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Publisher's version / Version de l'éditeur:

Cold Regions Science and Technology, 13, 2, pp. 107-19, 1987

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Statistical and Geometrical Definition of Snow Avalanche Runout

by D.M. McClung and K. Lied

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Reprinted from
Cold Regions Science and Technology
Vol. 13, No. 2, 1987
p. 107 - 119
(IRC Paper No. 1460)

Price \$3.50

NRCC 27715

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RÉSUMÉ

Deux modèles mathématiques de calcul de la course maximale d'une avalanche à partir d'un point de référence fixe sur la trajectoire sont présentés et comparés. Les modèles sont : (1) des estimations de limites de confiance concernant des analyses de régression des paramètres topographiques, et (2) un calcul des limites de confiance pour des estimations de la course à partir de la statistique des valeurs extrêmes. En utilisant ce dernier modèle pour définir la course, et des approximations polynomiales des profils des trajectoires des avalanches, on a construit un modèle géométrique de terrain. Ensemble, ces modèles redéfinissent le problème classique de répartition des zones d'avalanches en distinguant : (1) une estimation de la course, nécessitant une décision technique quant au choix de la limite de confiance, et (2) un problème dynamique nécessitant des estimations de la vitesse le long du plan incliné entre les positions de départ et d'arrêt, la géométrie étant déterminée par le modèle géométrique entre ces positions.

STATISTICAL AND GEOMETRICAL DEFINITION OF SNOW AVALANCHE RUNOUT

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(Received January 9, 1986; accepted in revised form March 27, 1986)

ABSTRACT

Two mathematical models for calculating the extreme avalanche runout from a fixed reference point on the path are presented and contrasted. The models are: (1) estimates of confidence limits on regression analyses of topographic parameters and (2) calculation of confidence limits on runout estimates from extreme value statistics. Using the extreme value statistics model to define runout, and polynomial fits to avalanche path profiles, a geometrical model for avalanche terrain was constructed. Taken together, these models redefine the traditional zoning problem by dividing it into (1) an estimation of runout distance, requiring an engineering decision on the choice of confidence limit, and (2) a dynamic problem requiring speed estimates along the incline between the start and stop positions, with the geometry specified by the geometrical model between these positions.

INTRODUCTION

The traditional method of determining maximum runout distances and impact pressures of snow avalanches for zoning purposes involves predicting jointly the avalanche runout and dynamics by selecting appropriate friction coefficients for an avalanche dynamics model. The actual physical modelling required for this approach involves many unknowns including: constitutive equations, boundary conditions and descriptive and quantitative features of flowing snow. These gaps in knowledge are significant; clearly the problem is far from solved.

The engineering aspects of zoning for avalanche-prone regions may be separated into two parts: (1) estimating runout distances for the extreme event on a path based upon statistical models that include topographic features and the choice of a confidence limit selected by an engineering decision and (2) defining speeds along the incline between the start position and runout position. In this paper, the first part of the problem is introduced and examples are given that illustrate the applicability and limitations of the method. The second part of the problem is considered by defining a terrain model. This model gives to extreme runout predictions a quantitative, geometrical interpretation, thereby providing path geometry between start and stop positions in a systematic way. The terrain model should prove useful for examining the effects of varying path parameters in avalanche dynamics modelling.

DESCRIPTION OF DATA AND VARIABLES

The data set used for the present analysis consists of topographic parameters for 212 avalanche paths from the maritime climate region of Western Norway. Extreme runout positions for time scales of at least 100 years were measured in the field, and additional parameters of the avalanche paths were determined. Papers by Lied and Bakkehøi (1980) and Bakkehøi, Domaas and Lied (1983) provide basic descriptions of the parameters.

The parameters used in the present analysis are defined in Fig. 1. The following equations provide

definitions of the first three variables, angles α , β , δ measured in degrees:

$$\tan \alpha = \frac{1}{X_0} \int_0^{X_0} \left(\frac{dy}{dx} \right) dx = \frac{H_0}{X_0} \quad (1)$$

$$\tan \beta = \frac{1}{X_\beta} \int_{\Delta X}^{X_0} \left(\frac{dy}{dx} \right) dx = \frac{H_\beta}{X_\beta} \quad (2)$$

$$\tan \delta = \frac{1}{\Delta X} \int_0^{\Delta X} \left(\frac{dy}{dx} \right) dx = \frac{\Delta H}{\Delta X} \quad (3)$$

The fourth variable is the starting zone angle defined by $\tan \theta$, the average slope in the first 100 m of vertical drop of the avalanche starting zone. The origin of geometry is the extreme tip of the runout (α point), and the β point (ΔH , ΔX) is that for which the slope angle first equals 10° proceeding downslope from the avalanche start position.

The β point is chosen as a reference for the definition of runout, and $\tan \delta$ is the average slope in the runout zone. Since β is a zero reference point, runout distances can be regarded as taking positive or negative values if the avalanche stop position is below or above the β point, respectively.

For eqns. (1–3), ΔX and ΔH are the horizontal reach and vertical distance, respectively, to the β point and X_0 and H_0 are the horizontal reach and

vertical drop, respectively, from the start position to the runout position.

The angle α was measured in the field using a clinometer, whereas most of the values for β , δ and θ were determined from maps with a scale of 1:5000 and contour intervals of 5 m. The α angles, which represent an estimate of the tip of the avalanche debris based on observations of damage from the dense core of the avalanche, do not generally include the effects of powder clouds which are observed to travel much further. Errors have probably occurred when damage from the powder portion of the avalanche was mistaken for damage from the tip of avalanche debris. Most of the angles in this analysis are accurate to the nearest degree, with only a few being determined to the nearest half-degree (Bakkehoi, Domaas and Lied, 1983).

No length parameters were measured for the present data set. However, three parameters with length dimensions were calculated from terrain profiles of the avalanche paths in the data set. These length scales were calculated by fitting the polynomial curve $y = ax^2 + bx + c$ to the avalanche terrain profiles. It was found that this polynomial provided an excellent fit to the avalanche terrain profiles. In most instances the correlation coefficient, R , was greater than 0.98. The three length parameters calculated were: $y'' = 2a$, H and L (Fig. 1) where H is the vertical distance from the avalanche start position to the position where $y' = 0$ and where L is defined as $H/\tan \alpha$, the approximate horizontal reach of the avalanches. Calculations show that H is very close to the vertical drop of the avalanches in most instances.

The values in Table 1 list some properties of the primary variables in this study. Histograms for these variables showed that most may be considered to have an approximately Gaussian distribution. Inspection of the value in Table 1 shows that the mean differs from the median most significantly for the variables δ , L and y'' . The constructed histograms showed that these variables appear to deviate the most from a Gaussian distribution. Some of the values in Table 1 are given to an accuracy greater than that of the variables they are calculated from. For example, the mean values and standard deviations of the angles are given in tenths of degrees. This was done for the purposes of analysis only and does not imply the angles are known with such accuracy.

Fig. 1. Definition of parameters used in the analyses in relation to an avalanche path.

TABLE 1

Primary variables in the analysis

Variable	Mean	Standard deviation	Median	Range	No. of avalanche paths
α (°)	31.1	6.4	30.3	18–49	212
β (°)	33.8	6.0	33.0	21–51	212
θ (°)	42.3	7.4	41.8	25–63	212
δ (°)	6.5	3.5	10	0–16	131
H (m)	839	250	831	342–1539	212
L (m)	1434	514	1343	563–3407	212
y'' (10^3 m^{-1})	0.752	0.417	0.680	0.14–2.18	212

REGRESSION ANALYSIS OF TOPOGRAPHIC PARAMETERS

With β as a reference point from which to calculate runout and α as the response variable, all the primary variables listed in Table 1 can qualify as predictor variables except δ and L . The use of δ as a predictor variable is limited to those few cases where the terrain in the runout zone angle is at a constant angle. For the present data set, 131 paths have known δ angles whereas the other primary variables are known for 212 avalanche paths. The variable L cannot be used as a predictor because it is calculated from a knowledge of α .

Correlation coefficients (R) were calculated for α with respect to the other variables in Table 1, giving: 0.919 (β), 0.388 (θ), -0.111 (δ), 0.245 (H), -0.519 (L) and 0.615 (y''). A dimensionless product variable Hy'' introduced by Lied and Bakkehøi (1980) has a correlation coefficient of 0.754 with respect to both α and β .

In this regression analysis, extreme runout is based upon a prediction of the minimum value of α for given values of the predictor variables. An examination of the correlation coefficients and an optimal regression analysis showed the best one-predictor model to be of the form $\alpha = f(\beta)$. Linear regression gave the following equation for all 212 avalanches

$$\hat{\alpha} = 0.985\beta - 2.24^\circ \quad (4)$$

with $R^2 = 0.845$ and the standard error, S , = 2.52° . The value of the F statistic is 1148 for eqn. (4).

A plot of the residuals vs. α and β showed that eqn. (4) provides biased estimates. This suggested a

transformation on α . Power law regression gave a good unbiased relationship

$$\hat{\alpha} = 0.730\beta^{1.06} \quad (5)$$

with $R^2 = 0.861$, $S = 0.0764$ and the F statistic = 1306. The correlation coefficient, $R = 0.928$, is identical in this case to the Spearman rank correlation coefficient, R_s , which shows the curvilinear aspect implied by the data.

Another transformation explored was $\sqrt{\alpha}$. For this case the regression equation is

$$\sqrt{\hat{\alpha}} = 0.0879\beta + 2.57 \quad (6)$$

with $R^2 = 0.853$, $S = 0.218$ and the F statistic = 1220. Plots of residuals showed that, although this equation removes some of the bias in estimates that was found in the linear model, it is not as good in that respect as eqn. (5). Equation (6) may prove useful in some applications.

Many other relationships were explored, including multiple regression equations. It was concluded that θ , H and y'' are not useful for improving the prediction of α through a multiparameter relationship containing β . Adding almost any variable to a multiple regression scheme will improve the multiple correlation coefficient and may reduce the standard error. However, extensive analyses with the present data set showed that adding predictor variables to β improves R^2 and the standard error only slightly, over the one-parameter models [eqns. (5) and (6)]. In all cases explored, the resulting equations were either overfitted or the changes in prediction were not significant enough when compared with the accuracy of the input data, and consequently, inclusion of other

predictor variables was not warranted. Equations (5) and (6) explain 85–86% of the variation of α through variations in β , which means that β has a dominating influence. It would be very difficult to improve upon this result by adding a second variable.

Adding δ as a predictor variable, along with β , slightly improves the prediction of α . In view of the geometrical connection between α , β and δ (Fig. 1), it would be surprising if this were not so. Multiple regressions gave the following relations analogous to eqns. (4–6) for 131 avalanche paths

$$\hat{\alpha} = 0.960\beta + 0.222\delta - 3.18^\circ \quad (7)$$

$$R^2 = 0.889, S = 1.79^\circ \text{ and } F = 512$$

$$\hat{\alpha} = \beta^{1.04} \exp[-0.296 + 0.00718\delta] \quad (8)$$

$$R^2 = 0.889, S = 6.06 \times 10^{-2} \text{ and } F = 514$$

$$\sqrt{\hat{\alpha}} = 0.0879\beta + 0.0198\delta + 2.42 \quad (9)$$

$$R^2 = 0.891, S = 0.163 \text{ and } F = 522$$

A plot of the residuals vs. response and predictor variables showed that eqns. (8) and (9) provide good unbiased predictions. Expression (9) is the best two-predictor equation found. For eqn. (9), the t statistic for the variable β is 32.1 and for δ it is 4.68. For the subset of 131 avalanche paths for which δ is known, calculations show that $\sqrt{\hat{\alpha}} = 0.0845\beta + 2.66$ with $R^2 = 0.872$, $S = 0.176$ and $F = 880$ for comparison with eqns. (6) and (9).

An attempt was made to derive better predictive equations for selected ranges of the parameters of the paths. For an accurate statistical analysis over the ranges of parameters in the present study, data from about 100 paths should be included. Since the present data set has 212 avalanche paths, data subsets above and below the median values of the parameters listed in Table 1 should be analyzed. Calculations for these subsets of 106 avalanche paths showed better predictive equations (in the sense that the standard errors were lower) for parameters in the range: $\beta \leq 33^\circ$, $H \geq 830.5$ m, $L \geq 1342.6$ m, and $\theta < 41.75^\circ$. Given a value of H , β is related to path length with lower values corresponding to longer paths. In addition, it is suspected that avalanche size and frequency is related to starting zone angle: lower starting zone angles mean larger and less frequent

avalanches. A glance at the ranges of parameters above shows consistently that the modelling is more accurate for longer paths and perhaps for larger avalanches. Equations analogous to eqns. (4–6) for these data subsets were derived. The results of these calculations showed that for $\beta \leq 33^\circ$ the standard errors were significantly lower but the fits (R^2) were not as good as for all the data. For $\theta < 41.75^\circ$ no improvement in fit was noted but the standard error was reduced over equations for all the data. For the 106 avalanche paths with $L \leq 1342.6$ m, the but the standard errors were larger.

The best results were found for the parameter L greater than the median value. For $L \geq 1342.6$ m the following results were derived analogous to eqns. (4–6) for 106 avalanche paths

$$\hat{\alpha} = 0.906\beta - 0.334^\circ \quad (10)$$

$$R^2 = 0.883, S = 1.79^\circ, F = 788$$

$$\hat{\alpha} = 0.855\beta^{1.01} \quad (11)$$

$$R^2 = 0.891, S = 0.061, F = 854$$

$$\sqrt{\alpha} = 0.0845\beta + 2.62 \quad (12)$$

$$R^2 = 0.885, S = 0.166, F = 799$$

For the 106 avalanche paths with $L \leq 1342.6$ m, the following equations were derived

$$\hat{\alpha} = 0.985\beta - 1.62^\circ \quad (13)$$

$$R^2 = 0.803, S = 2.94^\circ, F = 424$$

$$\hat{\alpha} = 0.803\beta^{1.04} \quad (14)$$

$$R^2 = 0.817, S = 0.085, F = 464$$

$$\sqrt{\hat{\alpha}} = 0.0851\beta + 2.73 \quad (15)$$

$$R^2 = 0.809, S = 0.249, F = 440$$

These analyses show consistently that the longer avalanche paths can be modelled more accurately. The parameter L seems to be the best scale of those chosen so far because it takes into account that better regression equations are available for lower β values and higher H values; that is, L accounts for both of these effects.

There are a number of reasons why longer paths should give better results. One reason is that errors in

determining parameters may be magnified for short paths. A second, and perhaps more likely reason, is that the position of the centre of mass of the deposit should provide a more consistent measure of runout than does the tip of the debris as used in the present study. For the shorter paths, the length of the debris deposit may become comparable to length scales such as L or H . When the length of the deposit is less than approximately 10% of L , the location of the centre of mass would be less important. Consultants using these models should recognize that application of the models would give runout distances too short for short avalanche paths.

Strictly speaking, L cannot be used as a predictor variable for the present data set since it is derived from a knowledge of α . However, a two-parameter model with β and L shows the influence of L indicated above. Multiple regression (for 212 avalanche paths) showed

$$\hat{\alpha} = 1.70\beta^{0.992} L^{-0.0823} \quad (16)$$

$$R^2 = 0.877, S = 0.0720, F = 747$$

which may be compared with eqn. (5). For typical values $\beta = 33^\circ$ and $L = 1000$ m, the value of α changes by more than 1° from eqn. (5). It appears that measured values of a horizontal reach length scale, or perhaps path length, may result in improvements in the regression schemes for future data sets. Equations (10–15) show the same trend as eqn. (6): given a value of β , increased values of L imply a lower α angles.

AN ALTERNATIVE SCHEME FOR AVALANCHE RUNOUT PREDICTION

Another method of estimating runout is to calculate the distance in metres from the β point to the measured runout position (α point). From eqns. (1–3) and Fig. 1 it can be seen that

$$\frac{\Delta X}{X_\beta} = \left(\frac{\tan\beta - \tan\alpha}{\tan\alpha - \tan\delta} \right) \quad (17)$$

and similarly

$$\frac{\Delta X}{H_\beta} = \left(\frac{1 - \tan\alpha/\tan\beta}{\tan\alpha - \tan\delta} \right) \quad (18)$$

For the present data set with known values for 131 avalanche paths, calculations show that $\Delta X/X_\beta$ has a mean value of 0.171, a standard deviation of 0.113 and a range of -0.121 to 0.523 . The parameter $\Delta X/H_\beta$ has a mean value of 0.276, a standard deviation of 0.197 and a range of -0.209 to 0.969 . Calculation of correlation coefficients and multiple regression analyses indicated that $\Delta X/X_\beta$ and $\Delta X/H_\beta$ are statistically independent of all the predictor variables to a good approximation. Histogram plots showed that neither of these quantities are Gaussian variables.

Calculation of Spearman rank correlation coefficients, R_s , showed that correlating $\Delta X/X_\beta$ with respect to β results in $R_s = -0.068$ and correlating $\Delta X/H_\beta$ with respect to β results in $R_s = -0.332$. These coefficients differ little from the ordinary correlation coefficients (-0.061 and -0.336 , respectively), but the Spearman coefficient does not employ Gaussian assumptions and is considered a better indicator. The negative correlation coefficients are consistent with the trend from the previous analysis concerning path length effects for the regression models. These and other calculations show that the assumptions of statistical independence with respect to predictor variables is better for $\Delta X/X_\beta$ than for $\Delta X/H_\beta$. This assumption is utilized in a model for extreme runout prediction in the next section.

STATISTICAL PREDICTION OF EXTREME AVALANCHE RUNOUT

The regression models and the parameter $\Delta X/X_\beta$ constitute alternatives for calculating extreme runout from the β point as a reference position. Both alternatives are developed in this section.

Calculation of extreme runout for the regression model depends on a prediction of minimum α . Specifying extreme runout corresponds to estimating the upper confidence limits on the distribution of β (ordinate) vs. α (abscissa). For the present analysis such upper confidence curves are calculated on a point-by-point basis given values of β . Alternative definitions are possible (e.g. Miller, 1980). For a linear model, the upper $P\%$ value of the confidence interval for predicting a minimum value of α , with a

value of $\beta = \beta_0$, is given by (Walpole and Myers, 1978)

$$\alpha_P = \hat{\alpha} - t_{(1-P/100)} S \sqrt{1 + X'_0 A^{-1} X_0} \quad (19)$$

where $t_{(1-P/100)}$ is a chosen value of the t distribution with $N - 2$ degrees of freedom, N is the number of avalanche paths, X'_0 is the row vector $(1 \ \beta_0)$, and X_0 is the corresponding column vector. The 2×2 matrix A^{-1} is the variance-covariance matrix of the estimated regression coefficients. With σ^2 as the variance of the regression equation, $A^{-1} \sigma^2$ has variances on the diagonal and covariances on the off-diagonal elements.

Detailed calculations have shown that for 210 degrees of freedom, eqn. (19) may be safely approximated by

$$\alpha_P = \hat{\alpha} - t_{(1-P/100)} S \quad (20)$$

The correction term $\sqrt{1 + X'_0 A^{-1} X_0}$ causes at most a 2% change to the multiplier $t_{(1-P/100)} S$.

For models obtained by transforming the response variable α , eqns. (19) and (20) are replaced by the similar expressions, with α_P and $\hat{\alpha}$ replaced by the transformed variables. With this change, the best one-predictor runout model is provided by eqns. (5) and (20)

$$\alpha_P = \frac{\beta^{1.06}}{1.370 \exp[t_{(1-P/100)}] [0.0764]} \quad (21)$$

Similar expressions may be derived for the two-predictor model. In this case the row vector X'_0 becomes $(1 \ \beta_0 \ \delta_0)$ and the variance-covariance matrix is a symmetric 3×3 matrix. The best two-predictor model is

$$\sqrt{\alpha_P} = 0.0879\beta + 0.0198\delta + 2.42 - [0.163] [t_{(1-P/100)}] \sqrt{1 + X'_0 A^{-1} X_0} \quad (22)$$

where A^{-1}

$$A^{-1} = \begin{bmatrix} 0.3887 - 1.002 \times 10^{-2} - 8.168 \times 10^{-3} & & \\ & 2.833 \times 10^{-4} & 1.156 \times 10^{-4} \\ (\text{symmetric}) & & 6.736 \times 10^{-4} \end{bmatrix}$$

for $N - 3 = 128$ degrees of freedom.

Calculations at the mean values $\bar{\beta} = 32.7^\circ$ and $\bar{\delta} = 6.5^\circ$ show that $\sqrt{1 + X'_0 A^{-1} X_0} = 1.03$ for the minimum correction to the approximate expression

$$\sqrt{\alpha_P} = 0.0879\beta + 0.0198\delta + 2.42 - [0.163] [t_{(1-P/100)}] \quad (23)$$

Equations (22) or (23) may be used in those restricted cases for which the runout zone has a constant or approximately constant known slope angle.

Equations (21) to (23) state that, given a value of β and a value of P between the limits $50 \leq P < 100$, $P\%$ of avalanche paths in the data set are estimated to have α angles greater than α_P . For the number of degrees of freedom in the data set, the t distribution is nearly indistinguishable from a Gaussian distribution. Calculations with $P = 99$ and $P = 50$ for which $t_{0.01} = 2.326$ and $t_{0.50} = 0.0$, are illustrated in Fig. 2 for eqn. (21).

The alternative to predicting runout using the regression model is to specify $\Delta X/X_\beta$ for long-running avalanches. Since this parameter is statistically independent of β , θ and δ to a very good approximation, values could be extrapolated beyond the mean value, independent of these parameters. The ratio $\Delta X/X_\beta$ is not a Gaussian variable; therefore, several other distributions were tried, including log-normal, Weibull and extreme value (Gumbel). It was found that the best model is provided by the first asymptotic extreme value distribution (Chow, 1964).

Fig. 2. Prediction of values of α for $P = 50$ and $P = 90$ for the regression model [eqn. (21)]. Extreme runout corresponds to the upper envelope of the data points when β (ordinate) is plotted vs. α (abscissa). The 212 avalanche paths are represented by (Δ) with some points representing multiple data.

The fit of the extreme values of $\Delta X/X_\beta$ to this distribution is nearly perfect. The 50 highest values of $\Delta X/X_\beta$ are plotted on extreme value probability paper in Fig. 3. Regression analysis of $\Delta X/X_\beta$ with the reduced variate $-\ln[-\ln(P/100)]$ gave $R^2 = 0.993$ with a standard error $S = 6.402 \times 10^{-3}$ where Weibull plotting positions were used to define the reduced variate (Chow, 1964).

Fig. 3. Plot of $\Delta X/X_\beta$ vs. the reduced variate for the 50 highest values of runout on extreme value probability paper using Weibull plotting positions.

The extreme value distribution gives the model

$$\left(\frac{\Delta X}{X_\beta}\right)_P = 0.1419 - 0.07978 \ln [-\ln(P/100)] \quad (24)$$

where $50 \leq P < 100$. The choice of $P = 50$ gives a mean value of 0.171. Equation (24) states that $P\%$ of avalanche paths in the data set (131 paths) have a horizontal reach less than $(\Delta X/X_\beta)_P$. For $P = 50$ and $P = 99$, the values predicted in eqn. (24) are 0.171 and 0.509, respectively, which may be compared with the Gaussian predictions of 0.171 and 0.434. The fact that the highest values of avalanche runout appear to obey extreme value statistics is not particularly surprising. Extreme value distributions are known to characterize flood data in hydrology work, and the physical analogy to extreme avalanche events seems reasonable.

The regression models and the extreme value model constitute alternative mathematical prediction schemes for extreme runout based on the data set. Specific applications may require one or the other. Experience indicates that the models are largely equivalent to within data accuracy.

Equations (21–24) may help to provide a mapping for extreme runout distances in zoning applications. The value of P in these equations is determined by a decision of the acceptable risk, based upon the value and use of the land and upon the desired level of safety, conditioned by the estimated return period for the climate regime in question. In many instances where land prices are high, $P = 99$ would provide unacceptable runout predictions because the very long distances implied would exclude much expensive land from consideration for construction.

More importantly, eqns. (21–24) provide a mathematical framework to examine measured runout on avalanche paths, in order to define those with an exceptionally long reach. Close examination of those paths with long runout may reveal patterns in terrain features that can provide a basis for more accurate runout prediction and zoning models in the future. Studies of avalanche paths outside the present data set have defined several types of avalanche terrain profiles that produce excessively long runout. Some examples are given in a later section.

GEOMETRICAL DEFINITION OF RUNOUT

The runout models defined thus far are derived from angles with no particular geometry implied. In this section, predicted avalanche runout is interpreted in terms of a terrain model that provides a geometrical definition. A geometrical model may be useful for future avalanche dynamics modelling; in addition, it gives a pictorial representation of the runout prediction models.

Regression analysis for the avalanche paths in the present study has shown that when the terrain profile is plotted with heights, y , versus horizontal distances, x , the polynomial $y = ax^2 + bx + c$ gives an excellent fit to the terrain. In the great majority of cases, $R > 0.98$ and additional terms are unnecessary to accurately describe the terrain. The terrain model given in this paper consists of defining a , b and c in terms of α , β and δ for a given confidence limit or value of P .

In defining the actual runout position in the terrain model, two different assumptions were employed: (1) the regression model given by eqn. (21); and (2) the prediction of maximum runout from extreme value statistics given by eqn. (24). Analyses

using these models showed that the latter model is preferable. For a given value of β , the extreme value statistics model gives a terrain model with runout distances that are consistent with the regression model predictions within data accuracy.

For the polynomial $y = ax^2 + bx + c$, the origin is chosen at the tip of the runout. In the present data set, the avalanches either stop above the valley floor or they run out onto a flat valley floor. Those avalanches that run up the opposite mountainside were excluded. Thus, the terrain model utilizes the polynomial expression for avalanches that do not reach the valley floor and a combination of the polynomial expression and a level straight-line segment for those avalanches that do reach the valley floor.

With the chosen origin of coordinates, the additive constant c in the polynomial may be ignored. The slope at the origin is: $dy/dx|_0 = b$; this quantity is zero for avalanches that reach the valley floor. The parameter $\tan\delta$ defines the average slope in the runout zone from the β point to the origin. For $\tan\delta \geq \frac{1}{2}\tan 10^\circ$, the stop position is at or above the valley floor. For $\tan\delta \leq \frac{1}{2}\tan 10^\circ$, $b = 0$, which suggests the use of the parabola $y = ax^2$ and a straight line segment as the terrain model for these values of δ .

For the polynomial $y = ax^2 + bx + c$ ($\delta \geq 5.04^\circ$)

$$\tan\delta = \tan\alpha - \tan\beta + \tan 10^\circ \quad (25)$$

For a runout position just at the valley floor, $\tan\delta = \frac{1}{2}\tan 10^\circ$, which suggests a definition of an angle α' such that $\tan\alpha' = \tan\beta - \frac{1}{2}\tan 10^\circ$ where α' defines the position of the valley floor. For the parabola $y = ax^2$, $a = \tan^2\alpha'/H$, where H is the vertical drop from start position to the valley floor. Similarly, it may be verified that

$$\frac{H}{H_\beta} = \frac{2\tan^2\alpha'}{\tan\beta(2\tan\alpha' - \tan 10^\circ)} \text{ or, with } \tan\beta = \frac{H_\beta}{X_\beta}$$

This last expression becomes

$$aX_\beta = \tan\beta - \tan 10^\circ \quad (26)$$

The horizontal reach from the β point to the valley floor, $\Delta X'$, is given by

$$\frac{\Delta X'}{X_\beta} = \frac{1}{2} \left(\frac{\tan 10^\circ}{\tan\beta - \tan 10^\circ} \right) \quad (27)$$

The runout position $(\Delta X/X_\beta)_P$ is given by eqn. (24). The length of a straight line segment, ΔX_0 ,

for P values that imply an avalanche reach beyond the valley floor ($\delta \leq 5.04^\circ$) is obtained as follows

$$\frac{\Delta X_0}{X_\beta} = \left(\frac{\Delta X}{X_\beta} \right)_P - \frac{1}{2} \left(\frac{\tan 10^\circ}{\tan\beta - \tan 10^\circ} \right) \quad (28)$$

For avalanches that reach the valley floor and beyond ($\delta \leq 5.04^\circ$), the average slope in the runout zone is (from eqns. (17) and (24))

$$\tan\delta = \frac{(\frac{1}{2}\tan 10^\circ)^2}{(\tan\beta - \tan 10^\circ)} \frac{1}{(\Delta X/X_\beta)_P} \quad (29)$$

Similarly, the α angle is defined by

$$\tan\alpha_P = \frac{(\Delta X/X_\beta)_P \tan\delta + \tan\beta}{(\Delta X/X_\beta)_P + 1} \quad (30)$$

For avalanches that do not reach the valley floor ($\delta > 5.04^\circ$), the polynomial $y = ax^2 + bx$ is used with the origin at the runout position. From eqns. (17), (24) and (25) it is easily shown that

$$\tan\delta = \left[\left(\frac{\Delta X}{X_\beta} \right)_P + 1 \right] \tan 10^\circ - \left(\frac{\Delta X}{X_\beta} \right)_P \tan\beta \quad (31)$$

and similarly

$$\tan\alpha_P = \left(\frac{\Delta X}{X_\beta} \right)_P \tan 10^\circ + \left[1 - \left(\frac{\Delta X}{X_\beta} \right)_P \right] \tan\beta \quad (32)$$

Using $dy/dx|_{\Delta X} = 2a(\Delta X) + b$ and eqn. (17), the parameter b is derived to be

$$b = 2\tan\delta - \tan 10^\circ = \left[2 \left(\frac{\Delta X}{X_\beta} \right)_P + 1 \right] \tan 10^\circ - 2 \left(\frac{\Delta X}{X_\beta} \right)_P \tan\beta \quad (33)$$

For $\delta \geq 5.04^\circ$, aX_β is given also by eqn. (26).

Another quantity of interest is the path length, S_β , along the polynomial $y = ax^2 + bx + c$ from the starting position to the 10° point on the path. Integration gives the following ratio

$$\frac{S_\beta}{X_\beta} = \frac{1}{4aX_\beta} \left[\tan\theta_0 \sec\theta_0 - \tan 10^\circ \sec 10^\circ + \ln \frac{\tan\theta_0 + \sec\theta_0}{\tan 10^\circ + \sec 10^\circ} \right] \quad (34)$$

where θ_0 , the initial angle on the curve, is $\tan\theta_0 = 2\tan\beta - \tan 10^\circ$.

Equations (25–33) define a complete system of equations for a geometrical terrain model, given chosen values of β and P and a length scale X_β or H_β with the runout position defined by extreme value statistics. These equations define avalanche terrain in a systematic way consistent with measured data on avalanche runout and terrain profile fits to actual avalanche paths. This terrain profile model may be of use in theoretical avalanche dynamics modelling. It will enable realistic values of terrain parameters to be introduced in an organized manner in studies of the application of the model to real situations.

It is of interest that the predictions of α_P from the terrain model are in good agreement with the one-parameter regression model described by eqn. (21). Figure 4 shows a comparison for 99%, 90% and 50% confidence limits. The differences are greater at the highest confidence limits, but they are generally less than 1° , which corresponds to data accuracy in the present study.

The regression model was used as an alternative to define the runout position for a different terrain model. Calculations showed that predicted runout exceeded that obtained from extreme value statistics for high values of P . This supports the choice of the extreme value statistics model for runout definition in the terrain model. This seemingly paradoxical result is due to a slight mathematical differences in the two runout models.

Fig. 4. Comparison of runout predictions for the two statistical runout models as a function of confidence limit (P value). The extreme value statistics model is used to define runout positions for the terrain model. These positions are used to calculate values of α_P for comparison.

The terrain model implies some interesting characteristics of runout which may or may not be correct. Since $(\Delta X/X_\beta)_P$ is independent of β in the model, from eqn. (34) the ratio of path lengths for $(X_\beta = \text{constant})$ $\beta = 20^\circ$ and $\beta = 50^\circ$ is $S_{20}/S_{50} = 0.665$. The implication is that $\Delta X_{20}/\Delta X_{50} \approx \frac{2}{3}(S_{20}/S_{50})$, independent of the confidence limit. For $H_\beta = \text{constant}$, $S_{20} = 2.937 H_\beta$ and $S_{50} = 1.348 H_\beta$, or equivalently, $\Delta X_{20} = 3.27 \Delta X_{50}$. The result indicates a strong path-length dependence. One possible physical interpretation is that rapid deceleration does not begin until the slope angle reaches nearly 10° , causing longer runout. These path-length effects are similar to, but much stronger than those shown for the regression models [e.g. eqn. (16)].

Path-length effects are shown (H_β scaled to unity) for the terrain model in Fig. 5 using the 99% confidence limit with $\beta = 20^\circ$, 33° and 50° . This figure also illustrates another feature of the terrain model: the maximum reach onto the valley floor is predicted for β angles, which depend on the confidence limit chosen. From eqn. (28), the angle, $\beta_{\max}$, which produces the maximum distance onto the valley floor, is defined by

$$\tan \beta_{\max} = \frac{\gamma + \sqrt{\gamma^2 - 2\gamma(\tan 10^\circ)}}{2} \quad (35)$$

where $\gamma = \tan 10^\circ \{ [2(\Delta X/\Delta X_\beta)_P + 1] / (\Delta X/\Delta X_\beta)_P \}$. For $P = 99$, this gives $\beta_{\max} = 30.8^\circ$; for $P = 90$, $\beta_{\max} = 38.8^\circ$; and for $P = 60$, $\beta_{\max} = 52.2^\circ$. These results are a mathematical feature of the model chosen, and are a consequence of the choice of origin and the polynomial curves.

Fig. 5. Illustration of profiles of y versus x predicted by the terrain model for $P = 99$. The solid lines (—) connect the start position and the β point. The dashed lines (---) connect the start position and the α point. Points on the predicted profiles are denoted by: ($\circ$, $\beta = 20^\circ$) ($\triangle$, $\beta = 33^\circ$) ($\square$, $\beta = 50^\circ$). The profiles are scaled by setting $H_\beta = 1$.

By inspection, Fig. 5 also shows that for $H_\beta =$ constant, a profile with $\beta = 20^\circ$ implies a value of ΔX , more than three times as large as for a profile with $\beta = 50^\circ$, which is why Lied and Bakkehøi (1980) consider profiles with low values of β to be of great importance in land use planning. Prime building land normally has slope angles below 10° .

EXAMPLES OF RUNOUT CALCULATIONS FOR EXCEPTIONALLY LONG RUNOUT

In this section, sample runout calculations are given for avalanches outside the present data set to illustrate predictions, limitations and problems of the modelling.

An interesting example is the Salezertobel avalanche in Switzerland discussed by Föhn and Meister (1981). These authors showed that runout distances based upon the tip of the debris from the extreme event for each year appear to obey extreme value statistics in direct analogy to flood data from hydro-

Fig. 6. (top): The Salezertobel avalanche profile. (bottom): Comparison of predictions of the regression model ($\diamond$) and the extreme value statistics model ($\circ$) plotted on the extreme value statistics line calculated by Föhn and Meister (1981) for runout distance (S) vs. mean return period.

logy. For this path, $X_\beta = 1770$ m, $H_\beta = 855$ m and $\beta = 25.8^\circ$. The calculations of Föhn and Meister show that measured runout has a good correlation ($R^2 = 0.936$) with respect to the reduced variate of the extreme value distribution for the longest running event for each year (46 years data). They defined runout (S) as the horizontal distance from 395 m before the β point. Figure 6 shows measured runout on the path, and the predictions of runout from the regression model (eqn. 21) and the extreme value statistics model (eqn. 24) for $P = 50, 90$ and 99 . These calculations indicate implied return periods of 17–24 years for $P = 50$, 60–70 years for $P = 90$, and 160–200 years for $P = 99$ for the Salezertobel avalanche. The data indicate that Salezertobel is a potentially long-running avalanche compared with the Norwegian data set. For a return period of about 100 years, the implied value of P is close to 95. These calculations imply some consistency in the modelling for the Salezertobel avalanche.

The remainder of the examples in this paper are chosen because they constitute cases which are troublesome with respect to the predictive equations. These examples may be regarded as worst cases. An example from Colorado, Ruby Peak, (depicted in Fig. 7) was provided to the authors by A.I. Mears (personal communication). In this profile $\Delta X = 855$ m, $X_\beta = 1315$ m, $\alpha = 16.7^\circ$ and $\beta = 23.8^\circ$. Calculations using the regression model give $\alpha_{99} = 17.6^\circ$; using the extreme value statistics model yields $(\Delta X/X_\beta)_{99} = 0.509$ vs. a measured value of 0.651. For the extreme value statistics model $(\Delta X/X_\beta)_{99.9} = 0.693$,

Fig. 7. Profile of the Ruby Peak avalanche from Colorado.

making this avalanche comparable to the one avalanche in a thousand based on the data set. There are two possible reasons for the unusual runout of this avalanche. In this profile, the terrain steepens again to a segment with a 22° slope after reaching the β point, potentially giving to the debris a velocity boost resulting in added runout. Special problems for runout calculations may result from profiles with multiple β points; for example, when a flat or gently sloping bench occurs high on an avalanche path. This condition would not suffice to define a β point. The definition of the β point implies that it is reached by a monotonic decrease in slope angle proceeding down-slope from the starting zone.

Another problem with avalanche paths in a continental climate, such as Ruby Peak, is that the maximum extent of runout identified by vegetation damage may be due to the powder portion of the avalanche rather than the tip of the debris from the core of the avalanche. This can result in runout being overestimated when compared with the current data set. It may not be possible to describe avalanches from regions with a continental climate, such as Ruby Peak, using the mathematical models developed from the present maritime data set. On the other hand, the 100-year avalanche predicted by the model may have a higher probability of occurring in regions with dry snow avalanches, such as those with a continental climate. It may well be that the return period for avalanches producing the same runout distance is shorter under continental climatic conditions.

Another avalanche that is difficult to model is the Iron-ton Park avalanche from Colorado (Fig. 8), described by Lang, Dawson and Martinelli (1979). For this path, $\Delta X = 265$ m; $X_\beta = 550$ m, and $\beta = 32.5^\circ$. Calculations using the runout models show that $\alpha_{99} = 24.5^\circ$ and $(\Delta X/X_\beta)_{99} = 0.509$ vs. the measured values of 24° and 0.482, respectively. These values show that Iron-ton Park has a long runout distance based on the criteria used in this paper. Two related reasons may explain the length of the runout. Firstly, the profile has a fairly abrupt transition at the valley floor. If the profile is steep until the valley floor has been reached, energy-absorbing, particle-locking effects may be delayed, producing longer runout. Secondly, the vertical fall and length of the path are near the minimum in the Norwegian data set. In this case the difference between the position

Fig. 8. Profile of the Iron-ton Park avalanche from Colorado.

of the center of mass and the tip of the debris may be substantial relative to vertical drop or horizontal reach. This effect has been evident in several other examples (not given in the present paper) with a vertical drop less than the lower limit in the present data set (350 m). These examples have shown excessively long runout compared with predictions for the present data set.

Yet another troublesome example is the Ariefa-2 avalanche (Fig. 9) reported by Frutiger (1980) and Buser and Frutiger (1980). The profile has a vertical fall of 550 m. The β angle estimated from the profile is 31.4° and the α angle is 20.8° . The ratio $\Delta X/X_\beta$ is 0.896 based on the profile. These values fall outside the predictions of the regression model and the extreme value statistics model for the one-in-one-hundred avalanche. The polynomial $y = ax^2 + bx + c$ shows an excellent fit ($R^2 = 0.996$) to the profile. Calculating the β point along the curve using $dy/dx = \tan 10^\circ$ gives an estimated β angle of 27.2° . Using the regression model with this figure gives $\alpha_{99} = 20.7^\circ$, whereas the implied value of $(\Delta X/X_\beta)$ is 0.442, which is near $P = 98$ for the extreme value statistics model. This example indicates that a β point defined by a polynomial fit to the profile may be a very consistent method of definition. In practice, it is suggested that a polynomial fit should be made to the profile, from which the β point can then be calculated. A comparison of the calculated β point with that determined from the profile can indicate a troublesome profile with respect to the proposed model.

Fig. 9. Profile of the Ariefa-2 avalanche from Switzerland showing the true β point and another β point calculated by fitting the polynomial $y = ax^2 + bx + c$ to the profile.

For most profiles the calculated and measured values of β are very similar. For instance, the Ironton Park and Ruby Peak avalanches have calculated β points within a few tenths of degrees of those estimated from the profile. For the Ariefa-2 avalanche the difference is close to 4° , which is the worst example found so far. The Ariefa-2 avalanche is an interesting case because casual inspection of the profile does not reveal it as troublesome. It is indeed an avalanche path with a long runout distance, but still appears explainable as the one-in-fifty avalanche from the point of view of the extreme value statistics modelling.

DISCUSSION AND CONCLUSION

Alternative methods for calculating snow-avalanche runout distances have been presented based upon statistical analyses of a data set from Western Norway. These models are: (1) a regression analysis based upon the relation between the Gaussian variables α and β and (2) predictions of extreme value statistics based upon the non-Gaussian variable $\Delta X/X_\beta$. The models are largely equivalent to within data accuracy.

One immediate use of the model predictions is apparent: the statistical mapping of runout distances given a value of β for the path, and a selection of values of P based on acceptable risk, avalanche return period and land prices at the location. In view of the uncertainties involved, it seems far more

rational to specify such a mapping of runout distances for a location rather than two distinct lines that define precisely the limits between safe areas and areas with restrictions on construction. This latter method is the most common way of specifying zones and is in contrast to that suggested in the present paper. The approach presented here is more consistent with modern engineering practice.

In most cases, the definition of values of P at a given location will be based somewhat on guesswork due to lack of data for the climate and other factors at proposed new building sites. A balance between risk and land value has to be carefully considered. Selecting a value $P = 99$ (the one-in-one-hundred avalanche path) may result in unacceptably long runout distances in many areas where land is expensive but the risk of damage would be fairly low.

A second, more tangible, use of the runout distance models is to provide a mathematical framework for defining avalanches with long runout distances with respect to a fixed reference point. This makes it possible to focus study on extreme or troublesome avalanches to refine runout distance estimates. The three examples given in the present paper illustrate such a procedure.

Given the runout distance, or values of β , P and a length scale (e.g. X_β), a geometrical model for snow avalanche terrain has been given based upon polynomial fits to actual profiles. This model provides a beginning to the second part of the engineering zoning question: specifying speeds along the profile between avalanche start and stop positions. The terrain model may enable definition of avalanche terrain in a rational manner that will allow the avalanche dynamics problem to be explored theoretically for variations in terrain features.

The two-part method suggested in the present paper with respect to the engineering zoning problem is in contrast to the traditional method. The traditional method is based upon selected friction coefficients in a dynamics model which are used to calculate speed estimates and runout distances simultaneously. Such calculations are not rational at the present time because of the unknowns regarding constitutive equations and boundary conditions for flowing snow, as well as criteria for effects such as locking and snow particle break-up during collisions. Mathematical description of such behaviour is not close

to being achieved with the accuracy required in modern zoning problems. The present paper represents a proposal to divide this complicated problem into parts which are more manageable.

Several checks and refinements must be made in future studies to improve runout distance calculations. The runout models are based upon data for time scales of at least 100 years. As such, the data should represent runout from large, dry snow avalanches whereas the data set used in this paper is from a maritime climate. It is essential to compare a data set from a continental climate with the present data set.

In the present paper some possible refinements have been pointed out in regard to the modelling. One particular need for future study is the analysis of short avalanche paths with a vertical drop less than 350 m. Although these occur very frequently, no studies exist at present that can be applied to them. This illustrates the need for modelling based on sound physical principles. Field experience clearly shows that other parameters such as starting zone size, and perhaps path length, have an influence on avalanche runout distance, but explicit appearance of these effects in modelling has yet to take place.

ACKNOWLEDGEMENTS

Discussions with, and data provided, by A.I. Mears and M. Martinelli, Jr. proved very useful with regard to the examples used in this paper. This paper is a

joint contribution from the Division of Building Research, National Research Council of Canada and the Norwegian Geotechnical Institute.

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