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EFFECT OF THICKNESS ON THE THERMAL PROPERTIES OF THICK SPECIMENS OF LOW-DENSITY THERMAL INSULATION

by C.J. Shirliffe

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SOMMAIRE

Un modèle à triple couche composé de deux couches superficielles et d'une couche centrale est créé pour expliquer les effets de l'épaisseur de l'échantillon sur les propriétés thermiques d'une épaisse isolation. Le modèle permet de préciser les effets du pouvoir émissif limite et de la densité de l'échantillon sur les mesures. Il en découle un rapport entre la résistivité thermique apparente et l'accroissement de la résistance par unité d'épaisseur. On donne des équations correspondant aux densités où survient la résistance thermique maximale de la couche centrale et de l'ensemble de l'échantillon et aux variations de conductivité thermique de la couche centrale et de l'échantillon avec le rapport de densité à densité où survient la conductivité minimale. Les densités où surviennent les extrêmes ne dépendent pratiquement pas de l'épaisseur de l'échantillon. Au contraire de la forme habituelle en U non symétrique, on propose une forme en V pour la variation de la conductivité thermique. L'ouvrage propose également des équations à utiliser pour l'ajustement des courbes par une courbure du sommet et un déplacement de position de la valeur minimale.

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C. J. Shirliffe¹

Effect of Thickness on the Thermal Properties of Thick Specimens of Low-Density Thermal Insulation

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ABSTRACT: A three-layer model comprising two surface layers and a core is developed to explain the effect of specimen thickness on the thermal properties of thick insulation. The model explains the effect of boundary emittance and specimen density on the measurements. A relationship between the apparent thermal resistivity and the increase in resistance per unit thickness is derived. Equations are given for the densities at which the maximum thermal resistance of the core of the specimen and whole specimen occurs and for the variation of thermal conductivities of the core and the specimen with the ratio of density to density at which the minimum conductivity occurs. The densities at which the extremes occur are almost independent of specimen thickness. A V-shape and not the customary nonsymmetrical U-shape is suggested for the variation of thermal conductivity. Equations for use in curve fitting that introduce curvature at the apex and shift the position of the minimum are suggested.

KEY WORDS: thermal insulation, thermal resistance, thermal conductivity, apparent thermal conductivity, thick specimens, thickness effect, low-density and thermal radiation equations, plate emittance, boundary emittance

The term "thickness effect" has been given to the influence of specimen thickness on the value of apparent thermal conductivity of low-density thermal insulation measured in test apparatus. The existence of the thickness effect has in the last year or two been accepted by the majority of individuals involved with the characterization, specification, or production of thermal insulation.

Few individuals actually understand the effect. Most often the effect on

¹ Research officer, Division of Building Research, National Research Council, Ottawa, Ontario, Canada.

thermal conductivity is partially understood. Techniques for identifying or quantifying the effect are not understood. The insistence of the community always to think in terms of thermal conductivity or resistivity has been one of the major impediments.

Lao and Skochdopole [1]² in 1976 attempted to convert equations in thermal conductivity to thermal resistance and to explain the effect in physical terms. This was perhaps necessary to reconcile the equations they developed with equations developed by Poltz [2] in 1962, applied by Jones [3] in 1968, interpreted in thermal resistance by Shirliffe [4] in 1972, and incorporated in the ASTM Test for Steady-State Thermal Transmission Properties by Means of the Guarded Hot Plate (C 177-76) and the ASTM Test for Steady-State Thermal Transmission Properties by Means of the Heat Flow Meter (C 518-76).

The concept must first be understood in terms of thermal resistance. It can then be interpreted as a thermal conductivity effect for thick specimens of normal insulations. The interpretation is successful only because most insulations attenuate thermal radiation almost completely by the process of absorption and re-emission. The equations³ that are normally coupled uncouple due to the requirement for a continuity in flux within the specimen and simplify to the same basic form [5]. The inherent, though unrecognized, assumption of uncoupling in the simplified analysis is therefore correct. The equations for specimens with less ideal forms of radiation attenuation also uncouple when the specimens are optically thick. The thin heat transfer in specimens, that is, specimens with thickness less than twice the mean free path length for thermal radiation, is governed and can be explained only by coupled radiation-conduction (and sometimes convection) equations except when the attenuation process is solely by absorption or scattering. Coupled equations are difficult to solve and involve parameters not normally measured. Fortunately, the process within these thin layers near the surfaces of insulation is of little interest except to the designer of the insulation itself.

When told about the thickness effect in low-density insulation, experts in heat-transfer phenomena immediately apply equations for conduction within the insulation and thermal radiation transfer from surface to surface. These equations show that thermal radiation increases the heat flow and produces a negative thickness effect. The immediate reaction is to blame the observed positive effect on errors in measurement, especially errors in surface temperature measurements due to contact resistance. Contact resistances in measurement apparatus are one or two orders of magnitude less than necessary to produce such effects. The analysis and "thinking" are in error. Thermal radiation is not transferred from surface

²The italic numbers in brackets refer to the list of references appended to this paper.

³Equations describing the flow of energy by radiation and conduction are interdependent when coupled. The radiation flux depends on the conduction flux and vice-versa.

to surface in the lowest-density (normal) thermal insulations.⁴ The specimens are optically thick. Scattering or absorption phenomena predominate. These phenomena do lead to a positive thickness effect of the correct magnitude.

Thick layers of thermal insulations act as three-layer materials. A layer termed as a surface layer with thickness of the same order of magnitude as the mean free path length for thermal radiation exists near each surface. The thermal resistance of this layer is approximately proportional to the inverse of the total hemispherical emittance of the plates or surface in contact with the insulation. In this one respect the thickness effect is a measurement-related phenomenon, but the phenomenon exists in almost all practical applications of insulation. The thermal resistance of these surface layers provides at least one and a half times the thermal resistance of an equal thickness of insulation in the core.

Measurements made to identify and quantify the transfer processes in thermal insulation should be made at two or more thicknesses or with two [6] or more apparatuses, each having plate emittance of a different but known level. One apparatus might have plates with an emittance of 0.9, another of 0.10 [6].

The thickness effect has a profound influence on most aspects of the characterization, the specification, and the application of thermal insulation.

Rearrangement of Equations

Many authors have developed equations for the heat transfer through, and therefore the apparent or effective thermal conductivity of, optically thick specimens of foam and fibrous insulations. The resulting equations are for the most part identical. The results of analysis by Stripens [7] and Bhattacharya [8] are typical examples. These equations for the apparent thermal conductivity of fibrous insulation can be used to give a simple explanation of the thickness effect.

The thermal resistance divided by the specimen thickness is normally termed the apparent or effective thermal conductivity. The equation developed for the apparent thermal conductivity of mineral fiber insulations is

$$k_a = \frac{4 \sigma T_m^3 L}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1\right) + N' L} + \frac{k_g L_F}{(1-f)(L_G + L_F)_{P,T}} + C\rho \quad (1)$$

⁴ Radiation "windows" in transmittance curves carry limited amounts of energy near room temperature.

$$k_a = \frac{4 \sigma T_m^3 L / N'}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1\right) \frac{1}{N'} + L} + \frac{k_g L_F}{(1-f)(L_G + L_F)_{P.T}} + C\rho \quad (1a)$$

where

σ = Stefan-Boltzmann constant for radiation,

T_m = average of the two plate temperatures [more precisely, T_m^3 should be replaced by $T_m(T_m^2 + \Delta T^2)$ where ΔT is T (hot plate) - T (cold plate)/2],

L = specimen thickness,

ϵ_1, ϵ_2 = emittances of the bounding surfaces,

N' = backscattering cross section or scattering power,

k_g = thermal conductivity of the gas phase,

f = volume fraction of the solid and is small as $1 - f$ is a weak function of density, $f = \rho/\rho$ (solid),

L_F = parameter related to fiber spacing, termed effective pore size ($= [\pi/4f] \times$ effective pore diameter)

L_G = parameter related to the mean free path length between molecular collisions in the gas (both L_F and L_G are functions of gas pressure and temperature),

C = constant related to fiber contact and binder content,

ρ = normal bulk density for the thermal insulation, and

ρ_s = density of the solid phase.

Equation 1a has the form

$$k_a = \frac{a_1 L}{b_0 + b_1 L} + c_0 + d_0 \rho \quad (2)$$

where a_1, b_0, b_1, c_0 , and d_0 are not strong functions of thickness and d_0 is not a strong function of density.

The thermal resistance by definition is

$$R = L/k_a \quad (3)$$

or

$$R = \frac{L(b_0 + b_1 L)}{a_1 L + (c_0 + d_0 \rho)(b_0 + b_1 L)} \quad (4)$$

$$= \frac{b_0 + b_1 L}{a_1 + (c_0 + d_0 \rho)(b_0 + b_1 L)/L} \quad (5)$$

$$= \frac{b_0}{M(L, \rho, p, T)} + \left(\frac{b_1}{M(L, \rho, p, T)} \right) L \quad (6)$$

where $M(L, \rho, p, T)$ is a functional representation. Equation 6 can be simplified by writing

$$R = R_0 + r_\infty L \quad (7)$$

The specimen resistance is the sum of two resistances, R_0 , the apparent resistance for $L = 0$ and $r_\infty L$ for large thicknesses; r_∞ is the resistivity for a specimen of infinite thickness. For thicknesses of air-filled thermal insulation of over 30 mm, both R_0 and r_∞ are essentially independent of specimen thickness.

In the derivation certain assumptions were made. It was assumed that the fibers have a negligible cross section for absorption P' compared with the scattering cross section N'

$$N' \gg P' \quad (8)$$

that N' is a constant for temperatures between T_H and T_C and equals $1/\lambda_r$, where λ_r is the mean free path length for photon-fiber collisions, and that L is large compared with λ_r . The term N' can be represented by a specific backscattering cross section

$$N'' = N'/\rho \quad (9)$$

Using definitions implicit in Eqs 1a and 2, and Eq 9

$$b_0 = \left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1 \right) \frac{1}{\rho N''}$$

$$b_1 = 1$$

and from Eq 6

$$\frac{R_0}{r_\infty} = \frac{b_0}{b_1} = \left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1 \right) \frac{1}{\rho N''} \quad (10)$$

and if $\epsilon_1 = \epsilon_2 = \epsilon$

$$\frac{R_0}{r_\infty} = \left(\frac{2}{\epsilon} - 1 \right) \frac{1}{\rho N''} \quad (11)$$

The ratio R_0/r_∞ is independent of thickness and inversely proportional to the bulk density. Further, it is a function of the plate emittance. With $\epsilon = 1$, that is, "black" plates, it equals λ_r , the mean free path length for photon-fiber collisions.⁵

For a given apparatus and insulation structure, the value of R_0 at density ρ_2 is related to R_0 at density

$$R_{02} = R_{01} \frac{\rho_2}{\rho_1} \cdot \frac{r_{\infty 2}}{r_{\infty 1}} \quad (12)$$

The function a_1 in Eq 5 is

$$a_1 = \frac{4\sigma T_m^3}{\rho N''}$$

so a_1 is independent of L .

The denominator in Eq 5, when expanded, can be written as

$$M = \frac{4\sigma T_m^3}{\rho N''} + \left[\frac{k_g L_F}{(1-f)(L_F + L_G)} + C\rho \right] \left[\left(\frac{2}{\epsilon} - 1 \right) \frac{1}{\rho N'' L} + 1 \right] \quad (13)$$

and since $b_1 = 1$, equations (6) and (7) yield

$$1/r_\infty = M$$

$$r_\infty = \frac{1}{M} \quad (14)$$

$$k_\infty = 1/r_\infty = M \quad (15)$$

Then using Eq 11

$$R = \frac{(2/\epsilon - 1)}{\rho N'' M} + \frac{L}{M} \quad (16)$$

But since $R = L/k_a$

$$\frac{1}{k_a} = \left(\frac{(2/\epsilon - 1)}{\rho L N''} + 1 \right) \frac{1}{k_\infty}$$

⁵ For thick specimens, this means that R_0 is much less than $r_\infty L$.

where

$$k_a = k_\infty / [1 + (2/\epsilon - 1)/\rho L N''] \doteq k_\infty [1 - (2/\epsilon - 1)/\rho L N'']$$

The quantities k_∞ and r_∞ are very weak functions of L since $1/\rho N'' L = \lambda_r/L < 1$ except when L approaches λ_r . For thick specimens they can be considered as constants for a given material.

The term R_0 can be expressed in terms of λ_r and r_∞

$$R_0 = \frac{(2/\epsilon - 1)}{\rho N'' M} \quad (17)$$

$$= \frac{r_\infty (2/\epsilon - 1)}{\rho N''} \quad (18)$$

$$= \lambda_r r_\infty (2/\epsilon - 1) \quad (19)$$

As emittance decreases, R_0 increases, indicating why different test apparatuses often give different values of R_0 .

The term R_0 is normally independent of L , but $\epsilon \rightarrow 0$ and $(2/\epsilon - 1) \rightarrow \infty$

$$R_0 = \frac{L}{\frac{k_g L_F}{(1-f)(L_F + L_G)} + C\rho} \doteq \frac{L}{k_{\text{air}}}$$

where k_{air} is the conductivity at the gas phase and what is termed r_∞ in Eq 7 approaches zero. When $\epsilon \rightarrow 0$ and the insulation is evacuated

$$R_0 \rightarrow \frac{L}{\rho C}$$

and

$$r_\infty \rightarrow 0$$

Substituting Eq 19 into Eq 7

$$R = (2/\epsilon - 1) r_\infty \lambda_r + r_\infty L \quad (20)$$

This can be rewritten by adding and subtracting a term

$$R = (2/\epsilon - 1) r_\infty \lambda_r + r_\infty \lambda_r + r_\infty (L - \lambda_r) \quad (21)$$

$$= 2(2r_\infty/\epsilon)(\lambda_r/2) + r_\infty [L - 2(\lambda_r/2)] \quad (22)$$

$$= 2 R_s + R_c = 2 R_{\text{surf}} + R_{\text{core}} \quad (23)$$

This is the three-layer model where the total resistance is the sum of the thermal resistances of two surface layers, each with thickness equal to half the mean free path length of the thermal radiation, and the core. For $\epsilon = 1$, the thermal resistivity is $2r_\infty$ or twice the thermal resistivity of the core. In effect, each layer acts as if it was twice as thick as it really is. At lower emittances the resistivity is even larger. If it were possible to have $\epsilon = 2$, the thermal resistivity would equal that of the core, $r_s = r_\infty$.

The surface layers could more realistically be assumed to have a thickness equal to the mean free path length for photon-fiber collisions. In this case Eq 22 becomes

$$R = 2 \left[\left(\frac{1}{\epsilon} + \frac{1}{2} \right) r_\infty \right] \lambda_r + r_\infty (L - 2\lambda_r) \quad (24)$$

For $\epsilon = 1$, the thermal resistivity of the surface layer is $3r_\infty/2$ or 1.5 times the thermal resistivity of the core. The surface layer would have the same resistivity of the core, if it were possible for the plate emittance, ϵ , to equal two.

Further assumptions are possible such as the thickness of the surface layers being $2\lambda_r$, in which case $r_s = 1.25 r_\infty$ when $\epsilon = 1$. No preference as to which combined thickness is correct and the related thickness of the layers on the cold and hot surfaces can be given without further study.

The most important aspect is that the three-layer model does have a theoretical basis for thick specimens. Since the theoretical equations are approximate in any event, the equations will have to be fitted to experimental data to find the numerical values of R_0 and r_∞ .

Equation 7 does not contain an assumption for the thickness of the surface layers, so this form is preferable. However, if sufficient data on R versus L are available for thin and thick specimens the thickness of the surface layers and the core could be found; thus Eq 22 is used.

The three-layer model has a second advantage. The approximate solution could be used for the core, and a more precise theoretical solution, which includes the coupling of radiation and conduction, could be derived for the thermal resistance of the surface layers.

Either the thermal resistance of the surface layers $2R_s$ or the constant R_0 can be measured for known emittance plates and corrected for plates with other emittance using the equation

$$R_0(\epsilon = \epsilon_2) = R_0(\epsilon = \epsilon_1) \cdot \frac{[2/\epsilon_2 - 1]}{[2/\epsilon_1 - 1]} \quad (25)$$

If R_0 , ϵ , and r_∞ are known, λ_r can be determined from Eq 19. Measurements of R_0 and r_∞ made at the National Research Council, Ottawa, yield typical values of λ_r in the range 1.7 to 6.9 mm.

Equations with the form of Eqs 13 and 14 but containing empirical constants and ρ , L , and ϵ can be derived when an empirical equation for apparent thermal conductivity of a material in terms of density, thickness, and emittance has already been determined.⁶

The density at which R reaches a maximum or k_a reaches a minimum is often of interest. The derivative of R with respect to ρ at constant thickness is

$$\frac{\partial R}{\partial \rho} = \frac{\partial R_0}{\partial \rho} + \frac{\partial r_\infty}{\partial \rho} \cdot L \quad (26)$$

Substituting Eq 19 in Eq 26, using

$$\frac{\partial \lambda_r}{\partial \rho} = \frac{\lambda_r}{\rho}$$

and setting $\partial R / \partial \rho = 0$ at the maximum

$$\frac{\partial r_\infty}{\partial \rho} = \frac{(2/\epsilon - 1) \lambda_r r_\infty}{\rho[(2/\epsilon - 1) \lambda_r + L]} = \frac{R_0 r_\infty}{\rho R} \neq 0 \quad (27)$$

or

$$\begin{aligned} \frac{\partial k_\infty}{\partial \rho} &= \frac{-(2/\epsilon - 1) \lambda_r k_\infty}{\rho[(2/\epsilon - 1) \lambda_r + L]} \\ &= \frac{-R_0 k_\infty}{\rho R} \end{aligned} \quad (28)$$

The thermal resistance does not reach a maximum and the apparent thermal conductance does not reach a minimum with respect to density when r_∞ and k_∞ reach a maximum and minimum, respectively. The density at which the thermal resistance reaches a maximum can be found from Eq 1. This density depends on the emittance of the bounding surfaces and specimen thickness. If the conductivity of the gas phase is assumed to be independent of density, the density at which R reaches a maximum and k_a reaches a minimum can be determined from Eqs 13 and 20

$$\rho_{\max}^* = \left[\left(\frac{4\sigma T_m^3}{N''C} \right)^{0.5} - \frac{(2/\epsilon - 1)}{N''L} \right] \quad (29)$$

⁶ Manufacturers normally fit such equations to data.

For glass fiber insulation $\rho_{\max}^*$ would fall between 50 and 80 kg/m³.

The equations can be simplified if the density at which k_{∞} reaches a minimum is used as a parameter. From Eqs 13 and 15

$$\frac{\partial k_{\infty}}{\partial \rho} = 0 = -\frac{4\sigma T_m^3}{\rho_m^2 N''} - \left(\frac{k_m'}{(1-f_m)} + C\rho_m \right) \frac{(2/\epsilon - 1)}{\rho_m^2 L N''} + \left[\frac{k_m'}{(1-f_m)^2 \rho_s} + C \right] \cdot \left[\frac{(2/\epsilon - 1)}{\rho_m L N''} + 1 \right] \quad (30)$$

where $k' = k_g [L_G/(L_F + L_G)]$.⁷ Subscript m indicates at minimum k_{∞} and subscript s indicates the solid phase.

$$\left[C + \frac{k_m'}{(1-f_m)^2 \rho_s} \right] \rho_m^2 - \left[\frac{k_m'}{(1-f_m)^2 \rho_s} \right] \left[\frac{(2/\epsilon - 1)}{L N''} \right] \rho_m - \left[\left(\frac{4\sigma T_m^3}{N''} \right) + \left(\frac{k_m'}{1-f_m} \right) \left(\frac{2/\epsilon - 1}{L N''} \right) \right] = 0 \quad (31)$$

When $\lambda_r \ll L$, the center term is negligible and f_m is considered a constant⁸

$$\frac{\rho_m}{\rho_s} \doteq \left[\frac{\frac{4\sigma T_m^3}{N''} + \frac{k_g(2/\epsilon - 1)}{\rho_s L N''}}{\rho_s C + k_g} \right]^{0.5} \quad (32)$$

For glass fiber insulation ρ_m would fall between 35 and 70 kg/m³. Combining Eqs 30 and 32

$$k_{\infty} = \frac{k'}{(1-f)} + \rho_m C \left(\frac{\rho_m}{\rho} + \frac{\rho}{\rho_m} \right) + \frac{(2/\epsilon - 1)}{\rho_m L N''} \rho_m C + \left(\frac{\rho_m}{\rho} \right) \left(\frac{k'}{(1-f)} - \frac{k_m'}{(1-f_m)} \right) + \frac{k_m' f_m}{(1-f_m)^2} \left(\frac{\rho_m}{\rho} \right) \left[\frac{(2/\epsilon - 1)}{\rho_m L N''} + 1 \right] \quad (33)$$

⁷ From Ref 8

$$k' \doteq k_G \left(1 - \frac{\pi D}{4} L_G \frac{\rho}{\rho_s} \right) \frac{\partial k'}{\partial \rho} \doteq \frac{-\pi D L_G}{4 \rho_s}$$

Both are small terms and have little effect on the results. The value of ρ_m becomes smaller.

⁸ The actual equation that must be satisfied would be a fourth-order equation in ρ_m . The reaction of k' would affect third-order and lower terms.

or if $\lambda_r \ll L$

$$\begin{aligned}
 k_{\infty} &= \frac{k'}{(1-f)} + \rho_m C \left(\frac{\rho_m}{\rho} + \frac{\rho}{\rho_m} \right) + \frac{k' f_m}{(1-f_m)^2} \left(\frac{\rho_m}{\rho} \right) \\
 &= k_g + \rho_m C \left(\frac{\rho_m}{\rho} + \frac{\rho}{\rho_m} \right) + k_g \frac{\rho_m}{\rho_s} \left(\frac{\rho_m}{\rho} + \frac{\rho}{\rho_m} \right) \quad (33a)
 \end{aligned}$$

Equation 33 indicates that k_{∞} is the sum of the conductivity of the gas phase and terms in ρ_m/ρ and ρ/ρ_m which include a secondary effect of the emittance of the bounding surfaces, the specimen thickness, and the conductivity of the gas phase. As the material is evacuated, the effects of the bounding surfaces, thickness, and of changes in gas phase increase. Both eqs 33 and 33a may be simplified by replacing $k'/(1-f)$ by k' and ρ_m/ρ in the last terms by $(\rho_m/\rho) + \rho/\rho_m$ with second-order errors.

Similar expressions for k_{∞} and k_a may be obtained in terms of the respective minimum values of each, $k_{\infty m}$ and k_{am} .

$$\begin{aligned}
 k_{\infty} - k_{\infty m} &= \frac{4\sigma T_m^3}{\rho_m N''} \left(\frac{\rho_m}{\rho} - 1 \right) + \left[\frac{k'}{(\rho/\rho_m)(1-f)} - \frac{k'_m}{(1-f_m)} \right] \\
 &\quad \left[\frac{2/\epsilon - 1}{\rho_m N'' L} \right] + (k' f_m + \rho_m C) \left(\frac{\rho}{\rho_m} - 1 \right) \quad (34)
 \end{aligned}$$

and

$$\begin{aligned}
 k_a - k_{am} &= \frac{4\sigma T_m^3}{\rho_m N''} \left[\frac{\left(\frac{\rho_m}{\rho} - 1 \right)}{\left[\frac{(2/\epsilon - 1)}{\rho_m N'' L} \left(\frac{\rho}{\rho_m} \right) + 1 \right] \left(\frac{(2/\epsilon - 1)}{\rho_m N'' L} + 1 \right)} \right] \\
 &\quad + (k' f_m + \rho_m C) \left(\frac{\rho}{\rho_m} - 1 \right) \quad (35)
 \end{aligned}$$

Both equations show dependence on ρ/ρ_m and ρ_m/ρ . Like terms in k' and k'_m were canceled as k' was assumed constant with respect to density in the differentiation and since k changes significantly only at high densities. Equation 35 simplifies when $\lambda_r \ll L$ and the relationship becomes clearer

$$k_a - k_{am} = \frac{4\sigma T_m^3}{\rho_m N''} \left(\frac{\rho_m}{\rho} - 1 \right) + (k' f_m + \rho_m C) \left(\frac{\rho}{\rho_m} - 1 \right) \quad (36)$$

A radiation and solid conduction term multiplies the linear function $\rho/\rho_m - 1$. Both terms are independent of specimen thickness. A plot of both Eqs 34 and 36 would yield V-shaped curves and not the rounded curve found in practice for apparent thermal conductivity.

Equations used to fit data should be expressed in terms of ρ_m/ρ and ρ/ρ_m combined with L and $2/\epsilon - 1$ and higher-order terms if necessary to ensure a reasonable fit to data.

Equation 33 can be used to find $k_{\infty m}$

$$k_{\infty m} = \frac{k_m'}{(1 - f_m)^2} + \rho_m C \left(2 + \frac{(2/\epsilon - 1)}{\rho_m L N''} \right) \quad (37)$$

or if $\lambda_r \ll L$

$$k_{\infty m} = \frac{k_m'}{(1 - f_m)^2} + 2\rho_m C \quad (38)$$

$$k_{\infty m} \doteq k_m' + 2 \frac{\rho_m}{\rho_s} + 2 \rho_m C \quad (38a)$$

For typical fiber glass insulation $k_{\infty m}$ would be expected to fall between 0.030 and 0.035 W/mK.

Similar expressions for k_a and k_{am} may be derived from Eq 1a and an expression for C when k_a is a minimum. The resulting equations are

$$k_a \doteq k' + \frac{4\sigma T_m^3}{\rho_m N''} \left[\left(\frac{\rho_m}{\rho} + \frac{\rho}{\rho_m} \right) - \frac{(2/\epsilon - 1)}{\rho_m N'' L} \left(\frac{\rho_m^2}{\rho^2} + \frac{2\rho}{\rho_m} \right) \right] \quad (39)$$

and

$$k_{am} \doteq k' + \frac{4\sigma T_m^3}{\rho_m N''} \left[2 - \frac{3(2/\epsilon - 1)}{\rho_m N'' L} \right] \quad (40)$$

If $\lambda_r \ll L$

$$k_a \doteq k' + \frac{4\sigma T_m^3}{\rho_m N''} \left(\frac{\rho_m}{\rho} + \frac{\rho}{\rho_m} \right) \quad (41)$$

and

$$k_{am} = k' + \frac{8\sigma T_m^3}{\rho_m N''} \quad (42)$$

Both $k_{\infty m}$ and k_{am} are independent of L , the specimen thickness, when $\lambda_r \ll L$. Note that k_{am} is somewhat smaller than $k_{\infty m}$.

Fitting Curves to Data

Equation 33 can be rewritten as

$$k_{\infty} = \left[k' + \frac{\rho_m C (2/\epsilon - 1)}{\rho_m L N''} \right] + \left[\rho_m C + \frac{k' \rho_m}{\rho_s} \right] \cdot \left[\frac{\rho_m}{\rho_s} + \frac{\rho}{\rho_m} \right]$$

with less than 0.1 percent added round-off error for glass fiber insulation. All terms then are constant or reach a minimum at $\rho = \rho_m$, which is an advantage in curve-fitting. Higher-order terms in $(\rho_m/\rho)^n + (\rho/\rho_m)^n$ could be added to adjust the curve to the data. Such adjustments are necessary since the equations do not account for the fiber diameter, the lay of the fibers, the layering of the fibers, or distribution of binder in fibrous insulations, nor for cell wall thickness, cell size distribution, cell shape, or open cell walls in cellular insulations. An equation with the same form as Eq 39 has already been suggested for cellulose fiber insulation [8].

Equation 39 suggests a second type of higher-order term to use in fitting equations for k_a to data. The term is $(\rho_m^2/\rho^2 + 2\rho/\rho_m)/L$, though $(2/\epsilon - 1)(\rho_m^2/\rho^2 + 2\rho/\rho_m)/L$ might be more useful if ϵ is expected to vary. Other higher-order terms of the same form are $\rho_m^3/\rho^3 + 3\rho/\rho_m$, $2\rho_m^3/\rho^3 + 3\rho^2/\rho_m^2$, $3\rho_m^2/\rho^2 + 2\rho^3/\rho_m^3$, and $2\rho_m/\rho + \rho^2/\rho_m^2$. These could be combined with other parameters as indicated by the data and theoretical considerations. The use of a polynomial in ρ/ρ_m , ρ^2/ρ_m^2 , ρ^3/ρ_m^3 and in ρ_m/ρ , ρ_m^2/ρ , ρ_m^3/ρ would be more difficult to combine with $1/L$ and $(2/\epsilon - 1)$ and would likely shift the position of the minimum in both k_a and k_{∞} .

Conclusions

The thickness effect, or the dependence of measured apparent thermal conductivity of low-density insulation on specimen thickness, does exist. Conventional equations for heat transfer within the insulation correctly predict the magnitude of the effect for thick specimens with powder, cellular, or fibrous structure. The three-layer model for insulation in which the specimen is divided into a core and two layers near the surfaces is a useful concept in understanding the effect. The thermal resistance of the layers near the surface depends on the test apparatus plate emittance. The conduction and radiation are uncoupled within the core, but coupled in the layers near the surfaces. Empirical equations may be necessary to relate the thermal resistance of these layers near the surface to the physical parameters, such as fiber diameter and lay of fibers, of the insulation. The

thickness of the layers near the surface must be determined empirically until some theoretical prediction is made. The thermal resistance of thick specimens may be corrected for surface emittance effects by simple equations.

The apparent thermal conductivity and resistivity of the specimen are different from the corresponding values for the core of the specimen. The apparent thermal properties of the specimen and the core reach minimum, or corresponding maximum, values at different values of density. Equations for each have been derived in terms of the ratio of density, ρ , to the density at which each reaches a minimum or a maximum, ρ_m , and the inverse of the ratio. The extreme values of apparent thermal conductivity and resistivity are predicted closely by the equations, but, since the equations fail to include the structure of the insulation, exact values cannot be found. The extreme values of the apparent thermal conductivity and thermal resistivity of thick specimens are not significantly affected by the specimen thickness.

Empirical equations must still be relied upon to interpolate or extrapolate the performance of a given type of insulation at different densities and thicknesses. The use of equations expressed in $\rho/\rho_m + \rho_m/\rho$ and other higher-order terms may prove more advantageous in the fitting of curves to data than the polynomials in ρ now used. Where sufficient data are available, the coefficients of the terms in the empirical equations should be examined to see if they can be correlated to the physical structure.

Equations of the form $R = R_0 + r_\infty L$ can be fit to experimental data and then used to determine the influence of the thickness effect on the characterization of the thermal properties of an insulation, stacking of insulation layers, and compression of a layer of insulation.

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