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DYNAMIC GROUND COMPLIANCE IN MULTISTOREY BUILDINGS

J. H. RAINER

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ACTION DYNAMIQUE RECIPROQUE ENTRE LE SOL ET LES STRUCTURES DANS LE CAS DES BATIMENTS A MULTIPLES ETAGES

L'article étudie les changements dans les caractéristiques du mode naturel de vibration des bâtiments à multiples étages appuyés sur des fondations flexibles. Le problème est d'abord formulé comme un problème standard de fréquence propre appliqué au cas d'amortissement proportionnel. Ensuite, des expressions générales des changements dans les fréquences naturelles et de la forme des modes naturels sont dérivées pour la théorie de vibration linéaire. Au moyen d'un exemple, il est démontré, cependant, que seul le premier mode correspond à la prédiction des changements de fréquence et de la forme des modes pour un large éventail de rigidité de fondation. Les modes supérieurs dévient de façon importante du comportement linéaire. Cette déviation est attribuée aux changements géométriques de la forme des modes naturels.

DYNAMIC GROUND COMPLIANCE IN MULTISTOREY BUILDINGS

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(Received 24 October 1970)

A study is presented of the changes in the characteristics of the natural modes of vibration for multistorey structures which are founded on flexible foundations. First a standard eigenvalue problem is formulated for the proportionally damped case. Then general relationships of changes in natural frequencies and mode shapes are derived for the linear vibration theory. By means of an example problem it is demonstrated, however, that only the first mode obeys the predicted changes of frequencies and mode shapes over a wide range of foundation stiffness. The higher modes are shown to deviate substantially from the linear behaviour. This deviation is ascribed to geometric changes in mode shapes.

1. INTRODUCTION

An understanding of the interaction between a structure and its supporting ground is of importance in the design of safer structures and foundations subjected to earthquake motions, and also in the interpretation of full-scale measurements of dynamic properties of structures. The influence of flexible foundations also needs to be considered in satisfying critical performance requirements of structures such as radio telescopes, nuclear reactor vessels and tower structures under dynamic loads.

Two approaches have commonly been used in the study of dynamic effects of flexible foundations: (i) calculation of changes of forces and deformations in a structure when subjected to arbitrary disturbances, as, for example, in references [1, 2]; (ii) determination of changes in natural modes of vibration with varying foundation stiffnesses, as in references [3, 4, 5]. Although the former approach appears to offer the most direct solution to the problem, it has the disadvantage that certain interaction effects are likely to be obscured by the random nature of the disturbance chosen. As the resonance frequencies of a structure are modified with varying base flexibilities, the shifts in frequencies can result in large changes of response—either amplification or reduction—depending on the frequency content of the particular disturbance chosen. It is thus possible that effects arising from a flexible foundation are overshadowed by response changes that are due merely to shifts in the resonance frequencies of the structure. The natural modes of vibration, on the other hand, are characteristic of the structure and its support conditions alone. The modal approach therefore offers a greater likelihood of arriving at generalizations regarding the effects of a flexible foundation on the dynamic behaviour of the structure. With a knowledge of the mode shapes and frequencies, the response to given disturbances can then be estimated by various procedures of modal superposition [6].

This paper presents a method of modal analysis for multistorey structures with rocking flexibility in the foundations. First, a standard eigenvalue problem is formulated, from which mode shapes and natural frequencies can be calculated. Second, relationships are derived to predict the changes in the normal modes and frequencies of vibration of multistorey structures with various amounts of foundation rocking flexibility if the fixed-base mode shapes and

frequencies are known. The range of applicability of the derived relationships is then investigated by means of an example problem.

A question that arises in connection with modal analysis is the influence of damping. For the exact applicability of conventional modal analysis, proportional damping is required, i.e., the damping matrix is to be a linear combination of the mass and stiffness matrix for the structure [7]. With the inclusion of foundation flexibility and foundation damping, this assumption will generally not be satisfied. For the small levels of structural damping present in most buildings and foundations, however, the influence of non-proportional damping on the mode shapes can be assumed to be small. Furthermore it is not yet possible to assign realistic values to foundation nor to structural damping.

In this analysis it has also been assumed that the foundation and structural properties are frequency independent. From results of previous theoretical studies for single-storey [8, 9] as well as multistorey structures [2], this assumption is judged to be reasonable.

2. MATHEMATICAL MODEL FOR EIGENVALUE PROBLEM

The type of structure under investigation is represented by the idealized model shown in Figure 1, which consists of n masses, m_1 to m_n , and n interstorey springs with stiffnesses k_1 to k_n . No restriction is placed on the nature of the interstorey stiffnesses: they may be of the

Figure 1. Multistorey structure with ground flexibility.

shear-type, flexural type, or a combination of both. This implies that all terms in the fixed-base stiffness matrix may be non-zero. The base mass m_1 rests on a flexible foundation whose rotational capacity is characterized by the rotational stiffness K_R , or conversely, the rotational flexibility f_R . The condition where the base mass m_1 is restrained from horizontal movement can be treated simply by eliminating the equation of motion and the coordinate that pertains to the base mass.

The equations of motion under arbitrary base disturbance are as follows. For relative horizontal displacements:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = -\mathbf{M}(\ddot{u}_g + \mathbf{h}\ddot{\theta}). \quad (1)$$

For rotation about the base:

$$\mathbf{d}^T \ddot{\mathbf{x}} + J\ddot{\theta} + C_R \dot{\theta} + K_R \theta = -d_0 \ddot{u}_g, \quad (2)$$

where bold capital letters denote square matrices, bold lower case letters denote column matrices, and the superscript T denotes the transpose of a matrix; $\mathbf{M}$ = mass matrix of structure with horizontal flexibilities only, $\mathbf{C}$ = damping matrix, $\mathbf{K}$ = stiffness matrix, $\mathbf{d}$ =

$\{m_1 h_1, m_2 h_2, \dots, m_n h_n\}^T$, $\mathbf{x} = \{x_1 \dots x_n\}^T$ = interstorey displacement vector, $\mathbf{h} = \{h_1 \dots h_n\}^T$ = storey heights above base, $d_0 = \sum_{i=1}^n m_i h_i$, $J = J_0 + \sum_{i=1}^n m_i h_i^2$, u_g = ground displacement, J_0 = sum of the mass moments of inertia of storey slabs about their own axis of rotation, C_R = rotational damping coefficient, K_R = rotational spring stiffness, θ = angular displacement coordinate and dots over a variable refer to differentiation with respect to time.

To formulate a standard eigenvalue problem for the calculation of natural frequencies and mode shapes, $\ddot{u}_g$ is set to zero. Under the assumption of proportional damping, the mode shapes are the same as for the undamped case; thus $\mathbf{C}$ and $\mathbf{C}_R$ can be omitted from the eigenvalue problem. However, the mass and stiffness matrices are required to be symmetrical for the whole system, which consists of equations (1) and (2). If equations (1) and (2) are re-written as

$$\mathbf{M}^* \ddot{\mathbf{x}}^* + \mathbf{K}^* \mathbf{x}^* = 0, \quad (3)$$

where

$$\mathbf{M}^* = \begin{bmatrix} \mathbf{M} & \mathbf{d} \\ \mathbf{d}^T & J \end{bmatrix}, \quad \mathbf{K}^* = \begin{bmatrix} \mathbf{K} & 0 \\ 0 & K_R \end{bmatrix}, \quad \mathbf{x}^* = \begin{bmatrix} \mathbf{x} \\ \theta \end{bmatrix},$$

then for sinusoidal free vibration, equation (3) becomes

$$[\Omega^{*2} \mathbf{M}^* - \mathbf{K}^*] \mathbf{x}^* = 0, \quad (4)$$

where $\Omega^{*2} = \text{diag}(\omega_1^{*2}, \omega_2^{*2}, \dots, \omega_N^{*2}, \omega_{N+1}^{*2})$ are the natural modal frequencies for the structure with foundation flexibility. Equation (4) is now in the form of a standard eigenvalue problem, which can be solved for the eigenvalues and mode shapes by available computer routines.

3. DETERMINATION OF MODE SHAPES FROM FIXED BASE PROPERTIES

As an alternative to the eigenvalue calculations given above, a method of analysis is presented to determine the mode shapes of multistorey buildings with foundation compliance, making use of fixed-base modal properties. The range of applicability of this method is outlined later in the example problem.

Under conditions of free vibration and with the assumption of proportional damping, $\mathbf{C}$ and $\mathbf{C}_R$ are omitted from the eigenvalue calculation, and $\ddot{u}_g = 0$. In order to facilitate the subsequent analysis, it is further assumed that moments of inertia, J_0 , of the masses about their own axis of rotation are negligible as compared to the terms $\sum m_i h_i^2$, so that $J = \mathbf{d}^T \mathbf{h}$. Equations (1) and (2) then become

$$\mathbf{M} \ddot{\mathbf{x}} + \mathbf{M} \mathbf{h} \ddot{\theta} + \mathbf{K} \mathbf{x} = 0, \quad (5)$$

$$\mathbf{d}^T \ddot{\mathbf{x}} + \mathbf{d}^T \mathbf{h} \ddot{\theta} + K_R \theta = 0. \quad (6)$$

Premultiplication of equation (5) by $\mathbf{d}^T \mathbf{M}^{-1}$ gives

$$\mathbf{d}^T \ddot{\mathbf{x}} + \mathbf{d}^T \mathbf{h} \ddot{\theta} + \mathbf{d}^T \mathbf{M}^{-1} \mathbf{K} \mathbf{x} = 0. \quad (7)$$

The first two terms in equations (6) and (7) are seen to be identical; therefore,

$$\mathbf{d}^T \mathbf{M}^{-1} \mathbf{K} \mathbf{x} = K_R \theta. \quad (8)$$

A change of coordinates of the left-hand side of equation (8) into the modal coordinates $\mathbf{q}$ of the fixed-base structure gives [2]

$$\bar{\mathbf{x}} = \Phi \mathbf{q}, \quad (9)$$

$$\bar{\mathbf{M}} = \Phi^T \mathbf{M} \Phi, \quad (10)$$

$$\bar{\mathbf{K}} = \Phi^T \mathbf{K} \Phi, \quad (11)$$

and

$$\mathbf{e}^T = \mathbf{d}^T \Phi. \quad (12)$$

Φ is the modal matrix for the fixed-base structure, whose columns consist of the mode shapes ϕ_i of the natural modes of vibration. For convenience the mode shapes ϕ_i are normalized so that the top-storey components $\varphi_{in} = 1$. Note that

$$\bar{\mathbf{M}}^{-1} \bar{\mathbf{K}} = \Omega^2 = \text{diag} (\omega_1^2, \omega_2^2, \dots, \omega_n^2); \quad (13)$$

thus, equation (8) becomes

$$\mathbf{e}^T \Omega^2 \mathbf{q} = K_R \theta. \quad (14)$$

For any mode i , equation (14) can be rewritten as

$$e_i \omega_i^2 q_i = K_R \theta_i \quad (15)$$

or

$$q_i/h_n \theta_i = (K_R/h_n e_i) (1/\omega_i^2). \quad (16)$$

If one recalls that $e_i = \mathbf{d}^T \phi_i$, where ϕ_i is the mode shape of the fixed-base structure, then $h_n e_i \omega_i^2$ can be interpreted as the fixed-base overturning moment of the i th mode. Consequently, equation (16) expresses the equality between the ratio of the modal coordinate q_i to the displacement of the top storey $h_n \theta_i$ due to rigid body rotation, and the ratio of rotational stiffness to the fixed-base overturning moment of the i th mode.

Equation (16) can also be written as

$$q_i/h_n \theta_i = \lambda_i^2/\omega_i^2, \quad (17)$$

where $\lambda_i = K_R/h_n e_i$ is the square of the rigid-body rocking frequency of the equivalent single-degree-of-freedom rocking system, and $h_n e_i$ = equivalent moment of inertia of the fixed-base mode i . If the fundamental mode shape is a straight line, then

$$\Phi_1 = \{\phi_{11}, \phi_{21}, \dots, \phi_{n1}\}^T = (1/h_n) \{h_1, h_2, \dots, h_n\}^T$$

and

$$h_n e_i = \mathbf{d}^T \mathbf{h} = \sum_{i=1}^n m_i h_i^2 = J, \quad (18)$$

which is the geometric transfer portion of the moment of inertia of the structure about the base.

4. CHANGES IN FREQUENCIES OF NATURAL MODES WITH BASE ROTATION

Equation (17) shows that the relation between the fixed-base frequency and the pure rocking frequency is exactly that derived by Balan *et al.* [11] for a single-storey structure with foundation rocking. It is further shown in reference [11] that the ratio of fixed-base natural frequency ω to the combined natural frequency ω^* is given by

$$\omega/\omega^* = [1 + \omega^2/\lambda^2]^{1/2}, \quad (19)$$

where λ is the rocking frequency. The same relationship was derived by Merritt and Housner [1]. Since equation (17) is derived for any mode i , equation (19) can also be generalized to any mode i . Thus

$$\omega_i/\omega_i^* = [1 + \omega_i^2/\lambda_i^2]^{1/2}. \quad (20)$$

The curve for equation (20) is plotted in Figure 2. With the aid of Figure 2 or equation (20) it is thus possible within their range of applicability to find the combined natural frequencies of structures with rotational foundation flexibility.

Figure 2. Frequency reduction for foundation rocking systems. Plot is given by $\omega_i / \omega_i^* = (1 + \omega_i^2 / \lambda_i^2)^{1/2}$.

Implicit in the derivation of equation (16) and consequently of equation (20) is the assumption that the mode shape of relative displacements remain constant with the incorporation of rotational ground flexibility. For the higher modes this assumption is satisfied only for small rotational flexibilities. This will be demonstrated in the following numerical example.

5. EXAMPLE PROBLEM

The flexibility matrix for horizontal loads applied in the short direction of the Administration Building of the Department of Agriculture, Ottawa, Canada, was used to investigate natural modes of vibration with various degrees of rotational foundation flexibility. The building is of flat slab construction with columns and shear walls, and has 13 full storeys and two penthouse floors. The flexibility matrix used is that of model 3b, Table 1 of reference [5],

Figure 3. Mode shapes with foundation rocking flexibility. (a) Fundamental mode. $\circ$ — $\circ$, Fixed base, $f_R = 0.01$, $f_R = 0.1$; —, $f_R = 0.1$; ---, $f_R = 0.01$. (b) Second mode. $\circ$ — $\circ$, Fixed base; $\bullet$ — $\bullet$, —, $f_R = 0.1$; $\blacktriangle$ — $\blacktriangle$, ---, $f_R = 0.01$. (c) Third mode. $\circ$ — $\circ$, Fixed base; $\bullet$ — $\bullet$, —, $f_R = 0.1$; $\blacktriangle$ — $\blacktriangle$, ---, $f_R = 0.01$. Note: all values of f_R should be multiplied by 2.51×10^{-12} rad/in lb.

corresponding to a combined cantilever shear-frame system. Further structural details may be obtained from reference [5].

The mode shapes of the first three modes obtained from the eigenvalue problem of equation (4) are shown in Figure 3 for various values of foundation flexibility f_R . The largest value of f_R considered is one for which the rotational deformation at the top floor is comparable to the relative displacement of the structure in the first mode.

From Figure 3(a) it may be observed that the relative displacements of the first mode remain essentially constant with increasing rotational flexibilities. Noticeable changes are

evident, however, in the mode shapes of relative displacements for the second and third modes presented in Figure 3(b) and (c), respectively. This shows that the assumption of constant mode shape is satisfied for the first mode throughout the full range of foundation flexibilities considered, but is not satisfied for the larger values of flexibilities for the second and third modes.

The validity of the proportionality expressed by equation (16) can be established quantitatively by evaluating equation (12) using the relevant quantities from the eigenvalue calculation. If in equation (12) the actual mode shapes of relative displacement ϕ_i associated with a combined mode are used, instead of the original fixed-base mode shape ϕ_{i0} , one obtains a measure of the deviation from the linearity expressed by equation (16). The values of θ_i/q_i vs. $e_i \omega_i^2/K_R$ obtained for the first three modes are plotted on a log-log scale in Figure 4.

Figure 4. Linearity test for example problem.

It may be observed that the first mode satisfies the proportionality of equation (16) throughout the range of values for f_R considered, whereas for the higher modes the proportionality applies only to small values of f_R . Furthermore, the region of applicability of equation (16) decreases with increasing mode number. This deviation from the linearity of equation (16) is a direct result of the changes in mode shapes of relative displacements as the foundation flexibility increases. This can be seen from the results presented in Figure 3(b) and (c).

The deviation from linearity also affects the changes in modal resonant frequencies as given by equation (20). Table 1 shows a comparison between values of the frequency ratio ω_i/ω_i^* from the results of the eigenvalue calculation and those computed from equation (20). Here $\bar{J}_i$ is the total moment of inertia for mode i ,

$$\bar{J}_i = e_i + J_0. \quad (21)$$

For the first mode the two values agree well up to the largest value of f_R considered, but substantial deviations are shown for the second mode after $f_R > 0.1 \times 10^{-2}$, and for the third mode after $f_R > 0.1 \times 10^{-3}$. With the aid of the comparisons of values in Table 1 and the curves of Figure 4, for practical purposes the limits of proportionality and consequently the limits of applicability of equations (16) and (20) can be assigned judiciously. Suggested limits are indicated in Figure 4 by the heavy lines. For the quantitative evaluation of the influence

TABLE 1

Comparison of predicted and computed natural frequencies

Mode <i>i</i>	Fixed-based natural frequency, ω_i ($\times 3.60$ rad/s)	Total equivalent inertia, J_i ($\times 3.07 \times 10^{10}$ in lb s ²)	Rocking flexibility, f_R ($\times 2.51 \times 10^{-12}$ rad/in lb)	Computed natural frequency, ω_i^* ($\times 3.60$ rad/s)	Square of fixed-based rocking frequency, λ_i^2 ($\times 3.60$ rad/s) ²	ω_i/ω_i^* from equation (4)	$[1 + \omega_i^2/\lambda_i^2]^{1/2}$
1	1.534	3.817	1×10^{-4}	1.533	2619	1.0007	1.0005
			1×10^{-3}	1.527	261.9	1.0048	1.0045
			1×10^{-2}	1.466	26.19	1.0464	1.045
			1×10^{-1}	1.090	2.619	1.408	1.380
			1×10^{-4}	6.674	18600	1.0002	1.0012
2	6.675	0.5378	1×10^{-3}	6.664	1860	1.0016	1.012
			1×10^{-2}	6.571	186	1.0158	1.115
			1×10^{-1}	5.959	18.6	1.120	1.55
			1×10^{-4}	15.325	39750	1.0001	1.003
			1×10^{-3}	15.308	3975	1.0013	1.030
3	15.327	0.2523	1×10^{-2}	15.047	397.5	1.0186	1.263
			1×10^{-1}	10.587	39.75	1.448	2.63

of rotational flexibility beyond these limits, the eigenvalue calculation of equation (4) or a Stodola iteration method can be used [5].

The results from the example problem are applicable to other multistorey buildings of similar construction. Jennings [12] has shown that the first three measured mode shapes of some eight multistorey buildings ranging from 9 to 44 storeys are very similar. The first mode can be approximated by a straight line and the inflection points of the second and third modes agree closely. For these buildings the behaviour of the fundamental mode of vibration with foundation rocking can therefore be predicted by the linear relationship of equation (16), whereas the applicability of equation (16) to the higher modes would be restricted to small foundation rocking flexibilities.

6. SUMMARY AND CONCLUSION

Two methods of analysis for the natural modes of multistorey buildings with rotational foundation flexibility are presented. The mode shapes and natural frequencies are computed by means of an eigenvalue formulation, and relationships are derived to predict the rotational component and changes in natural frequencies when the fixed-base mode shapes and frequencies are known. It is demonstrated that the latter method will predict the mode shapes and natural frequencies for the fundamental mode for a wide range of foundation flexibilities. Due to changes in mode shapes with increasing foundation flexibilities, however, higher modes show deviations from the linear relationships derived.

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