NRC Publications Archive Archives des publications du CNRC

The horn as a coupling element for acoustic impedance measurements Northwood, T. D.; Pettigrew, H. C.

This publication could be one of several versions: author's original, accepted manuscript or the publisher's version. /
La version de cette publication peut être l'une des suivantes : la version prépublication de l'auteur, la version
acceptée du manuscrit ou la version de l'éditeur.

Publisher's version / Version de l'éditeur:

Journal of the Acoustical Society of America, 26, pp. 503-506, 1954

NRC Publications Record / Notice d'Archives des publications de CNRC:

<https://nrc-publications.canada.ca/eng/view/object/?id=72bcf444-bed0-443a-9d32-860fef161b85>

<https://publications-cnrc.canada.ca/fra/voir/objet/?id=72bcf444-bed0-443a-9d32-860fef161b85>

Access and use of this website and the material on it are subject to the Terms and Conditions set forth at

<https://nrc-publications.canada.ca/eng/copyright>

READ THESE TERMS AND CONDITIONS CAREFULLY BEFORE USING THIS WEBSITE.

L'accès à ce site Web et l'utilisation de son contenu sont assujettis aux conditions présentées dans le site

<https://publications-cnrc.canada.ca/fra/droits>

LISEZ CES CONDITIONS ATTENTIVEMENT AVANT D'UTILISER CE SITE WEB.

Questions? Contact the NRC Publications Archive team at

PublicationsArchive-ArchivesPublications@nrc-cnrc.gc.ca. If you wish to email the authors directly, please see the first page of the publication for their contact information.

Vous avez des questions? Nous pouvons vous aider. Pour communiquer directement avec un auteur, consultez la première page de la revue dans laquelle son article a été publié afin de trouver ses coordonnées. Si vous n'arrivez pas à les repérer, communiquez avec nous à PublicationsArchive-ArchivesPublications@nrc-cnrc.gc.ca.

The Horn as a Coupling Element for Acoustic Impedance Measurements

T. D. NORTHWOOD AND H. C. PETTIGREW*

Division of Building Research, National Research Council, Ottawa, Canada

(Received February 15, 1954)

ANALYZED

Impedance tube methods of testing acoustical materials are restricted to frequencies below the value for which the wavelength is approximately equal to the largest lateral dimension of the tube. Above this frequency, extraneous modes of propagation interfere with measurements. The restriction is a serious one for typical acoustical materials that cannot adequately be sampled in less than 1 square foot. It has been found that the difficulty may be circumvented by using a horn to couple a large sample to a small impedance tube. Comparisons between horn and tube measurements have been made for a range of absorptive materials.

COMMERCIAL materials for sound absorption are most frequently fabricated in 1-foot squares, which may be mounted in a variety of ways. Generally the absorption properties of the material depend significantly on the type of mounting, and to measure the performance of the material it is therefore necessary to test whole units on their prescribed mountings.

One such measurement is the random-field sound absorption coefficient, as determined in a reverberation chamber. This provides data that are adequate for the solution of most problems involving the application of acoustical materials. However, the method has its drawbacks: it requires a large and elaborate establishment, a lengthy series of measurements, and a rather large sample (usually about 72 square feet). Hence, while it may be warranted for occasional standard tests on commercial products, it is too cumbersome to use in the design of materials or for quality control. Moreover, the random-field coefficient is not very simply related to the structure of the acoustical material and it is therefore difficult to use for design purposes.

A simpler approach is exemplified by the many methods of measuring the normal acoustic impedance of acoustical materials. Generally the procedure is to observe the sound field in a small cavity of which the sample material forms one boundary. However, the various methods all have the limitation that they are applicable only to samples whose dimensions are a fraction of a wavelength. For example, the impedance-tube method in its commonest form involves measuring the maxima and minima in the standing-wave pattern produced when a plane wave travelling down the tube is reflected from a sample mounted on the end of the tube. The method works well as long as only plane waves are propagated in the tube, and this is true if the wavelength is somewhat greater than the largest lateral dimension of the tube. Above this frequency higher-order modes may be excited and these will interfere with the plane-wave pattern on which the impedance measurements are based. Consequently, an impedance tube big enough to test a full-sized acoustic

tile is useful only for measurements below about 500 cycles.

The object of this paper is to present a test procedure having the relative simplicity of the impedance-tube method but suitable for measurements on 12-inch square acoustical units, complete with mountings, over a wide frequency range. Briefly, the procedure is to mount the large sample at the mouth of a horn and measure the impedance or absorption at the throat of the horn using a conventional impedance tube suitable for measurements over the required frequency range. Emphasis will be on measurement of the normal-incidence absorption coefficient, although the method is equally suitable for measuring the acoustic impedance.

The horn should fulfil two main requirements: first, there should be a simple relation between the mouth and throat impedances, so that the sample impedance is readily deduced from the measured throat impedance; second, the sound wave incident on the sample should be approximately a plane wave, so that the impedance measured is in fact the "normal" impedance of the sample.

The first requirement may be studied theoretically for horns which are reasonably simple mathematically. One type, the exponential horn, has been examined following the usual plane wave horn theory, and has been found suitable for the present application. Possibly an examination of other horn types might lead to a better one, although it appears from experimental work to date that the exact shape of the horn is not critical.

The second requirement is rather complicated to discuss since most horn theories begin with the assumption that plane waves are propagated, whereas it is found experimentally that the wave front at the mouth of an ordinary horn is curved. However, since only a plane wave can be propagated in the small-diameter impedance tube, minor departures from a plane wave in the horn mouth will not interfere with the actual impedance measurement; they will be serious only if an appreciable amount of energy is dissipated in a non-plane-wave mode, resulting in a high absorption that might erroneously be attributed to the normal-incidence absorption of the sample. For example, if a cross-resonance were excited at the horn mouth,

* Now at Canadian General Electric Company, Toronto, Canada.

FIG. 1. Magnitude of β_1/β_2 at lower frequencies, for $\beta_2=0.5$. (Note: $-bl/\pi$ corresponds to approximately 200 cycles per sec.)

grazing the sample, one might expect an anomalous peak in the measured absorption. Effects of this sort were observed in the present study and will be referred to later.

THE EXPONENTIAL HORN

Consider a horn whose area of cross section is defined by $S=S_0 \exp 2mx$, where S_0 is the throat area and x is distance along the axis from the throat. Following the usual plane-wave horn theory the throat and mouth impedances are related by:¹

$$Z_1' = \frac{Z_2' \cos(bl - \theta) + j \sin bl}{\cos(bl + \theta) + j Z_2' \sin bl}, \quad (1)$$

where Z_1' and Z_2' are the specific impedance ratios at throat and mouth, $b = (k^2 - m^2)^{1/2}$, $k = 2\pi f/c$ and θ is defined by $\cos \theta = b/k$, $\sin \theta = -m/k$. At the set of characteristic frequencies given by $bl = n\pi$ ($n = 1, 2, 3$, etc.), $Z_1' = Z_2'$. These, therefore, appear to be suitable frequencies for impedance measurements. Expanding the compound angles and substituting for $\cos \theta$ and $\sin \theta$ the expression becomes:

$$\frac{Z_1'}{Z_2'} = \frac{b/k \cos bl - m/k \sin bl + j/Z_2' \sin bl}{b/k \cos bl + m/k \sin bl + j Z_2' \sin bl}. \quad (2)$$

This can be examined numerically in the vicinity of the characteristic frequencies, to investigate such factors as the effect of frequency errors. However, it was found convenient to replace the impedance ratio with a parameter which is better behaved in the limiting case of a perfect reflector. To avoid confusion among the several conventions for handling wave problems, the relations will be given in detail.

The sound field in the impedance tube may be regarded as the sum of incident and reflected plane waves whose pressures and particle velocities are given by

$$\begin{aligned} p_i &= P_+ e^{jk(ct-x)}, & u_i &= p_i/\rho c, \\ p_r &= P_- e^{jk(ct+x)}, & u_r &= -p_r/\rho c. \end{aligned}$$

The absorption coefficient of the reflecting end is given by

$$\alpha_0 = 1 - |p_r/p_i|^2 = 1 - |\beta|^2, \quad (3)$$

introducing the complex reflection ratio $\beta = p_r/p_i$.

The impedance ratio is related to β by

$$Z' = \frac{p}{\rho c u} = \frac{p_i + p_r}{p_i - p_r} = \frac{1 + \beta}{1 - \beta}, \quad (4)$$

and the horn equation (2) may be rewritten in the form:

$$\frac{\beta_1}{\beta_2} = \frac{b/k \cos bl - (1/\beta_2)(m/k) \sin bl - j \sin bl}{b/k \cos bl - \beta_2(m/k) \sin bl + j \sin bl}. \quad (5)$$

When the horn is mounted on the end of the impedance tube, β_1 may be measured in the usual way; its magnitude is obtained from the ratio of sound pressure minima and maxima, and its phase from the positions of the minima with respect to the end of the tube. Then, in principle at least, β_2 can be calculated via Eq. (5), and the absorption coefficient or impedance may be obtained for the sample mounted on the horn mouth.

The operations are most readily performed at the characteristic frequencies for which $\beta_1 = \beta_2$ exactly, although it is possible to make approximate measurements of absorption at any frequency, using Eq. (5). Figure 1 shows how the absolute value of this expression varies for a typical case ($\beta_2 = 0.5$).

Assuming that measurements are made at the characteristic frequencies, the precision of the method in terms of possible frequency errors may be considered. Actually, it is wavelength rather than frequency which is the direct variable in Eq. (3), and wavelengths may be measured with considerable precision in the impedance tube. Assuming constant velocity of sound in the medium (i.e., constant temperature), the system including the oscillator may be calibrated initially in terms of wavelength, resulting in a series of oscillator settings corresponding to the "characteristic frequencies." With typical laboratory oscillators it is easy to reset to within 1 percent of the specified frequencies. Calculations show that, for the horn described below, and for absorption coefficients in the range 0.2 to 0.9, a frequency error of 1 percent would result in an error of less than 0.01 in the absorption coefficient.

FIG. 2. Schematic diagram of apparatus.

¹ G. J. Thiessen, J. Acoust. Soc. Am. 17, 558 (1950).

For measurements of acoustic impedance the phase of β_2 must also be determined. This necessitates precise control of the phase shift due to the horn, and hence of frequency or wavelength of the sound. For example, a frequency error of 1 percent would produce a corresponding error in wavelength and since the double traverse of the horn may be, say, ten wavelengths long, the accumulated phase error at the throat of the horn will be 10 percent. However, the wavelength may be checked with sufficient precision in the impedance tube. Experimentally, it is occasionally found necessary to make a slight second adjustment of the oscillator toward a characteristic frequency in order to make phase measurements accurate to 2 or 3 percent.

ABSORPTION BY THE HORN WALLS

For precise measurements with an impedance tube it is usually necessary to correct for losses in the tube itself, due to heat-conduction losses at the walls and to attenuation in the medium.

The conventional procedure is to observe the linear increase in successive pressure minima, progressing down the tube from the sample, and to extrapolate back

FIG. 3. Comparison of absorption coefficients obtained by horn and by direct impedance tube measurements, on a sample of $\frac{1}{2}$ -inch soft fiberboard.

to the face of the sample. An alternative that is found equivalent experimentally is to observe the minima with a rigid end on the tube and subtract these "tube corrections" from the minima obtained with the sample. Similarly, by terminating the horn in a rigid plate, a set of horn corrections may be obtained.

EXPERIMENTAL RESULTS

To test the practicability of the method, use was made of an available exponential horn of circular cross section having mouth and throat diameters of $11\frac{3}{4}$ inches and $2\frac{1}{4}$ inches and a length of about 32 inches. This was coupled to an impedance tube $2\frac{1}{4}$ inches in diameter by 40 inches long, which follows fairly closely the design of R. A. Scott.² The associated apparatus is shown schematically in Fig. 2.

Using a rigid termination on the horn mouth, the characteristic horn frequencies were determined by noting the frequencies for which there was zero phase shift at the throat (i.e., for which the first minimum was one quarter of a wavelength from the end of the

² R. A. Scott, Proc. Phys. Soc. (London) 58, 253 (1946).

FIG. 4. Comparison of absorption coefficients obtained by horn and by direct impedance tube measurements, on a sample of $\frac{3}{4}$ -inch Fibreglas tile.

impedance tube). These agreed very closely with calculated values. At the same time a horn-loss correction was obtained by noting the standing-wave ratio with the rigid end.

The observed horn losses were equivalent to absorption coefficients ranging from 0.04 to 0.10, over the frequency range 240 to 3000 cycles. The measuring procedure was to maintain a constant value for the voltage at the first maximum and to observe voltages at the minima. The horn-loss correction was made simply by subtracting the rigid-end minima from the minima obtained with the test samples.

Absorption measurements were made on large samples of typical acoustical materials mounted on the mouth of the horn and compared with measurements on small samples mounted directly on the impedance tube. A rigid backing was carefully maintained for both tests. In this way a correlation was obtained between horn and tube measurements, for materials with absorption coefficients ranging from 0.05 to 0.99. Good results were obtained with the simple horn for

FIG. 5. Comparison of absorption coefficients obtained by horn and by direct impedance tube measurements, on a sample of 2-inch Fibreglas Type PF.

frequencies below about 1000 cycles. For work above this frequency it was found necessary to introduce baffles in the mouth section to prevent a radial resonance at the face of the sample. An additional improvement was obtained by introducing an axial obstacle to increase the axial path length and produce a better approximation to a plane wave at the mouth. It was then possible to get good agreement between the two methods up to about 3000 cycles. Absorption data obtained for three materials covering a wide range of absorption coefficients are shown in Figs. 3, 4, and 5.

In addition to the above program, an extension was added to the horn to adapt it to samples 12 inches square. This led to a rather abrupt change in horn section and resulted in discrepancies at some frequencies

above 2000 cycles. However, no serious new problems were introduced, and useful measurements are being made up to this frequency. This makeshift assembly will shortly be replaced with a new horn, now being built, which terminates properly in a 12-inch-square mouth. It is expected that satisfactory measurements up to about 3000 cycles will then be obtainable.

The primary aim to date has been the measurement of the normal-incidence absorption coefficients, in order to meet a specific requirement. A few measurements of acoustic impedance have been made and these agree well with impedance-tube data. The impedance problem will be examined more carefully after a routine for the absorption measurements has been established.