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Calculation of Thermal Conductance Based on Measurements of Heat Flow Rates in a Flat Roof Using Heat Flux Transducers

by C.P. Hedlin

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RÉSUMÉ

On a mesuré, à l'aide de transducteurs de flux de chaleur, le flux de chaleur traversant un isolant rigide en fibre minérale de 60 mm d'épaisseur posé dans le toit plat d'une installation extérieure d'essai. Les différences de température en divers points de l'isolant ont été mesurées simultanément au moyen de thermocouples.

On a utilisé trois méthodes analytiques pour calculer la conductance thermique de l'isolant :

- a) le rapport entre le flux de chaleur et la différence de température;
- b) le taux de variation du flux de chaleur en fonction de la différence de température et de la température moyenne; et
- c) la

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Charles P. Hedlin¹

Calculation of Thermal Conductance Based on Measurements of Heat Flow Rates in a Flat Roof Using Heat Flux Transducers

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ABSTRACT: Heat flow rates through 60-mm-thick rigid mineral fiber insulation mounted in the flat roof of an outdoor test facility were measured with heat flux transducers. The temperature differences across the insulation were measured simultaneously with thermocouples.

Three different analytical procedures were used to calculate the thermal conductance of the insulation:

- (a) the ratio of heat flow rate to temperature difference,
- (b) the slope of heat flow rate as a function of temperature difference and mean temperature, and
- (c) transfer functions.

The results obtained with these three methods are compared.

The effect of ignoring thermal lag was estimated for the first two methods.

KEY WORDS: heat flux transducers, flat roofs, thermal insulation, transfer functions, thermal lag, thermal conductance, heat flow

Nomenclature

- A Thermal conductance at 0°C, W/m² · K
- °C Temperature
- C Thermal conductance

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C_r, C_s, C_t	Thermal conductance found by ratio, slope, and transfer function methods
E	Coefficient representing the variation in thermal conductance with mean insulation temperature, $W/m^2 \cdot K^2$
F	Scanning frequency
Q	Heat flow rate, W/m^2
T_T, T_B	Temperatures at the top and bottom surfaces of the insulation, respectively, $^{\circ}C$
ΔT	Temperature difference across insulation, $T_B - T_T$, K
T_m	Mean insulation temperature, $(T_T + T_B)/2$, $^{\circ}C$
I, J, K	Transfer coefficients for T_T, T_B , and Q

The field measurement of heat flow through building envelopes using heat flux transducers (HFTs) is characterized by a variety of conditions and requirements.

1. The heat flow varies continuously and unpredictably because of changes in the temperature difference and thermal storage effects.
2. The conditions in normal buildings are not ideal; there may be anomalies in the system, such as moisture, discontinuities in thermal insulation, and heat flow diversion by structural members. These cause special problems for measurement and analysis of data.
3. The measurement system should impose the least possible disturbance on the heat flow and, in some cases, moisture movement.
4. The data should be in a form that can be readily handled. Further, the analysis of heat flow data must produce results that have counterparts in the better established steady-state technology.
5. Highly conductive materials are unfavorable for HFT mounting sites; wide temperature swings should be avoided; some heat flux transducers are damaged by moisture. Thus, the choice of the site for an HFT may represent a compromise between the least unfavorable conditions and may require the purchase of heat flux transducers that are appropriate for the circumstances, for example, wet conditions.

This report contains a brief description of a field-type measurement system that is subject, in some degree, to all of the foregoing conditions. It also deals with some aspects of data analysis in which heat flow rates and temperature differences are used to estimate thermal conductance of insulation *in situ*.

Instrumentation

At the Prairie Regional Station (PRS), heat flow has been measured in flat roofs, using one make of heat flux transducer [1]. Two sizes (50 and 106 mm in diameter) were used. These transducers were calibrated by the manufac-

turer and recalibrated at intervals at the Division of Building Research, Ottawa, Canada.

Type-T thermocouples were used to measure temperature. The thermocouple was referenced to an aluminum block 89 mm in diameter and 152 mm long, embedded in thick insulation to keep it at a uniform, though not constant, temperature. In turn, this block was referenced to an automatic ice bath using a single thermocouple (Fig. 1). Thus, by adding millivoltages, each thermocouple was referenced to the ice point.

The output from the heat flux transducers and thermocouples was recorded by a digital data acquisition system, and the data were processed by computer.

The examples used in this paper are based on data obtained using a specimen of dry rigid glass fiber 60 mm thick, sealed in polyethylene.

The assembly is shown in Fig. 2. The supporting equipment consisted of two aluminum plates, each 400 mm square and 6.3 mm thick. One plate was secured to the underside of a 9.5-mm-thick plywood roof deck. The heat flux transducer was placed in a 6.3-mm-thick Bakelite holder machined to receive it. The exposed surface of the transducer was flush with the surface of the holder. The holder and transducer were placed on the lower aluminum plate. Plywood 6.3 mm thick was used as edge surround material. The insulation was clamped between the transducer/holder/plywood surface and the upper aluminum plate.

The holder and aluminum plate act as buffers against fluctuations in air temperature. With an unprotected transducer, air movements across the surface produce small variations in temperature. Because the temperature drop across the transducer is small, even minor disturbances in its surface temperature produce considerable "noise" in its output.

The thermocouples were located at the top and bottom surfaces of the insulation (inside the polyethylene) above the center of the heat flux transducer.

FIG. 1—(a) Data gathering system; (b) aluminum reference block.

FIG. 2—Roof cross sections showing locations of heat flux transducers (dimensions in millimetres).

The heat flow rates and temperatures were measured once every 20 min from January to June. Similar measurements have been made for other insulation specimens, but inclusion of more than one set of results was not considered necessary for the purposes of this discussion.

The temperature of the heat flux transducer varied with daily and seasonal temperature swings. The maximum daily average temperature difference, from winter to summer, was approximately 10 K. Since the HFT output is affected by temperature, a correction based on a laboratory calibration was applied to nullify the effect of this temperature variation.

The results were obtained in the Canadian prairie climate. References to winter and summer are made in that context, and conclusions drawn about daily or seasonal variations may not apply to other regions.

Calculation of Thermal Conductance

Three methods of calculating thermal conductance are discussed: (1) a ratio method, (2) a slope method, and (3) transfer functions. These are designated C_r , C_s , and C_t , respectively.

Ratio Method

For the ratio and slope methods, daily average values of heat flow rate, Q (W/m^2) and $\Delta T = T_B - T_T$ (where T_B and T_T are bottom surface and top surface temperatures, in degrees Kelvin), were used.

The daily averaging method to find T_B , T_T , and Q is based on the premise that there are diurnal repetitions of the heat flow and temperature conditions and that thermal mass effects are nullified by the averaging over a full cycle. In fact, a 24-h period does not normally produce closure of the cycle, nor is any other extended or shortened "day" likely to produce an exact repetition of conditions. Presumably, an interval selected so that ΔT or Q are the same at the beginning and end would be somewhat better in this respect than a fixed 24-h period. Each of these values represents one of the conditions describing the thermal state of the system. If one (or both) are equal at the beginning and end of the test period, other thermal conditions, such as temperature distribution, should also be closer to those of the initial state than would be the case at the end of an arbitrarily fixed 24-h period.

Nonclosure and thermal lag are illustrated in Fig. 3. The heat flow rate and temperature difference across the glass fiber specimen versus time of day are

FIG. 3—Hourly temperature differences, in degrees Celsius, and heat flow rates, in watts per square metre, for (a) a winter day and (b) a summer day.

plotted, using 24 of the 72 available values, for a winter day (Fig. 3a) and for a summer day (Fig. 3b). Both show a 24-h cycle. The daily variation is more pronounced for the summer day than for the winter one. Under some winter conditions, the cycle variation is hidden by long-term changes in temperature; the temperature difference at a selected time of day would not be repeated in the following 24-h period—and possibly not for a number of days if the temperature continued to drift in one direction. In summer the diurnal swing is normally so large that points somewhere in the cycle are almost certain to be repeated.

Q can be plotted against ΔT (Fig. 4). Conductance can be found as the ratio

$$C_r = \frac{Q}{\Delta T} \quad (1)$$

As Q and ΔT decrease, the error in conductance tends to increase, since measurement errors comprise increasingly large fractions of the measured values.

Figure 4b illustrates a curve that does not pass through the origin. An expanded graph, showing the region of the origin (Fig. 4c), demonstrates that a conductance calculated using $Q/\Delta T$ for Point 1 would give a plausible, but inaccurate, conductance value. At Point 2, the calculated conductance would be zero, at Point 3 it would be negative, and at Point 4 it would be infinitely negative. For data points lying to the left of the origin, conductances will be positive.

A consistent bias in the location of the curve with respect to the origin would occur if there were systematic errors. Such errors might stem from a variety of sources.

Systematic errors would occur if the measurement of temperature difference was consistently inaccurate. In this work, thermocouples were used. Comparison with a calibrated wire-wound resistance thermometer indicated that the thermocouples agreed to within 0.01 K. They were referenced to the

FIG. 4—Heat flow rate versus temperature difference—the effect of non-zero ordinate intercepts.

aluminum block; hence (assuming that the temperature of the block was uniform), the error introduced should be very small.

Heat flux transducers could be biased, so that the output for heat flow in one direction is different from that for the same heat flow rate reversed. Tests on the meters used at PRS did not reveal a detectable error of this type; however, such an error cannot be ruled out.

The thermal conductance is influenced by mean temperature, T_m , where

$$T_m = \frac{T_T + T_B}{2} \quad (2)$$

$$C = A + ET_m \quad (3)$$

Inward heat flow occurs during the warm part of the day. Therefore, for a given temperature difference, the rate of flow will be somewhat higher than the rate of outward flow, which occurs when the insulation is cooler. This would be expected to produce a negative intercept. Normally this does occur, although in the present case (Eqs 5, 9, and 10), the intercept was slightly positive. Scatter or measurement error may mask the temperature effect.

The existence of a negative (or positive) intercept should be construed as an inadequacy of the measurement and analytical procedures rather than as an apparent contravention of thermodynamic laws.

In the present case, it is discussed because, however precise the measurement and analyses, some intercept value is likely to exist in practice and affect the conductances calculated by the ratio

$$C_r = \frac{Q}{\Delta T} \quad (1)$$

In the case of moist insulation, a much greater (negative) intercept occurs than for dry materials [2]. It is not practical to include it in this discussion. It is mentioned here only to point out the existence of another factor that may complicate the development of equations to express practical heat transfer relationships.

If a finite intercept is caused by systematic measurement error, the conductance (C_r in Eq 1) may be found by applying a correction (equal and opposite to the intercept value) to the heat flow values. The difficulty may also be avoided by using the slope of Q versus the temperature parameter, as discussed in the following section.

The Slope Method

Thermal conductance may be found from the slope of an equation that relates heat flow and temperature difference. Thus, if sufficient data points

are available, an equation relating ΔT and Q can be found by least squares analysis.

Assuming Q to be a second-order function of ΔT

$$Q = a + b\Delta T + c(\Delta T)^2 \quad (4)$$

This is solved in Eq 5 using the data, without allowance for thermal lag.

$$Q = 0.038 + 0.587\Delta T - 0.00122(\Delta T)^2 \quad (\text{W/m}^2) \quad (5)$$

This is illustrated in Fig. 5.

$$C_r = \frac{Q}{\Delta T} \approx 0.587 - 0.00122\Delta T \quad (6)$$

if the first term on the right is ignored.

Equation 4 implies that the conductance is a function of ΔT . In fact, although ΔT may have a secondary effect, it is more appropriate to express it in terms of mean insulation temperature. An alternative relationship explicitly includes the effect of mean temperature, T_m .

If it is assumed that

$$Q = C\Delta T \quad (7)$$

FIG. 5—Heat flow rate, Q , in watts per square metre, versus temperature difference, ΔT , for 60-mm-thick mineral fiber specimen.

where C is the thermal conductance, and

$$C = A + ET_m \quad (2)$$

where A = the thermal conductance, C , at 0°C , then

$$Q = A\Delta T + E\Delta TT_m \quad (8)$$

Constants for Eq 8 are found in the following two ways.

1. Approximately, by multiple regression analysis. This results in a small intercept. The analyses were done using data points representing one-day averages. Normally, a number of days' data are averaged. However, one of the purposes of this study is to assess the effect of thermal lag; one-day averages were used to avoid the masking of thermal lag effects that occur when a number of days are averaged. When thermal lag was disregarded

$$Q = 0.047 + 0.550\Delta T + 0.00158\Delta TT_m \quad (\text{W/m}^2) \quad (9)$$

When thermal lag effect was included

$$Q = 0.008 + 0.551\Delta T + 0.00175\Delta TT_m \quad (\text{W/m}^2) \quad (10)$$

For the two analyses, the indexes of determination were 0.9996 and 0.9998, and the standard errors of estimate were 0.118 and 0.082 W/m^2 , respectively. Thermal conductances can be found either by ratio ($C_r = Q/\Delta T$) or by slope ($C_s = Q/\Delta T$). From Eq 9

$$C_r = \frac{0.047}{\Delta T} + 0.550 + 0.00158T_m \quad (\text{W/m}^2 \cdot ^\circ\text{C}) \quad (11)$$

$$C_s = 0.550 + 0.00158T_m \quad (\text{W/m}^2 \cdot ^\circ\text{C}) \quad (12)$$

From Eq 10

$$C_r = \frac{0.008}{\Delta T} + 0.551 + 0.00175T_m \quad (\text{W/m}^2 \cdot ^\circ\text{C}) \quad (13)$$

$$C_s = 0.551 + 0.00175T_m \quad (\text{W/m}^2 \cdot ^\circ\text{C}) \quad (14)$$

2. Alternatively, Eq 8 can be solved by using simultaneous equations. Based on a solution using 85 data points

$$Q = 0.551\Delta T + 0.00175\Delta TT_m \quad (\text{W/m}^2) \quad (15)$$

$$C_s = C_r = 0.551 + 0.00175T_m \quad (\text{W/m}^2 \cdot ^\circ\text{C}) \quad (16)$$

Transfer Functions

The heat flow rate can be represented as a function of the top and bottom temperatures using polynomial relationships [3]. One of these involves the use of transfer functions. The heat transfer rate, Q , is expressed in terms of temperatures. The time lag effect is accommodated by including earlier values of temperature and heat flow rate in the relationship.

$$Q_0 = I_0 T_{T_0} + I_1 T_{T_1} + \dots + J_0 T_{B_0} + J_1 T_{B_1} + \dots + K_1 Q_1 + \dots \quad (\text{W/m}^2)$$

The subscripts 0, 1, and 2 indicate the heat flow or temperatures occurring currently or one or two time steps earlier. (A time step might be an hour but does not have to be.)

In the present case, the following relationship was used

$$Q_0 = I_0 T_{T_0} + I_1 T_{T_1} + I_2 T_{T_2} + J_0 T_{B_0} + J_1 T_{B_1} + J_2 T_{B_2} + K_1 Q_1 + K_2 Q_2 \quad (17)$$

K_0 , the coefficient for Q_0 , is taken to be unity.

Equation 17 might be described as a 3-3-3 arrangement, since there are three coefficients each for the top and bottom temperatures and heat flux. Other arrangements were tried, including 5-3-3, 5-2-2, and 4-2-2. These arrangements weighted representation in favor of the top surface temperature, which varied more widely and erratically than the bottom surface temperature and heat flux. These did not produce better results, based on internal consistency and comparison with Eq 14 (or Eq 16).

Equation 17 contains eight coefficients, but ten data sets are required to determine the coefficients (data going back two time steps are required). In the present case, many data were available. A computer program that could handle up to 250 data sets was used to obtain best-fit results. Normally, far fewer than 250 data sets would be used.

Usually, 24 data sets per day were used.² These were obtained either by skipping two out of every three of the available numbers or by averaging three consecutive data sets to obtain 1-h values of temperature and heat flux. Normally 26, 50, 74 . . . data sets were used to find conductances for 1, 2, 3 . . . day periods.

An almost infinite variety of combinations exists, given the options for numbers of coefficients, size of time steps, numbers of data sets used per day, and so on. Perhaps other arrangements would have been better than those that were used.

²This was found to be a sufficient number of points. However, if fewer points are to be used, care should be taken to ensure that the results are accurate.

The transfer function approach is appropriate only for linear systems. A linear relationship between heat flow and temperature can be derived and used in Eq 17. It produces a modified expression for temperature which incorporates the effect of temperature on thermal conductance. The resulting conductances should all be equal to the conductance at $T_m = 0^\circ\text{C}$.

The development is as follows

$$Q = C_i(T_T - T_B) \quad (\text{W/m}^2) \quad (18)$$

and

$$C_i = A + ET_m = A + \frac{E(T_T + T_B)}{2} \quad (\text{W/m}^2 \cdot \text{K}) \quad (19)$$

$$C_i = A \left\{ 1 + \frac{E}{2A} (T_T + T_B) \right\} \quad (\text{W/m}^2 \cdot \text{K}) \quad (20)$$

$$Q = A \left\{ (T_T - T_B) + \frac{E}{2A} (T_T^2 - T_B^2) \right\} \quad (\text{W/m}^2) \quad (21)$$

$$= A \left\{ \left(T_T + \frac{E}{2A} T_T^2 \right) - \left(T_B + \frac{E}{2A} T_B^2 \right) \right\} \quad (\text{W/m}^2) \quad (22)$$

$$= A \{ T_T^* - T_B^* \} \quad (\text{W/m}^2) \quad (23)$$

where $T_T^* = T_T + (E/2A)(T_T^2)$ is a modified temperature, and Q is a linear function of the modified temperature.

Equation 17 then becomes

$$Q_0 = I_0 T_{T_0}^* + I_1 T_{T_1}^* + I_2 T_{T_2}^* + (J_0 T_{B_0}^* + J_1 T_{B_1}^* + J_2 T_{B_2}^*) + (K_1 Q_1 + K_2 Q_2) \quad (24)$$

Using the coefficients in Eq 16

$$\frac{E}{2A} = \frac{0.00175}{2(0.551)} = 0.0016 \quad (\text{K}^{-1}) \quad (25)$$

The transfer function coefficients differ widely, depending on the data used. Table 1a and b gives four examples based on time steps of 1 h using $E/2A = 0.0016 \text{ K}^{-1}$ to adjust the results to a 0°C base.

The value of A (the thermal conductance for $T_m = 0^\circ\text{C}$) was computed from these coefficients. Using the results for Day 167 and the J coefficients

$$A = \frac{\sum_0^2 J}{1 - \sum_1^2 K} = \frac{0.23026 + 0.07092 - 0.20904}{1 - 0.79591 - 0.04034} = 0.563 \quad (\text{W/m}^2 \cdot \text{K}) \quad (26)$$

The sums of the I and J coefficients should be equal and opposite in sign; thus, conductance can be found by using one or the other. However, the sums are not exactly the same, and an average of the two should be used. This gives $A = 0.561$, which is higher than the expected value of 0.551.

The linearizing procedure should make A independent of temperatures. Values of A were found using all of the data. Linear least squares fits gave the following results for $E/2A = 0.0016 \text{ K}^{-1}$

$$A = 0.551 + 0.0004 T_m \quad (27)$$

and for $E/2A = 0.0020 \text{ K}^{-1}$

$$A = 0.551 - 0.00008 T_m \quad (28)$$

Thus, the temperature effect was almost eliminated by the linearizing procedure.

Calculations were also made using $E/2A = 0$. The results are shown in Fig. 6 for two-day spans. The best-fit relationship gave

$$C_t = 0.550 + 0.00143 T_m \quad (29)$$

TABLE 1a—Transfer coefficients for four time periods in 1979, with heat flow data obtained using dry glass fiber 60 mm thick.

Days	I_0	I_1	I_2	K_1	K_2	J_0	J_1	J_2
11	-0.29677	-0.05547	-0.11214	0.39063	-0.21384	0.51479	0.00113	-0.07556
11 to 14	-0.20468	-0.20310	0.03950	0.44710	-0.11607	0.46094	0.06390	-0.15455
134 to 138	-0.21463	-0.13823	0.03269	0.43657	-0.01093	0.50904	-0.09508	-0.09424
167	-0.16877	-0.14822	0.22534	0.79591	0.04034	0.23026	0.07092	-0.20904

TABLE 1b—Interim calculations for a, based on coefficients in Table 1a.

Days	$1 - \sum K$	$\sum I$	$A = \frac{-\sum I}{1 - \sum K}$	$\sum J$	$A = \frac{\sum J}{1 - \sum K}$	Mean A
11	0.82321	-0.46438	0.564	0.44036	0.535	0.550
11 to 14	0.66897	-0.36828	0.551	0.37029	0.554	0.552
134 to 138	0.57436	-0.32017	0.557	0.31972	0.557	0.557
167	0.16375	-0.09165	0.560	0.09214	0.563	0.561

FIG. 6—Thermal conductances calculated using the transfer function, C_i , versus the mean insulation temperature, T_m . $E/2A = 0$.

where C_i is used in place of A to indicate that the data are not linearized. The fit is not good, but it does show an upward trend in conductance with increasing T_m .

Calculation of C Using Values Obtained Within a One-Day Period

In what follows, the data are used to inspect several factors in terms of their influence on $C_i = Q/\Delta T$ computed on the basis of one-day averages.

Three points are considered:

- (a) whether or not the effect of thermal lag is significant [4],
- (b) whether the "day" should be 24 h or variable and determined by some other criterion, and
- (c) the effect of the scan frequency, F , that is, the number of values used per 24-h period, on the accuracy.

There still remains the very important practical task of mounting a heat flux transducer on a building component and allowing for the time required for the resulting disturbance to settle out. The following method of calculating was used:

1. Thermal lag—The temperature values were paired with Q values measured three scans (60 min) later (as described earlier).

2. Variable day length—In finding a Q versus ΔT pair for a day, the computer first surveyed 100 scans (each representing 20 min) and found the maximum and minimum values of ΔT in that interval. The mean value, ΔT_0 , was used as the reference temperature difference for that day. Returning to the beginning of the 100-scan interval, the computer progresses until ΔT crosses ΔT_0 . It begins the averaging process at that point and continues until it reaches a crossing point one cycle later. In some cases, bobbling of ΔT produced a very short cycle. The computer program was arranged to ignore short cycles and normally to produce one pair of points per day. Even so, some sets comprised fewer than 40 and others more than 120 scans.

3. Scanning frequency, F —Using 72 scans per day, calculations were made for all the cases in which $72/F$ was an integer. Further, at each value of F , all the values were used, that is, for $F = 36$, one pair of Q versus ΔT values was found by averaging the 1st, 3rd, 5th, . . . 71st values and a second pair by averaging the 2nd, 4th, 6th, . . . 72nd values. Thus, for $F = 72$, there was 1 pair of values, for $F = 36$ there were 2 pairs, for $F = 24$ there were 3, and for $F = 18, 12, 9, \dots 1$ there were 4, 6, 8, . . . 72 pairs, respectively.

Effect of Thermal Lag and Day Length on C_r

Figure 7 shows conductances (C_r in Eq 1) versus ΔT (based on one-day averages) along with temperature scales showing T_m and ΔT . Also included in Fig. 7 is C_i (from Eq 12 in Fig. 7a and Eq 14 in Fig. 7b).

In Fig. 7a, no allowance was made for thermal lag, whereas a 1-h allowance was made in Fig. 7b. Visual inspection suggests that the inclusion of allowance for thermal lag produced a small improvement in the results.

Some comparisons were made using portions of the data. To eliminate extreme error values, scans in which the temperature difference was less than 2 degrees Kelvin were left out. Analyses were done for the remaining data exclusive of $\Delta T > 12$, $\Delta T > 20$, and for all the remaining data, that is, the ranges of the three sets of data were $2 < \Delta T < 12$; $2 < \Delta T < 20$; and $2 < \Delta T$. This demonstrates the increased error of the estimate of C_r at reduced values of ΔT .

The same process was repeated using variable day length (same ΔT at the beginning and end). This was limited to cases in which the scan frequency was between 40 and 120, inclusive. The results, including standard errors of estimate, are given in Table 2.

FIG. 7—Plots of C_r versus T and T_m : C_r is based on one-day averages of Q and T (not all points are shown): (a) without allowance for thermal lag and (b) with allowance. C_r represents conductance based on the curve of the best fit using data points derived from 24-h averages in Eqs 12 and 14.

TABLE 2—Standard error (SE, %) in C_r based on one-day averages of Q and ΔT for thermal lag corrections of zero and 1 h.^a

Conditions	ΔT max, °C	SE, %	
		Zero Lag	1-h Lag
72 scans/day	12	4.2	3.0
Fixed ΔT	12	3.3	3.3
72 scans/day	20	3.6	2.5
Fixed ΔT	20	2.9	2.8
72 scans/day	no limit	2.9	2.0
Fixed ΔT	no limit	2.5	2.4

^a Three ΔT max conditions were used. The calculations for fixed scan frequency and fixed beginning and ending ΔT . (In the latter case, scan frequencies below 40 and above 120 per "day" were rejected.) ΔT min = 2°C in all cases.

Using the data in Table 2, the effect of including or ignoring the influence of thermal lag on C_r can be estimated for fixed and variable day lengths. For the fixed day lengths, the standard error of estimate for C_r was reduced by 30 to 35% by including the effect of thermal lag in the computation. For variable day lengths, there was no apparent reduction when the effect of thermal lag was included.

Comparing standard errors of estimate for fixed versus variable-day lengths, the standard deviations in C_r were not significantly different. Based on these data and the analytical method used here, using cycles with the same temperature difference at the beginning and end did not appear to improve the result.

Effect of Reduced Scanning Frequency and Thermal Lag on C_r

The effect of reduced scanning frequency on C_r might be represented in a variety of ways. In this study, the maximum and minimum values of C_r are found for each value of $F < 72$. This difference is divided by 2 and expressed as a percentage of the mean value. The mean value is only approximately correct. Days 26, 137, and 174 are each represented by one point in Fig. 8, and each of those points lies at some distance from the presumed best value given by C_s .

As shown in Fig. 8, the maximum deviation from the mean conductance is affected by the thermal lag, temperature difference, and frequency of scan-

FIG. 8—Maximum deviations of C_r from the mean values for three days—scan frequencies from minimum of one per day: Day 26, winter; Day 137, spring; Day 174, summer.

ning. For Day 174, the deviation was very large for low scan frequencies. It decreased rapidly with increasing values of F , reaching 5% for 12 scans per day and 2% for 24 scans per day. For Day 137, the pattern was similar, except that a frequency of 4 scans per day reduced the deviation to 3% and 2% for the no-lag and 1-h lag calculation.

For Day 26, the maximum deviation from the mean conductance for one scan per day was 4 and 3% for no lag and 1-h lag, respectively, and was less than 2% for $F = 2$ for both cases.

Summary

1. The study of the thermal performance of buildings may require *in situ* measurement of thermal conductance of building components. This can be done by analyzing measured heat flow rates and temperatures.

The method of analysis will affect the result. In the present case, simple averaging was used to derive pairs of heat flow and temperature differences for part of the report. Even then, many possibilities exist. The effects of some of these variations were explored using heat flow temperature difference data for a glass fiber specimen. In addition, transfer functions were calculated and used to find thermal conductance.

2. Thermal conductance can be calculated using 24-h average heat flows and temperature differences. This assumes that the thermal conditions in the component being tested repeat themselves every 24 h. This is only approximately correct. In summer a well-defined daily cycle usually occurs, but exact repetition of conditions is unlikely. In winter the cycle is less well defined. Further, because of the thermal inertia of the system, heat flow lags behind the temperature difference that produces it. Scatter in the results is fairly constant for ΔT greater than 10 K but becomes much larger for ΔT less than 10 K.

3. Equations representing one-day averages of Q in terms of ΔT and T_m were found with and without correction for thermal lag. Q calculated with the two equations differed by less than 0.04 W/m^2 in the range from 0 to 40°C . Conductance differed by a maximum of 0.75% in the same range. Transfer functions can be used to express the heat flow rate through a specimen as a continuous function of time, given the appropriate temperature conditions applied to the specimen. Also, thermal conductance can be calculated from the transfer functions.

4. Thermal conductance can be defined as the ratio of heat flow rate to the temperature difference, C_r , or can be derived from the slope of a second-order relationship between the same two variables, C_s . Both have shortcomings. Errors in C_r become increasingly large as ΔT approaches zero. C_s can be calculated only when sufficient data points are available.

5. The effects of thermal lag and scanning frequency on conductances were studied for a winter day, a spring day, and a summer day.

In an effort to offset the effects of thermal lag, temperature differences were paired with heat flow rates that occurred 1 h later.

For thermal conductances calculated by the ratio $Q/\Delta T$, the smallest effect of scanning frequency occurred on the winter day, followed by the spring and summer days. The maximum variation from the conductance found by using all 72 values was calculated for scanning frequencies ranging from 72 to 1 per day. Maximum variations were less than 2% for 2 scans on the winter day, for 6 scans on the spring day, and for 24 scans on the summer day. Calculation of the conductances using a thermal lag of 1 h showed reductions in these conductance variations ranging up to about 50%.

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DISCUSSION

*D. L. McElroy*¹ (written discussion)—Have you considered radiation transfer to explain any of your results?

C. Hedlin (author's closure)—No effort was made to estimate the individual contributions of radiation or other heat transfer modes.

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D. A. Hagan² (written discussion)—How did the phase shift correction of Q versus T relate to the time constant of the component being tested, and did the resulting calculation of thermal conductivity convert to the same value as the uncorrected calculation?

C. Hedlin (author's closure)—In Fig. 3b, the heat flow rate was estimated to lag behind the temperature difference by about 1 h, based on the difference in time as each crossed the line when temperature difference and heat flow rate were zero. The calculations were made pairing temperature differences with simultaneous heat flow rates and also with heat flow rates occurring 1 h later.

Rough comparisons of the results with and without allowance for thermal lag were made for the ratio and slope methods. For the first, Fig. 7 shows that maximum deviations of C_r from the mean were somewhat (1 to 5%) less when thermal lag was taken into account. For the slope method, a least squares analysis of heat flow versus a combination of temperature difference and mean temperature gave standard errors of estimate of 0.118 and 0.082 W/m² when the lag was neglected or considered, respectively. Further, the heat flow rate calculated by comparison of the two equations (Eqs 9 and 10) at several points showed that estimated values of heat flow rate differed by less than 0.05 W/m². Equations 12 and 14 give the thermal conductances obtained when thermal lag was disregarded and when it was included.

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