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Simplified design methods for pipelines subject to transverse and longitudinal soil movements

B.B. Rajani, P.K. Robertson, and N.R. Morgenstern

Abstract: Pipelines are often subjected to transverse and longitudinal movements due to displacements in the ground caused by landslides. As landslide movements develop, pipelines can undergo transverse and longitudinal displacements and the resistance offered by the surrounding soil steadily increases depending on the soil characteristics. This resistance reaches an ultimate as the soil reaches failure and develops plastic strains. Two simple analytical solutions have been developed based on this concept: one for transverse movements and another for longitudinal movements. Nondimensional relationships have been developed and are presented in the form of charts. These charts permit hand calculations and rapid verification of structural design of the pipeline and, thus, assess the integrity of existing pipelines located in areas with ground instability. A knowledge of the soil strength and subgrade modulus is required along with pipeline geometry and pipe stiffness to apply the nondimensional relationships. The soil parameters can be measured in situ or estimated using empirical correlations. Worked examples are given to illustrate the proposed methods and to show how various pipe failure criteria can be included.

Key words: pipelines, landslide, transverse and longitudinal movements, design method.

Résumé : Les pipelines sont souvent soumis à des mouvements transversaux et longitudinaux dus au déplacements provenant de glissements de terrain. Lorsque les mouvements se développent, les pipelines peuvent avoir des déplacements transversaux et longitudinaux, avec une augmentation constante de la résistance offerte par le sol environnant en fonction de ses caractéristiques. Cette résistance atteint une valeur ultime quand le sol arrive à la rupture et développe des déformations plastiques. Deux solutions analytiques simples ont été développées à partir de ce concept, l'une pour les mouvements transversaux, l'autre pour les mouvements longitudinaux. Des relations adimensionnelles ont été développées et sont présentées sous la forme d'abaques. Ces abaques permettent des calculs à la main et une vérification rapide de la conception structurelle du pipeline et par conséquent permettent d'évaluer l'intégrité des pipelines existants et situés dans des zones instables. Pour pouvoir appliquer les relations adimensionnelles il faut connaître la résistance du sol et son module de réaction ainsi que la géométrie du pipeline et sa raideur. Les paramètres du sol peuvent être mesurés sur place ou estimés par corrélations empiriques. On donne des exemples résolus pour illustrer les méthodes proposées et pour montrer comment divers critères de rupture de tube peuvent être inclus.

Mots clés : pipelines, glissement de terrain, mouvement transversaux et longitudinaux, méthodes de conception.

[Traduit par la rédaction]

Introduction

In many parts of the world buried pipelines are required to cross unstable ground where landslides can occur. Therefore pipelines can be subjected to either transverse or longitudinal movements due to displacements in the ground caused

by landslides. As the landslide movements develop, the pipeline can undergo displacement and the resistance offered by the surrounding soil steadily increases depending on the soil characteristics. This transverse or longitudinal loading from soil movements can eventually cause failure of the pipeline.

Once an active unstable slope has been identified, often field monitoring techniques such as slope indicators and strain gauges are used to monitor ground movement and development of pipeline strain, respectively. Installation and monitoring of these sensors in remote areas can be very expensive and impractical, especially if long lengths of pipeline need to be monitored. On the other hand, not all

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Fig. 1. Transverse movement of a pipeline due to landslide.

active slopes pose a serious threat to pipeline integrity, since many variables may effect the pipeline-soil interaction, e.g., slide geometry, pipeline and soil properties. Consequently, a pipeline designer faces the challenge to first identify the active landslides that pose a threat to pipeline integrity and, subsequently, to determine how much landslide movement can be tolerated without jeopardizing serviceability of the pipeline. This concern would be equally valid for an existing pipeline or a new pipeline routed through a landslide-prone region.

NOVA Corporation of Alberta, Canada, initiated a research program in collaboration with the University of Alberta to study soil-pipeline interaction due to landslide movements. The research was performed in several phases. The first phase reviewed the available literature on spring-slider models for soil-pipeline interaction analyses and field testing techniques for determination of the required soil parameters. The second phase involved a parametric study to determine the sensitivity of pipeline integrity to variations in soil parameters for transverse landslide loading. The results of the second phase showed that the transverse pipeline deflection profile, due to the landslide loading, essentially follows a double curvature pattern on either side of the landslide interface. The study showed that the pipeline response was controlled predominantly by the soil strength and that soil stiffness had little influence. The study also showed that for the pipes analyzed (NPS-24 and NPS-48; NPS stands for "nominal pipe size" and the size is specified in inches) the pipelines could deflect large amounts before reaching failure, where failure was defined as 0.5% compression strain. Complex finite element calculations were performed in the second phase to study the sensitivity of pipeline integrity to variations in soil parameters. A preliminary attempt was made to identify a parameter that could characterize the soil-pipeline behaviour during landslide movement. However, further development

of this parameter was found to be difficult because of the complexities of the numerical studies required to encompass all the expected variables. Consequently, an analytical solution was investigated and developed as part of the third phase of the study to better understand the influence of all the main variables. As part of the fourth phase of the study, a similar analytical solution was developed for longitudinal loading.

This paper presents the results of this study on the transverse and longitudinal loading of pipelines. Analytical solutions previously developed for the uplift behaviour of a shallow pipeline embedded in an elasto-plastic medium are adapted to obtain the transverse response. The simple analytical solution for longitudinal soil loading is also described. The use of these solutions is illustrated using worked examples.

Analytical soil-pipeline interaction: transverse movement

During the second phase of this research project (Gale et al. 1989) complex numerical studies using the finite element code ADINA were performed to understand the basic pipeline-soil interaction for landslide movements. As previously mentioned, since the results of the second phase showed that the transverse pipeline deflection profile due to the landslide loading essentially follows a double curvature pattern, it suffices to analyze only half of the pipeline and prescribe the end of the pipeline at the edge of the landslide to half of the free-field landslide movement. Free-field landslide movement is defined here as the landslide movement that would take place in the absence of any restraints. A preliminary attempt was made to identify a parameter that could characterize the soil-pipeline behaviour during landslide displacement. For pipelines with the same material stress-strain behaviour, internal

pressure, and failure criteria, a soil-pipeline strength parameter (ϵ_{ss}) was defined based on the shear strength of the soil and the diameter and wall thickness of the pipeline. Hence, soils and pipelines with the same value of ϵ_{ss} could be expected to reach failure at the same magnitude of landslide movement.

Further development of this parameter was found to be difficult because of the complexities of the numerical studies needed to encompass all the involved variables. It was felt that to better understand the influence of the main variables, the development of an analytical solution was required. Consequently, an analytical solution was developed that traces the sequence of events that takes place as a result of the interaction between the pipeline and the surrounding soil as the landslide movement monotonically increases. These events can be summarized as follows: on initial landslide movement, the buried pipeline as well as the soil behave elastically; as the landslide movement increases, ultimate passive resistance will be developed in part of the surrounding soil medium but the pipe will remain elastic; on further landslide movement, a plastic hinge or wrinkles begin to develop in the pipeline depending its structural characteristics. The solution was based on an elastic pipeline embedded in an elastic-perfectly plastic soil, and full details are given by Rajani and Morgenstern (1993). The solution incorporates all the major variables that play a significant role in the soil-pipeline interaction. A schematic representation of the transverse movement of a pipeline due to a landslide is shown in Fig. 1.

The attractiveness of an analytical solution based on a pipe embedded in an elastic-perfectly plastic soil is the flexibility of the solutions for a broad range of applications. The resulting solutions depend on the total soil resistance per unit length of pipe (R) developed due to movement. Therefore, other solutions can be developed for different boundary conditions provided a suitable total soil resistance can be input to represent the given boundary condition (Rajani and Morgenstern 1993).

Figure 2 illustrates the concept of an elastic pipeline embedded in an elasto-plastic soil to represent the real field condition. The soil is assumed to be an elastic-perfectly plastic isotropic, homogeneous medium; whereas, the pipe is assumed to be a linear elastic beam. The key parameters that control the analytical solution are the soil stiffness, resistance per unit length of pipe represented by the soil subgrade modulus, k_s , the ultimate soil resistance, R , and the pipe stiffness represented by the flexural modulus, EI , of the pipe. The ultimate soil resistance is dependent on the soil strength and the prevailing boundary conditions. Typically, the ultimate force per unit length, R , resistance available for clay corresponding to the undrained state can be expressed by

$$[1] \quad R = \bar{N}_c b s_u \quad \text{cohesive soil}$$

where s_u is the undrained strength and $\bar{N}_c$ is a horizontal bearing capacity factor for cohesive soils. The pipeline is embedded at a depth, h , below the ground surface and has a diameter, b , and wall thickness, t . The embedment depth, h , is measured from the ground surface to the base of the pipeline, as shown in Fig. 1.

Numerical solutions developed by Rowe and Davis (1982) for cohesive soils can be used to evaluate the ultimate

Fig. 2. Spring-slider representation of soil-pipeline interaction. (a) Pipe displaced to the right. (b) Spring-slider representation. (c) Force-displacement relation.

soil resistance developed due to the transverse movement of the pipeline. Rowe and Davis (1982) showed that the ultimate soil resistance is dependent on the embedment depth and the degree of bonding between the back of the pipeline and the soil. Figure 3 shows the solutions developed by Rowe and Davis (1982). For an embedment ratio (h/b) of 2, the ultimate soil resistance, R , is essentially $4bs_u$ when the soil separates from the back of the pipe, and $9bs_u$ when no separation occurs.

The transverse (uplift or lateral) response of the pipeline to landslide movement was obtained by solving the general analytical solution described by Rajani and Morgenstern (1993) for particular values of pipeline-soil resistance factors, $\bar{N}_c$, developed by Rowe and Davis (1982) (Fig. 3). In this solution, the free-field landslide displacement is designated as $2w$, and considering that the pipeline displacement profile has a double curvature, the transverse displacement of the pipeline at the edge of the landslide is w , which is half of the free-field landslide displacement as shown on Fig. 1. Concurrently, it is also assumed that the sliding mass has sufficient transverse extent that it is adequate to analyze the pipeline-soil interaction near one edge of the landslide. It is noted that the general analytical solution described by Rajani and Morgenstern (1993) was first applied to evaluate the behaviour of the uplift response of pipelines. The load, P , applied to the pipeline is a theoretical load that causes the pipeline to deflect (w) at the edge of the landslide. The solution is generalized for pipeline deformation when embedded in an elasto-plastic soil. The resolution

Fig. 3. Variation of pipeline resistance with embedment ratio (data from Rowe and Davis 1982).

Fig. 4. Range of nondimensional load – displacement curves for typical pipeline embedments subject to transverse movement due to landslide.

of the analytical solutions for different values of $\bar{N}_c$ and transverse load can be summarized by two sets of characteristic curves, as explained in the appendix and shown in Figs. 4 and 5. Figure 4 shows the characteristic curves for transverse loading in terms of nondimensional end load, $\bar{P}$, and the associated nondimensional end displacement, $\bar{w}$. The nondimensional end load, $\bar{P}$, is defined as

$$[2] \quad \bar{P} = \frac{P\beta}{bs_u}$$

where P is applied load at the edge of landslide, β ($= \sqrt[3]{k_s b/4EI}$) is a parameter whose reciprocal is characteristic length of soil-pipeline, k_s is soil subgrade modulus, E is pipeline Young's modulus, and I is pipeline moment of inertia. The parameter β is a ratio of soil subgrade modulus and beam stiffnesses. The reciprocal of β is the characteristic length of the soil-beam system. Thus, the characteristic length is a measure of the interaction between the beam and the elastic foundation.

Fig. 5. Range of nondimensional maximum moment – displacement curves for typical pipeline embeddings subject to transverse movement due to landslide.

The nondimensional end (edge) displacement, $\bar{w}$, is defined as

$$[3] \quad \bar{w} = \frac{k_s w}{s_u}$$

Figure 5 shows the characteristic curves for transverse loading in terms of a nondimensional maximum moment in the pipe, $\bar{M}$, associated with various nondimensional end displacements, $\bar{w}$. The nondimensional moment $\bar{M}$ is defined as

$$[4] \quad \bar{M} = \frac{MB}{P}$$

where M is the maximum moment developed in the pipeline due to the applied end load P .

Typically large diameter pipelines in Alberta, Canada, are placed so that the soil cover is at least equal to the diameter of the pipeline. This means that the pipeline embedment ratio, h/b , defined as the ratio of depth to the base of the pipeline and the soil cover, is typically about 2 or very near this value. Based on solutions provided by Rowe and Davis (1982) (Fig. 3), the resistance factor $\bar{N}_c$ can vary from 4 to 9 depending on whether soil separation occurs or not at large displacement. Most pipelines in Alberta are placed in relatively stiff glacial till deposits. When these pipelines are subjected to transverse movements due to ground movements it is expected that separation will occur between the soil and the back of the pipe at large displacements. Initially, for small displacements the soil can be expected to move with the pipe (i.e., no separation). However, separation is likely with large movements. Consequently, the $\bar{N}_c$ value would be high (around 9) at small displacements, whereas, separation may occur with large displacements when the $\bar{N}_c$ value would be relatively low (around 4).

In general, the most severe ground loading conditions for a pipeline will be when the soil is strong and (or) stiff

Fig. 6. Relation between shear strength and liquidity index of Alberta tills.

relative to the pipe. A high soil resistance will produce large stresses in the pipe for any given ground displacement. The characteristic curves (Fig. 5) represent the complete generalized solution for an elastic pipeline embedded in an elasto-plastic soil for all combinations of soil and pipe stiffness, embedment ratios, pipe geometry, and soil strength.

Table 1. Range of values for pipeline subgrade modulus, k_s (adapted from Bowles (1977) and Poulos and Davis (1980)).

Soil	Range of k_s (MPa/m)
Loose sand	5–16
Medium sand	9–78
Dense sand	63–126
Clayey sand (medium)	31–78
Silty sand (medium)	24–47
Clayey soil (s_u in kPa)	
$s_u < 50$	0–15
$50 < s_u < 100$	15–30
$100 < s_u < 200$	30–62
$s_u > 200$	>62

Failure of the pipeline can be defined in terms of a maximum moment (M) or maximum stress, σ , where

$$[5] \quad \sigma = \frac{Mb}{2I}$$

and I is the pipeline moment of inertia.

The use of these characteristic curves requires the knowledge of the soil strength (s_u) and subgrade modulus (k_s) along with pipeline geometry and stiffness. In general, the pipeline characteristics are well defined in terms of b , t , h , E , and I . The soil parameters can be measured in situ or estimated using empirical correlations, such as shown in Fig. 6 (Rajani et al. 1992).

While the discussion of the analytical solution and corresponding characteristic curves has been in particular reference to cohesive soils, the characteristic curves are general enough to be applicable to pipelines buried in granular soils. The ultimate soil resistance for a frictional soil (sand) is given by (Committee on Gas and Liquid Fuel Lifelines 1984)

$$[6] \quad R = N_{qh} b \bar{\gamma}_s \left(h - \frac{b}{2} \right) \quad \text{frictional soil}$$

where N_{qh} is horizontal bearing capacity factor for frictional soil and $\bar{\gamma}_s$ is effective unit weight of soil. The comparison of eq. 1 for cohesive soils with eq. 6 for granular soils indicates that the characteristics curves (Figs. 4 and 5) can still be used for analysis of pipelines buried in granular soils if $\bar{N}_c$ and s_u are substituted by N_{qh} and $\bar{\gamma}_s (h - b/2)$, respectively.

For most soil conditions, the undrained rapid response will generally produce the highest resistance and will, therefore, be the most conservative. In general, the backfill material will have a lower strength than the surrounding natural ground. For preliminary design it would be more prudent to apply the shear strength of the natural ground, since this would produce larger calculated stresses in the pipe for a given ground movement. However, in critical regions where large transverse soil movements are anticipated the backfill material should be as weak as possible and the pipeline trench as wide as possible. This would

Table 2. Summary of soil properties and pipeline data for transverse analysis of pipelines.

Geotechnical properties		
Undrained shear strength	s_u	20 kPa
Subgrade modulus	k_s	6.5 MPa/m
Parameter	β	0.15 m^{-1}
Pipeline geometry		
Diameter of pipeline	b	1.22 m
Cover to base of pipe	h	2.12 m
Wall thickness	t	0.024 m
Elastic modulus	E	207 GPa
Moment of inertia	I	$1.61 \times 10^{-2} \text{ m}^4$
Ratio	b/t	50.8
Embedment ratio	h/b	1.74

allow maximum movement of the natural ground without producing high stresses and strains in the pipe.

Although not immediately apparent from Figs. 4 and 5, the soil strength has the most dominant affect on the pipeline response. The soil stiffness in terms of a subgrade modulus, k_s , has a large effect at small displacements (w) but a much smaller effect at large displacements when the soil reaches failure and becomes plastic. The soil subgrade modulus can be determined from either full-scale tests, plate loading tests, or estimated using empirical correlations. If no laboratory data are available, then the range of possible values can be estimated from Table 1, as suggested by Bowles (1977) and Poulos and Davis (1980). The soil subgrade modulus indicated in Table 1 does not account for pipeline size effects since its influence is probably overshadowed by the range of soil properties encountered along the linear extension of the buried pipeline. It will generally be sufficient to estimate k_s using Table 1 since the soil subgrade modulus values have only a small influence on the resulting design. The undrained shear strength can be determined from either laboratory or in situ tests or estimates can be obtained from index properties, as suggested in Fig. 6.

The current analytical solutions shown in nondimensional form in Figs. 4 and 5 are based on assumptions that the displacements and strains experienced in the pipeline are small. As long as the transverse displacements are small in comparison with the diameter of the pipeline, the dominant behaviour of the pipeline at the interface of the sliding mass is flexure. As soon as the transverse displacement is significant in comparison to the diameter of the pipeline, the mode of behaviour of the pipeline changes from flexure to extension. An indicator for performing a large deformation analyses is when the slope along a deformed pipeline exceeds $1/7$, i.e., $dw/dx > 1/7$. This requirement derives from the approximation used in the definition for curvature for the analysis of beams. The evaluation of the response for large displacements requires that the "cable-like" behaviour of the pipeline must be taken into account since considerable tension will be induced in the pipeline. The resulting extension and bending may be significant to cause plastic strains in the pipeline. While a large displacement solution is complex (Rajani et al. 1992), it is likely that

Table 3. Key displacement–stress history points for a 1.22 m pipeline subjected to transverse landslide movement.

Free-field landslide movement $2w$ (m)	Landslide edge movement w (m)	Nondimensional edge displacement $\bar{w}$	Nondimensional maximum moment $\bar{M}$	Maximum stress σ (MPa)
0.118	0.059	19	0.51	88
1.086	0.543	176	0.99	325
2.006	1.003	326	1.16	429

the small strain, small displacement formulation discussed here will be adequate for most design problems as long as landslide movements do not significantly exceed the pipeline diameter.

Worked example: transversal movement

A worked example is presented to illustrate the application of the characteristic curves shown in Figs. 4 and 5. The pertinent pipeline and soil characteristics are shown in Table 2.

The data for this worked example is similar to the one that served as the baseline case data set used for the parametric study of soil–pipeline interaction for pipelines crossing landslides carried out by Gale et al. (1989). There is no significance to the fact that undrained shear strength for the clay in this worked example does not represent a stiff clay typically encountered in Alberta.

Only half of the free-field landslide movement is assumed to occur at the edge of the landslide since the transverse pipeline deflection profile has a double curvature. A series of different edge displacement values can be assumed to calculate the corresponding maximum moment (M) and stress (σ) in the pipeline. For a free-field landslide movement ($2w$) of, for example, 1.08 m, the landslide edge movement, w , is 0.54 m and, consequently, the nondimensional displacement is $\bar{w} = k_s w / s_u = 176$. Using Fig. 4 and taking the characteristic curve $\bar{N}_c = 5$, the nondimensional end load is $\bar{P} = P\beta / bs_u = 8$. Hence, for this pipeline at this particular displacement, the end load, P , is 1.3 MN. Using Fig. 5 and taking the characteristic curve $\bar{N}_c = 5$, the nondimensional maximum moment is $\bar{M} = M\beta / P = 0.99$. Since P was already determined from Fig. 4 in the previous step, it is now possible to calculate the maximum moment, M , which is 8.59 MN-m, and corresponding stress ($\sigma = Mb/2I$) in the pipeline is calculated to be 325 MPa.

Subsequently, other key displacement–stress history points can now be identified, as indicated in Table 3. Yielding in steel commences typically in the range of 0.1 to 0.2% strain, which corresponds to a stress range of 200–400 MPa. In practice, loading imposed on the pipeline as a result of landslide movement is considered as a secondary loading condition and, typically, the compressive (wrinkling) strain is limited to about 0.5% (Canadian Standards Association 1992) for steel pipelines in the pipeline industry. This means that for landslide displacements greater than 1 m, the analysis needs to be modified to account for

the nonlinearity in the pipeline as opposed to that in the soil only.

As explained earlier, the level of strain developed at any particular position in the pipeline will depend on the magnitude of landslide movement as well as on the distance of the point from the edge of the landslide. Consequently, the analysis of this particular problem indicates that portions of the pipeline exceed the yield strain as soon as the landslide movement is greater than 1 m. Recent (Rajani 1992) model experiments have shown pipelines can undergo severe compressive strains, i.e., greater than 0.8% strains without rupture. The typical strain at first yield in tension for steel is about 0.10–0.15%.

Since the present analytical solution does not account for nonlinear (plastic) deformation of the pipeline, an approximate procedure is proposed. In practice, yielding of pipeline initiates at one particular section and, thereafter, yielding extends to adjacent sections on further deformation. Analyses of pipelines using practical pipeline and soil data indicate that the maximum stress generally develops within the first 5 diameters of the pipe from the edge of the landslide. An upper bound estimate of the strain can be obtained if it were to be assumed that the whole length of pipeline in the vicinity of the landslide edge yields at the same instant. Under these circumstances, the pipeline can be reanalyzed using a reduced pipe stiffness recognizing that the procedure proposed here is certainly not rigorous but provides a reasonable estimate. In this procedure, it is further assumed that the solution for a pipeline embedded in an elasto-plastic soil continues to hold even though the pipeline behaviour in tension and compression beyond yield or wrinkling is not the same. In the approximate procedure, an equivalent elastic modulus for the steel pipeline is used on the basis that failure is defined in terms of limiting critical compressive strain to 0.5% rather than the yield stress (Committee on Gas and Liquid Fuel Lifelines 1984; Canadian Standards Association 1992).

Since steel has an elastic modulus (E_1) of 207 GPa, yield stress (σ_y) for steel is 310 MPa if it is assumed that steel yields at a strain of 0.15%. Thus, the equivalent secant modulus (E_{sec}) of steel reduces to 62 GPa at a strain of 0.5%. Subsequently, the pipeline can be reanalyzed except that the equivalent secant modulus (E_{sec}) is used instead of the elastic modulus (E_1). Parameter β , the reciprocal of characteristic length, is 0.21 m^{-1} . Therefore, a free-field landslide displacement ($2w$) of 2 m will produce a nondimensional edge displacement ($\bar{w}$) of 325. Using Fig. 4 ($\bar{N}_c = 5$), the nondimensional end load ($\bar{P}$) is 9.2,

Fig. 7. Pipeline subjected to a longitudinal planar slide: (a) plan view, (b) section view.

which corresponds to an end load (P) of 1069 kN. Using Fig. 5 ($\bar{N}_c = 5$), the nondimensional moment ($\bar{M}$) is 1.16, and, therefore, $M = 5905$ kN·m and $\sigma = 224$ MPa. For a free-field landslide movement of 2 m, the maximum strain developed is 0.36% when a reduced secant modulus is used in the analysis. Therefore, this approximate analysis indicates that if the development of plastic or wrinkling strains is tolerated, landslide movements larger than 2 m can be accommodated before the tensile or compressive strain exceeds 0.5%.

Alternate applications of the analytical solution

Though the characteristic curves shown in Figs. 4 and 5 were developed for transverse movements of a pipeline embedded in a soil at shallow depth, they can be applied to analyse alternative beam-soil interaction type problems.

The solution can be modified to account for the uplift behaviour of pipelines or the lateral loading of vertical piles if the ultimate soil resistance values are modified to account for the different boundary conditions. An example of the solution for the uplift behaviour of pipelines is given by Rajani and Morgenstern (1993). For the uplift behaviour of pipelines, the equivalent resistance factor, $\bar{N}_c$, ranges

Fig. 8. Pipeline – planar slide force system.

from about 2 to 12 depending on the separation of soil beneath the pipe. For laterally loaded piles, the boundary conditions are different, with the free surface perpendicular to the pile. For laterally loaded piles, the resistance factor $\bar{N}_c$ ranges from 9 to 11. Hence, the curves shown in Figs. 4 and 5 can be applied to other applications that can be represented by an elastic pipe embedded in an elastoplastic material subjected to lateral loading.

Soil-pipeline interaction: longitudinal movement

When pipelines cross river valleys they are often constructed such that the pipeline follows the slope of the valley wall. If slope instability occurs in the valley wall, the pipeline will be subjected to longitudinal soil-pipeline interaction. This longitudinal interaction will induce longitudinal strains in the pipe that could lead it to failure.

The failure of the pipeline at the Simonette River Crossing in 1978 prompted an inquiry of the observed failure. Studies undertaken by M. Rizkalla and M. Moschopedis (private communication, 1989) concluded that failure was probably caused by a combination of operating, bending, and residual stresses. One of the mechanisms that was studied was the longitudinal stresses imposed on the pipeline as a result of longitudinal slide movement. The longitudinal stresses or strains were estimated by the use of a finite element model. However, during the design stage the estimate of probable stresses imposed on a pipeline, as a consequence of a landslide hazard, is generally not justified by making use of a relatively detailed model because of uncertainty in the soil properties and landslide geometry. In the following sections, the development of characteristic curves for longitudinal movements is described. A worked example is given to illustrate the application of the method to evaluate the longitudinal response of a pipeline.

A significant amount of axial force can be exerted on a pipeline as a consequence of the longitudinal movement of a sliding soil mass. As an initial approximation, it is reasonable to assume that the sliding mass is planar and has finite dimensions of width (W_s) and length (L_s), as indicated in Fig. 7. The anticipated mechanism envisaged

is that the movement of the sliding mass B and the firm embedment of the pipeline in a stable nonmoving mass A results in a net force at the interface to overcome the sliding resistance between the pipeline and the surrounding soil. A key design question follows: how large a slide movement can be tolerated before pipeline failure occurs if the pipeline and soil characteristics can be estimated reasonably well. The problem can be solved using two different but related approaches. One approach would be to use limit equilibrium to estimate the net force exerted at the interface of the moving-nonmoving soil masses. The second approach would be to estimate the net force at the interface (P_o) by looking at the force displacement interaction of the pipeline embedded in either the nonmoving soil mass A or the moving mass B.

Approach 1: Limit equilibrium analysis of sliding mass B

One approach would be to use limit equilibrium to estimate the net force exerted at the interface of the moving-nonmoving soil masses. It is often reasonable to assume that the slide is planar and that an estimate of the slide depth, h_s , can be made from conventional slope stability analysis. The side forces on the slide shall be neglected in the following analysis. The forces acting on a typical slice in the slide with the embedded pipeline within the sliding mass are shown in Fig. 8. The resolution of forces parallel and normal to the slope, λ , result in the following equations:

$$[7] \quad \Delta l_s f_s + T = W \sin \lambda$$

$$[8] \quad N = W \cos \lambda$$

where f_s is the resulting soil-pipeline resistance per unit length of pipeline, W is the weight of the sliding mass per unit length of pipeline, N and T are the soil forces acting on the failure plane in the slope, and Δl_s is the unit length of pipeline.

The failure criterion for the surrounding soil is assumed to be of the Mohr-Coulomb type and can be expressed as

$$[9] \quad \tau = c + \sigma' \tan \phi'$$

where c and ϕ' are the effective stress cohesion and friction angle parameters, respectively, of the soil along the failure plane in the slope. The following two equations relate the mass of the slide slice with geometry:

$$[10] \quad W = \rho g h_s b_s W_s$$

$$[11] \quad b_s = \Delta l_s \cos \lambda$$

If slide movement occurs then the driving force must equal to or exceed the resisting forces, i.e., factor of safety is one as in conventional stability analysis. The resulting soil-pipeline resistance per unit length of pipeline, f_s , is given by

$$[12] \quad f_s = \gamma_s h_s W_s (\sin \lambda \cos \lambda - \cos^2 \lambda \tan \phi') - c W_s$$

where γ_s is the soil unit weight, h_s is depth of the landslide, W_s is the slide width, λ is slope of soil sliding mass. If the soil strength for the soil along the slope failure plane is represented by the undrained shear strength ($\phi' = 0$, $c = s_u$), the expression becomes

$$[13] \quad f_s = \gamma_s h_s W_s \sin \lambda \cos \lambda - s_u W_s$$

Fig. 9. (a) Typical soil resistance vs. displacement at the soil-pipeline interface, and (b) stress vs. strain relation for typical pipeline steel.

where s_u is the undrained shear strength on the failure plane in the slope. Thus, the soil-pipeline resistance can be determined if the slide characteristics and soil properties can be reasonably estimated. If the slide length is L_s , the total force, P_o , at the interface of moving-nonmoving masses is

$$[14] \quad P_o = f_s L_s$$

This approach assumes that the slope is just at failure (i.e., factor of safety of 1.0) and that the full shear strength has not been developed between the soil backfill and pipe, i.e., there is no slip between the soil and the pipe. The limiting condition exists when the strength between the soil backfill and pipe is fully developed and slip occurs along the soil-pipe interface.

Approach 2: Axial soil-pipeline interaction analysis in stable mass A

The second approach would be to estimate the net force at the interface (P_o) by looking at the force displacement interaction of the pipeline embedded in either the nonmoving soil mass A or the moving mass B. If a force P_o is imposed at the slide interface that increases monotonically, then it will be supported by the development of soil-pipeline resistance. The resistance along the interface will depend on the developed displacements and, hence, the stress in the pipeline can be traced as a function of the net force, P_o . The soil is represented as an elastic-perfectly plastic material (Fig. 9a) and the pipeline itself is considered to follow

Fig. 10. Longitudinal pull on pipeline due to planar slide.

the nonlinear strain hardening characteristics of a typical steel pipeline (Fig. 9b). Typical ultimate axial resistance per unit length (Committee on Gas and Liquid Fuel Lifelines 1984), F_x , for a pipeline and the surrounding soils (clay and sand) can be expressed as

$$[15] \quad F_x = \pi b \alpha s_u \quad \text{for clay}$$

$$[16] \quad F_x = 0.5 \pi b \bar{\gamma}_s H (1 + K_0) \tan \delta \quad \text{for sand}$$

where s_u is the undrained shear strength of the soil around the pipe, b is pipeline diameter, α is the adhesion factor, K_0 is the coefficient of earth pressure at rest, δ is the interface friction angle between the pipeline and soil, $\bar{\gamma}_s$ is the effective unit weight of soil, and H is the depth from ground surface to the centre of the pipeline. The limiting elastic soil displacement, u_x , is typically 5–10 mm (Committee on Gas and Liquid Fuel Lifelines 1984). The axial subgrade modulus, k_x , is then given by

$$[17] \quad F = k_x u \pi b \quad \text{for} \quad u \leq u_x$$

where F is axial force per unit length, πb is the perimeter of the pipeline and u is the axial displacement. Equation 17 expresses the relationship between the soil resistance and axial displacement when the soil response is still elastic.

It is worth pointing out that in the first approach using limit equilibrium analysis, the failure takes place along a soil–soil interface since it is assumed that mass B (Fig. 7) is undergoing downward movement and, thus, it permits the evaluation of the overall mobilized soil–pipeline resistance per unit length to calculate the axial force exerted on the pipeline at the slide – stable ground interface. In approach 2, the soil–pipeline resistance is evaluated on the basis of on-going soil–pipeline interaction and accounts for the fact that the soil in the stable mass may be different from that in the sliding mass.

The overall pipeline response can be proportioned to three distinct stages of soil–pipeline interaction as the longitudinal movement is monotonically increased. These three stages are (1) both soil and pipeline response are elastic; (2) pipeline response is elastic but sufficient displacement has taken place as to develop ultimate soil resistance (soil is in the plastic state), and (3) both soil and pipeline are in an elasto-plastic state. The response will be characterized by load and displacement at the interface of the moving and nonmoving masses. The surrounding soil can be represented by axial springs.

Analytical solutions were developed for each stage identified above. Appropriate differential equations were established and solved for the corresponding boundary conditions. Full details are given in Rajani et al. (1992). These

analytical solutions were developed in terms of non-dimensional longitudinal displacement and load parameters, $\bar{u}_0 = \pi b k_x u_0 / F_x$ and $\bar{P} = \gamma P_0 / F_x$, respectively, where πb is the perimeter of the pipeline, k_x is the axial subgrade modulus, u_0 and P_0 are the axial displacement and load at the landslide interface, and F_x is the axial soil resistance defined in either eq. 15 or 16. In addition, the solutions are a function of the reciprocal of the axial characteristic length, γ ,

$$\gamma = \sqrt{\frac{\pi b k_x}{E_1 A}}$$

where E_1 is the elastic modulus for pipeline and A is the cross-sectional area.

(1) Soil and pipeline are both elastic

Figure 10 shows a free body diagram for axial response of a pipeline subjected to an end load, P_0 . As long as the longitudinal movement is less than the ultimate longitudinal movement required to induce unrecoverable strains in the soil, i.e., $u \leq u_x$; the following nondimensional equation results:

$$[18] \quad \bar{u}_0 = \bar{P}$$

where $\bar{u}_0$ is the nondimensional longitudinal edge displacement.

(2) Pipeline response is elastic and soil resistance is plastic

If the load level is high enough then the ultimate soil resistance will be developed along the pipeline–soil interface for a portion of the pipeline defined by $\bar{x}$. Hence, over the portion of the pipeline defined by $\bar{x}$, the soil response is elasto-plastic. The axis $s-s$, therefore, distinguishes between the elastic and plastic soil response portions (assume $\bar{x}_p = 0$ in Fig. 10). For the plastic soil response portion, the analysis is similar as before except that the axial resistance is independent of longitudinal displacements. The corresponding response can be described by the following non-dimensional equation:

$$[19] \quad 2\bar{u}_0 = 1 + \bar{P}^2$$

For loads less than $P_0 \leq F_x / \gamma$ the elastic displacement can be expressed in the same form as in [18].

(3) Pipeline response is elasto-plastic and soil resistance is plastic

The procedure for this phase of the analysis is similar to before except that we have an additional zone where the pipeline itself is undergoing strain hardening. The load–displacement response can then be followed at the interface of the moving–nonmoving masses and can be expressed in nondimensional form as indicated below:

$$[20] \quad \bar{u}_0 = -\frac{\kappa^2}{2} (1 - \eta) + \frac{\eta}{2} \bar{P}^2 + \kappa \bar{P} (1 - \eta) + \frac{1}{2}$$

where $\eta = E_1 / E_2$ is the ratio of elastic to hardening moduli of pipe, $\kappa = (\pi b \epsilon_y k_x / \gamma F_x)$ is the soil–pipeline stiffness parameter, ϵ_y is the pipeline strain at yield ($= \sigma_y / E_1$ (Fig. 9b)).

Equation 20 provides an opportunity to examine the relation between the longitudinal load and corresponding displacement for a range of parameters. The sensitivity of

Fig. 11. Nondimensional characteristic curve for axial load–displacement for longitudinal soil–pipeline interaction.

the relationship to the nondimensional parameters η and κ can also be easily examined. If typical values for elastic and hardening modulus for steel are used, then η will be about 80 and κ varies according to the undrained shear strength, adhesion factor, and other parameters that intervene in the definition of κ . However, for typical soil and steel properties, κ should vary in the range of 5–20 with lower values for stiffer soils. For the present study, κ was estimated to vary in the range of 10–15.

The resulting nondimensional axial load – displacement characteristic curves are shown in Fig. 11. Figure 11 shows three sets of curves that represent the three main conditions: (1) both soil and pipeline response are elastic; (2) pipeline response is elastic but sufficient displacement has taken place as to develop ultimate soil resistance (soil is in the elasto-plastic state); and (3) both soil and pipeline are in an elasto-plastic state. It is evident that the largest forces are produced by the somewhat trivial case when the soil and pipe are considered to be elastic. This is certainly too conservative for most pipeline designs. The next worst loading is when the pipe is assumed to be elastic but the soil has become plastic in the areas of large movement, i.e., close to the interface between the stable and sliding soil masses. The application of this condition (Fig. 11) would also result in a conservative pipeline design. A more realistic design would result from the application of the curve that represents the condition of a plastic soil and elasto-plastic pipe. This condition requires some knowledge of the κ parameter. However, Fig. 12 shows that when the soil and pipe are in the elasto-plastic state the κ parameter plays a minor role in the load–displacement response. Hence, it is reasonable to assume a value of $\kappa = 12$ for most designs.

Values of the axial subgrade modulus k_x can be estimated by assuming an axial displacement of 5 mm to reach the maximum axial resistance per unit length, F_x . Hence,

$$k_x = \frac{F_x}{\pi b} \frac{1}{0.005} \quad (\text{kN} \cdot \text{m}^{-2} \cdot \text{m}^{-1})$$

The adhesion factor, α , can be estimated from data obtained for axial pipeline behaviour and this data has been recently summarized by Sladen (1992).

Fig. 12. Nondimensional characteristic curve for axial load–displacement for different values of the soil–pipeline stiffness parameter, κ .

Comparison of proposed longitudinal method with other solutions

Rizkalla and McIntye (1991) presented a simplified expression relating the stress in a pipe due to longitudinal movements of a clay slope. This section compares the nondimensional analytical solution presented in eq. 19 with that obtained by Rizkalla and McIntye (1991).

For a clayey soil the nondimensional expressions defined earlier become (where $F_x = \pi b \alpha s_u$)

$$[21] \quad \bar{u}_0 = \frac{k_x u_0}{\alpha s_u}$$

$$[22] \quad \bar{P} = \frac{P_0 \gamma}{\pi b \alpha s_u}$$

For the case of an elastic pipe and plastic soil the relationship given in eq. 19 holds. Substituting [21] and [22] into [19], the resulting equation for a clayey soil is

$$[23] \quad u_0 = \frac{1}{2} \left(\frac{P_0^2}{F_x E_1 A} \right) + \frac{1}{2} u_x$$

where u_x is axial displacement when soil and pipe are both elastic (Fig. 9).

Rizkalla and McIntye (1991) have proposed a similar relationship for a clayey soil as follows:

$$[24] \quad \sigma = \left(\frac{u_0 F_x E}{A} \right)^{1/2}$$

Replacing the stress (σ) for the axial load (P) the above expression becomes

$$[25] \quad u_0 = \frac{P_0^2}{F_x E_1 A}$$

The expression given by Rizkalla and McIntye (1991) assumes an elastic pipeline and plastic soil. It also assumes that the displacement u_0 at the edge of the landslide is twice the landslide movement, since the soil is assumed to move equal amounts about either side of the slide interface. Hence, the expressions derived in this study (eqs. 19

Table 4. Summary of soil properties, landslide mass geometry, and pipeline data for longitudinal analysis of pipelines.

Geotechnical properties			
Unit soil weight	γ_s	17.2 MN/m ³	
Undrained shear strength	s_u	70 kN/m ²	
Adhesion factor	α	0.20	
Limiting elastic soil displacement	u_x	0.005 m	
Axial resistance/unit length	F_x	40.2 kN/m	
Axial subgrade modulus	k_x	2800 kN/m ² /m	
Pipeline geometry			
Diameter of pipeline	b	0.95 m	
Area of pipeline (wall)	A	0.067 m ²	
Wall thickness	t	0.024 m	
Yield stress	σ_y	367 MPa	
Axial characteristic length	γ	0.024 m ⁻¹	
Landslide characteristics			
Slope angle	λ	18°	
Height of landslide	h_s	15 m	
Width of landslide	W_s	100 m	
Length of sliding mass	L_s	150 m	
Depth of centreline of pipeline from surface	H	1.37 m	

and 23) are similar to the expression derived by Rizkalla and McIntyre (1991) and shown in eqs. 24 and 25. However, the expression derived in this study assumes a stable soil mass with the displacement at the interface derived only from the moving unstable soil mass. The factor of two that differentiates eqs. 23 and 25 is explained by the definition of u_o . Equation 23 also includes the very small amount of displacement (u_x) due to the response of the soil-pipeline interaction when both the soil and pipeline are assumed to be elastic, i.e., $\bar{u}_o = \bar{P}$.

Figure 11 shows that the sole application of eq. 23 and, hence, eq. 24 to analyze pipelines may result in an unduly conservative design. Since steel can be characterized as an elasto-plastic material with strain hardening (E_2), larger longitudinal displacements can be tolerated by the pipeline (eq. 20 shown in Fig. 11) before axial strains exceed the limits set by current practice or standards.

Worked example: longitudinal movement

The objective of this worked example is to illustrate the application of the proposed nondimensional characteristic curves for longitudinal movement of an unstable slope. It is important to note that, in general, pipelines will not be predominantly in a straight line going down slope (Fig. 13) but an overbend and a sidebend can also exist. Therefore, it will not always be possible to attribute the failure totally to longitudinal movement alone.

In the analytical procedure discussed above, the geometry of the mass movement (Fig. 14) can be idealized to a rectangular block and average dimensions can be established by judgment from some data on previous landslides in the

Fig. 13. Plan view of a typical pipeline traversing a potential landslide area.

region. Soil properties and landslide dimensions for this example are summarized in Table 4.

Two approaches can be used to estimate the condition of a pipeline. One approach is to estimate the displacement (u_o) required to yield the pipe. The other approach is to estimate the landslide geometry required to induce yielding of the pipe.

Approach 1

To estimate the landslide geometry required to yield the pipe requires the determination of the total force (P_o) based on limit equilibrium of the slide using eqs. 12 and 14.

If the yield stress is taken as $\sigma_y = 367$ MPa. Then based on the area of the pipeline, the total force P_o at yield is P_o (yield) = 25 000 kN. Based on eq. 12, the soil-pipeline resistance per unit length of slide is $f_s = 494$ kN/m. Therefore, using eq. 14, the length of landslide required to generate $P_o = 25$ 000 kN would be about 50 m. In circumstances where an existing pipeline has to be assessed, and if it is known that the estimated landslide length is exceeded, then a check may be required to see how much more landslide movement can be tolerated before exceeding strain limits dictated by regulatory standards. Typically the critical axial tensile strain limit is 0.5%, which exceeds the elastic strain proportionality limit of 0.1%. Consequently, the corresponding ultimate end load on the pipe corresponding to specific landslide size would be determined on the basis of the strain hardening modulus.

Approach 2

For the displacement approach, it is first necessary to estimate the total axial load (P_o) required to cause yield in the pipe. Based on the yield stress of $\sigma_y = 367$ MPa, the total force P_o for yield is $P_o = 25$ 000 kN. Therefore, the nondimensional axial load at yield, $\bar{P}_{yield} = \gamma P_o / F_x = 15$.

Assuming $\kappa = 12$ and $\eta = 80$, the nondimensional axial displacement corresponding to the calculated nondimensional axial load (Fig. 11) depends on whether or not the stress in

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List of symbols

A	cross-sectional area of pipe
b	pipeline diameter
c	cohesion of soil
E, E_1	elastic modulus for pipeline
EI	flexural modulus of the pipe
f_s	soil-pipeline resistance per unit length of pipeline
F	axial force per unit length
F_x	ultimate axial resistance per unit length
h	depth of pipeline embedment
h_s	depth of landslide
H	depth from ground surface to centre of pipeline
I	pipeline moment of inertia
k_s	soil subgrade modulus
k_x	axial subgrade modulus
K_o	coefficient of earth pressure at rest
L_s	length of sliding soil mass
M	nondimensional maximum moment developed in the pipeline
M	maximum moment developed in the pipeline
N, T	soil forces acting on the failure plane in the slope
$\bar{N}_c$	horizontal bearing capacity factor for cohesive soils
N_{qh}	horizontal bearing capacity factor for frictional soil
P	load applied at edge of landslide
P_o	force at interface of moving-nonmoving masses
$\bar{P}$	nondimensional end load
R	ultimate soil resistance
s_u	undrained shear strength
t	pipeline wall thickness
u	axial displacement
u_o	longitudinal edge displacement
u_x	axial displacement when soil and pipe are both elastic
$\bar{u}_o$	nondimensional longitudinal edge displacement
U_z	elastic displacement unit
w_x	transverse displacement along x-axis
w	transverse displacement at edge of landslide
$2w$	free-field landslide displacement
$\bar{w}$	nondimensional end displacement
W	weight of the sliding mass per unit length of pipeline
W_s	width of sliding soil mass
x	longitudinal coordinate axis
$\bar{x}$	region in plastic state
$\hat{x}$	point of maximum bending moment
α	adhesion factor
β	reciprocal of flexural elastic characteristic length
δ	interface friction angle between the pipeline and soil
Δl_s	unit length of pipeline
ϕ'	effective stress friction angle for soil
γ_s	unit weight of soil
$\bar{\gamma}_s$	effective unit weight of soil
γ	reciprocal of axial elastic characteristic length
η	ratio of elastic to hardening moduli of pipe
κ	soil-pipeline stiffness parameter
λ	slope of soil sliding mass
σ	stress in pipe
σ_y, ϵ_y	yield stress for steel and strain at yield
ϵ_{ss}	soil-pipeline strength parameter
τ	shear stress

Appendix

A sequence of events as a result of the transverse interaction between the beam and the surrounding foundation takes place as the load is monotonically increased and these can be described as follows:

- On initial application of the end load, P ($0 \leq P \leq P_1$), the embedded beam as well as the soil behave elastically. P_1 is the load level beyond which inelastic strains are induced in surrounding soil.
- As the load, P , is increased to a load level $P_1 \leq P \leq P_2$, ultimate passive resistance will be developed in part of the surrounding soil medium but the pipe will remain elastic. P_2 is the load level beyond which inelastic strains are induced in the soil as well as in the pipeline. Referring to Fig. 1, we define an axis $x-x$ that distinguishes two regions: region A where the medium is in an elasto-plastic state and region B where the medium is still elastic. The position of the axis $x-x$ will shift from initial position $s-s$ where it is initially coincident with the edge where the load or prescribed displacement is applied. The shift from the far edge to the axis $x-x$ is denoted by $\bar{x}$ at any particular loading stage.
- As the load, P , is further increased to, say, $P_2 \leq P \leq P_3$, the distance $\bar{x}$ increases when a plastic hinge begins to develop in the beam or wrinkles develop in the beam depending on the structural characteristics of the beam. P_3 is the load level beyond which the pipeline section is fully plastic.

The objective is to trace this load resistance behavior for the first two events. Only the relevant equations have been presented here but full details can be found in Rajani and Morgenstern (1993).

Stage $0 \leq P \leq P_1$ and $w \leq U_z$

As noted earlier, for a load $0 \leq P \leq P_1$ and as long as the displacement w does not exceed the limiting elastic displacement of the soil, i.e., $w \leq U_z$, the solution for a beam on elastic foundation is perfectly valid and the corresponding differential equation, boundary conditions, and solution (Hetenyi 1968) are:

$$[A1] \quad EI \frac{d^4 w}{dx^4} + k_s b w = 0$$

Solving the above differential equation with the appropriate boundary conditions, the displacement is given by

$$[A2] \quad w = \frac{2P\beta}{k_s b} e^{-\beta x} \cos \beta x$$

for $0 \leq x \leq \infty$. The displacement at the edge of the landslide ($x = 0$) in terms of nondimensional variables, eq. A2, is

$$[A3] \quad \bar{w} = 2\bar{P}$$

The maximum moment in nondimensional form, $\bar{M} = M\beta/P$, is easily identified by usual procedures as shown below:

$$[A4] \quad \bar{M} = e^{-\beta \hat{x}} \sin \beta \hat{x}$$

$$\text{where } \beta \hat{x} = \frac{\pi}{4} \quad \text{and} \quad \bar{P} \leq \bar{P}_1$$

where $\bar{P}_1 = \bar{N}_c/2$, and $\hat{x}$ is the point of maximum bending moment.

Stage $P_1 \leq P \leq P_2$ and $w \geq U_z$, $\epsilon_{\text{pipeline}} \leq \epsilon_y$

As soon as the beam displaces sufficiently so as to exceed the elastic displacement limit, U_z , then an ultimate passive resistance offered by the surrounding soil will be acting on that portion of the beam while the rest of the beam foundation is still elastic. It is also required that no plastic strains are induced in the pipeline during this loading stage, i.e., $\epsilon_{\text{pipeline}} \leq \epsilon_y$. The equilibrium equations for the two regions described earlier are

$$[A5] \quad \text{region A:} \quad -\bar{x} \leq x \leq 0 \quad EI \frac{d^4 w_A}{dx^4} = -R$$

$$[A6] \quad \text{region B:} \quad 0 \leq x \leq \infty \quad EI \frac{d^4 w_B}{dx^4} + k_s b w_B = 0$$

where $\bar{x}$ has been defined earlier. The pair of differential equation can be solved with the application of the appropriate boundary conditions. The solution is obtained in terms of the displacement at the point of application of the load which can be evaluated as a function of the applied load to determine what is commonly termed as the characteristic curve. The expression thus obtained can be conveniently expressed in nondimensional form:

$$[A7] \quad \bar{w} = \bar{N}_c \left(\frac{1}{2} + \frac{2}{3} \frac{\bar{P}}{\bar{N}_c} + \frac{8}{3} \frac{\bar{P}^4}{\bar{N}_c^4} \right)$$

During the second stage of loading, i.e., $P_1 \leq P \leq P_2$, the maximum curvature can develop on either side of the moving axis $x-x$ since different functions describe the displacement pattern as indicated in equations [A5] and [A6].

If $\bar{P}_1 \leq \bar{P} \leq 2\bar{P}_1$, then the maximum moments occurs to the left of axis $x-x$ and is given as

$$[A8] \quad \bar{M} = \frac{3}{2} \frac{\bar{P}}{\bar{N}_c} + 4 - \frac{2\bar{N}_c}{\bar{P}}$$

where $\beta \hat{x} = 1 - \bar{P}/\bar{N}_c$.

On the other hand if $2\bar{P}_1 \leq \bar{P} \leq \bar{P}_3$, the maximum moment is to the right of axis $x-x$ and is given as

$$[A9] \quad \bar{M} = \frac{e^{-\beta \hat{x}} \bar{N}_c}{2\bar{P}} \left[\sin \beta \hat{x} + \left(2 \frac{\bar{P}}{\bar{N}_c} - 1 \right) \cos \beta \hat{x} \right]$$

$$\text{where } \tan \beta \hat{x} = \frac{\bar{N}_c - \bar{P}}{\bar{P}}$$

It is important to note that the point ($\hat{x}$) of maximum moment increases as the load is monotonically increased. For specific values of the horizontal bearing capacity factor $\bar{N}_c$, the load-displacement (Fig. 4) and moment-displacement (Fig. 5) characteristic curves can be traced with the help of eqs. A3 and A7, and eqs. A4, A8, and A9, respectively.