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## **SIMULATION OF PROBABILISTIC AVERAGING IN ICE LOAD ESTIMATION**

**Chuanke Li<sup>1</sup>, Robert Frederking<sup>2</sup> and Ian Jordaan<sup>1</sup>**

**<sup>1</sup>C-CORE and Memorial University of Newfoundland, St. John's, Canada**

**<sup>2</sup>National Research Council Canada (NRC), Ottawa, Canada**

### **ABSTRACT**

This paper mainly focuses on establishing a probabilistic model to explore the relationship between local pressures and the global pressure. Both spatial and temporal correlations among the local pressures are considered in the study. A vector autoregressive model (VAR) is applied in this research to include these correlations. The global contact area is divided into a group of local areas. The model is used to illustrate the probabilistic averaging effect from local pressure to global pressure. The duration effect in which a higher load is likely to result from a longer interaction process is also showed using this model. The model is also used to explore the pressure-area relationship and pressure-aspect ratio relationship. The results imply that using the pressure-area relationship and the pressure-aspect ratio relationship alone may not be sufficient for global load estimation.

**KEY WORDS:** Probabilistic averaging; Autoregressive model; High pressure zones; Pressure-area; Pressure-aspect ratio

### **INTRODUCTION**

Both local pressure and global pressure need to be considered in the design of a structure under ice loading. Studies (for example, Jordaan et al. 2005) showed that the local design pressure is higher than the global pressure for a given structure. This is attributed to the averaging effect resulting in less randomness in the global pressure. The local pressure is defined as the pressure on a specific area of interest on the structure, such as areas between frames or pressures on panels or subpanels in field tests. The global pressure is the pressure on the global interaction area during ice-structure interactions. Analysis of field data by Jordaan et al. (2006) showed that the local pressures were random but correlated in both spatial and temporal spaces. The objective of this paper is to establish a probabilistic model to include the randomness as well as the temporal and spatial correlations on local pressures. The model is used to show the effect of probabilistic averaging and to investigate the relationship between

local pressures and the global pressure. A larger pressure or force is more likely to be encountered with a longer loading duration as the result of randomness. This is termed as the duration effect, which is also investigated using the proposed probabilistic model.

The probabilistic model is also used to explore the pressure-area relationship and the relationship between pressure and the aspect ratio, which have been extensively studied in ice engineering (for example, Sanderson 1988, Masterson and Spencer 2001).

## PROBABILISTIC MODELING

During an ice-structure interaction, the global interaction area can be separated into a fractured area with nearly no pressure on it and the effective contact area. The global load exerted on the structure is largely a summation of the loads exerted on a group of local areas within the effective contact area. The pressures on these local areas are spatially correlated. This is illustrated in Figure 1. Within a local area, a number of high pressure zones are randomly distributed, as shown in Figure 1. These high pressure zones carry most of the load exerted on that area. For a local area of interest, the pressure on that area is

$$P_l = \frac{\sum_{i=1}^m F_i}{A_l} \quad (1)$$

where  $P_l$  is the pressure on a local area,  $A_l$  is the area of interest in design,  $m$  is the number of the high pressure zone within the design area and  $F_i$  is the zonal force for the  $i^{\text{th}}$  high pressure zone.

The global pressure at a specific moment can be expressed as

$$P = \sum_{i=1}^n P_i \times \beta \quad (2)$$

where  $P$  is the global pressure,  $n$  is the total number of defined local areas within the effective area and  $\beta$  is the ratio between the effective contact area and nominal global interaction area.

The nature of the ice loading is associated with the number of *hpsz*'s (high pressure zones) on the area of interest and the zonal forces of each *hpsz* as described in Equation 1. With an increasing number of *hpsz*'s or a relatively large local area, the process of ice loading during an interaction tends to be a Gaussian process given by Equation 1 which is based on the central limit theorem. Thus, in this research the Gaussian process is applied for the simulation of the traces of local pressures. This approximation reduces the complexity of the problem which involves uncertainties in the number of *hpsz*'s, zonal forces and the high probability of correlation between these two regarding which little information from the past can be obtained. A first order space-time autoregressive model (space-time *AR*) is introduced and applied in the following sections to include the spatial and temporal correlations among the pressures of local areas. Thus, the problem of estimation of the global load or pressure becomes a problem of aggregating the individual components exerted on the local areas.

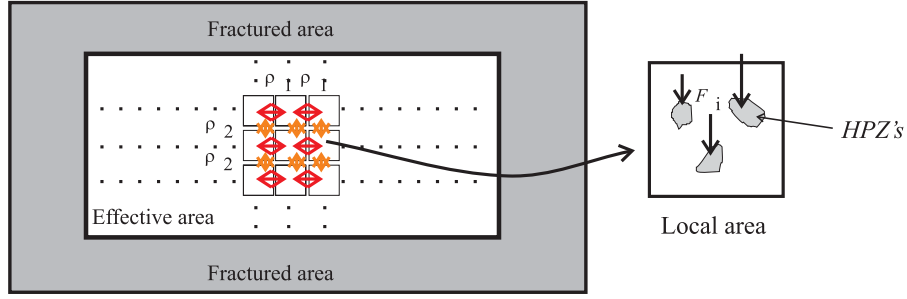


Figure 1 Descriptions of local and global interaction areas in the probabilistic model. The outer box represents the global interaction area, inner thick box represents the effective area and the inner thin boxes represent local areas. The symbols,  $\rho_1$  and  $\rho_2$  represent spatial correlations in the horizontal and vertical directions, respectively.

A general space-time *AR* model is the vector autoregressive model (*VAR*). The *VAR* model can solve problems with multiple orders of spatial and temporal correlation. A detailed description of the *VAR* model can be found in Luhkepohl (1993). Using the *VAR* model, the ice pressure traces on the local areas as shown in Figure 1 can be described as

$$p_{k,t} = \alpha_{k1,1} p_{1,t-1} + \alpha_{k2,1} p_{2,t-1} + \cdots + \alpha_{kK,1} p_{K,t-1} + \cdots + \alpha_{k1,p} p_{1,t-i+1} + \cdots + \alpha_{kK,i} p_{K,t-p+1} + \varepsilon_{k,t} \quad k = 1, \dots, K. \quad (3)$$

where  $k$  represents the  $k^{\text{th}}$  local area or panel,  $t$  represents the  $t^{\text{th}}$  time lag,  $\varepsilon_{k,t}$  is white noise and the subscript  $i$  represents the order of the *VAR* model.

Equation (3) can be simplified using a matrix representation as follows

$$p_t = A_1 p_{t-1} + \cdots + A_i p_{t-i+1} + \varepsilon \quad (4)$$

where  $p_t = [p_{1t}, \dots, p_{Kt}]'$ ,  $\varepsilon = [\varepsilon_{1t}, \dots, \varepsilon_{Kt}]'$  and  $A_i$  is the matrix representing the dependence of the present stage on the previous stages at different locations and is given as follows.

$$A_i = \begin{bmatrix} \alpha_{11,i} & \cdots & \alpha_{1K,i} \\ \vdots & \ddots & \vdots \\ \alpha_{K1,i} & \cdots & \alpha_{KK,i} \end{bmatrix}.$$

In this research, the first order *VAR* (*VAR*(1,1)) model, which takes into account the relationship of one lag in both space and time, is applied to investigate a few issues involved in ice load estimation.

## SIMULATIONS

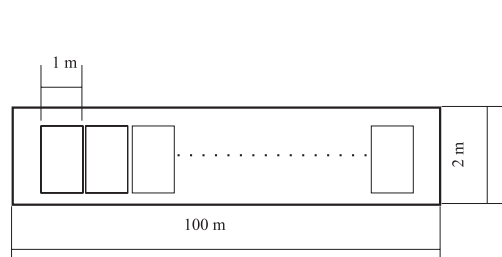
In the simulation, the global contact area is divided into a group of local areas. The load on

each local area is simulated using methodology described in the previous section. The global load will be the aggregation of the local loads.

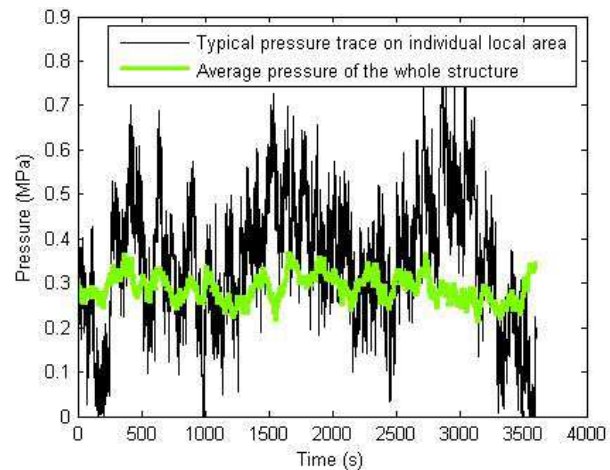
### Probabilistic averaging

In order to investigate the effect of probabilistic averaging, the pressures on local areas are simulated. Ice thickness is 2 m in the simulation and the generic structure length is 100 m. The ratios between the effective contact width and structure width, and the effective contact height and the ice thickness are both taken as 0.9, resulting in the ratio of the effective area and global interaction area equal to 0.81. The local area is 1.8 m in height and 1 m in width. This assumption is used through the whole paper. In total, 90 local areas are aligned in one row (see Figure 2(a)). The correlation coefficient is tentatively taken as 0.8 between the adjacent panels. The mean of the pressure on local areas is 0.37 MPa and the standard deviation is 0.15 MPa. The autocorrelation of one time lag is 0.93 and the length of one time lag is 1 second. Matrix  $A$  and the variance of the white noise can be derived from the autocorrelation and cross-correlation. Details can be found in Luhkepohl (1993).

The simulated load traces on a single local area and on the whole structure are compared in Figure 2(b). The comparison shows that the averaging results in a significant reduction on the variance of pressure on the whole structure (about 4.7% of the variance of single local area). The mean of averaging pressure trace is equal to the mean of the local pressure times the ratio of the effective area and global interaction area. Figure 2(c) gives details for a segment of load traces for two adjacent panels. The loading processes on the two local areas are approximately simultaneous because they are correlated. At each time lag, small differences between the two traces are visible. Comparison of the spatially correlated processes with a spatially independent process is given in Figure 2(d). The results showed that the variance of the correlated structure is about 4.3 times bigger than an independent structure. The autocorrelation between each time lag does not increase the variance of averaging pressure according to the spatially independent case. However, using a temporally independent process can cause an overestimation of loads because it introduces more uncertainties in time domain. Autocorrelation needs to be taken into account in ice load estimation. The results from simulations using the VAR(1,1) model agree well with the local averaging method by Vanmarke (1983). A detailed description of this method is given by Jordaan et al. (2006).



(a)



(b)

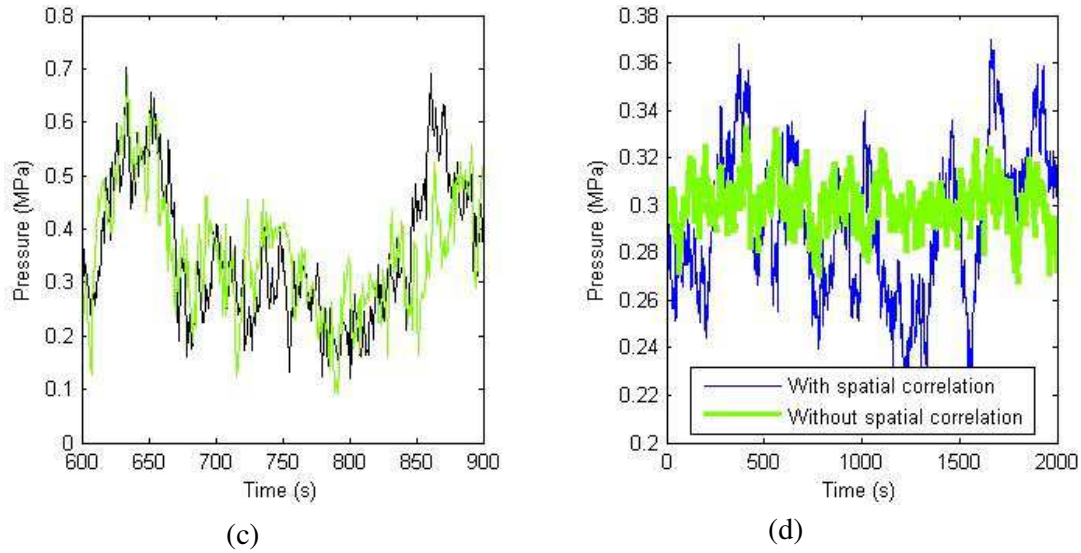


Figure 2 Averaging effects (a) A generic structure for probabilistic averaging (b) Pressure traces on a local area and on the whole structure (c) Pressure traces on adjacent local areas (d) Pressure traces for correlated structure and independent structure

### Duration effect

In this part, the same structure and ice thickness as the previous step are used. Simulations of global pressure on the whole structure were conducted with different loading durations. For each case, sufficient numbers of simulations were done to get the probabilistic parameters. The peak pressure of each simulation was extracted. Means and standard deviations of the peak pressure for different durations were obtained based on those simulations. A plot of peak pressure and loading duration was then plotted to investigate the duration effect, as shown in Figure 3. From Figure 3, it can be seen that the longer the interaction process, the higher the pressure the structure will experience. This is an important factor in comparing different groups of data. When using the previous information to estimate the design load, the life of the structure and the annual ice-structure impact rate should be taken into account.

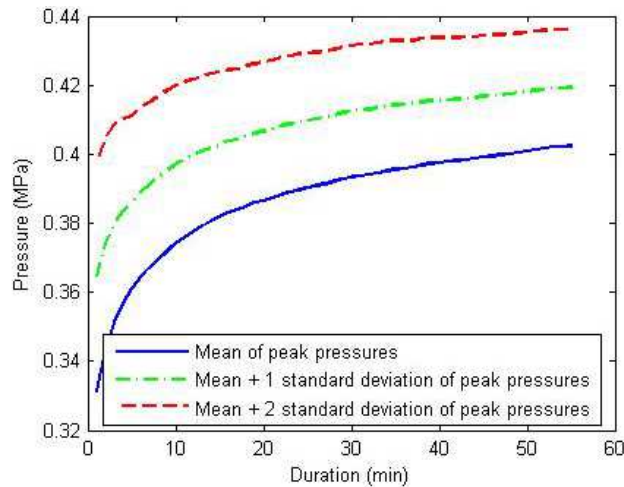


Figure 3 Pressure statistics for different durations

## Investigation of pressure-area and pressure-aspect ratio relationships

The first case investigated is the pressure-aspect ratio relationship with a fixed global interaction area. By changing the width and ice thickness, different aspect ratios can be obtained. By simulating the global pressure for each case and comparing the results of different aspect ratios, a relationship of pressure with aspect ratio can be obtained. The area is fixed at 200 m<sup>2</sup>. The dimensions of the individual area remain the same as the proceeding section. The correlation coefficient along the vertical direction is assumed to be 0.8 as well. The results from simulations are given in Figure 4. It shows that the pressure decreases with increasing aspect ratio. This is because the randomness decreases with decreasing aspect ratio. For example, in case #1 in Figure 4(b) (one row, which includes 90 local areas), the correlation between the first forty five continuous local areas and the second group is very small and negligible. However, in the case #2 in Figure 4(b) (two rows and each row includes 45 local areas), the correlation between those two groups is 0.8.

The second case investigated is the pressure-area with a fixed aspect ratio. A similar analysis as the first case is done. The aspect ratio is assumed to be 5. The results from simulations are given in Figure 5. The pressure decreases with increasing area with fixed aspect ratio. This can be attributed to the increased randomness in both horizontal and vertical directions with increasing areas, given a fixed aspect ratio.

Another case is to keep the structure width fixed and change the ice thickness. Here, we propose a generic structure of 40 m wide. The ice thickness varies from 2 m to 16 m. The results of pressure-area and pressure-aspect ratio relationship are given in Figure 6 (a) and 6(b). Figure 6(a) and 6(b) show different trends. The pressure-area relationship follows a traditional decreasing trend. This is because the reduced variance after averaging results from the increasing randomness in the vertical direction with increasing ice thickness. The pressure-aspect ratio relationship shows an increasing trend. This is again attributed to the reduced variance after averaging with increasing ice thickness while the aspect ratio decreases. This implies that using the pressure-aspect ratio curve may not be sufficient for design under some conditions. The pressure-area relationship dominates the scale effect in this case. However, for each specific area, the variance will change according to different aspect ratios (Figure 4). This needs to be considered in the design as well.

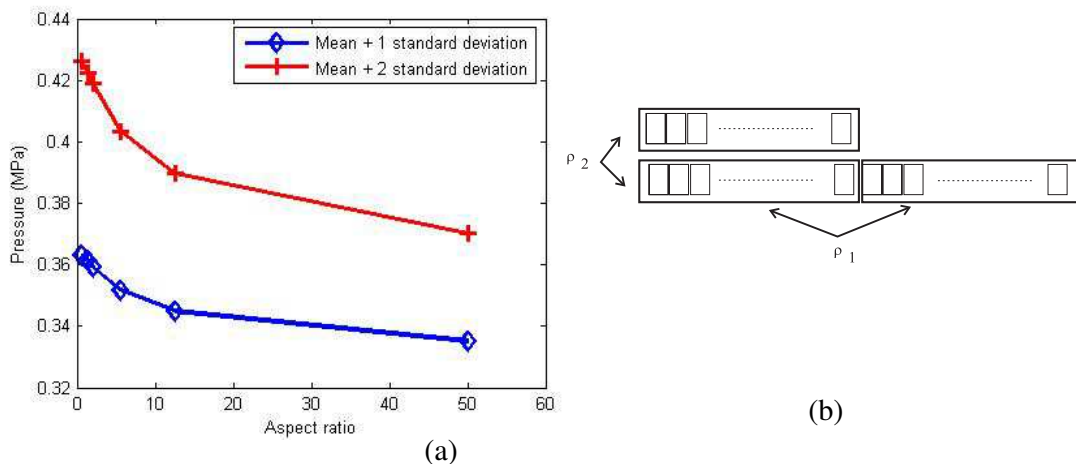


Figure 4 (a) Pressure-aspect ratio curve with fixed area (b) Different correlation structures for different aspect ratios.

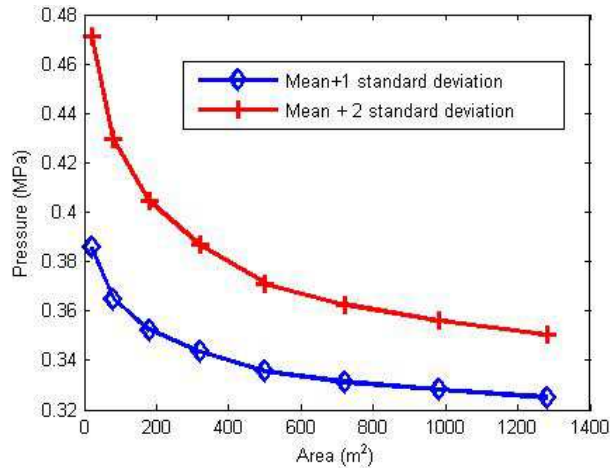


Figure 5 Pressure-area curve with fixed aspect ratio

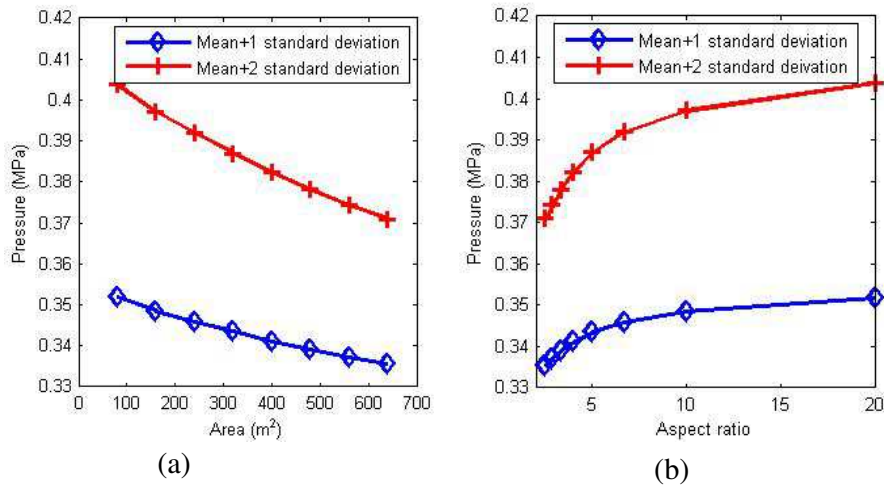


Figure 6 Simulations for fixed structure width (a) Pressure-area curve (b) Pressure-aspect ratio curve.

## DISCUSSION AND CONCLUSIONS

Uncertainties are always involved in ice-structure interaction. For example, the distribution of grain structures, properties and flaws are not uniform through an ice feature. These uncertainties and their consequences make the local pressure random though correlated to adjacent locations. An autoregressive model is introduced in this paper to study some aspects of ice load estimation. Both temporal and spatial correlations are considered in the model. The model assumes that the ratio of the effective contact area and the global interaction area is constant (0.81 is taken in this paper). This is suitable for relatively wide structures where edge effect is negligible. The global load is obtained by aggregating the local loads. The simulations showed that the variance of global load can be reduced significantly after averaging compared with local pressures. This indicates that using the variance of local pressures to estimate the global load may be conservative. A reduction factor needs to be applied in order to extrapolate the local data to global load. Details can be found in Jordaan et al. (2006). The simulations also showed that the temporal correlation has no effect on

averaging. Results from simulations showed that the longer the duration of the interaction is, the greater the probability of getting a higher load.

The autoregressive model was also applied to study the pressure-area and the pressure-aspect ratio relationships. The pressure-area curve with fixed aspect ratio and pressure- aspect ratio curve with fixed area follow a traditional decreasing trend. It is interpreted with the reduced variance by averaging effect with either increased randomness or reduced correlations. However, with a fixed structure width and various ice thicknesses, the pressure-aspect ratio curve follows an increasing trend. This results from the averaging effect by the increased randomness with increasing ice thickness. Overall, the pressure-area curve and pressure-aspect ratio curve can be interpreted using the averaging effect and different correlation structures. Using either the pressure-area curve or the pressure-aspect ratio curve alone may not be enough for ice load estimation. The results regarding this point are tentative and preliminary. More aspects need to be studied such as the size of *hpz*'s which may scale with ice thickness. This may change some findings.

Questions that arise are “will we have the same estimation on contact areas such as  $5\text{ m} \times 1\text{ m}$  and  $50\text{ m} \times 10\text{ m}$  (constant aspect ratio)? Or will we have the same load estimation from contact structure such as  $90\text{ m} \times 2\text{ m}$  and  $18\text{ m} \times 10\text{ m}$  (constant area)?” Using the pressure-aspect ratio curve for the first case and using the pressure-area curve for the second case result in the same load estimations for the two cases, respectively. This research is meant to illustrate the problem rather than to try to answer these questions outright. More effort is needed to sufficiently and properly answer these questions.

## ACKNOWLEDGEMENT

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