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YIELD DISPLACEMENTS IN MULTISTORY ASEISMIC DESIGN

BY

G. N. BYCROFT

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YIELD DISPLACEMENTS IN MULTISTORY ASEISMIC DESIGN

By G. N. BYCROFT

ANALYZED

ABSTRACT

This paper investigates the plastic displacements occurring when certain types of elastoplastic multistory shear structures are subjected to large earthquakes. It is directed toward finding a variation of the horizontal stiffness and strength up the structure in order to minimize yield displacements and distribute them over the structure. The analysis has been made on an analog computer using a white noise source to simulate the earthquake disturbance.

INTRODUCTION

It is now generally believed that structures designed in accordance with normal seismic codes must yield slightly during severe earthquakes. The large dissipation of energy involved appears to be the factor that keeps the motion within safe limits. This paper is one of a series of related studies¹ of the response of elastic and elastoplastic structures to earthquakes.

The first of these papers considers the yield displacements occurring when a damped, single degree of freedom, elastoplastic structure is subjected to one particular earthquake, viz., El Centro, 1940. The second paper shows that a random excitation appears as a suitable simulation of an earthquake, and in particular suggests that a "standard large earthquake" be represented by random ground accelerations which are "white" in nature, of duration half a minute, and having a spectral density of $1.0 \text{ ft}^2/\text{sec}^4/\text{cps}$. The technique of using a white noise source in conjunction with an analog computer is here used to determine the yield displacements in a multistory elastoplastic structure when subjected to the "standard large earthquake," and is directed toward finding a variation of the stiffness which will minimize and distribute yield displacements.

The structure considered here is a multistory framed structure deflecting only in shearing between the floors. The shearing rigidity is assumed to have an idealized elastoplastic characteristic, i.e., the hysteresis loop is a parallelogram. Viscous damping between floors and also relative to the base is introduced artificially in order to make the damping ratio of the various modes of vibration approximately equal. This conforms with what little is known about damping in buildings.

A large number of parameters are involved, and in order to keep the size of the investigation within tolerable limits many simplifications must be introduced. For convenience on the computer, a five-story structure has been analyzed. Simple interpolation will give a good picture of what happens with a greater or smaller number of stories. The masses of the floors are taken as equal. This will be approximately true for a number of buildings. Further simplifications will be mentioned as they arise.

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¹ G. N. Bycroft, M. J. Murphy, and K. J. Brown, "Earthquake Yield Spectra," *Proc. A.S.C.E.*, Vol. 85, No. EM4 (October, 1959), pp. 43-64, and G. N. Bycroft, "White Noise Representation of Earthquakes," *Proc. A.S.C.E.*, Vol. 86, No. EM2 (April, 1960), pp. 1-16.

Fig. 1. Analog computer diagram of n -th story.

FORMULATION OF THE PROBLEM

- $W(t)$ = the "white" ground acceleration representative of a "standard large earthquake."
 x_n = horizontal displacement of the n -th floor relative to the base.
 m = mass of each floor.
 k_n = the linear stiffness between the n -th and $(n-1)$ -th floors.
 y_n = the relative displacement of the n -th floor to the $(n-1)$ -th floor at which yield just occurs.
 u_n = the algebraic sum at any instant of all the previous yield displacements of the n -th floor relative to the $(n-1)$ -th floor. The signs of the yield displacements are those of their velocities.
 c_n = damping coefficient between the n -th and $(n-1)$ -th floors.
 C_n = damping coefficient of the n -th floor relative to the base.

The equations of motion are,

$$\begin{aligned}
 m\ddot{x}_n = mW(t) + & \left[k_{n+1} (x_{n+1} - x_n - u_{n+1}), |x_{n+1} - x_n - u_{n+1}| \leq y_{n+1} \right] \\
 & \left[k_{n+1} y_{n+1} \operatorname{sgn} (\dot{x}_{n+1} - \dot{x}_n), |x_{n+1} - x_n - u_{n+1}| \geq y_{n+1} \right] \\
 & - \left[k_n (x_n - x_{n-1} - u_n), |x_n - x_{n-1} - u_n| \leq y_n \right] \\
 & - \left[k_n y_n \operatorname{sgn} (\dot{x}_n - \dot{x}_{n-1}), |x_n - x_{n-1} - u_n| \geq y_n \right] \\
 & + c_{n+1} (\dot{x}_{n+1} - \dot{x}_n) - c_n (\dot{x}_n - \dot{x}_{n-1}) - C_n \dot{x}_n.
 \end{aligned} \tag{1}$$

The boundaries are satisfied by making $x_0 = 0$ and $x_6 = x_5$.

This equation may be simplified by putting

$$\omega_n = \sqrt{k_n/m} \quad (2)$$

i. e., the angular frequency of the n -th story separately.

$$G_n = k_n y_n / mg \quad (3)$$

i.e., the fraction of gravity at which the n -th floor, considered separately, yields.

$$R = C_n / 2\omega_n m, \quad r = C_n / 2\omega_n m \quad (4)$$

i.e., the damping ratio of each floor referred to the stiffness below it is assumed equal. The equation of motion is now

$$\begin{aligned} \ddot{x}_n + & \left[\begin{array}{l} \omega_n^2 (x_n - x_{n-1} - u_n), |x_n - x_{n-1} - u_n| \leq y_n \\ G_n g \operatorname{sgn} (\dot{x}_n - \dot{x}_{n-1}), |x_n - x_{n-1} - u_n| \geq y_n \end{array} \right] \\ & - \left[\begin{array}{l} \omega_{n+1}^2 (x_{n+1} - x_n - u_{n+1}), |x_{n+1} - x_n - u_{n+1}| \leq y_{n+1} \\ G_{n+1} g \operatorname{sgn} (\dot{x}_{n+1} - \dot{x}_n), |x_{n+1} - x_n - u_{n+1}| \geq y_{n+1} \end{array} \right] \\ & + 2\omega_n R(\dot{x}_n - \dot{x}_{n-1}) - 2\omega_{n+1} R(\dot{x}_{n+1} - \dot{x}_n) + 2r\omega_n \dot{x}_n = W(t). \end{aligned} \quad (5)$$

The computer variables are limited to 100 volts. If S is the maximum displacement expected, X_n the computer variable corresponding to x_n , Y_n to y_n , and U_n to u_n , then the scaled equation is

$$\begin{aligned} \ddot{X}_n = & \frac{100}{\omega_n^2 S} W(t) + \frac{2R\omega_{n+1}^2}{\omega_n^2} \dot{X}_{n+1} + \frac{2R\omega_{n-1}}{\omega_n} \dot{X}_{n-1} - \left(2R \frac{\omega_{n+1}}{\omega_n} + 2R + 2r \right) \dot{X}_n \\ & + \frac{\omega_{n+1}^2}{\omega_n^2} \left[\begin{array}{l} (X_{n+1} - X_n - U_{n+1}), |X_{n+1} - X_n - U_{n+1}| \leq Y_{n+1} \\ \frac{G_{n+1} g 100}{\omega_{n+1}^2 S} \operatorname{sgn} (\dot{X}_{n+1} - \dot{X}_n), |X_{n+1} - X_n - U_{n+1}| \geq Y_{n+1} \end{array} \right] \\ & - \left[\begin{array}{l} (X_n - X_{n-1} - U_n), |X_n - X_{n-1} - U_n| \leq Y_n \\ \frac{G_n g 100}{\omega_n^2 S} \operatorname{sgn} (\dot{X}_n - \dot{X}_{n-1}), |X_n - X_{n-1} - U_n| \geq Y_n \end{array} \right]. \end{aligned} \quad (6)$$

Figure 1 shows the computer diagram satisfying this equation. The elastoplastic characteristic is provided by the diode limited integrators.

Fig. 2. Damping of the modes $\rho = 1.5$.

Fig. 3. Periods of the modes.

Fig. 4. Interstory yield displacements $\rho = 0$, $\lambda = 0.10$.

STIFFNESS AND STRENGTH VARIATION

As has already been mentioned, the large number of parameters and the many possible combinations of them necessitate the consideration of some special cases. In general the relationship between the stiffness k_n and the strength or yield value G_n will be a complicated one depending on the nature of the structure. The assumption made here is that the variation of G_n up the height of the building will be proportional to that of the stiffness. This will be true for pin-jointed structures having the same geometrical configuration of cross-ties up the height. In order to determine the effect of changing the stiffness up the structure it is necessary to assume some type of variation. The simplest possible variation is a linear one and this has been taken here. In particular, if Z_n is the position of the point midway between the n -th and $(n - 1)$ -th floors, measured from the top of the structure, and h is the total height, then

$$k_n \propto (1 + \rho z_n/h) \quad (7)$$

$$G_n \propto (1 + \rho z_n/h) \quad (8)$$

The parameter ρ determines the stiffness and strength variation with height. This investigation is concerned with finding, if possible, a suitable value of ρ .

DAMPING

Very little is known about the damping of structures. The few measurements made on actual structures suggest that the damping ratios of the various modes of vibration are approximately equal and vary between 0.05 and 0.20.

Viscous damping is the easiest type to simulate. If only viscous damping between the floors were considered, it would be found that the damping of the modes is proportional to the frequency of these modes. The higher modes would thus be much more heavily damped than the fundamental. If viscous damping relative to the base only were considered, the damping of the modes would decrease with frequency. A combination of the two forms allows equal damping ratios of the modes to be approximately realized.

If

T_m = the period of the m -th mode of vibration,

λ_m = the damping ratio of the m -th mode,

$x_{m,n}$ = the maximum deflection of the n -th floor when vibrating in the m -th mode,

it may be shown that

$$\frac{\lambda_m}{(R + r)} = \frac{2\pi \left[R/r \sum_{n=1}^5 \omega_n (x_{m,n} - x_{m,n-1})^2 + \sum_{n=1}^5 \omega_n x_{m,n}^2 \right]}{T_m (R/r + 1) \left[\sum_{n=1}^5 \omega_n^2 (x_{m,n} - x_{m,n-1})^2 \right]} \quad (9)$$

Fig. 5. Interstory yield displacements $\rho = 10$, $\lambda = 0.05$. Fig. 6. Interstory yield displacements $\rho = 10$, $\lambda = 0.10$. Fig. 7. Interstory yield displacements $\rho = 10$, $\lambda = 0.20$.

For any particular configuration of stiffness k_n and hence ω_n in the structure, the periods of the various modes and their shapes were found experimentally by exciting the system with an oscillator at the mode resonances. Figure 2 shows equation (9) plotted as a function of the ratio R/r for one particular configuration of stiffness denoted by $\rho = 1.5$. It is noticed that if R/r is chosen to be 2.5, say, then the damping ratio of the first three modes is approximately the same, although the fourth and fifth are higher. As the lower harmonics are the only ones that really matter, this situation was regarded as satisfactory. For each arrangement of stiffness-variation considered, a set of curves similar to figure 2 was constructed and a suitable ratio R/r decided on. Equation (9) then determines R to give any required mode damping ratio λ_m .

RESULTS

The family of structures considered has various fundamental periods, mode damping ratios, and values of the parameter ρ . The parameter ρ determines a particular stiffness configuration and fixes the ratios between the periods of the harmonics. Figure 3 shows the ratio of period of the m -th mode to the period of the bottom story, itself, as a function of ρ . All members of the family have been given the same base yield value of $G_1 = 0.5$. The factor G_1 is with reference to the mass of the bottom floor only. This means that, with respect to the total mass of the structure, all the members have the same base yield acceleration of 0.1 g . This is a value commonly used in aseismic design. Actually, because safety factors are always introduced, it will mean that this set of structures would have been designed at the base to 0.05 to 0.07 g , but, because of the implicit safety factors, yield at 0.1 g .

The yield values at other points in the structure are then determined by equation (8). The maximum yield displacement between stories has been measured. This is defined as the maximum relative displacement measured from the initial zero position minus the elastic part of the displacement. Since each burst of noise gives a different result in a statistical fashion, the average of the maximum yield displacements taken over ten runs is shown. In the case $\rho = 3.0$ the average of twenty runs was taken.

Figure 4 shows a completely uniform structure, $\rho = 0$. The average maximum yield displacements are plotted against the fundamental period of the building T_1 . The elastic part of the displacement, which has the same value at each floor, is also shown. Addition of the elastic and plastic parts gives the total maximum relative displacement between floors. Naturally, in this case, most of the yielding takes place at the base and the actual values of the base yield are large.

A building designed so that the stiffness and strength taper off quickly toward the top is given by $\rho = 10$ and is shown in figures 5, 6, and 7 for three different damping ratios.

A whipping effect occurs, causing the major yield to be at the top, and again the values are large. Figure 5 shows the actual points obtained from an average of ten bursts of noise. The smooth curves only are drawn for the other cases.

It is required to find a design between these extremes which will minimize and distribute the yield displacements more evenly.

It may be shown, both analytically and by simulation on a computer, that in a structure subject to the same assumptions made here, except that it is linearly

Fig. 8. Interstory yield displacements $\rho = 1.5$, $\lambda = 0.05$.

Fig. 9. Interstory yield displacements $\rho = 1.5$, $\lambda = 0.10$.

elastic, equal strain up the structure is obtained approximately when ρ has values between 1.5 and 3.0. This result is largely independent of period, damping, and the number of stories. Furthermore, the value of ρ is not critical. Unfortunately, the elastoplastic case is much more complicated. Not only is a simple linear variation inadequate, but the required variation will be a function of period, damping, and

Fig. 10. Interstory yield displacements $\rho = 1.5$, $\lambda = 0.20$.

Fig. 11. Interstory yield displacements $\rho = 3.0$, $\lambda = 0.05$.

base yield value. Yield displacements for several different values of ρ were measured. These results showed that the displacements in the elastoplastic case depend more critically on the stiffness variation up the structure than in the elastic case. This is not surprising. Figures 8, 9, and 10 show the case $\rho = 1.5$. The difference in the yield displacements is large.

Most buildings will have fundamental periods less than 1.5 sec. In this period range and with the simple variation assumed it was found that the best general

Fig. 12. Interstory yield displacements $\rho = 3.0$, $\lambda = 0.10$. Fig. 13. Interstory yield displacements $\rho = 3.0$, $\lambda = 0.20$. Fig. 14. Ratio of plastic to elastic strain $\rho = 3.0$, $\lambda = 0.10$.

value for the elastoplastic case is given by $\rho = 3.0$. Figures 11, 12, and 13 show this situation. Depending on the damping and fundamental period, certain floors are under- or overdesigned. When $\lambda = 0.20$ the design is very reasonable except for the top floor, which is overdesigned. When $\lambda = 0.05$ and 0.10 , the third and fourth floors are underdesigned.

The seismic design of an elastoplastic structure will depend on the amount of relative displacement that may be permitted between floors as well as the amount of plastic deformation the materials may safely endure. The former is limited by the amount of distortion that the structure's cladding can stand without damage and

Fig. 15. Comparison of elastic and elastoplastic cases $\rho = 3.0$, $\lambda = 0.10$.

will be largely constant up the height of the structure. The latter is a rather difficult problem in the fatigue of materials. It will depend on the ratio of the plastic to the elastic strain. This ratio for $\rho = 3.0$ and $\lambda = 0.10$ is shown in figure 14. The longer the period the better, but this is limited by the permissible relative displacements.

Figure 15 compares the average maximum total relative displacement in the elastoplastic and elastic cases for $\rho = 3.0$ and $\lambda = 0.10$. The elastic case was simulated on the computer by removing the bias on the integrators. In order to avoid confusion on the graph the first three floors of the elastic case are shown. The interesting point is that here the total relative displacements are closely the same in the two cases. Housner² assumed this in some work on the limit design of structures. A probability curve of the maximum total relative displacements was constructed from two hundred runs and is shown in figure 16.

² G. W. Housner, "Limit Design of Structures to Resist Earthquakes," *Proceedings, First World Conference on Earthquake Engineering*, San Francisco, June, 1956.

Fig. 16. Probability distribution of maximum total relative displacements $\rho = 3.0$, $\lambda = 0.10$.

CONCLUSIONS

It appears that structures designed in accordance with existing codes must undergo plastic deformation during severe earthquakes. It should be the object of aseismic design to minimize and distribute these yield displacements over the entire structure. The achievement of an optimum design, however, is found to be very complicated. The relative displacements depend more critically on the stiffness and strength configuration than in the completely linear case. Thus, any particular design may be comparatively weak at some point in its height although adequate elsewhere.

Any particular proposed design could be analyzed in this fashion in order to determine weak points, but generalized conclusions are difficult. For the much simplified family of structures considered here a linear variation denoted by $\rho = 3.0$ roughly approximates a suitable design. The probability curves are very similar for the different stories and show only a small probability that the maximum total relative displacement is greater than twice the mean.

Bending, bending combined with shear, and rotation of the base of the structure could also be simulated on an analog computer. It is suggested that this type of analysis could be used to investigate many other aspects of aseismic design.

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