NRC Publications Archive Archives des publications du CNRC

Performance of ductile iron pipes. II : sampling scheme and inferring the pipe condition

Kleiner, Yehuda; Rajani, Balvant

This publication could be one of several versions: author's original, accepted manuscript or the publisher's version. / La version de cette publication peut être l'une des suivantes : la version prépublication de l'auteur, la version acceptée du manuscrit ou la version de l'éditeur.

For the publisher's version, please access the DOI link below. / Pour consulter la version de l'éditeur, utilisez le lien DOI ci-dessous.

Publisher's version / Version de l'éditeur:

[https://doi.org/10.1061/\(ASCE\)IS.1943-555X.0000098](https://doi.org/10.1061/(ASCE)IS.1943-555X.0000098)

Journal of Infrastructure Systems, 19, 1, pp. 120-128, 2012-02-06

NRC Publications Record / Notice d'Archives des publications de CNRC:

<https://nrc-publications.canada.ca/eng/view/object/?id=62f4fa25-a046-4079-99fa-f465fb184bd1>

<https://publications-cnrc.canada.ca/fra/voir/objet/?id=62f4fa25-a046-4079-99fa-f465fb184bd1>

Access and use of this website and the material on it are subject to the Terms and Conditions set forth at

<https://nrc-publications.canada.ca/eng/copyright>

READ THESE TERMS AND CONDITIONS CAREFULLY BEFORE USING THIS WEBSITE.

L'accès à ce site Web et l'utilisation de son contenu sont assujettis aux conditions présentées dans le site

<https://publications-cnrc.canada.ca/fra/droits>

LISEZ CES CONDITIONS ATTENTIVEMENT AVANT D'UTILISER CE SITE WEB.

Questions? Contact the NRC Publications Archive team at

PublicationsArchive-ArchivesPublications@nrc-cnrc.gc.ca. If you wish to email the authors directly, please see the first page of the publication for their contact information.

Vous avez des questions? Nous pouvons vous aider. Pour communiquer directement avec un auteur, consultez la première page de la revue dans laquelle son article a été publié afin de trouver ses coordonnées. Si vous n'arrivez pas à les repérer, communiquez avec nous à PublicationsArchive-ArchivesPublications@nrc-cnrc.gc.ca.

Performance of ductile iron pipes: sampling scheme and inferring pipe condition

Yehuda Kleiner and Balvant Rajani

National Research Council of Canada, Institute for Research in Construction
1200 Montreal Road, Ottawa, Ontario K1A 0R6, Canada

Abstract:

Ductile iron (DI) pipes have been used in North America since the late 1950s. This paper, the second of two companion papers, describes how understanding gained on the geometry of external corrosion pits is used to devise a sampling scheme and to infer the condition of a ductile iron buried water mains.

A companion paper (Kleiner et al. 2013) describes the exhumation of varying lengths of ductile iron pipes in four North American water utilities. The exhumed pipes were cut into sections, sandblasted and tagged. Soil samples extracted along the exhumed pipe were also provided. Pipe sections were scanned for external corrosion using a laser scanner to produce corrosion pit data sets. Statistical analyses were performed on geometric properties of corrosion pits such as pit-depth, pit-area and pit-volume. These analyses were developed further to assess the impact of the different soil characteristics on these pit properties. This paper, describes the investigation of appropriate sampling schemes to represent the statistical properties of ductile iron pipe corrosion. With known statistical properties, an approach is developed to infer the condition of the pipe

Keywords: Ductile iron pipe, corrosion pits, pipe sampling scheme, pipe condition assessment

1. Introduction

The ultimate goal of the investigation described in Kleiner et al. (2013) was to enable a statistical inference on the structural condition of a ductile iron (DI) pipe based on pit geometrical properties, coupled (possibly) with soil properties, both obtained from a few sample rings along the pipe. As noted in the conclusions of the Kleiner et al. (2013), no compelling evidence was found that soil samples would improve the ability to predict corrosion pit properties in any significant or consistent manner. Consequently, soil properties are not considered in the current

discussion of sampling and statistical inference on pipe condition. However, it would be fairly straightforward to extend the proposed method in this paper to include soil properties, should further research show the existence of such evidence.

As in any circumstance where an attempt is made to characterize a certain population based on sample data, the larger the sample the more accurate the character estimation and the higher the cost of this characterization. Consequently, there is a need to strike a balance between accuracy and cost. Additionally, specifically for buried water mains, it is important to determine what should be the sampled pipe ring length (definition of a pipe ring is provided in Kleiner and Rajani, 2013) and if sampling should be completely random or planned to represent the entire pipe length.

This paper describes the investigation of various sampling schemes conducted towards the recommendation for the preferred one as well as the manner with which sampling results can be used to make statistical inferences about the condition of the pipe. A key assumption is made, namely that Kleiner et al. (2013) provide sufficient evidence to support the notion that the right-truncated Gumbel is the underlying distribution for pit-depth maxima in rings, while Weibull is underlying distribution for pit-area and pit-volume in rings.

The remainder of this paper is organized as follows. The second section describes the investigation conducted on different sampling schemes. The third section describes the computation of confidence bounds for the discerned parameters of the probability distributions. The fourth section describes the process of making a statistical inference about ductile iron (DI) pipe condition and the fifth section provides summary and conclusions.

Sampling

The literature is rife with various types of sampling schemes that vary between completely random to some combination of randomness and systematic arrangement, e.g., stratified sampling, cluster sampling, quota sampling and others. The appropriate underlying population in the current context is that of rings in a pipe, as described in Kleiner et al. (2013). Simple random sampling of say, n samples, would entail the selection of n sample rings whose locations are determined randomly (with assumed uniform distribution) along the entire pipe. This type of sampling is appropriate if the geographical location along the pipe has no bearing on the

properties of the ring population. Notwithstanding the earlier conclusion about the lack of evident impact of soil properties on pit geometry in our data sets (Kleiner and Rajani, 2013), the prevailing inclination of most (if not all) experts in the field is to assume that the geographical location along the pipe has a bearing on corrosion (possibly through soil properties or other location-dependent factors such as stray currents, proximity to other metallic structures, etc.). Consequently, a simple so-called stratified sampling, where the pipe is divided into equal-length segments (or “sites”) and a ring(s) is (are) selected at random from each site. In this way, the entire length of the pipe is represented by the sample. It must be remembered that there is always a tradeoff between accuracy, which requires many samples, and costs to acquire these samples. The goal is to find a sampling scheme that provides acceptable accuracy at a minimal cost.

The notation used herein is as follows. The candidate pipe is divided into n equal-length sites, a location is selected at random at each site and k adjacent rings are then selected at that location. The reason why the selected k rings are adjacent is because the cost of scanning k adjacent rings at a given site is probably not significantly affected by the value of k (provided k is not exceedingly large). Consequently, a sample comprises $n*k$ rings. Each ring has three geometrical pit properties, i.e., pit-depth maximum, pit area and pit-volume. Pit-area is not considered in the sampling investigation because it was deemed less useful than pit-depth maximum (potential leaks) and pit-volume (loss of mass). It is important to understand that the sampled items in any sampling scheme are assumed to be independent and identically distributed (*iid*). If there is a reason to believe that the sample items are not *iid* then their correlation must be accounted for in the analysis to obtain valid results. In our proposed sampling scheme it is reasonable to assume that the corrosion properties in the rings are *iid* for $k = 1$ (i.e., one sample per site) provided n is not very large. However, multiple rings in a given site ($k > 1$) may not be *iid*, especially if they are adjacent.

Table 1 is reproduced here from Kleiner et al. (2013). It reflects the fact that the P-values of the probability distribution fit into the ring populations were quite low for a wide range of ring lengths but tended to increase somewhat as the ring lengths increased. This means that the shorter ring populations tended to better follow the selected probability distribution. For this and for practical reasons (easier and cheaper to scan shorter rings in the field), our investigation was narrowed down to the comparison of ring lengths of 25, 50, and 100 mm only.

Table 1. Statistical analysis results of pit geometries in ring populations (no soil impact)**Kansas City (mean wall thickness 10.89 mm)**

Ring		Max. pit depth: trunc-Gumbel			Pit area: Weibull			Pit volume: Weibull		
Length (mm)	Count	Scale	Location	P-value	Scale	Shape	P-value	Scale	Shape	P-value
25	2718	1.11	1.12	0.000	67	0.27	0.000	89	0.26	0.000
50	1380	1.16	1.33	0.000	169	0.32	0.000	226	0.31	0.000
100	645	1.23	1.61	0.000	451	0.38	0.000	607	0.36	0.000
150	456	1.29	1.85	0.000	862	0.44	0.000	1178	0.42	0.000
300	181	1.43	2.23	0.050	2215	0.53	0.000	3091	0.51	0.000
450	95	1.56	2.56	0.003	4013	0.61	0.000	5766	0.57	0.000
600	84	1.71	2.62	0.006	4519	0.59	0.001	6252	0.56	0.001

Louisville (mean wall thickness 8.89 mm)

Ring		Max. pit depth: trunc-Gumbel			Pit area: Weibull			Pit volume: Weibull		
Length (mm)	Count	Scale	Location	P-value	Scale	Shape	P-value	Scale	Shape	P-value
25	2738	1.04	0.94	0.000	38	0.24	0.000	50	0.23	0.000
50	1440	1.14	1.19	0.000	111	0.27	0.000	148	0.26	0.000
100	684	1.28	1.49	0.000	313	0.32	0.000	432	0.30	0.000
150	455	1.37	1.74	0.000	596	0.35	0.000	844	0.33	0.000
300	220	1.62	2.38	0.000	1965	0.47	0.000	2959	0.44	0.000
450	141	1.83	2.80	0.000	3615	0.55	0.000	5608	0.50	0.000
600	78	1.68	2.99	0.003	4480	0.60	0.000	6916	0.55	0.000

Calgary (mean wall thickness 8.44 mm)

Ring		Max. pit depth: trunc-Gumbel			Pit area: Weibull			Pit volume: Weibull		
Length (mm)	Count	Scale	Location	P-value	Scale	Shape	P-value	Scale	Shape	P-value
25	2297	1.45	1.41	0.000	13	0.29	0.000	21	0.27	0.000
50	1192	1.56	1.99	0.000	41	0.37	0.000	69	0.34	0.000
100	585	1.69	2.71	0.000	117	0.43	0.000	207	0.40	0.000
150	388	1.87	3.19	0.000	205	0.45	0.000	370	0.41	0.000
300	189	2.09	4.22	0.000	532	0.48	0.000	1008	0.44	0.000
450	83	2.06	4.59	0.007	824	0.47	0.001	1651	0.42	0.002
600	83	2.64	5.70	0.014	1289	0.51	0.002	2609	0.46	0.002

It is challenging to estimate inter-correlations between adjacent rings obtained at the same site and subsequently incorporate them into the inference process. This aspect was considered to be beyond the scope of this research effort. Instead, a large number of sampling simulations were carried out to gain some insight into possible trends, and these results were compared.

Figure 1 illustrates results obtained from these simulations for Calgary data on pit-depth maxima. One thousand (1000) sampling simulations were run for each of the sampling schemes (e.g., 1000 simulations for 4 sites with 1 ring per site, 1000 simulations for 5 sites with 1 ring per site, and so on) and the histograms, means and standard deviations (SDs) were computed for the truncated Gumbel distribution parameters. A few observations can be made:

- *Location parameter:* The simulation mean value for the location parameter in all the sampling schemes is greater than that of the true population location parameter, probably due to the long right tail (see example case in Figure 1). However, the simulation mean values tend to converge towards the true population value as the number of sites and the number of rings per site increases, which is what would be expected.
- *Scale parameter:* The simulation mean values for the scale parameter approximate well the true population scale parameter.
- As expected, the simulation SD values for location and scale parameters decrease as the number of sites increases and the number of rings per site increases.
- For a given number of sites, the simulation SD, for both scale and location parameters, decreases somewhat as k (the number of adjacent rings per site) increases. For a given number of total rings (e.g., 8 rings), the simulation SD is smaller for the scheme in which k is smaller (e.g., 8 sites with 1 ring per site will have a smaller simulation SD than 4 sites with 2 rings per site). This is expected due to the fact that adjacent rings are not *iid* and that the population is better represented by more sites.
- The same general behavior described above was also observed in the data sets for other cities, including 25 and 100 mm ring lengths.

A comparison of the performance of various ring length samples has to be done on a relative basis. Two criteria were used to make this comparison for accuracy and precision. Accuracy was measured as the ratio between the simulation mean value for the parameter and the true population parameter (referred to here as SimToPop ratio). Precision was measured as the ratio between the simulation parameter SD and the simulation parameter mean (this ratio is also known as the coefficient of variation or CoV). Figure 2 shows the results of these comparisons in terms of SimToPop ratio and CoV for pit-depth maxima in the Kansas City data set (recall that

Figure 1 Calgary pit-depth maxima data: comparisons of various sampling schemes

Scale parameter SimToPop ratio

# sites	Rings per site		
	4 x 25 mm	2 x 50 mm	1 x 100 mm
4	0.97	0.93	0.95
5	0.98	0.95	1.01
6	0.97	0.97	0.97
7	0.99	0.96	0.95
8	0.98	1.00	0.96
9	0.98	0.97	0.96
10	1.00	0.97	1.00

Scale parameter SimToPop ratio

# sites	Rings per site		
	8 x 25 mm	4 x 50 mm	2 x 100 mm
4	0.96	0.96	1.00
5	0.99	0.97	0.98
6	0.97	0.98	0.94
7	0.98	0.97	0.97
8	0.99	0.98	0.96
9	0.97	0.97	0.97
10	0.99	1.00	0.97

Location parameter SimToPop ratio

# sites	Rings per site		
	4 x 25 mm	2 x 50 mm	1 x 100 mm
4	1.08	1.05	1.13
5	1.04	1.06	1.12
6	1.04	1.06	1.06
7	1.04	1.03	1.05
8	1.03	1.05	1.04
9	1.03	1.03	1.05
10	1.03	1.03	1.04

Location parameter SimToPop ratio

# sites	Rings per site		
	8 x 25 mm	4 x 50 mm	2 x 100 mm
4	1.07	1.05	1.10
5	1.06	1.04	1.08
6	1.03	1.04	1.04
7	1.02	1.03	1.03
8	1.03	1.03	1.04
9	1.02	1.03	1.03
10	1.02	1.02	1.02

Figure 2. Kansas City pit-depth maxima: comparisons of various sampling schemes

right-truncated Gumbel distribution is assumed for pit-depth maxima). It should be noted that the total length of rings per site is the same in every comparison, i.e., 4 adjacent rings of 25 mm were compared to 2 adjacent rings of 50 mm and to 1 ring of 100 mm, or 8 adjacent rings of 25 mm were compared to 4 adjacent rings of 50 mm and to 2 rings of 100 mm. A few observations on these comparisons are noted:

- *Scale parameter - accuracy* (two tables on the top left in Figure 2): accuracy obtained from the simulations for the scale parameter in the pit-depth maxima was quite high in all cases, as manifested by the SimToPop ratios, which were all quite close to unity. The sampling scheme with 25 mm rings did slightly better but this was not significant. No significant improvement in the SimToPop ratio was noted as the number of sites increased.
- *Scale parameter - precision* (two charts on the top right in Figure 2): while the precision of the sampling scheme with 25 mm and 50 mm rings was comparable, the sampling scheme with 100 mm rings displayed relatively poorer precision (higher CoV). In all cases, the increase in the number of sites led to an improvement in the precision of the scale parameter obtained from the simulations. For a given number of locations, precision improves if more adjacent rings are sampled (despite the fact that these adjacent rings are not *iid*).
- *Location parameter - accuracy* (two tables on the bottom left in Figure 2): accuracy obtained from the simulations for the location parameter in the pit-depth maxima was comparable (and quite high) in the sampling scheme with 25 mm and 50 mm rings and somewhat inferior in the sampling scheme with 100 mm rings, especially in the schemes with the fewer sites. All sampling schemes showed clear improvement when the number of sites increased.
- *Location parameter - precision* (two charts on the bottom right in Figure 2): the precision of the sampling scheme with 25 mm and 50 mm rings was comparable, however the precision of the sampling scheme with 100 mm rings was lower when only one ring per site was involved. In all cases, the increase in the number of sites led to an improvement in the precision of the location parameter obtained from the simulations. For a given number of locations, precision improves if more adjacent rings are sampled (despite the fact that these adjacent rings are not *iid*).

Shape parameter SimToPop ratio			
# sites	Rings per site		
	4 x 25 mm	2 x 50 mm	1 x 100 mm
4	1.26	1.33	1.73
5	1.19	1.29	1.51
6	1.15	1.22	1.41
7	1.12	1.21	1.25
8	1.10	1.14	1.21
9	1.11	1.12	1.18
10	1.09	1.10	1.16

Shape parameter SimToPop ratio			
# sites	Rings per site		
	8 x 25 mm	4 x 50 mm	2 x 100 mm
4	1.14	1.24	1.36
5	1.13	1.17	1.26
6	1.11	1.17	1.24
7	1.09	1.11	1.16
8	1.06	1.10	1.13
9	1.06	1.08	1.13
10	1.05	1.08	1.11

Scale parameter SimToPop ratio			
# sites	Rings per site		
	4 x 25 mm	2 x 50 mm	1 x 100 mm
4	2.55	2.42	2.03
5	2.16	2.01	1.77
6	1.95	1.75	1.69
7	1.85	1.71	1.51
8	1.59	1.53	1.47
9	1.65	1.53	1.34
10	1.55	1.40	1.34

Scale parameter SimToPop ratio			
# sites	Rings per site		
	8 x 25 mm	4 x 50 mm	2 x 100 mm
4	1.97	1.96	1.74
5	1.86	1.86	1.67
6	1.63	1.60	1.59
7	1.51	1.46	1.47
8	1.37	1.36	1.32
9	1.42	1.30	1.34
10	1.35	1.38	1.29

Figure 3. Louisville pit-volume: comparisons of various sampling schemes.

Figure 3 shows the results of these comparisons in terms of SimToPop ratio and CoV for pit-volume in the Louisville data set (recall that Weibull distribution is assumed for pit-volume). A few observations on these comparisons are noted:

- *Shape parameter – accuracy*: (two tables on the top left in Figure 3): Accuracy obtained from the simulations for the shape parameter in the pit-volume was not nearly as good as that obtained for pit-depth maxima. It is quite clear that the accuracy deteriorated as the ring length increased. The accuracy improved as the number of sites increased and also improved when the number of rings per site doubled.
- *Shape parameter – precision*: (two charts on the top right in Figure 3): The same trends observed for the accuracy measure of deterioration and improvement can also be observed for the precision obtained from the simulations for the shape parameter.
- *Scale parameter – accuracy*: (two tables on the bottom left in Figure 3): Accuracy obtained from the simulations for the scale parameter in the pit-volume was relatively low. It seems that sampling schemes with longer ring samples are preferred for the estimation of pit-volume scale parameter, as their accuracy is significantly better than that of sampling schemes with shorter ring samples. Also, the accuracy improves across the board when the number of sites increases.
- *Pit volume scale parameter – precision*: (two charts bottom right in Figure 3): It is interesting to note that while there are substantial differences in accuracy, precision was comparable for all sampling schemes with different ring lengths. The precision obtained from the simulations for the scale parameter was rather low but the precision improved significantly as the number of sites increased.

Confidence bounds for distribution parameters

The likelihood ratio confidence bounds (LRB) is a commonly used method to find the confidence bounds of probability distribution parameters that are based on a relatively small sample (Kováčová, 2009; Reliasoft, 2008). The confidence bounds are calculated by finding, for a given confidence level, the extreme values of the parameters that satisfy the following equation (for a two parameter distribution):

$$LR = -2 \ln \frac{L(\phi_1, \phi_2)}{L(\hat{\phi}_1, \hat{\phi}_2)} = \chi_{a,k}^2 \quad (1)$$

where $L(\phi_1, \phi_2)$ is the likelihood function of the unknown parameters of the distribution, $L(\hat{\phi}_1, \hat{\phi}_2)$ is the likelihood function of the parameters estimated from the sample, and $\chi_{a,k}^2$ is the chi-square statistic with probability a and d degrees of freedom. If the required confidence level is A then $a = A$ for a two-sided bound and $a = 2A - 1$ for a one-sided bound. Equation (1) can be simplified to

$$LL(\phi_1, \phi_2) = LL(\hat{\phi}_1, \hat{\phi}_2) - \frac{1}{2} \chi_{a,d}^2 \quad (2)$$

where $LL(\phi_1, \phi_2)$ is the log-likelihood function of the unknown parameters of the distribution and $LL(\hat{\phi}_1, \hat{\phi}_2)$ is the log-likelihood function of the parameters estimated from the sample. Equation (2) cannot generally be solved analytically and an iterative numerical search method must be employed. For the Weibull probability distribution (n samples), equation (2) becomes:

$$\sum_1^n \ln[f(\alpha, \beta)] = \sum_1^n \ln[f(\hat{\alpha}, \hat{\beta})] - \frac{1}{2} \chi_{a,d}^2 \quad (3)$$

$$\text{where } f(\alpha, \beta) = \frac{\beta}{\alpha} \left(\frac{x_i}{\alpha} \right)^{\beta-1} \exp[-(x_i / \alpha)^\beta]$$

For the right-truncated Gumbel probability distribution (n samples), equation (2) becomes (x_o is the right truncation value):

$$\sum_1^n \ln[f(\alpha, \lambda)] = \sum_1^n \ln[f(\hat{\alpha}, \hat{\lambda})] - \frac{1}{2} \chi_{a,d}^2 \quad (4)$$

$$\text{where } f(\alpha, \lambda) = \frac{\exp[-\frac{x_i - \lambda}{\alpha}] - \exp[-\frac{x_o - \lambda}{\alpha}]}{\alpha \cdot \exp[-\exp(-\frac{x_o - \lambda}{\alpha})]}$$

As demonstrated in Kleiner et al. (2013), the probability distribution of ring properties (pit-depth maxima, pit-volume) depends on the ring length, when a given pipe is virtually “sliced” into rings of equal length to form a presumably homogeneous population. This results in a different set of distribution parameters for each ring length. Thirty-nine (39) random samples were generated for ring lengths of 25, 50 and 100 mm to test how ring length influences the confidence bounds for distribution parameters. Using the notation defined earlier, the samples comprised $n*k$ rings where $n = 10, 11, 12, \dots, 49$ (# of sites); and $k = 1$ (# of rings/site). Figure 4 illustrates the comparisons of the results (scale and location parameters) for pit-depth maxima for the Calgary data set. As expected, it is clear that in general, the higher the number of rings in the sample the narrower the confidence interval. Additionally, it appears that the 95% confidence interval for the 25 mm ring population are narrower than for the longer rings, which may lead towards a higher predictability of pit-depth maxima parameters in the shorter rings. This, however, is based on samples taken from a relatively small length of exhumed pipe and would require more extensive tests to validate.

Figure 5 illustrates similar comparisons for pit-volume (Weibull distribution) for the Calgary data set. Again, the general trend of bounds become closer (interval narrower) as the sample size increases is apparent. However, the predictability in short rings does not seem to be higher for the scale parameter and just slightly higher for the shape parameter. Here again, the conclusion is based on a relatively small length of exhumed pipe and therefore cannot be taken as definitive until more rigorous testing is conducted.

Inference about pipe condition

Two classes of inferences can be constructed based on the statistical properties of corrosion pit geometries identified in this research. The first class addresses the presence of corrosion through-holes causing leaks and the second addresses the remaining structural resiliency of the pipe. Since the predominant failure mode in DI pipes is corrosion, we focused in this research on the former, i.e., presence of corrosion through-holes causing leaks. The inference about the structural condition of the pipe is discussed in general terms as a possibility for future research.

Inference about leaks. Return period (or recurrence interval) usually refers to the estimated time between subsequent occurrences of an event (e.g., earthquake, intense storm, etc.). The principles for the computation of recurrence of events in time can also be used to compute

recurrence of events in space, e.g., the recurrence of a corrosion hole in a given length of pipe. Indeed, principles of return period have been used by others (e.g., Aziz, 1956; Hawn, 1977; Hay, 1984) in the context of corrosion modeling.

Figure 4. Calgary pit-depth maxima: parameter confidence bounds for various ring lengths

Figure 5. Calgary pit-volume: parameter confidence bounds for various ring lengths.

Let a pipe of length L and wall thickness x_o be divided into a population of rings of length l , and let the pit-depth maxima of this ring population be distributed as right-truncated Gumbel. An imminent leak in the pipe is defined as a situation where the remaining wall thickness is smaller than, Δ . The probability of a leak in any ring can be computed by

$$P(x > x_o - \Delta) = 1 - KF(x_o - \Delta) = 1 - K \exp\left[-\exp\left(-\frac{x_o - \Delta - \lambda}{\alpha}\right)\right] \quad (5)$$

where $K = 1 / \{\exp[-\exp(-\frac{x_o - \lambda}{\alpha})]\}$

and where λ is the location parameter and α is the scale parameter.

The expected number of leaks is given by

$$\left(1 - K \exp[-\exp(-\frac{x_o - \Delta - \lambda}{\alpha})]\right) \frac{L}{l} \quad (6)$$

The probability of no leaks in the pipe is given by

$$\left(K \exp[-\exp(-\frac{x_o - \Delta - \lambda}{\alpha})]\right)^{\frac{L}{l}} \quad (7)$$

The probability of at least one leak in the pipe is given by

$$1 - \left(K \exp[-\exp(-\frac{x_o - \Delta - \lambda}{\alpha})]\right)^{\frac{L}{l}} \quad (8)$$

Using the principles of likelihood ratio confidence bounds discussed earlier, it can be shown that the confidence bounds Z_{min} and Z_{max} on the expected number of leaks in the pipe can be determined by finding, for a given number of samples (n) and confidence level (α), the extreme values Z that satisfy the equation:

$$\sum_{i=1}^n \ln[f(\alpha, \lambda(Z))] = \sum_{i=1}^n \ln[f(\hat{\alpha}, \hat{\lambda})] - \frac{1}{2} \chi_{\alpha, k}^2 \quad (9)$$

$$\text{where } f(\alpha, \lambda) = \frac{\exp[-\frac{x_i - \lambda(Z)}{\alpha}] - \exp(-\frac{x_i - \lambda}{\alpha})}{\alpha \cdot \exp[-\exp(-\frac{x_o - \lambda(Z)}{\alpha})]}$$

Figure 6 illustrates the expected number of leaks and their 95% confidence bounds computed for various ring length and various size samples, using the Calgary data set. It is important to note that the population for the number of leaks represented by the horizontal line in Figure 6 is the theoretical (computed) expected number of leaks based on the parameters of the respective ring populations, i.e., 25, 50 and 100 mm ring populations. It is not the actual number of perforations

detected through scanning of the pipe. The reason for doing this requires a somewhat detailed explanation. As mentioned earlier, the right-truncated Gumbel probability distribution was found to fit well the histogram of pit-depth maxima in all the data sets. However, the fitting exercise was carried out where the data on actual perforated rings were excluded from the analysis. This approach was taken because the perforated state is a so-called “absorbing state”, meaning that the count of perforated rings continually grows over time until (given enough time) all rings will perforate. This situation cannot be captured using a conventional probability distribution.

Consequently, the methods proposed here are suitable to estimate the number of expected leaks in a pipe that is not overly deteriorated and has not already experienced several leaks. The three data sets for this study came from rather deteriorated pipes (50 mm ring populations from pipes in Calgary, Kansas City and Louisville had each experienced 28, 12, and 20 perforations, respectively) and therefore the number of actual perforations could not be accurately estimated with this method. It is possible that this method could be modified or an alternative method found to suit deteriorated pipes as well, but this effort would require more research and is not addressed here.

Based on Figure 6 (as well as on similar tests implemented on the other data sets), it is clear (as expected) that confidence bounds become narrower as the number of rings in the sample increases, but it cannot be said with any certainty which ring length (25, 50 or 100 mm) is preferable. This aspect too needs to be further investigated in future research.

Inference about structural condition. In reality, loss of ferrous material due to corrosion in a ring will occur as some combination between uniform (wall thickness diminishing uniformly around the ring) and localized pitting. In small diameter ductile iron pipes, uniform corrosion conditions is the more likely to lead to circumferential (bending) mode of failure, while significant localized corrosion is more likely to cause longitudinal failure.

The margins of safety associated with various corroded ring geometries can be determined from criteria used to design ductile iron pipes. These criteria can be subsequently used to define threshold values for pit depth and uniform corrosion that correspond to imminent failure (i.e., factor of safety of 1) for specific operating conditions. The ability to make inference on the structural condition of a pipe based on the statistical properties of the pit-depth maxima, pit-area and pit-volume in a population of rings requires the following:

Figure 6. Calgary data: confidence bounds for the expected number of leaks for various ring lengths.

- Ability to create a simplified (and approximate) expression for the geometrical property of the remaining wall of the ring, using maximum pit depth, area and volume.
- Ability to use this simplified geometric property to compute the approximate remaining safety factor for various operating conditions.
- Since pit-depth maxima, pit-area and pit-volume are not statistically independent, there is a need to derive the joint probability of observing a certain combination of pit-depth maxima, pit-area and pit-volume in a single ring.

With these aforementioned abilities, it could be possible to make reasonable statistical inferences about the ductile iron pipe propensity to fail structurally. These issues, however, require more research that was beyond the scope of this project.

Summary and conclusion

The companion paper, Kleiner et al. (2013) and this paper described some aspects of a research project on the long term performance of ductile iron pipes (full research report can be found at Kleiner and Rajani, 2012). Four pipes, each about 100 m long were exhumed in four different cities in North America and their exterior surfaces scanned for corrosion. Various soil properties were measured at discrete locations along the pipes.

Statistical analyses of corrosion pit data were carried out using three different approaches. In the first approach, individual corrosion pits on the pipe exterior surfaces were mapped (pit was defined based on the threshold depth). The underlying (or parent) distributions of corrosion pit-depth maxima, pit-area and pit-volume were investigated. The right-truncated Gumbel probability distribution was found to fit the observed distribution of pit-depth maxima quite well. The observed distributions of pit-area and pit-volume were too unbalanced to warrant the fitting of any theoretical probability distribution.

In the second approach, analysis was performed on pit-depth maxima, pit-area and pit-volume attributed to so-called pipe rings. Each of the exhumed pipes was virtually sliced into rings (or sections) of equal length (e.g., 25, 50, 100 mm and so on) and the pit-depth maxima, total corroded area (pit-area) and total volume lost to corrosion (pit-volume) were calculated for each

ring. The right-truncated Gumbel probability distribution was found to consistently provide the best fit to the observed frequency distribution of pit-depth maxima. The Weibull distribution consistently provided the best fit to the observed frequency distribution of pit-area and pit-volume.

In the third approach, analysis was also performed on the corrosion pit properties (maximum depth, area, volume) of rings, except that the analysis considered soil properties to examine whether one or more soil properties could “explain” variations in corrosion pit geometry on pipe rings. Soil properties included resistivity, redox potential, chloride, sulphide and in one case sodium concentrations, pH and % fines (clay). In addition, information on the location of metallic appurtenances along the pipe was included in the analysis. Multi-covariate probability distributions were assumed, where covariates comprised various soil properties associated with ring location as well as ring distance from known appurtenances. No single soil property or combination thereof appeared to emerge as statistically significant in a consistent manner in data sets obtained from the different cities. No compelling evidence was found to show that soil properties improved the ability to predict corrosion pit properties in any significant or consistent manner. This apparent lack of impact of soil properties on corrosion pit geometry seems to contradict a large body of work in the literature.

While the modeling part of any research is an important foundation, upon which all processes are created, the sampling and inference parts are the anchors that tie this research to reality.

Appropriate sampling schemes based on probability distributions that best fitted the various corrosion pit properties of rings were investigated. These sampling schemes also required the identification of the optimal ring length for sampling. Two approaches were used, one analytical and one simulation based. The analytical approach consisted of the derivation of confidence bound for the discerned distribution parameters, using the likelihood ratio confidence bounds method. The simulation based approach comprised the generation of 1000 sampling simulations for a very large number of sampling schemes and the subsequent examination and comparison of the simulation results.

Considering that the sampling scheme and corresponding inferences were tested on a few exhumed pipes, it is difficult to draw definitive conclusions and make definite recommendations without investigating more pipes from geographically diverse locations. As a preliminary

conclusion to assess ductile iron pipe condition, it appears preferable to sample shorter rings (10 - 50 mm) and take minimum of 7 to 10 samples (sites, in 100 m long pipe). This approach may appear as a costly endeavor for a 100 m pipe but it is likely that the number of required samples to assess a longer pipe will not increase substantially.

The ultimate goal of this research was to develop an ability to make inferences about the condition of the pipe. Two classes of inferences were proposed based on the statistical properties of corrosion pit geometries identified in this research. The first class addresses the presence of corrosion through-holes causing leaks and the second addresses the remaining structural resiliency of the pipe. The statistical inference about leaks was described in detail, while the inference about the structural condition of the pipe was discussed in general terms as a possibility for future research.

Several issues were identified as requiring further research to possibly transform this entire approach into a viable non-destructive, non-intrusive pipe evaluation technique. Specifically:

- Rigorously explore and compare all possibly sampling schemes for performance as well as cost. Performance evaluation should be done, where accuracy and precision are considered in conjunction with costs corresponding to false positives and false negatives, to arrive at a cost-effective sampling scheme.
- Explore modifications or alternative models to accommodate highly deteriorated pipes.
- Pursue the abilities (outlined in the previous section) toward the development of a statistical inference mechanism about the remaining structural integrity of the pipe.
- Develop hardware (e.g., keyhole technology vacuum excavation and robotic arm) capable of quick and inexpensive pipe sampling.

Acknowledgement

This research project was co-sponsored by the Water Research Foundation (WaterRF), the National Research Council of Canada (NRC) and water utilities from the United States, Canada and Australia.

References

- Aziz, P.M. (1956). "Application of the statistical theory of extreme values to the analysis of maximum pit depth data for aluminum". *Corrosion*, 12, pp. 35-46.
- Hawn, D.E. (1977). "Extreme value prediction of maximum pits on pipelines". *Material Performance*, **16**, pp. 29-32.
- Hay, L. S. (1984). "The influence of soil properties on the performance of underground pipelines". *M.Sc.(Agriculture) thesis*, Dept. Soil Science, University of Sydney, Sydney, Australia.
- Kleiner, Y. and Rajani, B. (2012) "Long-term performance of ductile iron pipe", *Research report to be published by the Water Research Foundation*, Denver, CO.
- Kleiner, Y. and Rajani, B. (2013) "Performance of ductile iron pipe: characterization of external corrosion patterns". *Submitted for publication in Journal of Infrastructure Systems, ASCE. New York, NY.*
- Kováčová, M. (2009). "Reliability likelihood ratio confidence bounds". *Aplimat – Journal of Applied Mathematics*, 2(3), pp. 217-225.
- Reliasoft (2008). "Life data analysis reference". *Reliasoft corporation* .Tucson, AZ.