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DIRECT-ENERGY DENSITY IN TRANSMISSION- LOSS MEASUREMENTS

BY
R. J. DONATO

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MESURE DES PERTES DE TRANSMISSION ACOUSTIQUE ET DENSITE D'ENERGIE SONORE DIRECTE

SOMMAIRE

L'auteur attire l'attention sur les erreurs qu'on peut commettre en tenant compte de la puissance acoustique transmise directement à une enceinte réceptrice. Il établit des formules pour évaluer la densité moyenne d'énergie acoustique directe à travers une pièce et aussi pour calculer sa variation selon la distance du mur rayonnant. L'auteur établit une comparaison avec les niveaux d'énergie acoustique de réverbération dans la pièce selon les distributions présumées des parois absorbantes.

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Direct-Energy Density in Transmission-Loss Measurements*

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Attention is drawn to the errors that might arise in allowing for the direct energy in a receiving enclosure. Formulas are developed for the mean direct-energy density throughout the room and also for its dependence on the distance from the radiating wall. Comparisons with the reverberant levels in the room are made for various assumed distributions of wall absorption.

INTRODUCTION

THE normal Buckingham formula¹ used for the calculation of the transmission loss through a wall makes no separation of the direct from the reverberant energy in the receiving enclosure. Beranek,² in his formula, introduces a term to account for the direct energy. In laboratory measurements of transmission loss, where the receiving room has a low absorption, this term is relatively unimportant compared to the reverberant term. In field testing, however, this is not necessarily the situation. Beranek's term is of doubtful validity as it is derived on the assumption that the direct field is uniform throughout the receiving room and is produced by a plane wave traveling normal to the transmitting wall. Should this be the case, the only wall receiving energy is the one opposite the test panel and the same argument might then be extended to subsequent reflections. These reflections would bounce back and forth between the test wall and the opposite wall so that conditions are closer to a channel than to an approximately cubic enclosure. London's³ approach to the problem is more realistic in that he shows that the direct field will vary throughout the enclosure, but, extending his argument into a test procedure, necessitates taking measurements close to the transmitting panel.

The analysis given below shows how an expression

for the average energy density of the direct field may be obtained, assuming a cosine law of emission, and enables the usual procedure of spatial averaging to be carried out.

I. THEORY

Before developing the analysis, we briefly review the argument used by Beranek. The energy density within the receiving room is split into two parts: a direct term E_d and a reverberant term E_r . If the total power radiated into the enclosure is W , then, for the power sent into the reverberant field, we may put

$$W_r = W(1 - \bar{\alpha}), \quad (1)$$

where $\bar{\alpha}$ is the absorption coefficient of the room surfaces. This is related to the energy density of the reverberant field by the expression,

$$E_r V \bar{\alpha} / t_1 = W(1 - \bar{\alpha}), \quad (2)$$

where t_1 is the mean time between collisions of a sound-ray path with the boundary of the enclosure and V is the total room volume. Similarly, we write for the energy density of the direct wave,

$$E_d V / t_2 = W, \quad (3)$$

where, again, t_2 is a mean free time. The total energy density E_t in the receiving-room enclosure is given by

$$E_t = E_d + E_r = (W/V)t_2 + [W(1 - \bar{\alpha})/V\bar{\alpha}]t_1. \quad (4)$$

Beranek fixes a value for t_2 equal to L_z/c , where L_z is the room dimension perpendicular to the transmitting panel and c is the velocity of sound. For t_1 , we make the customary substitution $4V/cS$. Thus,

$$E_t = W/cS_w + 4W(1 - \bar{\alpha})/cS\bar{\alpha}, \quad (5)$$

* This paper is a contribution from the Division of Building Research of the National Research Council, Canada, and is published with the approval of the Director of the Division.

¹ E. Buckingham, Natl. Bur. Std. (U. S.) Sci. Paper No. 506, 193-219 (1925).

² L. L. Beranek, *Acoustics* (McGraw-Hill Book Co. Inc., New York, 1954), p. 321.

³ A. J. London, J. Res. Natl. Bur. Std. (U. S.) 26, 419-453 (1941).

where S_w is the area of the transmitting panel and S the total wall area. To obtain the transmission coefficient τ , we utilize the relation

$$W = \frac{1}{4} \tau S_w c E_{inc}, \quad (6)$$

where E_{inc} is the energy density in the source room. Then

$$1/\tau = (E_{inc}/E_t) \{ \frac{1}{4} + (S_w/S)(1-\bar{\alpha})/\bar{\alpha} \}. \quad (7)$$

To make this equation identical to that of Buckingham,³ we must choose $t_2 = t_1 = 4V/cS$. Then Eq. 7 becomes

$$1/\tau = (E_{inc}/E_t) S_w / S \bar{\alpha}. \quad (8)$$

To resolve the discrepancy between Eqs. 7 and 8, we determine the average density of the direct component throughout the enclosure, assuming a random phase distribution over the transmitting panel. Since the incident energy is assumed to be diffuse, the energy emitted from the panel is also assumed to obey a cosine law of emission.

II. ANALYSIS

Although the problem can be solved for the three-dimensional case, the integrals involved in the computation become cumbersome. To facilitate the calculations, we solve the problem for the two-dimensional case and then superimpose two similar solutions to obtain that for the corresponding three-dimensional case.

The expression to be solved for the average energy density is

$$\int_0^{2L_x} \int_{-L_y}^{L_y} \int_{-L_y}^{L_y} x(r)^{-2} dx dy dy_0, \quad (9)$$

where y_0 is coordinate of the radiating element, $2L_x$ and $2L_y$ are the receiving-room dimensions, and x, y are coordinates of a point within the room. Substituting $r^2 = x^2 + (y - y_0)^2$, we find after integrating with respect to x that the integral becomes

$$\frac{1}{2} \int_{-L_y}^{L_y} \int_{-L_y}^{L_y} \{ \log[4L_x^2 + (y - y_0)^2] - \log(y - y_0)^2 \} dy dy_0. \quad (10)$$

We neglect constants throughout and write the integral to be solved as

$$\int_{-L_y}^{L_y} \int_{-L_y}^{L_y} \log\{A^2 + (y - y_0)^2\} dy dy_0, \quad (11)$$

where A may be $2L_x$ or 0. Integrating with respect to y yields

$$(L_y - y_0) \log[A^2 + (L_y - y_0)^2] + (L_y + y_0) \log[A^2 + (L_y + y_0)^2] - 4L_y + 2A \left[\tan^{-1} \left(\frac{y - y_0}{A} \right) \right]_{-L_y}^{L_y}. \quad (12)$$

Each of these terms is integrated over y_0 and the result is

$$4L_y^2 \log(A^2 + 4L_y^2) - 12L_y^2 - A^2 \log(A^2 + 4L_y^2) + A^2 \log A^2 + 8AL_y \tan^{-1}(2L_y/A). \quad (13)$$

We now subtract the value of this expression with $A=0$ from the value with $A=2L_x$, and thus obtain the integral

$$I = 4L_y^2 \log[(L_x^2 + L_y^2)/L_y^2] + 4L_x^2 \log[L_x^2/(L_x^2 + L_y^2)] + 16L_x L_y \tan^{-1}(L_y/L_x). \quad (14)$$

If we combine Eq. 13 with a similar expression, obtained when L_x is substituted for L_y , the average energy density $\bar{E}$ becomes

$$\bar{E} = \text{const} \times \{ \log[(L_x^2 + L_y^2)/L_y^2] + (L_x/L_y)^2 \log[L_x^2/(L_x^2 + L_y^2)] + 4(L_x/L_y) \tan^{-1}(L_y/L_x) \} \times \{ \log[(L_x^2 + L_y^2)/L_x^2] + (L_y/L_x)^2 \log[L_y^2/(L_x^2 + L_y^2)] + 4(L_y/L_x) \tan^{-1}(L_x/L_y) \}. \quad (15)$$

If $L_x \ll L_y$, L_x , then $\bar{E} = 2\bar{W}/c$, where $\bar{W}$ is the power transfer per unit area from room to room.³ Thus,

$$2(\bar{W}/c) = \text{const}[4(L_x/L_y)(\pi/2) \times 4(L_x/L_x)\pi/2] \quad (16)$$

or

$$\text{const} = (\bar{W}/2\pi^2 c)(L_y L_x / L_x^2). \quad (17)$$

Equation 15 can then be written as

$$\bar{E} = (\bar{W}/2\pi^2 c) [(L_y/L_x) \log[(L_x^2 + L_y^2)/L_y^2] + (L_x/L_y) \log[L_x^2/(L_x^2 + L_y^2)] + 4 \tan^{-1}(L_y/L_x) \times [(L_x/L_x) \log[(L_x^2 + L_y^2)/L_x^2] + (L_y/L_y) \log[L_y^2/(L_x^2 + L_y^2)] + 4 \tan^{-1}(L_x/L_x)]. \quad (18)$$

For a cubic room $L_x = L_y = L_z$, then

$$\bar{E} = (\bar{W}/2\pi^2 c) [\log 2 + \log(\frac{1}{2}) + \pi]^2 = \bar{W}/2c. \quad (19)$$

Comparing Eq. 19 with Eq. 5, we may now write

$$1/\tau = (E_{inc}/E_t) \{ \frac{1}{8} + (S_w/S)[(1-\bar{\alpha})/\bar{\alpha}] \}. \quad (20)$$

In a room where dimensions L_x, L_y, L_z are in the ratio $(4)^{1/3}:(2)^{1/3}:1$ (recommended to reduce redundancy of modes),

$$\bar{E} = (\bar{W}/2\pi^2 c) \{ (2)^{-1} \log(2^{1/3} + 1) + (2)^{1/3} \log[2^{1/3}/(1 + 2^{1/3})] + 4 \tan^{-1}(2)^{-1/3} \} \times \{ (4)^{-1/3} \log(4^{1/3} + 1) + (4)^{1/3} \log[4^{1/3}/(1 + 4^{1/3})] + 4 \tan^{-1}(4)^{-1/3} \}. \quad (21)$$

This gives a value of $\bar{E}$ equal to 0.72 times that of Eq. 19.

It is also possible to express the energy density averaged over $2L_y, 2L_x$ as a function of the distance $2L_x$ from the vibrating wall by using Eq. 18 to derive

$$d(\bar{E} \cdot 2L_x)/d(2L_x). \quad (22)$$

The resulting function, normalized to the energy density close to the wall, is shown in Fig. 1 for the case

FIG. 1. Normalized mean energy density over planes parallel to a square wall. ●: Mean density over plane. ○: London's results for axial energy density from circular panel.

of a square wall. London's results are also shown for the case where the energy density is measured only on the axis of symmetry perpendicular to a circular panel. If London's formula is used to obtain the mean energy density along this axis, the constant term in Eq. 20 will be approximately $\frac{3}{16}$ (in the place of $\frac{1}{8}$), when L_x is the panel radius. Figure 1 shows that the results obtained by differentiating Eq. 18 for a cubical room lie fairly closely to an exponential law of energy density versus distance from the wall.

III. NONUNIFORM ABSORPTION DISTRIBUTION

Apart from the difference over the mean free path for the direct wave, there is another uncertainty in the Beranek model. W_r has been assumed to be equal to $W(1-\bar{\alpha})$, where $\bar{\alpha}$ is the average wall absorption. To be more accurate, one should write

$$W_r = W(1-\alpha_1), \quad (23)$$

where α_1 is the absorption of the wall opposite the transmitting panel. Making this substitution, Eq. 20 becomes

$$1/\tau = (E_{inc}/E_t) \{ (\text{DIRECT TERM}) + (S_w/S)[(1-\alpha_1)/\bar{\alpha}] \}. \quad (24)$$

It is interesting to see what the order of magnitude of the error would be between using Eqs. 20 and 24, for example, if most of the absorption is on the transmitting surface itself. We consider only the reverberation term and represent the absorption coefficient of the transmitting wall by α_2 . It is now easy to show that the fractional error between the reverberation terms is

$$(S_w/S)[(\alpha_2-\alpha_1)/(1-\alpha_1)]. \quad (25)$$

If α_2 is high, say 0.8, and α_1 low, say 0.05, the error becomes $S_w/S(0.80)$. If we accept a maximum error of 0.5 dB in the estimate of transmission loss, then S_w/S must not be greater than 0.15, suggesting a room of approximately cubical shape. If the transmitting surface is the ceiling, the error is increased as the height of the room is reduced. This estimate of the error is pessimistic, and would be reduced if the direct energy term were considered. For this case, the sign of the error is such that the transmission loss of the wall is underestimated. The sign changes, however, when $\alpha_2 < \alpha_1$, i.e., when the transmitting wall has a lower absorption coefficient than the others.

IV. CONCLUSIONS

Buckingham's formula does not separate the direct from the reverberant energy. It can be rearranged, however, by writing

$$(S_w/\bar{\alpha}S) = (S_w/S) + (S_w/S)[(1-\bar{\alpha})/\bar{\alpha}]. \quad (26)$$

Thus, in the place of Beranek's $\frac{1}{4}$ term, Buckingham has a factor equal to S_w/S , which in the case of a cube would have the value $\frac{1}{6}$. These constant terms are in the same ratio as the values assumed for the mean free time associated with the direct-energy component. Buckingham's term would be correct if the sound were originating from all the wall surfaces, and Beranek's would hold only if the direct sound issued in a parallel beam for the radiating wall. Neither of these situations really applies. In Beranek's case, it is difficult to imagine diffuse incident sound being converted to a parallel beam when transmitted through a panel. If the angular distribution is preserved, we find the factor of $\frac{1}{4}$ to be reduced to $\frac{1}{8}$ for a cubically shaped receiving room and somewhat less when the room dimensions are in the ratio $(4)^{\frac{1}{2}}:(2)^{\frac{1}{2}}:1$.

In most cases, however, the term may be looked upon as a small correction whose importance arises only when the absorption of the receiving room is high. Figure 1 shows the relative magnitudes of the direct and reverberant energies, at different distances from a square wall, for mean absorption coefficients of 0.1 to 0.5.