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TRANSFORMATION OF MONO- AND MULTI-CHROMATIC WATER WAVES PROPAGATING FROM A QUASI-DEEPWATER TO A SHALLOW WATER REGION

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KEYWORD

3D numerical model, Mono- and multi-chromatic waves, Shallow sea bottom, Wave transformation, Wave breaking

ABSTRACT

A vertically integrated 3D numerical model that uses the concept of depth average velocity distribution with enhanced dispersion characteristics investigates the propagation and transformation of mono- and multi-chromatic waves from a quasi-deep region to shallow water region. A finite difference method has been employed for the numerical computation that follows ADI (Alternating Direction Implicit) algorithm. The local sea bottom is uneven and turns into a moderately shallow water due to presence of a ridge-like bottom near the coast. The bottom configuration is from a location off Nova Scotia coast in Canada. Detail results and discussions are presented.

INTRODUCTION

The basic features of the wave field are transformed when it moves from a deepwater region to a shallow water region. Researchers in this topic include: Peregrine (1967), Abbotts et al. (1978), Flaten and Rygg (1991), Sato et al. (1992), Karambas and Koutitas (1992), Nwogu (1993), Madsen and Sørensen (1991), Madsen et al (1997a and 1997b), Bayram and Larson (2000). In recent years there have been an increase in use vertically integrated models to study wave propagation, transformation, breaking and other hydrodynamic processes due to this change in bathymetry. It has been reported by many researchers that the approximation by the vertically integrated model such as, the Boussinesq type numerical model gives results that agree well with physical experiments and field

observations. The strength of such models is that they can be useful to study the wave transformation including breaking (Svendsen et al., 1996) for a wide range from deep sea to swash zone (Sørensen et. al., 1996) with high confidence.

In this study a Boussinesq equation based numerical model has been developed and employed for the computation of the propagation and transformation of mono- and multi-chromatic waves. In some cases monochromatic wave breaking is considered. In this study waves are allowed to propagate from a deep to a shallow region towards the coast. An enhanced dispersion relation has been invoked in the simulation.

MODEL FORMULATION

GOVERNING EQUATIONS

The wave transformation model used in this study is based on the Boussinesq-type equations with improved dispersion characteristics as reported by Madsen et al. (1991) and Madsen and Sørensen (1992). The basic continuity equation and equation of motions for x and y directions can be read as follows:

$$\eta_t + P_x + Q_y = 0 \quad (1)$$

$$P_t + \left(\frac{P^2}{D} \right)_x + \left(\frac{PQ}{D} \right)_y + gD\eta_x + \varepsilon P + \frac{f_d}{2D^2} P \sqrt{P^2 + Q^2} =$$

$$\nu \left(P_{xx} + P_{yy} \right) + \left(B + \frac{1}{3} \right) h^2 \left(P_{xt} + Q_{yt} \right)$$

$$+ Bah^3 \left(\eta_{xxx} + \eta_{yyy} \right) + hh_x \left(\frac{1}{3} P_{xt} + \frac{1}{6} Q_{yt} \right) + hh_y \left(\frac{1}{6} Q_{yt} \right)$$

$$+Bgh^2 \left[h_x (2\eta_{xx} + \eta_{yy}) + h_y \eta_{xy} \right] \quad (2)$$

$$\begin{aligned} Q_t + (PQ/D)_x + (Q^2/D)_y + gD\eta_y + \varepsilon Q + \frac{f_d}{2D^2} Q \sqrt{P^2 + Q^2} = \\ \nu (Q_{xx} + Q_{yy}) + \left(B + \frac{1}{3} \right) h^2 (Q_{yyt} + P_{xt}) \\ + Bah^3 (\eta_{yyy} + \eta_{xxy}) + hh_y \left(\frac{1}{3} Q_{yt} + \frac{1}{6} P_{xt} \right) + hh_x \left(\frac{1}{6} P_{yt} \right) \\ + Bgh^2 \left[h_y (2\eta_{yy} + \eta_{xx}) + h_x \eta_{xy} \right] \end{aligned} \quad (3)$$

where η is the instantaneous water surface elevation, P the depth integrated mass component (flux) in the x -direction, Q the depth integrated mass component (flux) in the y -direction, h the local still water depth, $D (=h+\eta)$ the local instantaneous water depth, g the gravitational acceleration, ε the boundary dumping function varies linearly along the width of the sponge layers and null elsewhere, ν the eddy viscosity describes the momentum exchange due to turbulence, f_d the energy dissipation coefficient and the subscripts x , y and t denote the differentiation with respect to space and time. A sponge layer is a numerical energy-dumping zone in the model. It is usually deployed around the computational domain to reduce or eliminate wave reflection from the domain boundaries. For better performance its width would be at least twice of the incident wavelength. The parameter B is an important factor in the dispersion relation that depreciates the computational error in the wave celerity and group velocity. See Madsen and Sørensen (1992) for details.

The expression for ν can be given by the following expression (Sato *et al.*, 1992 and Bayram and Larson, 2000):

$$\nu = \frac{\alpha_D g d \tan \delta}{\sigma^2} \sqrt{\frac{g}{d} \frac{\hat{Q} - Q_r}{Q_s - Q_r}} \quad (4)$$

where α_D is a coefficient (2.5 in the surf zone and null elsewhere), $\tan \delta$ the bottom slope, d the mean water depth and σ is the angular frequency. $\hat{Q}$ is the flow amplitude, Q_s the wave induced flow inside the surf zone and Q_r is the flow amplitude of the reform waves and are expressed as follows:

$$Q_s = 0.4(0.57 + 5.3 \tan \delta) \sqrt{gd^3} \quad (5)$$

$$Q_r = 0.135 \sqrt{gd^3} \quad (6)$$

The wave number would be evaluated from the following dispersion relation:

$$\frac{c^2}{gh} = \frac{1 + Bk^2 h^2}{1 + (B + \frac{1}{3})k^2 h^2} \quad (7)$$

where B is equal to 1/15 corresponds to the frequency dispersion obtained from Padé's (2,2) expansion of the Stoke's first-order theory, k is the wave number and c is the wave celerity.

Usually long period waves inherit considerable reflection. Numerically a sponge layer having a width equivalent to twice of the incoming wavelength or more can absorb most of the wave energies. But physically this job is still neither easy nor

economic for a very long period wave. Energy dumping event in a sponge layer follows the following relation:

$$\varepsilon_j = \frac{\gamma \varepsilon_m}{2(\sinh \gamma - \gamma)} \left[\cosh(\gamma X_j / F) - 1 \right] ; \quad j = 1, 2 \quad (8)$$

where the term ε_j is the boundary damping function and is null elsewhere apart from the boundary, $\varepsilon_m = \sqrt{gh}$, γ is a coefficient and its value is assumed to be 3.0 in this study, F is the width of the sponge layer and X_j is the horizontal distance along the sponge layer in the x and y directions, respectively.

The following expression (Goda, 2000) is used to compute the breaking height of a monochromatic wave:

$$H_b = 0.17 L_o \left[1 - \exp \left\{ -1.5 \frac{\pi h_b}{L_o} \left(1 + 15 s^{\frac{4}{3}} \right) \right\} \right] \quad (9)$$

where, H_b is the breaking wave height, h_b is the breaking water depth and s is the bottom slope.

The frequency spectrum for the wind waves proposed by Goda (1988) is used in this work to generate multi-chromatic waves. The equation can be given as follows:

$$S(f) = 0.205 H_{\frac{1}{3}}^2 T_{\frac{1}{3}}^{-4} f^{-5} \exp[-0.75(T_{\frac{1}{3}} f)^{-4}] \quad (10)$$

where, $f = 1/T_{1/3}$ is the frequency.

DISCRETIZATION OF THE GOVERNING EQUATIONS

A Finite difference numerical method has been employed for the solution of the governing equations. The discretization is done following the mesh shown in Fig. 1.

Figure 1: The computational mesh

The above governing equations 1, 2 and 3 have been discretized (see Zaman *et al.*, 2001) in such a way so that the solution could be obtained by ADI algorithm. Discretized governing equations are shown in the appendix.

RESULTS AND DISCUSSIONS

The above-mentioned numerical model has been applied to study the propagation of waves with different characteristics over a bottom of varying elevations. Figure 2 shows the bottom configuration of the computational domain. The bottom configuration is from a location off the Nova Scotia coast in Canada. Five numerical wave gauges (NG) are used to capture the time series of surface elevations at different locations shown in the domain as NG-1 to NG-5 in Figure 2. The locations coordinates of these numerical wave gauges in the domain are shown in Table 1 (see also Figure 2). In Figure 2 it is shown that the water depth at the incident boundary is 40m. Due to the up-hill type sea bottom the water depth at the NG-3 location reduced to 10m, which is pretty shallow wrt to incident wavelengths used here (99m to 260m). A sponge layer around the domain is utilized for waves energies dumping purposes. Equation 8 describes the characteristics of the sponge layer employed.

Table 1: Location of the numerical wave gauges

Numerical wave gauge	X - coordinate	Y - coordinate
NG-1	50	80
NG-2	150	80
NG-3	200	80
NG-4	250	80
NG-5	300	80

Table 2: Wave parameters and resulted wave steepness

T	h / L_o	H_o / L_o	H / L	Increase of wave steepness
8s	0.413	0.050	0.075	32.73 %
10s	0.286	0.035	0.055	36.75 %
12s	0.221	0.027	0.045	39.71 %
15s	0.166	0.020	0.033	39.22 %

In the case of monochromatic waves of period, $T=8s$, 10s, 12s and 15s are used separately to study their propagation and transformation over the above-mentioned bottom topography. The wave conditions are shown in Table 2. In all cases the incident wave height H_o is 4.935m but the incident wavelength L_o varies with the respective incident wave period. Here H and L are the resulted wave height and wavelength. The water depth at the incident boundary is 32.2m while the depth at the offshore boundary is 40.0m. In all figures surface elevations and wave heights are normalized by the incident wave height H_o . Figures 3 to 7 show the comparisons of the surface profiles at the locations NG-1 to NG-5 (see Figure 2 and Table 1) for waves of $T=8s$, 10s, 12s and 15s. From Figures 3 to 7 it may be observed that with the decrease of the relative water depths (h/L_o) considerable nonlinearities developed in the wave profiles especially over the shallower region. The crests become larger and steep while the troughs get smaller and flatten. Figure 8 shows the distribution of the wave heights over the domain in the horizontal X direction. It may be observed from this figure that substantial reflection takes place from the ridge surface towards the upstream side. It may also be noted

that in front of the ridge the steepness of the wave increases noticeably with the decrease of the relative water depths. The increase of the wave steepness is shown in the last column of Table 2.

Figures 9 to 13 show the normalized surface profiles at locations NG1 to NG5 for varying incident wave steepness. In this computation the incident wave period T is 8s, wavelength L_o is 98.7m and wave height H_o varies from 4.935m to 12.337m. Four different cases are considered in this computation shown in Table 3.

Table 3: Wave parameters for varying steepness

	T	L_o (m)	H_o (m)	H_o / L_o
Case-1	8s	98.7	4.935	5.0%
Case-2	8s	98.7	7.402	7.5%
Case-3	8s	98.7	9.870	10.0%
Case-4	8s	98.7	12.337	12.5%

In this particular computation wave breaking has been taken into consideration and Equation 9 is used for this purpose. From Figures 9 to 13 it maybe observed that with the increase of the wave steepness waves break according to Equation 9. It maybe seen from Figures 11 to 13 at NG-3 to NG-5 that the surface elevations become smaller for waves of higher steepness. This is because after the breaking the rest of the wave energy keeps propagating with their equivalent amplitude, which is always smaller. Figure 14 shows the distribution of the wave heights for Case-1 to Case-4. For Case-1 the wave does not break but for Case-2, Case-3 and Case-4 wave breaking takes place.

In the computations of multi-chromatic waves three different categories of waves are utilized. Table 4 describes the wave parameters. In the table $T_{1/3}$, $H_{o1/3}$ and $L_{o1/3}$ are the significant wave period, significant wave height and significant wavelength of the incident wave.

Table 4: Multi-chromatic wave parameters

$T_{1/3}$	$h / L_{o1/3}$	$H_{o1/3}$
10s	0.286	4.935
12s	0.221	4.935
15s	0.166	4.935

Figure 15 shows the distribution of the normalized significant wave heights for these three groups of waves. It may be observed that the longer the wave period the larger the wave heights over the upstream side of the domain.

CONCLUSIONS

A three-dimensional Boussinesq model with enhanced dispersion characteristics has been examined to study the propagation of mono- and multi-chromatic waves over a mild to steep bottom topography. For the case of monochromatic waves it is observed that with the increase of the shallowness (decrease of relative water depth) wave reflection increases and considerable nonlinearities in the wave profile develop. Wave steepness is observed to enlarge greatly over the upstream side of the domain from top of the ridge. This model is capable of simulating breaking waves over steep bottom. For the case of

multi-chromatic waves it is found that with the increase of the significant wave period the wave height over the upstream side of the domain increases.

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Figure 2: Bathymetry of the computational domain

Figure 3: Normalized surface elevations at NG-1
($H_0=4.935$, $h/L_0=0.413$, 0.286 , 0.221 and 0.166 , $h=40m$ at the incident boundary)

Figure 4: Normalized surface elevations at NG-2
($H_o=4.935$, $h/L_o=0.413$, 0.286 , 0.221 and 0.166 , $h=40m$ at the incident boundary)

Figure 5: Normalized surface elevations at NG-3
($H_o=4.935$, $h/L_o=0.413$, 0.286 , 0.221 and 0.166 , $h=40m$ at the incident boundary)

Figure 6: Normalized surface elevations at NG-4
($H_o=4.935$, $h/L_o=0.413$, 0.286 , 0.221 and 0.166 , $h=40m$ at the incident boundary)

Figure 7: Normalized surface elevations at NG-5
($H_o=4.935$, $h/L_o=0.413$, 0.286 , 0.221 and 0.166 , $h=40m$ at the incident boundary)

Figure 8: Distribution of the normalized wave heights in the X spatial direction
($H_o=4.935$, $h/L_o=0.413$, 0.286 , 0.221 and 0.166 , $h=40m$ at the incident boundary)

Figure 9: Normalized surface elevations at NG-1
($H_o=4.935$, $h/L_o=0.413$, $T=8s$, $h=40m$ at the incident boundary)

Figure 10: Normalized surface elevations at NG-2
($H_o=4.935$, $h/L_o=0.413$, $T=8s$, $h=40m$ at the incident boundary)

Figure 11: Normalized surface elevations at NG-3
($H_o=4.935$, $h/L_o=0.413$, $T=8s$, $h=40m$ at the incident boundary)

Figure 12: Normalized surface elevations at NG-4
($H_o=4.935$, $h/L_o=0.413$, $T=8s$, $h=40m$ at the incident boundary)

Figure 13: Normalized surface elevations at NG-5
($H_o=4.935$, $h/L_o=0.413$, $T=8s$, $h=40m$ at the incident boundary)

Figure 14: Distribution of the normalized wave heights in the X spatial direction
($H_o=4.935$, $h/L_o=0.413$, $T=8s$, $h=40m$ at the incident boundary)

Figure 15: Distribution of the normalized wave heights in the X spatial direction
($H_{o1/3}=4.935m$, $h/L_{o1/3}=0.286$, 0.221 and 0.166 , $h=40m$ at the incident boundary)