) four distinct scenarios (A_s , B_s , C_s , and D_s) of how the calculated fuzzy factors of safety (FS_{dc}) decrease with the increase in unsupported bedding length and corrosion pit depth. Each scenario in Fig. 5 has a TFN (shaded area) representing the membership to (minimum, most likely, maximum) fuzzy factors of safety. The long tail (positively skewed) in the TFN of the factor of safety scenario A_s , with no loss of wall thickness (no corrosion) and no loss of bedding support, diminishes towards scenarios B_s , C_s , and D_s with reduced wall thickness and increased unsupported length. In accordance with the central limit theorem (Melchers 1987), the product of a large number of random variables approaches a logarithm-normal distribution, irrespective of the individual distributions of the random variables. The observed TFNs demonstrate an analogous behaviour in the possibilistic analysis described here. The long tail in the membership for fuzzy factor of safety (of scenario A_s) indicates that in a new pipe most input-

Fig. 5. Variation of fuzzy factor of safety with remaining pipe wall thickness and unsupported bedding length.

parameters play a significant role in the determination of the safety of the pipe at the design stage. As the pipe deteriorates, however, corrosion plays a dominant role as indicated by the short tails of the TFNs representing FS in scenarios C_s and D_s. The fuzzy factor of safety decreases noticeably as the remaining wall thickness decreases. The situation (scenario C_s, with remaining wall thickness equal to 50% of original thickness) is exacerbated when the water mains experience loss of bedding support (scenario D_s, with unsupported length of 0.5 m), which would likely happen if the pipe developed an undetected leak.

Figure 5 also provides the possibility, Π , and necessity, N , measures for the possibility of failure. Scenario A_s indicates a very low possibility of failure. Scenario B_s indicates that, although failure is entirely possible, it is only 16% sure. Scenarios C_s and D_s indicate 51% and 100% surety of failure, respectively.

The second type of sensitivity analysis is not restricted to assessing the influence of two specific variables but is broad in the sense that it assesses the outcome of variations in any of the input data-parameters (Cullen and Frey 1999). This type of analysis also serves to identify the important parameters to prioritize additional data-collection efforts. It can also highlight parameters that can be de-emphasized or perhaps eliminated altogether without having a major impact on responses predicted by the model.

Several methods are available (Cullen and Frey 1999) to conduct a sensitivity analysis, each with advantages and disadvantages. A sensitivity analysis is commonly carried out using random sampling (Monte Carlo type simulations) where the probability distributions for input data can be either assumed or derived from observations. Thereafter, the rank correlation method (Cullen and Frey 1999) is applied to the results of the Monte Carlo simulations to identify input data-parameters that dominate the output. The rank correlation method involves the determination of correlation coefficients,

which measures the strength of the linear relationship between two variables. In statistics, procedures can be either parametric, which invariably requires that variables have normal distributions, or nonparametric, where assumptions about the distributions of variables are not essential. The Pearson product moment correlation (parametric) coefficient quantifies the relationship between the variables in the raw or transformed metric, i.e., it measures the extent of linearity of a relationship between two random variables. The non-parametric correlation (Spearman rank correlation coefficient, ρ_s) measures the strength of the relationship between the ranks of the data, i.e., it measures the extent of monotonicity of a relationship between two random variables. The Spearman rank correlation coefficient is used in the sensitivity analysis here because the input data-variables may be normally distributed; this condition cannot always be guaranteed for output (factor of safety). The Spearman rank correlation coefficient (ρ_s) between two sets of variables is obtained from the summation of the difference of ranks (d_r) squared, i.e., $\rho_s = 1 - 6 \sum d_r^2 / R(R-1)$, where R is the number of ranked values of iterations-realizations.

The sensitivity analyses based on the rank correlation method were applied in the context of fuzzy input data generated randomly (1000 iterations) using the α -cut concept of fuzzy sets. The procedure for the sensitivity analysis is schematically depicted in Fig. 6, and the general pseudo-code procedure is explained as follows:

FOR $j = 1$ to R , where R is the number of iterations in the sensitivity analysis

FOR each input, i to S , where S is the number of input data-parameters, Monte Carlo type random sampling, $\alpha_{i,j}$ level (uniformly distributed), compute $x_{i,j}$ at $\mu = \alpha_{i,j}$, where $x_{i,j}$ is the input data-parameter

NEXT i

Fig. 6. Schematic diagram for Monte Carlo type possibilistic sensitivity analysis procedure. In the tornado graph, the percent contribution sum up to 1, the negative sign is shown to represent negatively correlated data.

IF failure criterion = “in-plane failure” THEN

$$FS_{ip} = f(x_{i,j}, \dots, x_{S,j})$$

IF failure criterion = “distortion failure” THEN

$$FS_{de} = f(x_{i,j}, \dots, x_{S,j})$$

NEXT j

Rank FS after R iterations ($FS_{(1)} \dots FS_{(j)} \dots FS_{(R)}$), determine the Spearman rank correlation, $\rho_1 \dots \rho_s$, normalize and plot the $\rho_1 \dots \rho_s$ on a tornado graph

Sensitivity analyses were conducted for two cast iron pipe sizes, namely 150 mm (6 in.) and 500 mm (20 in.) buried at a depth of 1.52 m (60 in.). Each pipe had two different re-

maining wall thicknesses (t_{min}) of 75% t and 50% t and unsupported lengths (b). The unsupported lengths were assumed to be proportional to the pipe diameter (D) and to increase as the remaining pipe wall thickness decreases, i.e., for small-diameter (150 mm) mains, unsupported lengths of 1D and 4D with remaining pipe wall thicknesses of 75% t and 50% t , respectively; and for large-diameter (500 mm) mains, unsupported lengths of 1D and 3D with remaining pipe wall thicknesses of 75% t and 50% t , respectively. Pipe sizes were selected to investigate the influence of input data-parameters on the behaviour of the small- and large-diameter pipes. Results of the sensitivity analysis are plotted in a tornado graph. The tornado graphs facilitate visualization of the positive or negative influences (percent contribu-

Fig. 7. Sensitivity of factor of safety input variables for 150 and 500 mm diameter cast iron mains in accordance with the in-plane failure criterion.

Fig. 8. Sensitivity of factor of safety input variables for 150 and 500 mm diameter cast iron mains in accordance with the distortion energy failure criterion.

tion) on the factor of safety, as a consequence of the variation (increase only) in one specific input data-parameter. Sensitivity of each specific input data-parameter is shown in the form of tornado graphs in Fig. 7 (in-plane

energy failure criteria) and Fig. 8 (distortional energy failure criteria) for both sizes and with remaining wall thicknesses of 75% t and 50% t . As expected, Figs. 7 and 8 show that the factor of safety increases as the input data-parameters (e.g.,

Fig. 9. Profiles of structural capacity for 150 mm cast iron mains installed in 1930 in Calgary, Alta.

remaining pipe wall thickness (t_{min}), fracture toughness) that contribute to the increase in pipe structural resistance, and this occurs independent of the failure criterion. On the other hand, the factor of safety decreases as stress drivers (e.g., differential temperature, traffic load, frost load, and internal pressure) increase. Figures 7 and 8 show that external loads (q) have a greater significance in terms of reduction in factor of safety on large-diameter mains than on small-diameter mains. It is also evident (Fig. 8) that an increase in unsupported length (b) leads to a reduction in the factor of safety

that is much more significant on small-diameter mains than on large-diameter mains. The influence of external loads (q) and unsupported length (b) heightens as the remaining wall thickness decreases. The sensitivity analyses also show that some variables, e.g., soil modulus of elasticity (E_s), internal pressure (p_i), and transient pressure (p_t), have a very low to an insignificant influence on the factor of safety. This outcome indicates that the uncertainties present in these variables have little or no practical influence on the structural safety of buried pipes these sizes and buried at these depths.

It can also be argued that the analyses would not be unduly inaccurate if these variables were represented as constants “crisp” rather than as fuzzy numbers.

Case study

The City of Calgary has selectively inspected (Brander 2001; Brander and Ng 2002) its cast iron mains within the water-distribution network using a commercially available nondestructive technology (NDT) based on remote field eddy current (RFEC). This effort was largely driven by the need to implement a proactive management system to optimize repair and replacement. The specific RFEC technology (Brander and Ng 2002; Hartman and Karlson 2002) used by the city estimated the three most significant remaining wall thicknesses (pit depths) and the associated level of confidence in the estimates expressed in terms of signal quality indices (SQI) that range from 1 (excellent), to 2 (normal-average), to 3 (low). The confidence level in the estimate for remaining wall thickness varies because the technology relies on the interpretation of signature patterns (output) of a magnetic response rather than measurement of a specific physical feature such as pit depth and pit geometry. The uncertainty in the measurements is taken into account by using the SQI indices to fuzzify the measured remaining wall thicknesses.

The possibilistic analysis was applied to a specific 150 mm (6 in.) diameter $\times$ 13 mm (0.51 in.) wall thickness cast iron pipe of length 309 m (1014 ft.), installed in 1930, to illustrate how its condition could be assessed. Figure 9a shows the profile of the remaining pipe wall thicknesses of the three most significant pits (also referred to as graphitization in cast iron) along the pipe length as estimated by the NDT inspection technology. The possibilistic pipe-soil interaction model was applied using input data-parameters (Table 2) to each of the pits identified. Figure 9b shows the profile of fuzzy factors of safety based on the distortion energy failure criterion (FS_{de}) along the pipe length. The FS_{de} (based on biaxial stresses) is used because the sensitivity analyses showed that unsupported length and temperature differential are the most important considerations for small-diameter pipes. The pits are assumed to have the same diameter as their depth, since the NDT used does not estimate the aerial extent of the graphitization. The fuzzy factors of safety (minimum, most likely, maximum) in Fig. 9b should not be read in terms of ordinates but rather in terms of membership values (μ values, which are perpendicular to the plane), i.e., $\mu_{(\min. FS)} = 0$, $\mu_{(\text{most likely } FS)} = 1$, $\mu_{(\max. FS)} = 0$. For clarity, only representative FS_{de} and corresponding possibility Π and necessity N values are shown in Figs. 9c and 9d, respectively. For example, the maximum and minimum values of fuzzy factors of safety values of 2.10 and 0.06 at a distance of 215 m along the pipe, location A in Fig. 9d, have memberships of 0 while the most likely FS value of 0.75 has a membership of 1. The interval between the minimum and maximum values of fuzzy factors of safety reflects the uncertainties present in the input data-parameters. It can also be observed that the interval between the minimum and maximum factors of safety lies between 0 and 1.2 at the three locations (Fig. 9d as B_1 (at 65 m), B_4 (at 182 m), and B_5 (at 270 m)) where the remaining wall thickness is less than 25% of the original pipe wall thickness. The membership functions for the fuzzy

Fig. 10. Variation of possibility of failure with degree of optimism (*orness*).

factors of safety also do not have long tails; as explained earlier, this occurs largely as a result of the fact that corrosion and loss of bedding support have an overwhelming influence on pipe safety compared to other input data-parameters.

An alternative way to assess the significance of fuzzy factors of safety is to determine the possibility Π and necessity N of failure at each location along the pipe length. As discussed earlier, this requires the prior definition of critical factor of safety as a condition state of imminent failure. It is generally accepted that failure in civil engineering structures is imminent when $FS < FS_{cr}$ where $FS_{cr} = 1$. Figures 9c and 9d show the profiles of possibility Π and necessity N of failure for each pit along the pipe length. A close look at Fig. 9d indicates that the necessity of failure is particularly high at five locations (B_1 – B_5), and if the pipe has not already failed at these locations, near 50%–100% certainty exists that pipe failure is imminent at these locations.

The condition state of the pipe (Fig. 9a) depicts a snapshot of the current condition. If the pipe were to continue to corrode at a certain rate, then the next wave of corrosion pits leading to pipe failures would likely occur at locations C_1 , ..., C_5 (shown in Fig. 9d). Subsequent wave(s) of corrosion pits would likely lead to future pipe failures as the pits grow over time.

The crisp (or pignistic) decision-making level of possibility (risk) of failure can be derived by incorporating the decision-maker's attitude λ_d in the analysis. To illustrate this concept, the possibility (risk) of failure at locations B_2 (at 70 m) and A (at 210 m) are shown in Fig. 10. The corresponding possibility intervals $[N, \Pi]$ at these locations are $[0.49, 1]$ and $[0.15, 1]$, respectively. It can be shown that for locations B_2 and A at $\lambda_d = 0$, i.e., for the lowest possible optimism, the possibility of failure reduces to $\Pi = 1$ in both cases. Similarly, for $\lambda_d = 1$, i.e., for the highest possible optimism, the possibility of failure reduces to $N = 0.49$ and 0.15, respectively. For any other intermediate degree of optimism λ_d , the possibility (risk) of failure can be derived representing the preference of a decision-maker.

Conclusions

A previously developed Winkler-type pipe–soil interaction (WPSI) model was cast in a possibilistic framework to handle uncertainties encountered in the input data–parameters (using triangular fuzzy numbers) and to determine the fuzzy factors of safety (FS). Fuzzification of input data–parameters in a possibilistic framework provided a systematic approach to incorporate and propagate uncertainties throughout the solution process. The dual measures of “possibility” and “necessity” computed in this study can be integrated with failure consequences to allow the designer–owner–operator to decide the level of risk that he or she is willing to take and thus define different repair and maintenance strategies at different risk tolerances (Fig. 1).

A Monte Carlo type random sampling coupled with rank correlation method was performed to identify input data–parameters that dominate the FS. This type of analysis enables the identification of critical parameters and the interaction between them. As expected, independent of the failure criteria, input parameters that affect the structural resistance of the pipe (i.e., remaining wall thickness, residual strength (biaxial), and fracture toughness) are positively correlated with the FS. On the other hand, input data–parameters that contribute to the generation of stresses (i.e., unsupported length, temperature differential, and external load (including the effect of frost load)) exhibit negative correlation with the distortion factor of safety. Sensitivity analyses also confirm that large-diameter mains are more sensitive to external loads, and small-diameter mains are sensitive to the extent of loss of bedding support. The sensitivity analyses strongly suggest that reducing pit depth (graphitization) growth by using effective corrosion control can be the single most effective way to decelerate the breakage growth rate of existing pipes. This observation corroborates the experience of utilities and corrosion engineers.

The decision to renew or replace specific pipe segments or the whole pipe will depend on the physical parameters (corrosion or pit growth rate(s), density of the corrosion pits along the pipe length, hydraulics, and water quality), economics (costs of repair, renewal or replacement, discount factor, etc.), and attitude of a decision-maker. The proposed approach is a step towards the formulation of a method to facilitate the decision-making process of when to repair, renew, or replace individual pipe lengths based on their current condition state.

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List of symbols

a_1	constant (geometric factor)	K	stress concentration factor
a_n	lateral dimension of a pit	K_q	fracture toughness
A_s, B_s, C_s, D_s	scenarios	L	pipe length
A_i	fuzzy set (number)	L_1	multiplier
A_i^α	fuzzy set (number) at α -cut level	p_i	pipe internal pressure
b	unsupported pipe length	p_i'	working pressures
b_1	constant (geometric factor)	p_t	transient water pressure
B_b	trench width	P	internal bursting pressure necessary to cause pipe failure
d	corrosion pit depth	q	overburden load
d_r	difference of ranks	R	number of Monte Carlo type iterations
D	outside pipe diameter	s	constant (residual ultimate strength)
E_p	elastic modulus of pipe	S	number of fuzzy input parameters
E_s	elastic modulus of soil	t	pipe wall thickness
FS	factor of safety	$t_{\min}$	remaining wall thickness
FS_{cr}	critical factor of safety	TFN	triangular fuzzy number
FS_{de}	distortion energy factor of safety	u	any value defined of the universe of discourse
FS_{ip}	in-plane factor of safety	U	universe of discourse
H	trench depth	w	external bearing load
		w'	working load
		W	three-edge crushing (ring) load
		x	dummy uncertain quantity (subset of set A_i)
		α	alpha-cut level
		α_d	orness (decision-maker's attitude)
		α_t	thermal coefficient
		α_{ur}	constant (residual ultimate strength)
		β	geometric factor (fracture toughness)
		ΔT	maximum temperature difference between water temperature and surrounding soil
		γ_s	soil submerged unit weight
		λ_d	degree of orness
		μ	fuzzy membership function
		μ_{FS}	membership function of $FS\mu_{FS_{cr}}$
		μ_x	membership function
		ν_p	pipe Poisson's ratio
		ν_s	soil Poisson's ratio
		N	necessity
		Π	possibility
		ρ_s	Spearman rank correlation coefficient
		σ_1, σ_2	biaxial stresses
		σ_{bu}	burst tensile strength
		σ_r	rupture modulus
		σ_u	tensile strength
		σ_{ur}	residual ultimate strength
		σ_x	axial stress
		σ_x^a	total axial stress
		σ_x^f	stress in the longitudinal direction due to flexural action
		$\sigma_x^{p_i}$	axial stress due to internal pressure
		σ_x^T	axial stress due to temperature differential
		σ_x^w	axial stress due to external loads
		σ_θ	hoop stress
		σ_θ^f	hoop stress due to the Poisson's effect of longitudinal bending
		$\sigma_\theta^{p_i}$	hoop stress due to internal pressure
		σ_θ^T	hoop stress due to temperature differential
		σ_θ^{Total}	total hoop stress
		σ_θ^w	hoop stress due to overburden loads