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Generic Dynamic Simulation For Mechanical Systems

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Abstract

The goal of the machine simulator research project at IMTI is to develop a generic mechanical system simulator. With classical dynamics modeling techniques, dynamics equations for different multibody systems must be derived individually and separately prior to the simulation and control. This is only suitable to systems with fixed topologies or configurations. Therefore there is a need to develop a generic approach for arbitrary multibody systems. In this paper, we derive a generic form of dynamics equation by using recursive kinematics and dynamics. A topology matrix that defines the configuration of 3D open branch multibody systems is expressed explicitly in the equation. This leads to a system-independent form applicable to branch structures of arbitrary complexity. Based on this generic form of dynamic equation for open branch, we also developed constraint dynamics modeling and numerical algorithm. The practical applicability in real time simulation has been demonstrated with a few application examples.

Keywords: *Machine simulation, mechanical system, multibody dynamics, real time simulation*

1. Introduction

Designers and manufacturers are presently facing challenges to stay competitive and deliver products with high quality and low costs. Technologies such as virtual prototyping and virtual manufacturing are becoming enabling technologies to speed up the process by evaluating over virtual models much before the physical product are made and tested. Machine simulation is a vital component in virtual prototyping and manufacturing.

Conventional machine simulation is based on the traditional dynamics modeling techniques. These techniques demand dynamics modeling to be done prior to simulation. While the dynamics modeling obtained is specific and only suitable to this system. Thus, the simulation of every new configuration of machine or a new product has to undergo the process of deriving dynamics modeling repetitively. This is a highly time consuming and expensive process.

In this paper we present the methodology and results of our core project: Machine Simulator, in which the primary goal is to develop a generic system. Developing a generic system requires new techniques for multibody dynamics. The idea is to derive a set of body-independent and system-independent dynamics equations, so that it can be used in arbitrary mechanical systems without the change of the formulation.

2. Dynamics For Generic Mechanical System

2.1 Topology Of Arbitrary System

Parts (bodies) in a mechanical system are linked in a certain topologic relationship, called a configuration. This relationship can be presented by topologic matrix A^{IM}[1]. We use the same definitions of topologic matrix A^{IM} and attachment vector $\mathbf{\Lambda}_{e_i}$ as for 2D in, by extending $\mathbf{\Lambda}_{e_i} \in \mathbf{R}^{S\times 1}$ to $\mathbf{\Lambda}_{e_\xi}(\mathbf{I}_3) \in \mathbf{R}^{3S\times 3}$ for the 3D case [2].

2.2 Recursive Kinematics

Without loss of generality, for a rigid body e_i, let $\mathbf{R}_i^{COM} \in \mathbf{R}^3$ and $\mathbf{R}_i^{HP} \in \mathbf{R}^3$ be the position vector of the center of mass (COM) and Hang Point (HP) (shown in Figure 1 (a)) in global coordinates (inertial frame) respectively. Also, let $\mathbf{r}_i^{HP} \in \mathbf{R}^3$ and $\bar{\mathbf{r}}_i^{HP} \in \mathbf{R}^3$ be the vector from COM to HP in global coordinates and COM body-fixed coordinates, respectively, and $\mathbf{A}_i \in SO(3)$ be the transformation matrix of COM body-fixed frame of body e_i relative to the global frame. Finally, let $\mathbf{v}_i^{HP} \in \mathbf{R}^3$ be the unit vector of the axis of Hang Point joint of e_i, and $\boldsymbol{\omega}_i$ and $\boldsymbol{\alpha}_i \in \mathbf{R}^3$ be the angular velocity and acceleration vectors of e_i in global coordinate, respectively. We have the following equations

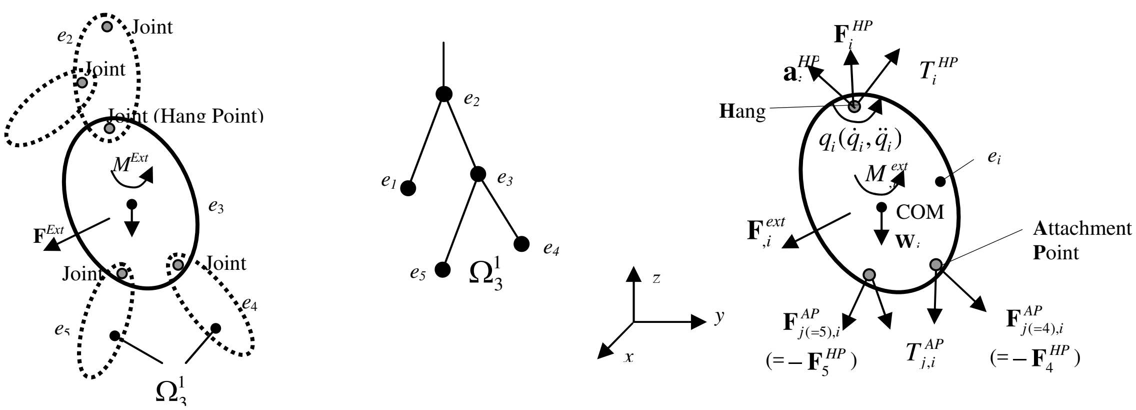
$$\mathbf{R}_i^{HP} = \mathbf{R}_i^{COM} + \mathbf{r}_i^{HP} = \mathbf{R}_i^{COM} + \mathbf{A}_i \bar{\mathbf{r}}_i^{HP} \tag{1}$$

$$\boldsymbol{\omega}_i = \boldsymbol{\omega}_u + \mathbf{v}_i^{HP} \dot{q}_i \tag{2}$$

$$\mathbf{a}_i^{HP} = \mathbf{a}_i^{COM} + \boldsymbol{\alpha}_i \times \mathbf{r}_i^{HP} + \boldsymbol{\gamma}_i^r \tag{3}$$

$$\boldsymbol{\alpha}_i = \boldsymbol{\alpha}_u + \mathbf{v}_i^{HP} \ddot{q}_i + \boldsymbol{\gamma}_i^q \tag{4}$$

where q_i and $\dot{q}_i$ are the HP joint value of e_i and its time derivative with respect to its parent body e_u, respectively; and $\boldsymbol{\gamma}_i^r = \boldsymbol{\omega}_i \times \left(\boldsymbol{\omega}_i \times \mathbf{r}_i^{HP}\right)$ and $\boldsymbol{\gamma}_i^q = \left(\boldsymbol{\omega}_i \times \mathbf{v}_i^{HP}\right)\dot{q}_i$



(a) Multibody system (b) Branch structure (c) A rigid body

Figure 1 Branching type multibody system graph

If we number the rigid bodies in the order of u, i, j, k, n, p, r, s from the layer of body e_i to the lowest layer, we can have recursive equations for any body in the path, say body e_p

$$\boldsymbol{\alpha}_p = \boldsymbol{\alpha}_u + \sum_{\xi=i}^{p} \mathbf{v}_\xi^{HP} \ddot{q}_\xi + \sum_{\xi=i}^{p} \boldsymbol{\gamma}_\xi^q \tag{5}$$

$$\mathbf{a}_p^{COM} = \mathbf{a}_i^{HP} + \boldsymbol{\alpha}_u \times \left(\mathbf{d}_{p,i} - \mathbf{r}_p^{HP}\right) + \sum_{\xi=i}^{n} \left(\mathbf{v}_\xi^{HP} \times (\mathbf{d}_{p,\xi} - \mathbf{r}_p^{HP})\ddot{q}_\xi - \boldsymbol{\gamma}_\xi^q \times \left(\mathbf{d}_{p,\xi} - \mathbf{r}_p^{HP}\right) + \boldsymbol{\gamma}_{\xi+1,\xi}^d\right) \tag{6}$$

where $\mathbf{d}_{p,i}$ is the vector of from the HP of i to HP of p and $\boldsymbol{\gamma}_{\xi+1,\xi}^d = \boldsymbol{\omega}_\xi \times \left(\boldsymbol{\omega}_\xi \times \mathbf{d}_{\xi+1,\xi}\right)$

2.3 Dynamics Equation

Applying Newton-Euler's equation [3] [4] to rigid body e_i, we have

$$m_i \mathbf{a}_i^{COM} = \sum_{\xi} \mathbf{F}_{\xi,i}^{ext} + \mathbf{W}_i + \mathbf{F}_i^{HP} + \sum_{\forall j, e_j \in \Omega_i^1} \mathbf{F}_{j,i}^{AP} \tag{7}$$

$$\mathbf{J}_i^{\theta\theta} \boldsymbol{\alpha}_i = \sum_{\eta} \mathbf{M}_{\eta,i}^{ext} + \sum_{\xi} \left(\mathbf{r}_{\xi,i}^{ext} \times \mathbf{F}_{\xi,i}^{ext} \right) - \boldsymbol{\omega}_i \times (\mathbf{J}_i^{\theta\theta} \boldsymbol{\omega}_i) + \mathbf{r}_i^{HP} \times \mathbf{F}_i^{HP} + \sum_{\forall j, e_j \in \Omega_i^1} \left(\mathbf{r}_{j,i}^{AP} \times \mathbf{F}_{j,i}^{AP} \right) \mathbf{T}_i^{HP} + \sum_{\forall j, e_j \in \Omega_i^1} \mathbf{T}_{j,i}^{AP} \tag{8}$$

where, m_i, $\mathbf{J}_i^{\theta\theta}$ are the mass and inertia tensor, respectively, of body e_i in global coordinates, $\mathbf{F}_{\xi,i}^{ext}$ and $\mathbf{r}_{\xi,i}^{ext}$ are the ξ^{th} external force and position vector from COM to its application position in global coordinates for body e_i. $\mathbf{M}_{\mu,i}^{ext}$ is the μ^{th} external moment, $\mathbf{W}_i$ is gravity force vector; $\mathbf{F}_i^{HP}$ and $\mathbf{T}_i^{HP}$ are the joint force and torque vectors at HP applied to e_j, respectively. Finally, $\mathbf{F}_{j,i}^{AP}$ and $\mathbf{T}_{j,i}^{AP}$ are the joint force and torque at the j^{th} Attachment Point joint (**AP**) applied to e_i by the attachment body e_j and $\mathbf{r}_{j,i}^{AP}$ is the vector from the **COM** to j^{th} **AP** in global coordinates**.**

Eliminating $\mathbf{F}_i^{HP}$ by substitution of Equation (7) into(8), the dynamics equation for body e_i can be expressed as

$$\mathbf{J}_i^{\theta\theta} \boldsymbol{\alpha}_i = m_i \mathbf{r}_i^{HP} \times a_i^{COM} - \boldsymbol{\gamma}_i^J + \mathbf{T}_i^{Ext} - \sum_{\forall j, e_j \in \Omega_i^1} \left(\mathbf{d}_{j,i} \times \mathbf{F}_j^{HP} \right) + \mathbf{T}_i^{HP} - \sum_{\forall j, e_j \in \Omega_i^1} \mathbf{T}_j^{HP} \tag{9}$$

where, $\mathbf{T}_i^{Ext} = \sum_{\eta} \mathbf{M}_{\eta,i}^{ext} + \sum_{\xi} \left(\left(\mathbf{r}_{\xi,i}^{ext} - \mathbf{r}_i^{HP} \right) \times \mathbf{F}_{\xi,i}^{ext} \right) - \mathbf{r}_i^{HP} \times \mathbf{W}_i$ and $\boldsymbol{\gamma}_i^J = \boldsymbol{\omega}_i \times (\mathbf{J}_i^{\theta\theta} \boldsymbol{\omega}_i)$.

For an ideal joint without damping, combining recursive equation (5), (6) and (9), we have a dynamic equation in terms of the joint acceleration $\ddot{q}_i$, extended moment of inertia J_i and four wrench terms as

$$J_i \ddot{q}_i = F_i^v + F_i^{Ext} + F_i^{Ha} + F_i^{Att} \tag{10}$$

where F_i^v is the Coriolis and centrifugal wrench term, F_i^{Ext} is the external force wrench term, F_i^{Ha} is the coupling term with the motion of parent body e_u, and F_i^{Att} is the coupling term for the attachment bodies

$$J_i = \mathbf{v}_i^{HP^T} (\mathbf{J}_{sys} - \mathbf{X}^{ii}) \hat{\boldsymbol{\Lambda}}_{e_i} (\mathbf{I}_3) \mathbf{v}_i^{HP} \tag{11}$$

$$F_i^v = -\mathbf{v}_i^{HP^T} \mathbf{J}_i^{Eff} \boldsymbol{\gamma}_i^q - m_i \left(\mathbf{v}_i^{HP} \times \mathbf{r}_i^{HP} \right)^T \boldsymbol{\gamma}_i^r - \mathbf{v}_i^{HP^T} \boldsymbol{\gamma}_i^J \tag{12}$$

$$F_i^{Ext} = \mathbf{v}_i^{HP^T} \mathbf{T}_i^{Ext} \tag{13}$$

$$F_i^{Ha} = -\left(\mathbf{v}_i^{HP} \times \left(\mathbf{K}^i \hat{\boldsymbol{\Lambda}}_{e_i} \right) \right)^T \mathbf{a}_i^{HP} - \mathbf{v}_i^{HP^T} \mathbf{J}_i^{Eff} \boldsymbol{\alpha}_u \tag{14}$$

$$F_i^{Att} = -\mathbf{v}_i^{HP^T} \left(\mathbf{Q}_{sys}^i \boldsymbol{\Lambda}_{e_i} \right) - \mathbf{v}_i^{HP^T} \left(\boldsymbol{\Phi}_{sys} \boldsymbol{\Lambda}_{e_i} \right) \tag{15}$$

where $\boldsymbol{\Lambda}_{e_\xi}$, $\hat{\boldsymbol{\Lambda}}_{e_\xi}$, $\boldsymbol{\Lambda}_{e_\xi}(\mathbf{I}_3)$ and $\hat{\boldsymbol{\Lambda}}_{e_\xi}(\mathbf{I}_3)$ are the attachment vectors of e_ξ, which reflects the topology of the system. In this equation, J_i is not only the function of mass and inertia of the body itself but of all the attached bodies as well. Equation (10) is a generic dynamic equation for arbitrary mechanical system since the topology of the system is expressed explicitly in the equation.

We have developed a numerical algorithm to solve the dynamics equations for generic system. More details about the numerical algorithm and validation can be found in [2] and [5]. Based on equation we have also developed point constraint dynamics and algorithm.

3. Applications

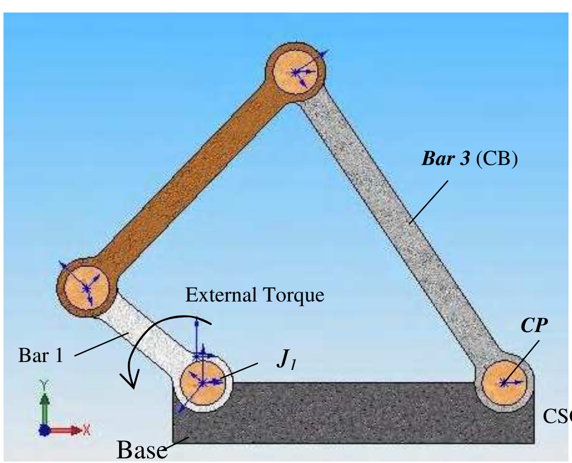
3.1 Simulation of a Robot

This application shows the use of current generic approach for open branch system, a puma robot as show in Fig 3. The robot is driven by separate motors for each joint using either torque control or velocity control. In the demonstration process, the robot is assembled component by component in real time, which gives the idea how the techniques work for changeable mechanical system.

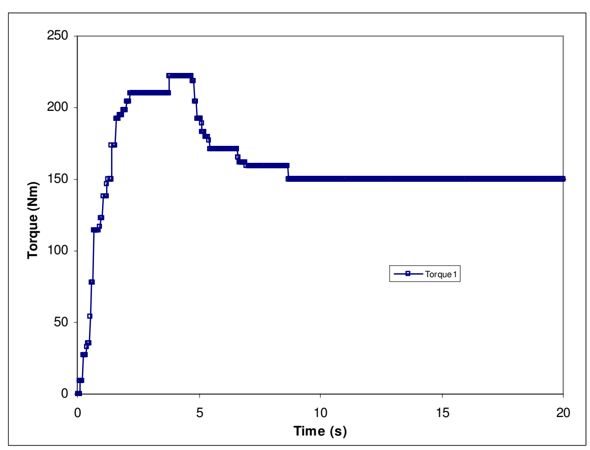
Figure 3 Puma robot model

3.2 Simulation of a Four-Bar Mechanism

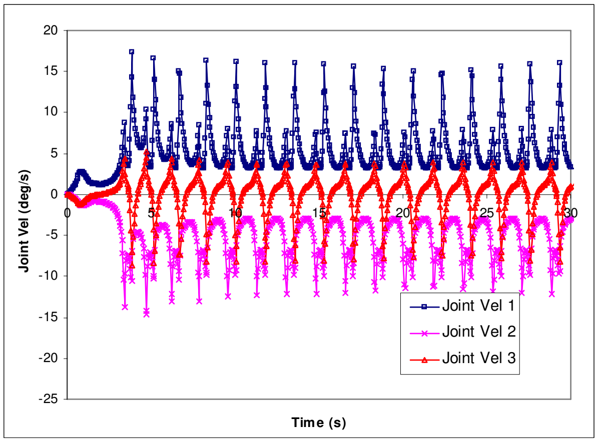
With two constraint cases, we demonstrate the validation of the constraint modeling and numerical algorithm discussed in previous sections. The first application is a four-bar mechanism with Point-Point constraints, Figure 4 (a). The Bar 1 is connected to base with a rotate joint J_1 and is driven by an external torque. Bar 3, the constraint body C, has a connection to constraint source object at CP. This connection is physically a revolute joint too. Figure 4 (b), (c) and (d) shows the external torques and joint velocity and constraint force with time, respectively.



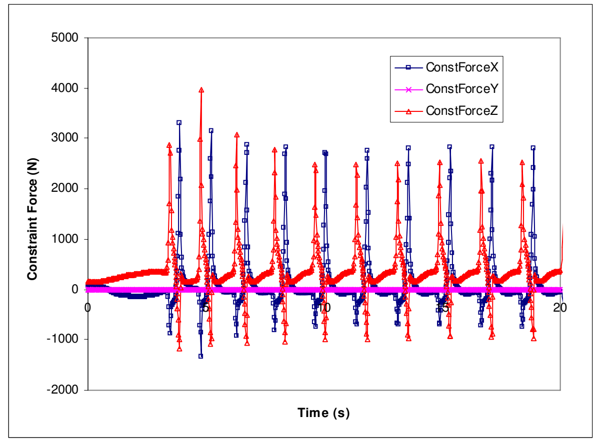
(a) Four-bar mechanism



(b) External torque



(c) Joint velocities

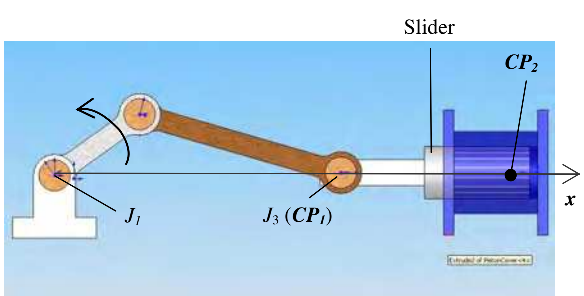


(d) Constraint force

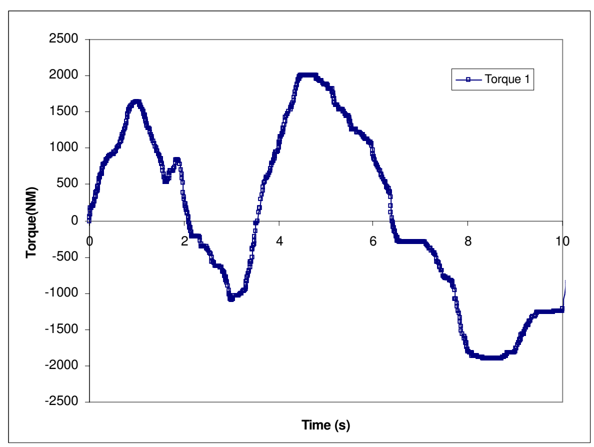
Figure 4 Four-bar mechanism simulation

3.3 Simulation of a Crank-Slider

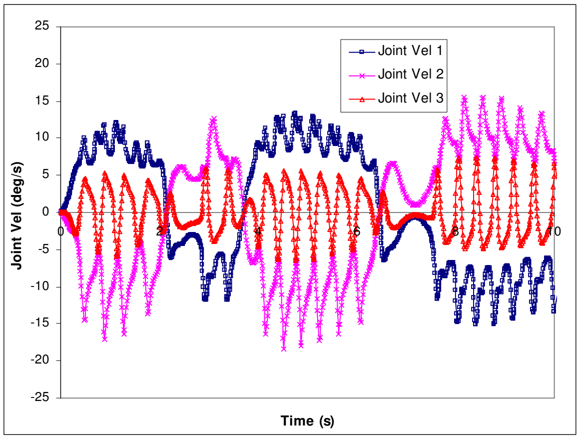
The third example is a Crank-Slider mechanism as shown Figure 5 (a). A crank connected to the base with a rotational joint J1 and is driven by an external torque. In this example, there are two constraints. The point (CP1) at the center of the joint J3 between the Link and Slider and is constrained to move along the x-axis. Another point CP2 on the Slider is also allowed moving along x-axis. This is the Point-Line constraint case. Figure 5 (b), (c) and (d) shows the external torques and joint velocity and constraint force with time, respectively.



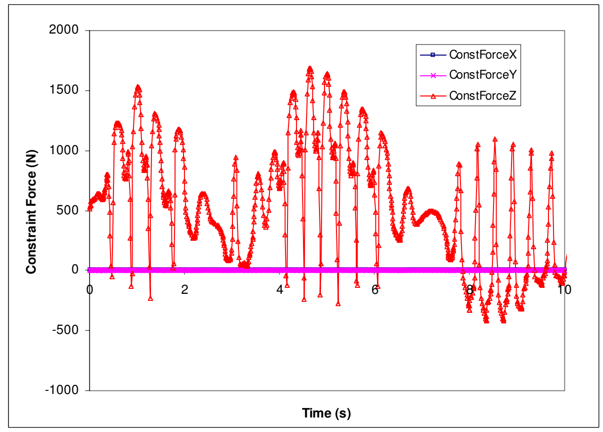
(a) Crank-Slider mechanism



(b) External torque



(c) Joint velocities



(d) Constraint force

Figure 5 Crank-Slider mechanism simulation

4. Conclusions

The generic dynamics equation present in this paper is a generic approach for multibody systems that allows arbitrary mechanical systems to be simulated, without having to re-derive system dynamics equations repetitively. The use of the incidence matrix that defines the configurations of 3D open branch multibody systems leads to a system-independent form applicable to branch structures of arbitrary complexity. The generic dynamics equation is used for each individual body in the system to form a system of dynamics equations. We have developed 3D dynamics equation for open branch multibody systems connected by rotation joints such as pin, universal and sphere by using recursive kinematics and dynamics. Based on the open branch equation, we developed constraint dynamics modeling and associated numerical algorithm.

Utilizing the C++ dynamic equation solver developed in IMTI, the real time machine simulator is a platform for generic mechanical system simulations. We have demonstrated several simulations applications such as puma robot, four-bar mechanism and crank-slider mechanism. The simulation have shown the procedure of the assembly or reconfiguration of the systems in real time, and demonstrated the applicability of the techniques in real time simulations for reconfigurable systems. Further research is being conducted for more complex systems.

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