Dynamic loading and response of footbridges (1987)
Rainer, J. H.; Pernica, G.; Allen, D. E.

Publisher's version / Version de l'éditeur:

Canadian Journal of Civil Engineering, 15, 1, pp. 66-71, 1988-02

National Research Council Canada
Conseil national de recherches Canada

Institute for Research in Construction
Institut de recherche en construction

Dynamic Loading and Response of Footbridges (1987)

by J.H. Rainer, G. Pernica and D.E. Allen

Reprinted from
Canadian Journal of Civil Engineering
Volume 15, No. 1, February 1988
p. 66-71
(IRC Paper No. 1557)

NRCC 29320

Canada

This paper is being distributed in reprint form by the Institute for Research in Construction. A list of building practice and research publications available from the Institute may be obtained by writing to the Publications Section, Institute for Research in Construction, National Research Council of Canada, Ottawa, Ontario, K1A 0R6.

Ce document est distribué sous forme de tiré-à-part par l'Institut de recherche en construction. On peut obtenir une liste des publications de l'Institut portant sur les techniques ou les recherches en matière de bâtiment en écrivant à la Section des publications, Institut de recherche en construction, Conseil national de recherches
K1A 0R6.

Dynamic loading and response of footbridges

J. H. Rainer, G. Pernica, and D. E. Allen
Structures Section, Institute for Research in Construction, National Research Council of Canada, Ottawa, Ont., Canada K1A 0R6

Received February 17, 1987
Revised manuscript accepted July 30, 1987

Dynamic forces were measured during walking, running, and jumping, using an instrumented platform. The results are expressed as sinusoidal force amplitudes normalized by the subject's weight and are plotted versus frequency. The maximum dynamic loads for walking were found to be nearly twice as large as those recommended in the 1983 Ontario Highway Bridge Design Code or the British Standard BS5400, and those for running or jumping, more than six times as large.

Responses of footbridges are calculated using a simple formula based on the dynamic loading due to one person, the response of a simple span at resonance, and limited duration of excitation. Good agreement was obtained with the measured response of two 17 m experimental spans subjected to human excitation, for both the first and second harmonics of the step rate. The resonant vibrations of the spans can be substantially reduced by resonant dampers.

Key words: footbridges, dynamic loads, dynamic response, design criteria, resonant dampers.

Les forces dynamiques ont été mesurées durant des activités de marche, de course et de saut, à l'aide d'une plate-forme instrumentée. Les résultats sont exprimés sous forme d'amplitudes de la force sinusoïdale normalisées par le poids du sujet; on a ensuite tracé la courbe en fonction de la fréquence. En ce qui a trait à la marche, les charges dynamiques maximales se sont révélées deux fois plus élevées que celles recommandées par l'Ontario Highway Bridge Design Code de 1983 ou la norme britannique BS5400. Les charges dynamiques des deux autres activités étaient six fois plus élevées.

Le comportement des passerelles a été calculé à l'aide d'une formule simple basée sur le chargement dynamique dû à une personne, la réponse d'une portée simple en condition de résonance et la durée limitée de l'excitation. Une bonne similitude a été obtenue avec la mesure du comportement de deux portées expérimentales de 17 m de long, pour la première et la deuxième harmoniques du taux de pas. Les vibrations résonnantes des portées peuvent être considérablement réduites par des amortisseurs de résonance.

Mots clés : passerelles, charges dynamiques, réponse dynamique, critères de conception, amortisseurs de résonance.

[Traduit par la revue]

Can. J. Civ. Eng. **15**, 66–71 (1988)

Introduction

Since the primary purpose of footbridges is the conveyance of pedestrians, such bridges need to be safe and to exhibit acceptable behaviour for users. Walking, running, and jumping produce dynamic forces and, as a consequence, a vibrational response, which can annoy or alarm people. Serviceability will therefore be a major design consideration.

A number of studies have dealt with the problem of vibrating footbridges (Tilly *et al.* 1984; Blanchard *et al.* 1977; Wheeler 1982; Matsumoto *et al.* 1978). Design requirements and guidelines have been established, such as the British Standard BS5400 (BS5400 1978), the 1983 Ontario Highway Bridge Design Code (OHBDC 1983), and those used in Australia (Wheeler 1982). The OHBDC bases its design requirements and recommendations largely on BS5400; both codes specify a dynamic design load of 180 sin $(2\pi ft)$ N. The resulting vibrations shall not exceed $0.5\sqrt{f_0}$ m/s^2 in BS5400, and less than that in OHBDC 1983. Wheeler (1982) presented a study of footbridge vibrations in Australia in which the pedestrian model was a sequence of measured single-step force pulses. The acceptance level was based on the "unpleasant" boundary contour by Kobori and Kajikawa (1974).

This paper presents the results of force measurements from persons walking, running, and jumping, and a comparison between calculated and measured responses of two test structures. Comparisons are made with available design procedures for acceptability of vibrations on footbridges.

Note: Written discussion of this paper is welcomed and will be received by the Editor until May 31, 1988 (address inside front cover).

Forces induced by pedestrian motion

A number of previous studies of forces induced by human activity can be found in the following references: Harper *et al.* (1961), Nilsson (1976, 1980), Ohlsson (1982), and Tuan and Saul (1985). These employed the forces produced by one step and then formed a simulated walking sequence by adding single-step results with appropriate time delays. The present investigation measures directly the continuous dynamic forces produced by one or more persons walking, running, or jumping.

Test procedure and analysis

The effects of walking and running were determined for a platform 17 m long by installing load cells at a centre support. A plan view and elevation of the platform are shown in Fig. 1. Further details of the construction and measurement techniques are presented in Rainer and Pernica (1986).

The tests were conducted by playing prerecorded pulses at the desired walking or running rate through loudspeakers, and requesting the subject to walk or run along the span at the pulse rate using a stride of his own choosing. The lowest natural frequency of the instrumented floor strip was 12 Hz. Useful results were limited to frequencies below about 10 Hz, since beyond that frequency the dynamic amplification becomes large and rather sensitive to damping.

Results of force measurements

Walking

A typical record of the force induced at the centre support due to a single person walking is shown in Fig. 2*a*. The record contains only the dynamic portion of the induced forces, since the static component has been eliminated by a 0.2 Hz high pass

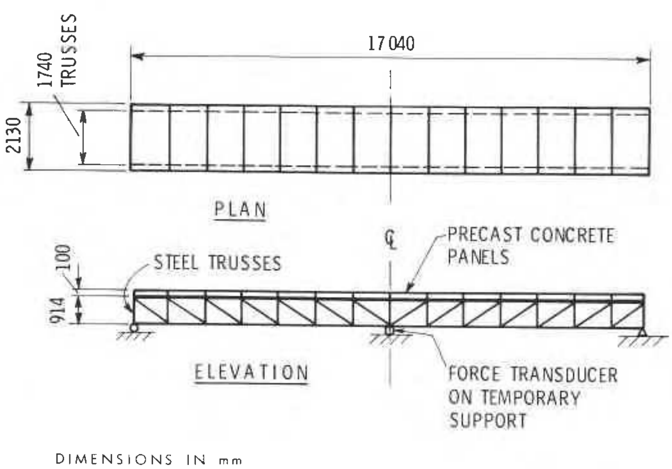

FIG. 1. Instrumented platform (span 1).

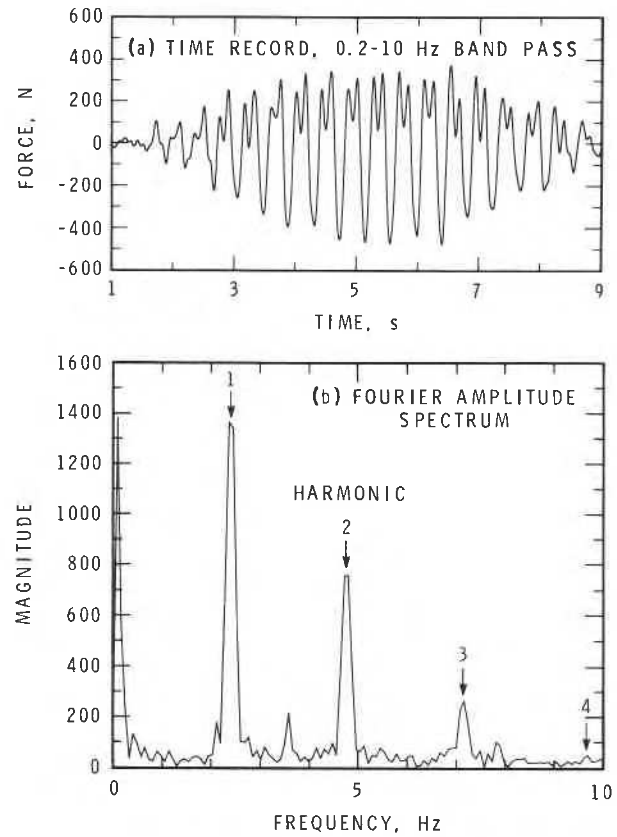

FIG. 2. Forces measured at centre support due to subject A walking at 2.4 steps/s.

filter. These forces are bounded by a parabolically shaped envelope that corresponds to the static influence line for the mid-span support of the test platform. At frequencies well below the lowest resonant frequency of the platform (12 Hz), these measured forces at the centre support represent the forces applied to the platform by the moving person, modified by the influence line for the centre support force. For frequency components closer to the resonance frequency of the platform, dynamic amplifications occur in the measured forces. The loading parameters presented here have been corrected for this dynamic effect and for the influence line envelope, and thus represent the force components generated by the moving person

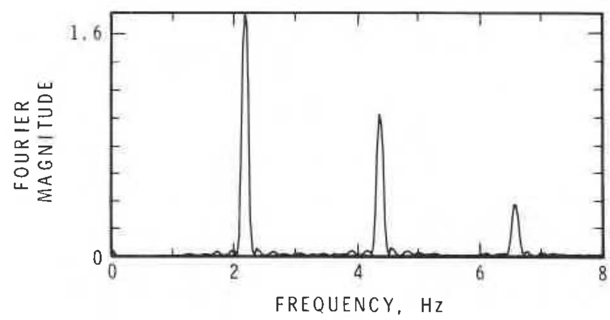

FIG. 3. Fourier amplitude spectrum of forces due to jumping at 2.2 jumps/s.

on the supporting structure. The Fourier amplitude spectrum of this force record is shown in Fig. 2*b*. The spectrum shows that the force produced by one person walking consists of distinct frequency components at integer multiples (harmonics) of the footstep rate, with amplitudes that decrease with increasing frequency. The first three or four harmonics comprise the main dynamic components of walking forces. As the time record in Fig. 2*a* shows, the forces are periodic functions of the footstep rate.

Running

The forces from one person running are similar to those for walking, except that they are truncated below zero during the time the runner is airborne. The Fourier spectrum of the force record again contains discrete frequency components at harmonics of the footstep rate.

Jumping

The jumping forces consist of a sequence of isolated pulses separated by a section of zero force (Allen *et al.* 1985). The Fourier spectrum of the force record (Fig. 3) again consists of harmonic components similar to that for walking.

Mathematical representation of forces

The above analysis of force records indicates that the forces ($F(t)$) from walking, running, and jumping can be represented by

$$[1] \qquad F(t) = P\left(1 + \sum_{n=1}^{N} \alpha_n \sin(n2\pi ft + \phi_n)\right)$$

where P = static weight of person; α = Fourier amplitude or coefficient; n = order of harmonic of the footstep rate, $n = 1, 2, 3 \ldots$; f = footstep rate in steps per second; t = time; ϕ = relative phase angle; N = total number of harmonics.

The dynamic component of the activity force in [1] is represented by the summation term, which is a Fourier series with Fourier coefficients α_n at the discrete frequencies nf. These correspond to the centre amplitudes of the various harmonics of the force record, as in Fig. 2*a*, normalized by the person's weight (P).

Variation of dynamic forces with step frequency

The key parameters in [1] that describe the dynamic force are the Fourier coefficients (α_n) and the footstep frequency (f). In a manner similar to that used to describe rhythmic forces (Allen *et al.* 1985; Supplement to the National Building Code of Canada 1985), the Fourier coefficients (α_n) are called "dynamic load factors," which are defined as the ratio of the force amplitude of each harmonic to the weight of the person. The variation of α_n with step frequency was studied for walking rates from 1.0 to

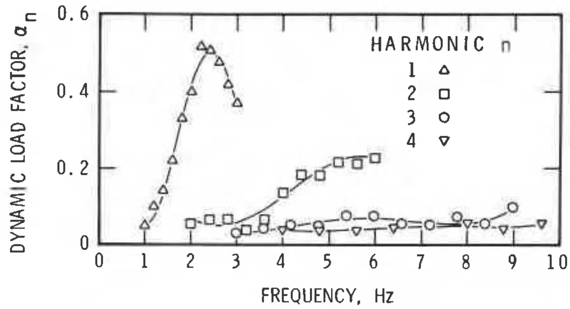

FIG. 4. Averaged dynamic load factors for walking, subjects A, B, and C.

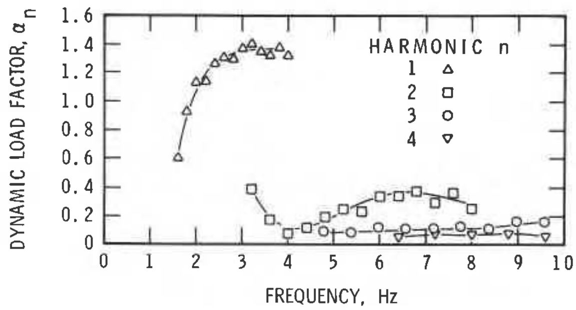

FIG. 5. Averaged dynamic load factors for running, subjects A, B, and C.

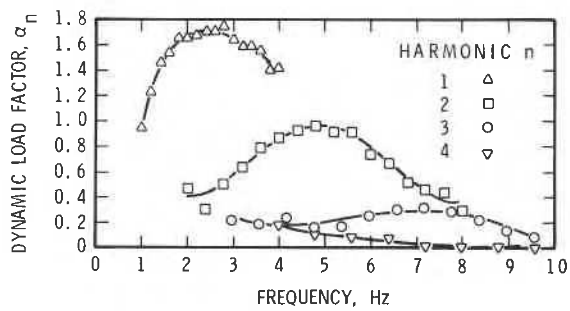

FIG. 6. Averaged dynamic load factors for jumping, subjects A, B, and C.

3.0 Hz, running rates between 1.6 and 4.0 Hz, and jumping rates from 1.0 to 4.0 Hz. The results of each activity for the test subjects were averaged and are shown in Figs. 4–6. Results for individual test subjects are similar to the averaged ones and are presented by Rainer and Pernica (1986) for walking and running. Figure 4 shows that for walking, the dynamic load factor of the first harmonic (α_1) is the largest, at 2.4 Hz, and reaches an averaged maximum of 0.52. For running, Fig. 5 shows a maximum averaged value for α_1 of about 1.4 between 2.8 and 4.0 Hz, whereas for jumping, Fig. 6 shows the maximum averaged α_1 to be about 1.75 between 2 and 3 Hz.

Not all of these rates are equally likely to occur in practice, however. Walking rates are generally within 1.7–2.3 steps/s, with a mean rate of 2.0 (Matsumoto *et al.* 1978), whereas running or jogging rates are 2–3 steps/s.

For groups of two or four people jumping to a common timing signal, the peak value for α_1 was little different than that for a single person jumping, although the peaks for the second harmonic are about 15% lower. For eight persons jumping, however, the dynamic load factor for the first harmonic was about 15% lower, and that of the second harmonic (α_2), 50%

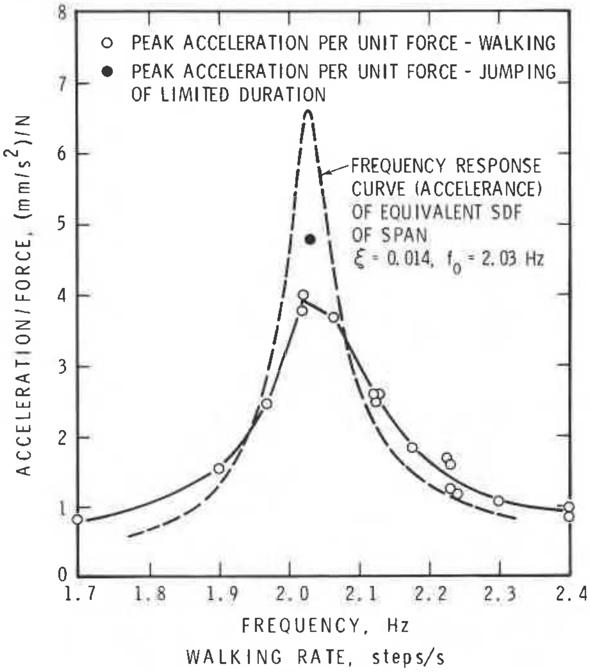

FIG. 7. Peak acceleration per unit force and frequency response curve for span 2.

lower than that for a single male. The third harmonic (α_3) was reduced even more relative to that for one person.

Response of test spans to pedestrian movement

Two simply supported test spans were located side by side for dynamic testing. Span 1, the trussed span shown in Fig. 1, but with the centre support removed, had a fundamental natural frequency (f) of 4.17 Hz. Span 2, a structure of equal length and decking but using two wide-flange beams as the main structural members, had a fundamental natural frequency (f) of 2.05 Hz. A single test subject weighing 735 N was used to excite the spans at selected step frequencies as given by an audible pulse. The response of each span was measured by two vertical accelerometers at mid-span; their signals were added, recorded, and analyzed. The dynamic load factors for walking and running for the test subject were reported by Rainer and Pernica (1986) under "subject A."

Span 2, f = 2.05 Hz

The peak acceleration response of span 2 to different walking rates is shown in Fig. 7. This plot has been normalized by the force input, i.e., by the weight of the test subject times the dynamic load factor applicable to each walking rate. The peak response occurs when the walking rate coincides with the fundamental frequency of the span. These results were compared with the steady state frequency response curve of the span in the form of "accelerance" (acceleration/force input) at mid-span, shown in Fig. 7 by the dashed line. At resonance, the normalized response from walking is considerably lower than that for steady state, whereas below and above resonance, the peak response for walking is slightly larger. The difference at resonance is caused by the limited duration and changing amplitude of the effective walking force (Fig. 2*a*). Away from resonance, the larger walking responses can be attributed to beating between the forced response at the walking frequency and the simultaneously excited vibration at the resonance

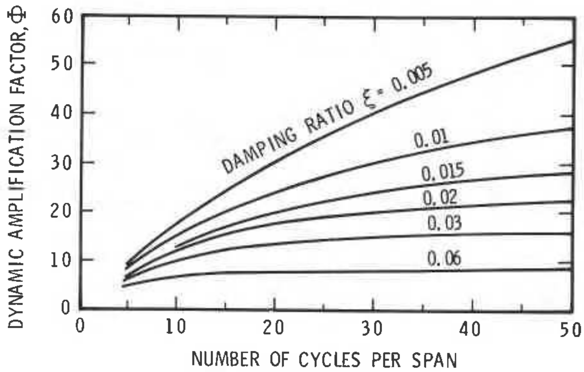

FIG. 8. Dynamic amplification factor for resonant response due to sinusoidal load moving across simple span.

frequency of the span. It is the response at resonance that is maximum, however, and only this will be addressed here.

Calculation of resonant response

Walking

To calculate the peak dynamic response at mid span, the span is modelled as an equivalent single-degree-of-freedom (SDF) oscillator. The amplitude of the sinusoidal excitation at the walking frequency is $F = \alpha P$. As the pedestrian crosses the span, the effective force variation across the span is enveloped by a curve corresponding to the fundamental mode shape with maximum αP occurring at mid-span. When such a force variation is applied to the SDF vibration model of the span, the maximum response is given by a dynamic amplification factor (Φ), similar to, but smaller than, the dynamic amplification factor for steady state excitation. The factor Φ for simply supported spans as a function of the number of cycles applied and various damping ratios has been calculated and is presented in Fig. 8. This amplification factor decreases with an increase in the damping ratio and increases monotonically towards the steady state solution with increasing number of cycles of excitation, i.e., with the number of steps it takes the person to traverse the span. Thus, the peak acceleration response (a) at the fundamental frequency is given by

[2] $$a = (2\pi f)^2 \frac{\alpha P}{k} \Phi$$

$\alpha P/k$ is the static displacement at mid-span due to the dynamic force amplitude, and multiplicaion by $(2\pi f)^2$ converts the displacement to acceleration.

An alternate expression for [2] is obtained by substituting $(2\pi f)^2 = k/m$:

[2*b*] $$a = \frac{\alpha P}{m} \Phi$$

where m = mass of equivalent SDF oscillator. For a uniform simply supported beam, $m = 17/35$ times the total mass (M) of the span, which is often approximated as $m = 0.5M$. Thus,

[3] $$a = \frac{2\alpha P}{M} \Phi$$

Equation [3] indicates that the acceleration response of a span when excited at its natural frequency is directly proportional to the dynamic load (αP) and the amplification factor (Φ), and inversely proportional to the total mass of the span.

For calculating the response of span 2 to walking, the following data were used: $\alpha = 0.41$ (subject A, Rainer and Pernica 1986); $P = 735$ N; $\Phi = 20$ for damping ratio of 1.43% and 19 steps (Fig. 8); k = measured stiffness of span at centre, 897 000 N/m; f = resonance frequency of span, 2.05 Hz.

Substitution in [2] gives a peak acceleration of 1.06 m/s^2, or 11% of gravity. This compares well with the maximum measured response at resonance of 1.14 m/s^2. Figure 9 shows the response of span 2 to the test subject walking at a rate that coincides with the natural frequency of the span.

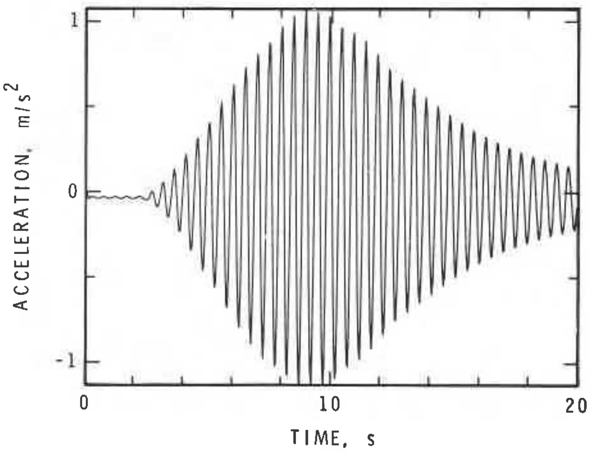

FIG. 9. Response of span 2 to walking at 2.06 steps/s.

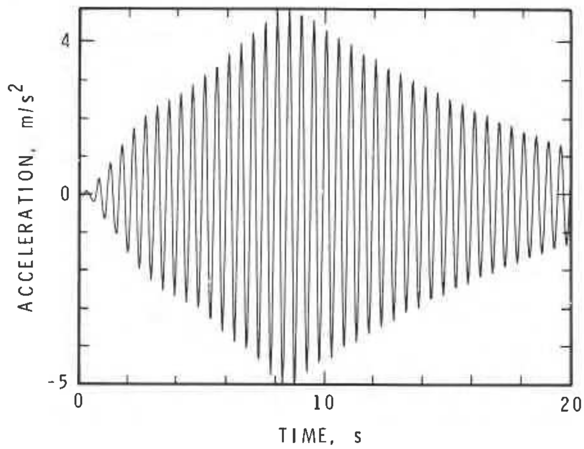

FIG. 10. Response of span 2 due to jumping at resonance frequency.

Running

The response of span 2 to running is, as for walking, given by [2] or [3]. With a dynamic load factor $\alpha = 1.1$ at 2.05 Hz (subject A, Rainer and Pernica 1986), the calculated peak acceleration is 3.14 m/s^2. The measured acceleration for the test subject running at 2.05 steps/s is 3.3 m/s^2. When the running rate does not coincide with the resonance frequency of the span, the response is significantly reduced; at 3.0 steps/s, a peak acceleration of 0.4 m/s^2 was measured.

Jumping

Span 2 was excited at the resonance frequency by the test subject jumping at centre span, and the response is shown in Fig. 10. Since this represents a steady state excitation, much greater response can be expected than from walking or running. At the peak acceleration of 5 m/s^2 (and up to 6 m/s^2 for other tests) the excitation was stopped, but this was evidently not the limit to which the span could have been excited. Using $\alpha = 1.7$ at 2.05 Hz (subject A, Rainer and Pernica 1986), this response of 6.0 m/s^2 corresponds to an acceleration-to-force ratio of (4.8 mm/s^2)/N and is shown in Fig. 7 by the solid circle. An upper-bound response, corresponding to the peak of the frequency response curve in Fig. 7, can be calculated from the relation applicable to steady state excitation:

$$[4] \quad a = (2\pi f)^2 \frac{\alpha P}{k} \frac{1}{2\xi}$$

or, with the substitution as for [3]:

$$[5] \quad a = \frac{\alpha P}{M\xi}$$

where ξ = critical damping ratio.

For span 2 this gives a peak acceleration of 8.0 m/s^2 or a peak dynamic displacement of 49 mm. The possibility that the damping ratio changes at these large amplitudes needs to be considered. It seems plausible, however, that a response close to this full resonance can be reached. Such a response level, often referred to as "vandal" excitation, should thus be considered in the design procedure and a check on stresses carried out.

Span 1, f = 4.17 Hz

The results from the force measurements presented in Figs. 4–6 indicate substantial components also at twice and, to a lesser degree, at 3 or 4 times the walking, running, or jumping rate (Rainer and Pernica 1986). Thus a resonant condition could also be produced at these higher harmonics. This was investigated on span 1. Walking rates were chosen so that the second harmonic of the forces falls near the natural frequency of the span. The measured peak accelerations are shown in Fig. 11. A peak acceleration of nearly 0.8 m/s^2 occurred at a walking rate of 2.08 steps/s, which corresponds to an excitation frequency of 4.16 Hz provided by the second harmonic of the walking forces.

The response to walking of span 1, having a natural frequency of 4.17 Hz, will now be determined by the proposed procedure, [2], or [3]. For the test subject walking, the dynamic load factor α_2 of the second harmonic at 4.17 Hz is 0.20; for 36 cycles, double the number of steps, the amplification factor Φ = 31, from Fig. 8 for the applicable damping ratio β = 1.2%. Substitution in [2] gives

$$a = 4\pi^2(4.17)^2 \frac{(0.20)(735)}{3720 \times 10^3}(31) = 0.84 \text{ m/s}^2$$

This compares with a measured peak response by the test subject of 0.80 m/s^2 for walking.

Comparisons with BS5400 and OHBDC

The procedure used here and the methods outlined in BS5400 (1978) and OHBDC (1983) need to be compared using a consistent set of assumptions. Although the stated forcing function in BS5400 and OHBDC is F = 180(sin $2\pi ft$) N to represent forces from walking, this is not used explicitly. Instead, the acceleration is calculated from an equivalent static deflection (w_s) due to a person weighing 700 N:

$$[6] \quad a = 4\pi^2 f^2 w_s K\psi$$

where f = natural frequency of span; K = configuration factor; ψ = dynamic response factor.

The stiffness k at the centre of span 2 was measured as 897 N/mm, so the deflection due to a 700 N person is w_s = 0.780 mm, K = 1.0 for a simple span, and ψ = 7.5 from the graphs in

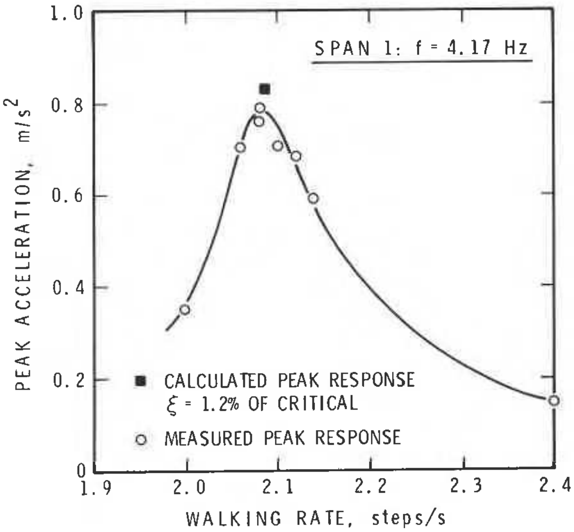

FIG. 11. Peak responses of span 1 to walking.

OHBDC (1983) and BS5400 (1978). Thus the calculated acceleration (a) is 0.97 m/s^2.

The response calculation given by [2] or [3] results in a peak acceleration (a) of 1.06 m/s^2, using f = 2.03 Hz, α = 0.41, P = 735 N, and Φ = 19 for 18 steps at 1.4% damping. This compares favourably with the measured peak response of 1.05 m/s^2 for the fundamental mode.

Although the final answers from BS5400 and OHBDC and the method proposed here are in good agreement, the values for the parameters used are quite different (Table 1).

The differences in α and the dynamic factors Φ and ψ largely compensate one another in the two calculation methods. The proposed method, however, has a wider application in that it would permit a designer or investigator to substitute different parameters that are applicable to other specific loading cases. For example, [2] and [3] apply to walking and running for one or more people in step, and [4] and [5] apply to jumping. These formulas can also be used with other than the first harmonic of the forcing function, as, for example, when the natural frequency of the span coincides with the second harmonic of the excitation.

The criterion given by BS5400 is that the peak response should not exceed $0.5\sqrt{f}$ m/s^2. For span 1, $0.5\sqrt{4.17}$ = 1.02 m/s^2, and for span 2, $0.5\sqrt{2.02}$ = 0.72 m/s^2. The permissible values in the OHBDC are 0.75 and 0.44 m/s^2 for span 1 and span 2, respectively. Thus the observed accelerations for walking on spans 1 and 2 just exceed the criteria under BS5400, whereas the accelerations on both spans substantially exceed the criteria under OHBDC. A subjective assessment by the investigators would support the OHBDC result, although this perception would not necessarily be applicable to in-service behaviour.

Response reduction by resonant dampers

Resonant dampers were constructed with natural frequencies of 2.06 and 2.14 Hz, applied at mid-span of span 2. A series of walking tests was performed and the resulting peak accelerations for various damper properties plotted in Fig. 12. Comparison with results without the damper shows major reductions in peak accelerations, well below the criteria quoted above. A similar observation was made when a resonant damper was

TABLE 1. Response calculations and measured values for span 2

Method	α	P (N)	Dynamic factor	Peak acceleration (m/s^2)
Proposed method	0.41	735	Φ = 19 (Fig. 8)	1.06
		700		1.01
BS5400, OHBDC	180/700 = 0.257	700	ψ = 7.5*	0.97
Measured		735		1.05

*From British Standard BS5400 (BS5400 1978) and the Ontario Highway Bridge Design Code (OHBDC 1983).

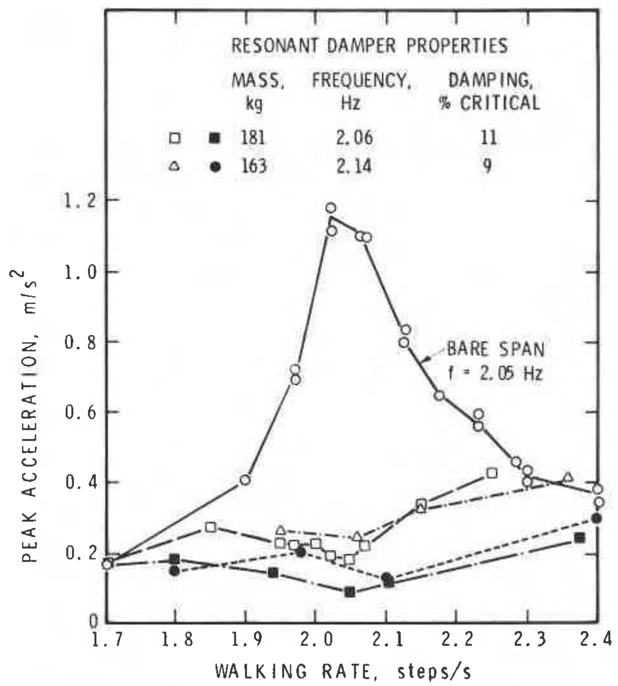

FIG. 12. Response reduction due to resonant damper.

applied to span 1. Thus, the objectionable vibrations at the resonant frequencies of the spans can be drastically reduced by an appropriately sized resonant damper. This confirms what other investigators (Tilly *et al.* 1984; Wheeler 1982; Matsumoto *et al.* 1978) have reported. Footbridges could therefore be designed with a resonant damper, or provision can be made for using one as a retrofit measure, should objectionable vibrations occur. The latter strategy may be a cost-effective solution, particularly in view of uncertainties in estimating damping and calculating the natural frequency of the structure.

Conclusions

Dynamic forces for walking, running, and jumping have been presented for the range of step frequencies usually associated with these activities. The loading function primarily consists of up to four harmonic components with frequencies of integer multiples of the footstep rate and decreasing amplitudes for the higher frequencies.

A procedure is described for evaluating the response of simple spans to walking, running, and jumping excitations at the resonance frequencies. The peak acceleration response at the frequency resonance of two test spans is computed. This is similar to the method in BS5400 and OHBDC, except that the latter two do not fully reflect the actual loading functions. The new procedure permits the designer to employ other loading functions that may be applicable to footbridges, as, for example, where excitation arises from the second harmonic component of pedestrian movement. The predicted peak accelerations are verified by tests on two laboratory footbridges. It is suggested that the response to jumping also be considered in design.

Acknowledgements

This paper is a contribution of the Institute for Research in Construction, National Research Council of Canada.

ALLEN, D. E., RAINER, J. H., and PERNICA, G. 1985. Vibration criteria for assembly occupancies. Canadian Journal of Civil Engineering, **12**: 617–623.

BLANCHARD, J., DAVIES, B. L., and SMITH, J. W. 1977. Design criteria and analysis behaviour of bridges. Department of Transport, Crowthorne, England, TRRL supplementary report SR275, pp. 90–106.

BS5400. 1978. Steel, concrete and composite bridges. Part 2. Specification for loads. British Standards Institution, London, England.

HARPER, F. C., WARLOW, W. J., and CLARK, B. L. 1961. The forces applied to the floor by the foot in walking. Building Research Station, National Building Studies Research Paper 32, HMSO, London, England.

KOBORI, T., and KAJIKAWA, Y. 1974. Psychological effects of highway bridge vibrations on pedestrians. Transactions, Japan Society of Civil Engineering, **6**: 41.

MATSUMOTO, Y., NISHIOKA, T., SHIOJIRI, H., and MATSUZAKE, K. 1978. Dynamic design of footbridges. International Association for Bridge and Structural Engineering, Proceedings, P-17/78, pp. 1–15.

NILSSON, L. 1976. Impact loads produced by human motion. Part 1: Document D13:1976. Swedish Council for Building Research, Stockholm, Sweden.

——— 1980. Impact loads produced by human motion. Part 2: Document D20:1980. Swedish Council for Building Research, Stockholm, Sweden.

OHBDC. 1983. Ontario Highway Bridge Design Code 1983 and Commentary. Ministry of Transportation and Communications, Toronto, Ont.

OHLSSON, S. 1982. Floor vibrations and human discomfort. Department of Structural Engineering, Chalmers University of Technology, Göteborg, Sweden.

RAINER, J. H., and PERNICA, G. 1986. Vertical dynamic forces from footsteps. Canadian Acoustics, **14**: 12–21.

SUPPLEMENT TO THE NATIONAL BUILDING CODE OF CANADA. 1985. Commentary A, serviceability criteria for deflections and vibrations. National Research Council of Canada, Ottawa, Ont., pp. 146–152.

TILLY, G. P., CULLINGTON, D. W., and EYRE, R. 1984. Dynamic behaviour of footbridges. International Association for Bridge and Structural Engineering, Surveys, S-26/84, pp. 13–24.

TUAN, C. Y., and SAUL, W. E. 1985. Loads due to spectator movements. ASCE Journal of Structural Engineering, **111**: 418–434.

WHEELER, J. E. 1982. Prediction and control of pedestrian induced vibration in footbridges. ASCE Journal of the Structural Division, **108**(ST9): 2045–2065.