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EFFICIENT REPRESENTATION OF THE VARIANT PSF OF STRUCTURED LIGHT SYSTEM

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ABSTRACT

This paper proposes a novel and efficient representation of the point-spread function (PSF) well suited for structured light systems used in digital profilometry. For this particular application the PSF is spatially variant over the working volume and the current parametric as well as non-parametric representations are insufficient. We validate our method using a standard data set and imagery acquired using an off-the-shelf multimedia projector and an industrial camera.

Index Terms— Point Spread Function, Deconvolution, Structured Light System

1. INTRODUCTION

Blurring has been identified as a significant limitation of digital projector profilometry [1]. In order to use deconvolution algorithm to improve the performance of a Structured Light System (SLS), a PSF model and well suited representations are needed. A geometric model of out-of-focus was used in depth-from-defocus methods [2]. However, as will be shown the blurring induced by the Point Spread Function (PSF) observed within SLS diverges significantly from the geometric model where the PSF only varies along the optical axis. One can verify that the PSF varies along all axes by looking at Fig. 1 where points q and q' are both in focus in the camera, while only q' is in focus with the projector.

This paper is aimed at providing an efficient non-parametric representation of the PSF well suited for the spatially variant PSF of a SLS. The approach includes two stages. In the off-line stage, the representation is computed using a set of PSF samples. In the on-line stage, the PSF at a 3D point can be reconstructed using the off-line results and by computing some coefficients. This representation is well suited for iterative deconvolution algorithms since it allows very efficient implementation using a GPU. Spatially variant PSF estimation for pre-conditioning of projector images has been proposed [3, 4, 5]. However, this application makes the underlying assumption that the camera is in-focus; we do not. Our proposed decomposition could speedup the iterative image pre-conditioning proposed in [5], the Gray

Fig. 1. Illustration of the depth of field of a SLS.

code decoding algorithm proposed in [6] and many others. There exists a large body of literature on non-parametric PSF. Many approaches have been designed to cope with spatially variant PSFs [7, 8, 9, 10]. Some of them assume that the PSF is piecewise constant [8, 10, 11], others use coordinate transformations [12, 13], while others use interpolation of the PSF across the image [7, 9, 10]. All these spatially variant PSF representations are intended for a 2D domain and may become less attractive in higher dimensionality. Here our problem requires that we work in higher dimensions, to account for the variation of the PSF over the working volume. Furthermore, when the SLS is used as a 3D scanner, piecewise constant representations introduce small undesirable gaps in the model. Also, many parametric PSF models are presented in [14]. The remainder of this paper is divided as follows: in Sec. 2 the image formation model is described; the PSF representation is presented in Sec. 3; experimental results are discussed in Sec. 4.

2. IMAGE FORMATION MODEL

In the image formation process illustrated in Fig. 1, three degradations of the projected images occur. The first and third degradations occur when the light goes through the projector and camera lenses and can be computed by applying a convolution (assuming the use of non-coherent light). Those degradations induced by the lenses take the form of a linear low-pass filter, and are induced by the Seidel aberrations, the

Fig. 2. Distribution of the PSF samples in the working volume. Each dot is a PSF sample and 3 samples are shown. Note how spatially variant they are.

mis-focus and ultimately the diffraction (see [15] for more details). The second degradation occurs when the light interacts with the object surface. We assume that this interaction can be approximated by a linear system. This simplification is acceptable when the effect of the surface is negligible compared to that of the lenses. Thus, the entire system can be modeled as a linear system. The PSF varies with the position of the surface and its orientation, and we assume here a non-parametric representation, i.e. as a $m \times n$ 2D array. Figure 2 illustrates a working volume with some PSF samples all having the same orientation. We sampled the PSF of a SLS and we observed that it varies significantly over the domain, but it does so smoothly. The parametric representation, frequently used for camera, such as pillbox, Gaussian, generalized Gaussian and sum of Fermi-Dirac functions (see [14]) are not well adapted for SLS. This results from the requirements of the system geometry.

3. PSF REPRESENTATION

The PSF in the working volume can be represented as $f(x, y, z, \mu, \nu)$ of R^5 into R^{mn} which takes a 3D position (x, y, z) and a normal $(\mu, \nu, 1)$ and return a 2D PSF of size $m \times n$. We use the convention that the Z axis coincides with the optical axis of the camera, and that the case $(\mu, \nu, 0)$ does not occur. We define f such that

$$f(x, y, z, \mu, \nu) = \sum_i \mathbf{u}_i \mathbf{v}_i^T g_i(x, y, z, \mu, \nu) \quad (1)$$

where a pairs of column vectors $\mathbf{u}_i$ and $\mathbf{v}_i$ are a base that is common to the entire working volume of the system and

$g_i(x, y, z, \mu, \nu)$ is a scaling coefficient for base i at point (x, y, z) with normal $(\mu, \nu, 1)$. In the proposed approach only the g_i 's are interpolated. This decomposition is similar to a Karhunen-Loève decomposition except that our bases are linearly separable. The approach requires the off-line precomputation of the bases. Given s PSF samples $\mathbf{P}_j$ of size $m \times n$, the b pair of vectors $\mathbf{u}_i$ and $\mathbf{v}_i$ of size n and m respectively are obtained by minimizing

$$\sum_{j=1}^s \|\mathbf{P}_j - \sum_{i=1}^b \lambda_i^j \mathbf{u}_i \mathbf{v}_i^T\|^2 \quad (2)$$

where λ_i^j is the scaling coefficient for base i and PSF sample j (i.e. $\lambda_i^j = g_i(x_j, y_j, z_j, \mu_j, \nu_j)$). The minimization in $\mathbf{u}_i, \mathbf{v}_i$ and λ_i^j for all i and j of Eq. 2 is over-parametrized and non-linear. Thus, the number of unknowns is $b(s + n + m)$ and we use the Levenberg-Marquardt (LM) algorithm to minimize Eq. 2. The next subsection presents a specialized implementation of the LM algorithm that can achieve a speedup of three orders of magnitude over the standard implementation: the computation of 8 bases and 4000 PSFs of size 30×30 can be accomplished in a few minutes rather than days on a 4-core 2.8 GHz Opteron.

3.1. Details

The main step of the LM algorithm is solving the augmented normal equation

$$[\mathbf{J}^T \mathbf{J} + \mathbf{I}\alpha] \delta = \mathbf{J}^T \epsilon \quad (3)$$

where $\mathbf{J}$ is the Jacobian, α is a parameter of the method and ϵ is the error vector (see [16] for more details). In the following discussion we assume that the parameter vector (i.e. the vector of unknowns) denoted $\mathbf{p}$ is $(\mathbf{u}_1, \dots, \mathbf{u}_b, \mathbf{v}_1, \dots, \mathbf{v}_b, \lambda_1, \dots, \lambda_b)$ where $\lambda_k = (\lambda_k^1, \dots, \lambda_k^s)$. In our approach, the matrix multiplication $\mathbf{J}^T \mathbf{J}$ can require large amounts of time. However, the regular structure of $\mathbf{J}$ allows the matrix $\mathbf{J}^T \mathbf{J} + \mathbf{I}\alpha$ to be directly computed without performing matrix multiplications. This structure is shown in Fig. 3. As an example, the sub-matrix $\mathbf{C}$ is block-diagonal of size $bs \times bs$. Furthermore, each block is of size $b \times b$ and all blocks are identical, thus making the computation of $\mathbf{C}^{-1}$ easy. We denote the blocks as $\mathbf{C}'$ and $\mathbf{C}'_{i,j} = (\mathbf{u}_i \cdot \mathbf{u}_j)(\mathbf{v}_i \cdot \mathbf{v}_j)$. The reader may verify that all entries of the matrix $\mathbf{J}^T \mathbf{J} + \mathbf{I}\alpha$ have slightly different forms involving the dot products $\mathbf{u}_i \cdot \mathbf{u}_j$, $\mathbf{v}_i \cdot \mathbf{v}_j$ and $\lambda_i \cdot \lambda_j$ for all $0 < i \leq j$ and $0 < j \leq b$. There are six different forms and once all the dot products are pre-computed, each non-zero element of the matrix $\mathbf{J}^T \mathbf{J} + \mathbf{I}\alpha$ takes either 1 or 2 scalar multiplications to compute (the diagonal elements also require one addition).

Furthermore, the structure present in the Jacobian allows to solve the augmented normal equation more efficiently than the standard $O(b^3(s + m + n)^3)$. This is done similarly as for

Fig. 3. Illustration of the structure of $\mathbf{J}^T \mathbf{J} + \mathbf{I} \alpha$ for $b = 2$, $m = 4$ and $n = 3$. The zero entries are color in white.

the sparse LM presented in Annex 6 of [16]. The parameter vector $\mathbf{p}$ is split into two vectors $\mathbf{p}_A = (\mathbf{u}_1 \dots \mathbf{u}_b, \mathbf{v}_1 \dots \mathbf{v}_b)$ and $\mathbf{p}_B = (\lambda_1 \dots \lambda_b)$. The matrix $\mathbf{J}$ and vector δ are split similarly in $\mathbf{J}_A$, $\mathbf{J}_B$, δ_A and δ_B . First, δ_A is obtained by solving

$$((\mathbf{A} - \mathbf{B}\mathbf{C}^{-1}\mathbf{B}^T)) \delta_A = \mathbf{J}_A^T \epsilon - \mathbf{B}\mathbf{C}^{-1} \epsilon. \quad (4)$$

and then $\delta_B = \mathbf{C}^{-1}(\mathbf{J}_B^T \epsilon - \mathbf{B}^T \delta_A)$.

The values used to initialize the LM algorithm are obtained using the following step. A Karhunen-Loève decomposition of the PSF samples is performed. The base with the largest singular value is decomposed using a singular value decomposition into $\mathbf{UDV}$. The two vectors $\mathbf{u}_1$ and $\mathbf{v}_1$ are the line and column of $\mathbf{U}$ and $\mathbf{V}$ associated with the largest singular value. With the fixed $\mathbf{u}_1$ and $\mathbf{v}_1$, the coefficients λ_1^j are computed by minimizing

$$\sum_{j=1}^{j \leq s} \|\mathbf{P}_j - \lambda_1^j \mathbf{u}_1 \mathbf{v}_1^T\|^2. \quad (5)$$

Then, we compute $\mathbf{P}'_j = \mathbf{P}_j - \lambda_1^j \mathbf{u}_1 \mathbf{v}_1^T$ for all j and $\mathbf{u}_2$ and $\mathbf{v}_2$ are computed similarly by replacing $\mathbf{P}_j$ by $\mathbf{P}'_j$.

3.2. interpolation of the coefficients in the volume

In the off-line calibration stage, a planar calibration target with fiducial markers is swept across the volume of the calibrated SLS. It is possible to recover the distance and normal of the target [16]. In this paper, we approximate the PSF by projecting a grid of white one-pixel wide dots which are assumed to be infinitely small¹ (see [5]). Each projector dot is viewed as a blob in the camera image. Using the position of the centroid of a blob and the plane equation of the target, it is possible to project this position onto the plane [16]. Thus, the 3D position and orientation of the PSF samples can be represented using a 2D coordinate system, assuming the plane equation is known. We fit the coefficients λ_i^j associated to the 2D coordinate of each PSF sample on a same plane to a NURBS surface. This is done using a global surface interpolation with regularly spaced control points and we use cubic base functions [17]. This enables the implementation of the evaluation of a point on the NURBS surface using a GPU.

¹Note that a edge-based method such as [14] could be used.

In the on-line stage (i.e. when deconvolution is performed), we obtain the coefficients associated to a given 3D point and normal (i.e. $g_i(x, y, z, \mu, \nu)$ for all i) by taking the N closest planes and projecting the given 3D point on each plane using the camera's center of projection. It is then possible to evaluate each NURBS associated with a plane and we compute a weighted average of the coefficients.

4. EXPERIMENTAL RESULTS

The spatially variant PSF of Hubble is a standard data set that can be downloaded from ftp://ftp.stsci.edu/software/stdas/testdata/sims/star_cluster. It contains 25 PSFs of 60×60 pixels. We compare our method with the one of [8] which we name *Kronecker* representation. This decomposition is the most similar to ours. Figure 4-top-left shows a graph of the running time with respect to the number of bases required to compute both bases and coefficients for our approach. On the top-right of the figure, the number of bases required to have an error equal or smaller than the one obtained by using the Kronecker decomposition is shown. Note that a base in the Kronecker approach is used only for one PSF, while, for our approach, the bases are used to represent each PSF and a coefficient is needed for each pair of PSF and base. Thus, better metrics are the memory space requirement and execution time needed to perform a convolution using the bases ratio of the top-right part of Fig 4. Note that our approach takes significantly less memory and is also faster. Those graphs were computed assuming a bi-linear interpolation and the relative efficiency of our method increases with the degree of interpolation. A deconvolution algorithm based on Conjugated Gradient or Lucy-Richardson requires 2 convolutions per iteration. Using a 2048×1536 camera, a 6 bases decomposition and 29×29 PSFs, more than 20 iterations per second could be performed using an 8800GTX NVIDIA GPU.

4.1. Structured Light System PSF data set

This dataset was acquired using a 1024×768 DLP projector and a C-CAM FCI 14000 monochrome camera equipped with a Nikkor 55mm f/2.8 macro lens. A volume of $300 \text{ mm} \times 500 \text{ mm} \times 200 \text{ mm}$ was swept by a plane on which 38×22 sample PSFs were projected. A total of 10032 samples were acquired. The distance between the adjacent PSF is 12 mm along the X and Y axes and 10 mm along the Z axis. Figure 2 shows the volume and some PSF samples, whilst Table 1 contains a comparison of different methods for representing the PSF for two different representation error levels (assuming tri-cubic interpolation). In another experiment, three planes of PSF samples were acquired. Each plane contained 38×22 samples and the inter-plane distance was 8 mm. The samples on the first and last planes were used to compute interpolated PSF samples for the center plane. The interpolated PSFs

Fig. 4. Top left) time required to compute the bases used in our representation. Top right) Number of bases required by our representation to obtain an error equal or smaller than that of [8]. Bottom) ratio of memory (left) and execution time (right) required to perform deconvolution using the bases ratio of top right.

method	Mean error			
	less than 1.0×10^{-4}		less than $.5 \times 10^{-5}$	
	time	memory	time	memory
Ours	10.9	12.5	10.2	11.6
Karhunen-Loève	155	14.7	133	12.6
Kronecker	100	100	100	100

Table 1. The values are normalized such that Kronecker is 100.

were compared to the one measured on the center plane. The RMS between the interpolated PSF and the measured one is 3.98×10^{-4} and the RMS between a 6-bases decomposition and the measured sample for the center plane is 2.63×10^{-4} . Thus the interpolation error is close to the representation one.

When a out-of-focus SLS scanned in 3D an object with an intensity edge, a geometric distorsion is introduced in the model. Fig 5 shows an example of artefact induced while scanning a flat surface with intensity edges. The use of our efficient PSF representation, the algorithm of [9] and phase shift patterns [18] allows a reduction of the artefact from 4.5 mm to 0.25 mm.

5. CONCLUSION

This paper presented a novel and efficient non-parametric representation of the PSF well suited for the spatially variant PSF of a structured light system. This representation is well adapted for iterative deconvolution algorithms and can be used for other systems.

6. REFERENCES

- [1] M.J. Baker, J. Chicharo, Jiangtao Xi, and Enbang Li, “Accuracy limitations introduced by digital projection sources in profilometric optical metrology systems,” in *Conf. on Opto-electronic and Microelectronic Materials and Devices*, 2004.
- [2] S.K. Nayar, M. Watanabe, and M. Noguchi, “Real-time focus range sensor,” in *Int. Conf. on Comput. Vision*, 1995.

Fig. 5. Left) a cross section of the 3D model (In green deconvolved and in blue the normal result) Right) Intensity image.

- [3] M.S. Brown, Peng Song, and Tat-Jen Cham, “Image pre-conditioning for out-of-focus projector blur,” in *IEEE Conf. on Comput. Vision and Pattern. Recogn.*, 2006.
- [4] Yuji Oyamada and Hideo Saito, “Defocus blur correcting projector-camera system,” in *ACIVS*, 2008, pp. 453–464.
- [5] Li Zhang and Shree Nayar, “Projection defocus analysis for scene capture and image display,” *ACM Trans. Graph.*, vol. 25, no. 3, pp. 907–915, 2006.
- [6] M.-A. Drouin and Guy Godin, “Deconvolution-based structured light system with geometrically plausible regularization,” in *The 2008 International Congress on Image and Signal Processing (CISP2008)*, June 2008.
- [7] A. Boden, D. Redding, R. Hanisch, and J. Mo, “Massively parallel spatially-variant maximum likelihood restoration of Hubble space telescope imagery,” *J. Opt. Soc. Am. A*, vol. 13, pp. 1537–1545, 1996.
- [8] Julie Kamm and James G. Nagy, “Kronecker product and SVD approximations in image restoration,” *Linear Algebra and its Applications*, vol. 284, no. 1-3, pp. 177–192, 1998.
- [9] T. Lauer, “Deconvolution with a spatially-variant PSF,” in *Astronomical Data Analysis II*, 2002, pp. 167–173.
- [10] James G. Nagy and Dianne P. O’Leary, “Restoring images degraded by spatially variant blur,” *SIAM Journal on Scientific Computing*, vol. 19, no. 4, pp. 1063–1082, 1998.
- [11] H. J. Trussell and B. R. Hunt, “Image restoration of space-variant blurs by sectional methods,” *IEEE Trans. Acoust. Speech, Signal Processing* 26, pp. 608–609, 1978.
- [12] A A Sawchuk, “Space-variant image restoration by coordinate transformations,” *J. Opt. Soc. Am.* 64, pp. 138–144, 1974.
- [13] S R McNow and B R Hunt, “Approximate shift-invariance by warping shift-variant systems,” in *The Restoration of HST Images and Spectra II*, 1994, pp. 181–187.
- [14] C. D. Claxton and R. C. Staunton, “Measurement of the point-spread function of a noisy imaging system,” *Journal of the Optical Society of America. A, Optics, Image Science, and Vision*, vol. 25, no. 1, pp. 159–170, January 2008.
- [15] Warren J. Smith, *Modern optical engineering: the design of optical systems*, McGraw-Hill, 1990.
- [16] R. I. Hartley and A. Zisserman, *Multiple View Geometry in Computer Vision*, Cambridge University Press, 2004.
- [17] Les Piegl and Wayne Tiller, *The NURBS book (2nd ed.)*, Springer-Verlag, New York, NY, USA, 1997.
- [18] Joaquim Salvi, Jordi Pages, and Joan Batlle, “Pattern codification strategies in structured light systems,” *Pattern Recogn.*, vol. 37, no. 4, pp. 827–849, 2004.