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WHITE NOISE REPRESENTATION OF EARTHQUAKES

BY

G. N. BYCROFT

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WHITE NOISE REPRESENTATION OF EARTHQUAKES

By G. N. Bycroft¹

SYNOPSIS

This investigation is intended to show that white noise is a suitable representation of earthquake motion. It appears that a white-noise source used in conjunction with an analog computer is a convenient method of analyzing structures subjected to complex ground motions and of assigning probabilities to the deformations arising. Until more statistical information is available about earthquakes, it is suggested that a "standard large earthquake" be represented by "white" ground accelerations having a flat spectral density of $0.75 \text{ ft}^2/\text{sec}^4/\text{cps}$ and a duration of $1/2$ min.

INTRODUCTION

One of the major difficulties in determining suitable aseismic designs lies in the lack of knowledge of destructive earthquakes. Accelerograms of a few large earthquakes are available and these records show very complex ground motions entirely different from each other. They are different in energy content or "size" and different in duration and shape. Earthquakes of the future

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will not necessarily bear any relation to those in the past and no prediction of the severity and character of future earthquakes can be made. Two earthquakes having the same energy content will have different effects on any particular structure. Each individual earthquake, of course, has a specific character and a specific effect on a particular structure, but this character is not predictable.

Many investigations of structural vibration due to ground motion have been made on a deterministic basis, using actual records of earthquakes and determining the deformations that would occur in various types of structures. This approach would be satisfactory if a large enough number of records were available and the results displayed on a probability basis. Until this is possible recourse to another mode of attack must be made.

Earthquakes originate as ill-defined slip-faults followed by innumerable reflections, refractions, and attenuations by the complex terrain through which it travels. Intuitively, one feels that the essential characteristic of the earthquake is its irregularity. Many writers^{2,3,4} have suggested simulating earthquakes by a random assemblage of impulses. In order that white noise should simulate an earthquake it must have the same effect on the kinds of dynamic systems that arise in structures. A linearly-elastic and elasto-plastic structure are considered herein.

LINEAR ELASTIC STRUCTURE

The response of a linear structure of many degrees of freedom to ground motion may be written in terms of the spectrum of that ground motion. Let

$\alpha(t)$ = the ground acceleration;

$v(x, t)$ = velocity of some point of the structure;

T_n = period of the n^{th} harmonic;

$K(x, T_n)$ = harmonic "weighting" factor and is a function of the structure only;

p = number of degrees of freedom; and

λ_n = fraction of critical damping of the n^{th} harmonic.

then,

$$v(x, t) = \sum_{n=1}^p K(x, T_n) \int_0^t \alpha(\tau) e^{-\frac{2\pi\lambda_n}{T_n}(t-\tau)} \sin \frac{2\pi}{T_n}(t-\tau) d\tau \quad (1)$$

Eq. 1 states that the response of a multi-degree-of-freedom structure is the sum of the responses of the various modes or harmonics. The earthquake excitation occurs only in the integral factor and consequently this is the only factor we need consider.

² "Response Spectrum Techniques in Engineering Seismology," by D. E. Hudson, Proceedings, World Conf. on Earthquake Engrg., San Francisco, Calif., June, 1956.

³ "Some Applications of Probability Theory in Aseismic Design," by E. Rosenblueth, Proceedings, World Conf. on Earthquake Engrg., San Francisco, Calif., June, 1956.

⁴ "Response of Tall Buildings to Random Shakes," by A. Cemal Eringen, Tech. Report No. 12, Office Naval Research, Div. of Engrg. Sciences, Purdue Univ., December, 1957.

The velocity spectrum of the ground motion is defined as the maximum value of this integral, that is,

$$V(\lambda, T) = \text{Max} \int_0^t \alpha(\tau) e^{\frac{-2\pi\lambda}{T}(t-\tau)} \sin \frac{2\pi}{T}(t-\tau) d\tau \dots (2)$$

and is a function of period and damping only.

Spectra of several large earthquakes have been calculated and are well known.⁵ The spectra are all different in detail but have the feature in common that they fluctuate rapidly, with respect to period T , about a mean line which starts at the origin and rises asymptotically to a constant value. G. W. Housner drew⁶ smooth mean curves through these oscillatory spectra and found that these smooth curves have almost the same shape for different earthquakes. The actual scale of the spectra varies according to the size of the earthquake. The full lines in Fig. 1 are taken from Housner's work⁶ and show the shape of these common average spectra, and the particular scale of V shown is that of the largest earthquake recorded, viz., El Centro, Calif., 1940.

The writer used an analog computer to construct mean spectra of bursts of "white noise" for comparison with the mean spectra of Fig. 1. "White noise" is defined as a random signal having a gaussian probability of amplitude distribution and a spectral density which is constant for all frequencies. The spectral density at any particular frequency may be regarded as the average power passing when the signal is filtered by a narrow band pass filter centered at the particular frequency. In practice, of course, one encounters "band-limited white noise," that is, the spectral density is only constant up to a certain frequency and is then rapidly cut off. As long as this frequency is large compared with the frequencies of the system being considered, the response of the system is unaffected by the value of the cut-off point. The computer used provides a band-limited white noise having a spectral density flat from 0 to 35 cycles.

The equation of motion of the mechanical oscillator subjected to an acceleration $\alpha(t)$ is simply,

$$\ddot{x} + 2\omega\lambda x + \omega^2 x = \alpha(t) \dots (3)$$

in which x is the displacement of the mass relative to its zero position and ω is the angular frequency of the oscillator. Variables in the computer are represented by voltages having an allowable maximum of 100 volts. If $\dot{x}$ is to have an expected maximum of V ft per sec the mechanical equation is scaled to

$$\left[\frac{100}{V\omega} \ddot{x} \right] + 2\lambda \left[\frac{100}{V} \dot{x} \right] + \left[\frac{100}{V} \omega x \right] = \frac{100}{V\omega} [\alpha(t)] \dots (4)$$

where the terms in square brackets are the respective computer voltages.

⁵ "Spectra Analyses of Strong Motion Earthquakes," by J. C. Alford, G. W. Housner and R. R. Martel, Earthquake Research Lab., Tech. Report to Office of Naval Research, Calif. Inst. of Tech., Pasadena, August, 1951.

⁶ "Limit Design of Structures to Resist Earthquakes," by G. W. Housner, Proceedings, First World Conf. on Earthquake Engrg., San Francisco, Calif., June, 1956.

FIG. 1.-EARTHQUAKE AND NOISE SPECTRA.

If $W(t)$ is the actual voltage of the noise source and X the computer variable in volts corresponding to x , we may write the equation as

$$\ddot{X} + 2\lambda \dot{X} + X = \frac{100k}{V\omega} W(t) \dots\dots\dots (5)$$

where the factor k will be chosen so that $k W(t)$ in volts is numerically equal to some representative ground acceleration $a(t)$ in ft per sec².

The actual velocity $\dot{x}$ in feet per second is given by the computer volts $\dot{X}$ multiplied by $V/100$, the acceleration by $\ddot{X} V\omega/100$ and the displacement by $X V/100\omega$.

Ideally, a probability distribution should be given to the size and duration of the earthquake as well as to its configuration. However, with the available information, this cannot be done satisfactorily. The available records of large earthquakes show that their duration was generally of the order of 20 sec to 30 sec. A duration of 25 sec has been selected here as representative of large earthquakes.

For various values of period T and damping λ , twenty separate bursts of white noise of 25 sec duration were fed into the circuit. The system was reset to zero after every burst. Each burst of noise gives a different value of the maximum velocity acquired by the oscillator.

The maximum value of $\dot{x}$ during each burst of noise is the required spectrum for that particular burst of noise. The mean of the maximum velocity of the twenty runs was taken in each case. This gives the variation of the mean maximum velocity for bursts of white noise as a function of period and damping. However, this mean still contains the scaling factor k , and the value given to k will determine the size of the earthquake being represented. This variation is shown by the circles in Fig. 1. The factor k was chosen so that the scales matched at an arbitrary point viz., $T = 3.0$ and $\lambda = 0.20$. With this value of k the position of the other points was determined. It is seen that the variation of the mean spectra of the noise with both period and damping is very similar to Housner's common average earthquake spectra. The scales could have been matched at any other point but the value of k would not be very different.

From this it appears that "white" ground accelerations are a suitable representation of earthquakes. Further, by choosing the scales so that the mean-noise results fit the averaged spectra of the largest earthquake recorded, we arrive at some kind of a definition of a standard large earthquake. Individual bursts of noise will give results above or below the average just as the actual spectrum of ElCentro fluctuates above and below the smooth curves of Fig. 1.

The spectral density of the noise source used was approximately 2.90 volt²/cps and the factor k came to 0.51. Thus a large earthquake may be represented by ground accelerations which are "white" in character and have a spectral density of 0.75 ft²/sec⁴/cps.

Fig. 2 shows a schematic diagram of the computer, where circles are potentiometers, triangles are summers, and double-based triangles are integrators.

ELASTO-PLASTIC STRUCTURE

It is interesting to compare the effect of white noise and an actual earthquake on a non-linear system. The most important type of non-linearity that

FIG. 2.—COMPUTER DIAGRAM FOR LINEAR DAMPENED OSCILLATOR.

FIG. 3.—ELASTO-PLASTIC CHARACTERISTIC.

will occur in a structure during severe ground motion will be plastic yielding. Fig. 3 shows the assumed relation between restoring force F and displacement x . Starting from zero with positive velocity, the restoring force is proportional to displacement until the yield point A corresponding to a displacement x_0 is reached. The restoring force remains constant at this value until a change in sign of the velocity occurs at some point B. The displacement A B will be called a yield displacement. The force F now decreases along a line parallel to the elastic part of the characteristic and stays on this line until C or perhaps B again is reached. If the velocity stays negative the reverse yield point C is reached and the force stays constant at this opposite value until the velocity becomes positive again and the characteristic proceeds from D to E. Areas enclosed by loops represent hysteretic energy loss.

In a recent paper⁷ the writer considers the yield displacements that occur when an oscillator with this yielding stiffness characteristic is subject to the 1940 El Centro earthquake. These yield displacements determined for this specific earthquake are now compared with the yields arising when the same system is subjected to bursts of "white" ground acceleration of spectral density $0.75 \text{ ft}^2/\text{sec}^4/\text{cps}$ simulating the standard large earthquake.

The equations of motion may be derived from a consideration of the elasto-plastic characteristic shown in Fig. 3. There are separate equations depending on whether the motion is taking place in the elastic or the plastic part of the characteristic. On the plastic part of the characteristic the retarding force is constant and has the sign of the velocity shown by $\text{sgn } \dot{x}$. On the elastic part of the characteristic the restoring force must be referred to the changing zero position caused by the yielding. The equations of motion are:

$$\ddot{x} + 2 \omega \lambda \dot{x} + \omega^2 (x - \mu) = \alpha(t), \quad |x - \mu| \leq x_0 \dots \dots (6a)$$

and

$$\ddot{x} + 2 \omega \lambda \dot{x} + \omega^2 x_0 \text{sgn } \dot{x} = \alpha(t), \quad |x - \mu| \geq x_0 \dots \dots (6b)$$

where μ is the algebraic sum of all previous yield displacements up to the time considered and the sign of the yield displacement taken as that of $\dot{x}$ at the time. Thus, in Fig. 3, when working on the linear portion D E, the value of μ is A B - C D.

It will be more convenient now if the yield point is specified in terms of equivalent acceleration rather than the displacement x_0 . Let G be the equivalent fraction of gravity and then

$$\omega^2 x_0 = G g \dots \dots \dots (7)$$

If S is the expected maximum value of x , and X is the scaled computer variable corresponding to x , then the scaled equations become:

$$\ddot{X} + 2 \lambda \dot{X} + (X - U) = \frac{0.51 W(t) 100}{\omega^2 S}, \quad |X - U| \leq X_0 \dots (8a)$$

$$\ddot{X} + 2 \lambda \dot{X} + \frac{100 G g}{\omega^2 S} \text{sgn } \dot{X} = \frac{0.51 W(t) 100}{\omega^2 S}, \quad |X - U| \geq X_0 \dots (8b)$$

⁷ "Earthquake Yield Spectra," by G. N. Bycroft, M. J. Murphy and K. E. Brown, Proceedings, ASCE, October, 1959.

FIG. 5.—MAXIMUM YIELD DISPLACEMENTS $G = 0.06$, $\lambda = 0.03$. FIG. 6.—MAXIMUM YIELD DISPLACEMENTS $G = 0.06$, $\lambda = 0.10$.

FIG. 7.—MAXIMUM YIELD DISPLACEMENTS $G = 0.06$, $\lambda = 0.20$. FIG. 8.—MAXIMUM YIELD DISPLACEMENTS $G = 0.12$, $\lambda = 0.10$.

FIG. 9.—MAXIMUM YIELD DISPLACEMENTS $G = 0.12, \lambda = 0.10$. FIG. 10.—MAXIMUM YIELD DISPLACEMENTS $G = 0.12, \lambda = 0.20$.

FIG. 11.—MAXIMUM YIELD DISPLACEMENTS $G = 0.24, \lambda = 0.03$.

FIG. 12.—MAXIMUM YIELD DISPLACEMENTS $G = 0.24, \lambda = 0.10$.

These figures, of course, compare the mean of the maxima of ten runs of noise with the maximum of one specific earthquake. Ideally, the comparison should be with the average of several earthquakes of equal spectral density. Each burst of noise is of the same energy level but gives a different value of maximum yield lying above or below the averaged values shown. Presumably one individual burst of noise could be found which would correspond to this one particular earthquake considered.

The probability distribution of the maximum yield was determined in three cases from one hundred runs of noise. These are shown in Figs. 14 to 16, and show reasonable probabilities of individual bursts of noise giving values of up

FIG. 13.—MAXIMUM YIELD DISPLACEMENTS $G = 0.24, \lambda = 0.20$.

to four times the average. This easily accommodates the differences shown between the average noise values and the one particular earthquake considered.

CONCLUSIONS

"White noise" appears to be a reasonable simulation of earthquake excitation for engineering design purposes. The technique of simulating a complex structure on an analog computer and feeding it with bursts of noise having an energy level representative of a large earthquake is a straight forward method of analyzing a structure and of assigning probabilities to the various displacements. Using this technique it will be possible to simulate multi-story structures deflecting in bending and shear and incorporating yield characteristics.

The probability values given in Figs. 14 to 16 illustrate an important point. Each burst of noise is of the same energy level but the resulting displacements for individual shots can differ by factors of 10 and more. Thus two earthquakes of the same content energy or magnitude but of different configurations may have vastly different effects on any particular structure. Also,

FIG. 14.—PROBABILITY DISTRIBUTION OF MAXIMUM YIELD DISPLACEMENTS $G=0.12$, $\lambda=0.10$, $T=0.75$.

FIG. 15.—PROBABILITY DISTRIBUTION OF MAXIMUM YIELD DISPLACEMENTS
 $G = 0.12$, $\lambda = 0.03$, $T = 2.5$.

FIG. 16.—PROBABILITY DISTRIBUTION OF MAXIMUM YIELD DISPLACEMENTS $G = 0.24$, $\lambda = 0.10$, $T = 0.20$.

one particular earthquake may seriously affect a structure with a certain set of parameters but have little effect on another one. This may well be part of the explanation of the inconsistencies noted in seismic damage.

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