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PRECISION AND ACCURACY OF GUARDED HOT PLATE METHOD

by M. Bomberg and K.R. Solvason

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RÉSUMÉ

Résumé de l'étude effectuée pour déterminer la précision et la justesse que l'on peut atteindre avec le GHP (guarded hot plate) de 450 mm du CNRC comportant un élément de chauffage principal de 300 mm et un carter chauffant de protection de 75 mm, et avec un GHP de 600 mm comportant un élément de chauffage principal de 300 mm et un carter chauffant de protection de 150 mm. Seules les incertitudes attribuables aux instruments ont été analysées.

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PRECISION AND ACCURACY OF GUARDED HOT PLATE METHOD

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ABSTRACT

This paper presents a summary of an investigation of the precision and accuracy achievable with the NRCC 450 mm square GHP with a 300 mm main heater and a 75 mm guard ring heater and a 600 mm GHP with 300 mm main and a 150 mm guard ring heater. Only the uncertainties that may be attributed to the instrument are discussed here.

INTRODUCTION

This paper deals with instrument errors as calculated or measured on "ideal" specimens and illustrates the precision to which the thermal resistance of a stable specimen can be determined at the National Research Council of Canada (NRCC) laboratories.

The basic measurement errors, those of temperature, thickness and power, are listed in Table 1. These uncertainties may be compared with the uncertainty in repeating measurements on the same specimen. The agreement is satisfactory since the repeatability precision determined for both 450 and 600 mm square guarded hot plate (GHP) on the two sigma level was less than about 0.2%.

Having established the basic measurement error, the other errors of the GHP method, i.e., unbalance error, E_g , and specimen edge heat loss error, E_e , can be analyzed.

Unbalance Error

The lateral heat flow across the gap and through the specimen

Table 1. Estimated Standard Errors, in Per Cent, and in Thermal Resistance Determined with GHP Apparatus on 25 mm Thick Specimen. Hot and Cold Temperatures:
($T_c = 13 \pm 2^\circ\text{C}$, $T_h = 35 \pm 2^\circ\text{C}$)

Source of Error	Repeatability	Uncertainty of the Tests	
		Standard	Differential
Power on main heater			
voltage divider	-	0.06	0.06
potentiometer	0.02	0.15	0.15
standard resistors	-	0.03	0.03
potentiometer	0.002	0.06	0.06
Temperature difference			
cold plate thermocouples	-	0.50	-
potentiometer	0.04	0.04	-
hot plate thermocouples	-	0.30	-
potentiometer	0.02	0.04	0.04
differential measurements	-	-	0.50
Thickness measurements	0.04	0.04	0.04
Metering area (at specified temperature)	-	0.001	0.001
Unbalance error	0.02	0.06	0.06
The uncertainty estimated with two standard deviations	0.13	1.2	1.1

can be expressed as follows¹:

$$Q_g = \Delta T_g (C_1 + C_2 \lambda_x) \quad (1)$$

where

C_1 is the total conductance across the gap from the metering area to the guard area (W/K). It is the sum of all the conductances multiplied by the areas for the heat flow paths across the gap, i.e., through the air gap between the face plates, through the heater core insulation or other mechanical connections between main and guard areas, and through thermocouple and thermopile wires crossing the gap.

$C_2\lambda_x$ is the lateral heat flow through the specimens per unit of temperature difference (W/K). The heat flow is assumed to be proportional to the lateral conductivity of the specimens, λ_x , (W/mK).

C_2 then is the effective area for heat flow parallel to the plate divided by the effective length of the heat flow path (m^2/m or simply m).

Equation (1) is identical to an equation of Woodside and Wilson² for isotropic materials ($\lambda_x = \lambda_y$) and for test conditions where the specimen contribution to the heat flow described by Eq. (1) is constant, and thus the parameter $C_2 \lambda_x$ becomes a constant.

C_1 and partly C_2 can be considered characteristic of the plate, both are proportional to the main heater perimeter and roughly inversely proportional to gap width. C_1 is also influenced by the size, length, and number of thermocouple and thermopile wires crossing the gap; the thickness of the face plates; and the thermal conductance of any material that bridges the gap. The value of C_1 may only be constant if the edge of the main heater face plate is isothermal and the inside edge of the guard ring face plates is isothermal. Since C_1 represents the sum of the heat flow through at least three paths, a ΔT may exist on one path even though the mean ΔT for the whole gap is zero.

The heat flow through the metered area of the specimen normal to the plate is:

$$Q = \Delta T \cdot A \cdot \frac{\lambda_y}{L} \quad (2)$$

where

$\Delta T = T_h - T_c$, the temperature difference between the hot and cold plates (K)

A = the area of the main heater, that is, to the centre of the gap (m^2)

λ_y = thermal conductivity of the specimen in the direction normal to the plate (W/mK)

L = thickness of the specimen (m).

The power, Q' , supplied to the main heater differs from Q by the lateral heat flow, Q_g .

$$Q' = Q + Q_g \quad (3)$$

The unbalance error then is:

$$E_g = \frac{Q_g}{Q} = \frac{Q_g}{Q' - Q_g} \quad (4)$$

When Q_g is very small compared with Q' , Eq. (4) can be simplified to:

$$E_g = \frac{Q_g}{Q'} \quad (5)$$

In this report, the unbalance error, E_g , is defined as a ratio of heat flow across the gap between metering and guard ring heaters to the power supplied to the metering area. The uncertainty of the unbalance error is $\pm 0.06\%$. The repeatability of the unbalance error is $\pm 0.02\%$.

Error Due to Specimen Edge Losses

When the ambient air temperature is maintained at the mean temperature of the specimen, half of the specimen edge temperature is higher and half lower than the ambient; the result is heat exchange, and a distortion of heat flow lines. Any difference between the average heat flux in the metering volume of the specimen and the heat flux on the main heater causes so-called edge loss error.

The error caused by multidimensional heat flow may be reduced in the following ways:

- (1) By limiting sample thickness relative to the width of the guard heaters.
- (2) By using additional edge insulation.
- (3) By limiting unbalance between the main and guard heaters since unbalance creates additional distortion of the heat flow lines.
- (4) By maintaining ambient air temperature equal to the specimen mean temperature and, therefore, limiting the difference between heat loss and heat gain at the specimen edges.

The edge loss error depends on specimen thickness, edge insulation, and ambient air temperature. Thus, the accuracy with which it can be determined depends largely on how well all of the other errors can be evaluated, on the testing technique, and on the characteristics of specimens used for the determination. These factors are discussed further in the experimental analysis.

Experimental Analysis of Edge Loss Error

Let us assume that the unbalance errors have already been determined, necessary corrections introduced, and the only error of the GHP apparatus is the edge loss error.

Edge losses are expected to depend on spatial distribution of the ambient temperature (variation between sides or elevation), ambient temperature variation with time, difference between mean specimen and mean ambient air temperatures as well as the level of edge insulation applied in the test.

The effect of edge insulation and ambient air temperature was checked on 25 mm thick polystyrene specimens tested in the 450 mm GHP with a 300 mm metering area. Figure 1 shows the thermal resistance of a 150 mm polystyrene stack composed of six specimens divided with paper septa.

The stack of polystyrene specimens was tested with and without different edge insulations ($R_e = 1.7, 2.4, \text{ and } 2.9 \text{ m}^2\text{K/W}$).

Fig. 1. Apparent thermal resistance of 150 mm thick polystyrene/paper stack, tested on 450 mm GHP as a function of edge insulation. A sum of thermal resistance of 25 and 50 mm layers is shown also.

Figure 1 shows that the lack of edge insulation introduced significant errors for the 150 mm thick polystyrene specimen. The difference between values obtained without edge insulation and with insulation having an $R_e = 2.9 \text{ m}^2\text{K/W}$ tested at ambient temperature equal to mean specimen temperature was 4.6%. Although the presence of some edge insulation ($R_e = 1.7 \text{ m}^2\text{K/W}$) increased the apparent thermal resistance by 4.3%, the additional increase of edge insulation to $R_e = 2.9 \text{ m}^2\text{K/W}$ was not very effective and resulted in a further increase of only 0.3%. The test results with the ambient temperature at 16°C and 32°C differed from those with an ambient temperature of 24°C by about -7% and +7% respectively. This indicates that for thick specimens ambient temperature must be controlled fairly close to specimen mean temperature.

The apparent resistance ($3.694 \text{ m}^2\text{K/W}$) obtained on the 149.4 mm specimen with $R_{2.9}$ edge insulation and with ambient air temperature equal to specimen mean temperature was about 3.3% lower than the sum of the individual R values for the six 25 mm thick layers as reported in Table 2.

Thermal resistance of the same material was determined in the other GHP using other pieces of polystyrene to build up the test specimen for the 600 mm square plate.

In addition to a standard Kraft paper septum, a special septum was used, i.e., the paper was coated with a few layers of paint known to be opaque to long wave radiation. Table 3 shows that the difference in results for the coated and uncoated septum did not exceed the variability of test results.

The thermal conductivity from Table 3 is 0.03985 W/mK , i.e., about 1.8% higher than the value 0.03912 W/mK determined on the 25 mm thick specimens.

Another test, labeled "Lateral Heat Flow Test" conducted on a 150 mm thick polystyrene specimen in the 600 mm GHP. The plates, cold plate, guard ring and main heater plates were all maintained at about 35°C, and the ambient air temperature was maintained at about 10°C. The heat loss from the edge of the specimen may, therefore, be assumed symmetric, half from the cold plate and half from both the guard ring and the main heater plates. The main heater output, corrected for the small temperature difference across the specimen (0.04°C) was 0.057W (Table 4), and this output represents the edge loss associated with a 26.4°C temperature difference between the heater plate and ambient.

In a standard test, the specimen is exposed to a temperature difference that varies from $(T_h - T_c)/2$ to zero or to a mean temperature difference of $(T_h - T_c)/4$. The percentage error indicated by this heat flow can then be expressed as:

Table 2. Apparent Thermal Resistance of Separate or Stacked 25 mm Polystyrene Specimens at 24°C Tested With Paper Septum on 450 mm GHP Apparatus

Test Code	Specimens Tested	Thickness mm	R m ² K/W	λ W/mK
60-74	3+4/5+8	50.00	1.272	0.03930
-85	3+4/5+8	50.07	1.272	0.03936
-76	7+10/9+12	48.87	1.247	0.03921
-77	1+2+3+4/5+6+8+11	100.14	2.510	0.03990
-129	1+2+3+4/5+6+8+11	100.12	2.509	0.03991
-127	1+2+3+4/5+6+8+11	100.15	2.527	0.03964
-128	1+2+3+4/5+6+8+11	100.10	2.520	0.03971
-92	4/8	25.04	0.640	0.03913
-93	3/8	25.02	0.641	0.03903
-94	4/5	24.97	0.637	0.03923
-91	3/4	24.97	0.638	0.03913
-96	3/4	24.99	0.637	0.03922
-97	3/4	24.97	0.638	0.03913
-116	1/2	24.89	0.637	0.03910
-117	1/6	25.02	0.640	0.03910
-118	1/11	24.82	0.634	0.03911
-119	2/11	24.84	0.635	0.03912
-120	2/6	25.02	0.640	0.03907
-73	1+2/6+11	49.91	1.272	0.03925
-134	1+3+4/2+5+8	75.03	1.892	0.03965
-68	1+2+3+4+7+10/5+6+8+9+11+12	149.42	3.694	0.04045
-121*	1+2/6+11	49.85	1.234	0.04041
-122*	1+2/6+11	49.84	1.236	0.04031

Averages: at 25 mm $\lambda_a = 0.03912$ (average 3/4 $\lambda_a = 0.03916$)

with paper
septum: at 50 mm $\lambda_a = 0.03931$ at 100 mm $\lambda_a = 0.03979$
at 75 mm $\lambda_a = 0.03965$ at 150 mm $\lambda_a = 0.04045$

without paper
septum: at 50 mm $\lambda_a = 0.0404$

Densities of the specimens:

1	15.7 kg/m ³	7	15.1 kg/m ³
2	15.7 kg/m ³	8	15.7 kg/m ³
3	15.8 kg/m ³	9	15.0 kg/m ³
4	15.7 kg/m ³	10	14.6 kg/m ³
5	15.2 kg/m ³	11	15.2 kg/m ³
6	15.6 kg/m ³	12	14.5 kg/m ³

Table 3. Apparent Thermal Resistance of 150 mm Polystyrene/Septum Stack Tested on 600 mm GHP at 24°C Mean Temperature

Test Code	Septum	R m ² /W	λ W/mK
360-164	Paper	3.745	0.03984
-180	"	3.757	0.03969
-181	"	3.746	0.03981
	Paper	Mean	0.03978
360-173	Paper and Paint	3.754	0.03980
-174	"	3.731	0.04005
-175	"	3.720	0.04014
-177	"	3.774	0.03956
-178	"	3.763	0.03968
	Paper and Paint	Mean	0.03985 W/mK
	"	Grand Mean	0.03982
	"	St. Deviation	0.00018

Mean thermal conductivity at 25 mm thickness

$$\lambda = 0.03912 \text{ W/mK (from Table 3)}$$

Difference between mean measured on 150 and 25 mm thickness

$$\frac{0.03982 - 0.03912}{0.03912} = 1.8\%$$

Table 4. Lateral Heat Flow Through Polystyrene/Paper Stack With 150 mm Thickness

Specimen Thickness mm	T _h °C	T _c °C	$\frac{T_c - T_h}{R} 2A$ W/m ²	Main Heater, 2Q W	Total Flux, Q W	T _{amb} °C	T _h - T _a K
149.5	35.05	35.09	0.02	0.055	0.057	9.5	25.6

$$E'_e = \frac{1}{4} \frac{\Delta T_o}{\Delta T_\ell} \frac{Q_\ell}{Q_o} 100 \quad (6)$$

where

E'_e = maximum edge loss error in a standard test due to lateral heat flow, %.

ΔT_ℓ = temperature difference between plates and the ambient air, °C.

ΔT_o = temperature difference between main heater and cold plate, $(T_h - T_c)$ °C.

Q_o = heat output on main heater, W/m², in a standard test.

Q_ℓ = as above but in a lateral heat flow test.

Substituting data from Tables 4 and 6, one obtains the maximum edge loss error:

$$E_e = \frac{1}{4} \times \frac{22.8}{25.6} \frac{0.057}{1.093} 100 = 1.2\%$$

This would suggest that the 1.8% error indicated by Table 3 consists of about 1.2% edge loss and about 0.6% unbalance error.

Experimental Analysis of Unbalance Errors

Table 5 shows the results of unbalance tests on 450 mm GHP. In the test, the guard ring temperature was controlled from 0.5 to 0.6°C above and below the main heater temperature. The temperature difference was calculated from the differential thermopile output. The values for $C_1 + C_2\lambda$, shown in Table 6, were calculated from the difference in heat flow for the positive and negative test values. Heat flows were corrected to the same temperature difference for the tests with plus and minus guard ring unbalance, ΔT_g . $C_1 + C_2\lambda$ values calculated in this manner are shown in Table 5.

Two conclusions may be drawn from the results shown in Table 5, namely:

- (1) Unbalance errors are dependent on the specimen thickness, as shown by an approximately 16% increase in $C_1 + C_2\lambda$ between 25 mm and 100 mm thicknesses.
- (2) the unbalance test may be performed on specimens subjected

Table 5. Calculations of $(C_1+C_2\lambda)$ on 450 mm GHP from Differences in Q^1 for Plus and Minus Unbalance Using Polystyrene/Paper Stacks

Specimen		Conditions		Results				
Code	Thick-ness	ΔT	ΔT_g	Q^1 (1)	$Q^1_1 - Q^1_2$	$C_1 + C_2 \lambda$	R_o (2)	λ
3/4	24.99	22	-0.593	6.206				
		22	0.578	6.615	0.409	0.350	0.6374	0.0392
3-4/5-8	50.00	22	-0.520	3.022				
		22	0.505	3.401	0.378	0.369	1.272	0.0393
1-3-4/ 2-5-8	75.03	33	-0.573	3.001				
		33	0.591	3.482	0.481	0.413	1.894	0.0396
1-2-3-4/ 5-6-8-11	100.13	22	-0.574	1.391				
		22	0.570	1.859	0.468	0.409	2.514	0.0398
1-2-3-4- 7-10-12- 9-11-8- 6-5	149.42	22	-0.683	0.802				
		22	0.954	1.477	0.675	0.412	3.772	0.0396

(1) Test heat flow corrected to same ΔT for each pair.(2) R-value from Q^1 interpolated to $\Delta T_g = 0$.Table 6. Calculation of $(C_1+C_2\lambda)$ on 600 mm GHP from Difference in Q^1 on Polystyrene/Paper Stack With 150 mm Thickness

Specimen Thickness mm	Conditions		Results				
	ΔT	ΔT_g	Q^1 (1)	$Q^1_1 - Q^1_2$	$C_1 + C_2 \lambda$	R_o (2)	λ
	K	K	W	W	W/K	m ² K/W	k/mK
149.22	22.46	-0.665	1.55631				
	23.09	0.661	0.63270	0.92361	0.697	3.740	0.0399
avg. 22.8			$Q_o = 1.0931$				

(1) Test heat flow corrected to the same ΔT for each pair.(2) R-value from Q^1 interpolated to $\Delta T_g = 0$.

to a significant distortion of heat flow lines without affecting $C_1 + C_2\lambda$, as shown by results on 75, 100 and 150 mm thick specimens. The additional specimen edge loss in tests on the thick specimens did not appear to alter the mean to indicated gap-temperature-difference ratio.

The values of $(C_1 + C_2\lambda) = 0.35$ to 0.41 W/K indicate that when ΔT_g is controlled to ± 0.01 K the maximum probable error on heat input is about ± 0.004 W. The percentage error related to the polystyrene specimens used in the evaluation and tested at a 22°C temperature difference would be 0.06% at 25 mm thickness and about 0.39% at 150 mm thickness.

Table 6 shows the calculation of $(C_1 + C_2\lambda)$ for 600 mm GHP for the same specimens of polystyrene as those used in 450 mm GHP verification. The number $(C_1 + C_2\lambda)$ being 70% higher indicates more "thermal bridging" through the gap than in 450 mm GHP. For 0.01°C unbalance on 150 mm polystyrene/paper stack, the probable error becomes 0.66%.

COMPUTER ANALYSIS OF GHP ERRORS

A commercial* stress-strain and thermal analysis program, ANSYS, was used to simulate the heat flux and temperature distribution through a specimen in a GHP. The simulations were mainly concerned with edge loss errors and did not analyze the heat transfer across the air gap.

Two different models were examined:

- (i) three-dimensional model with boundary conditions of uniform but different heat flux in main and guard heaters, model 1
- (ii) three-dimensional model with isothermal plate surfaces, model 2.

The calculations were performed on a quarter of the specimen divided into one thousand parallelepipeds and a quarter of the metal plates divided into one hundred parallelepipeds. The specimen thermal conductivity was assumed to be uniform throughout and equal to 0.03912 W/mK. The air gap resistance was selected in such a way that it produced the same sensitivity to temperature unbalance as had been found by experiment.

*Cybernet computer network was used via Control Data Canada Ltd.

Model 1 was based on uniform but different heat fluxes supplied under the face plate areas of the heater plate. The cold plates were assumed to be isothermal. A trial and error procedure was required to set the heat fluxes. The heat flux applied to the main heater plate was such that it would produce the required temperature gradient in one-dimensional heat flow and the heat flux initially applied to the guard ring heater was about 15% higher. When the program was executed, an unbalance between metering and guard areas was calculated and another value of the guard heater flux tried, until an unbalance of less than 0.02K was obtained.

The first question examined with model 1 was whether there was a significant difference between the integrated temperature difference over the gap and that indicated at a central point on each side, i.e., four points only. For 150 mm polystyrene in the 600 mm GHP with 22K difference across the specimen, a temperature variation of 0.002K was indicated, but the difference between the integrated ΔT_g and ΔT_g indicated at the center of the gap was less than 0.001K. It indicates that the choice of whether the thermocouples are placed at the center or 1/3 of the air gap side is perhaps less important than previously expected.

Table 7 shows R-values measured and calculated with the three-dimensional programs for a 150 mm thick specimen in the 450 mm GHP. Model 2 indicates very small errors, i.e., 0.3%. This model is based on isothermal plate temperatures that are assumed to extend to the center of the gap and, therefore, heat is supplied to all the specimen, including the area at the air gap so that the average heat flux entering the central part of the specimen, q_o , and the average heat flux on the metering area, q , are equal. Thus, the apparent thermal resistance $R_{app} = \frac{\Delta T}{q}$ and the thermal resistance of the central part of the specimen $R_o = \frac{T}{q_o}$ are equal.

Model 1 does not simulate heat generation in the air gap layer as the heat flux at each of the boundary points is separately formulated in the input data. An input flux of 5.738 w/m² was selected since it would give a 22K temperature difference for a thermal resistance of 3.834 m²K/W.

Table 7 shows how the calculated average heat flux, q_o , entering the specimen varies with the level of unbalance. Using this heat flux estimate for calculation of thermal resistance R_o , one obtains a value close to that of model 2 and only slightly affected by the unbalance. The apparent thermal resistance based on heater output divided by the metering area, q , however, shows a strong dependence on the unbalance.

It is not certain that model 1 simulates a real GHP better than

model 2, particularly as the apparent thermal resistance exceeds the true R-value. It does indicate, however, a different ratio between the heat flux entering the specimen and the average heat flux on the metering area from that calculated for model 2.

Table 8 shows measurements and calculation for the 600 mm GHP. The thermal conductivity measured on 150 mm is 1.8% greater than that determined on 25 mm thick layers. The error of 1.8% on 600 mm GHP corresponds to the error of 3.3% obtained on 450 mm GHP for these

Table 7. Measured and Calculated R-Values of 150 mm Thick Polystyrene Specimen Tested in 450 mm GHP Compared To Measurements on 25 mm Thick Specimens

Description	Heat Flux on the			Thermal		
	Main Heater		Meter- ing Area	Resistance		
	Input	Into Speci- men		$\text{m}^2\text{K/W}$		
	q_{in} W/m^2	q_o W/m^2	q W/m^2	ΔT K	$R_o = \frac{\Delta T}{q_o}$	$R_{\text{app}} = \frac{\Delta T}{q}$
Measured on 25 mm thick ($\lambda=0.03912$) and recalculated for 150 mm thickness	-	-	-	-	-	$0.15/0.03912$ $= 3.834$
Measured on 149.4 mm thick ($\lambda=0.04045$) and recalculated for 150 mm thickness	-	-	6.090	22.58	-	$0.15/0.04045$ $= 3.708$
Calculated with model 2 (isothermal plates)	5.868	5.752	5.752	22.00	3.825	3.825
Calculated with model 1 for $\Delta T_g = 0.013\text{K}$	5.738	5.712	5.623	21.79	3.815	3.875
Calculated with model 1 for $\Delta T_g = 0.053\text{K}$	5.738	5.589	5.623	21.29	3.810	3.786
Calculated with model 1 for $\Delta T_g = 0.112\text{K}$	5.738	5.425	5.623	20.62	3.800	3.667

specimens. The calculations with both models 1 and 2 indicate insignificant errors.

Calculations of edge loss errors in a standard test based on the results of "lateral heat flow test" indicate an error of 1.1%. A computer simulation using model 2 on the other hand gives 0.9% error, i.e., good agreement. Thus, both measurements and calculations as done in the tests labelled "lateral heat flow tests" may be useful for assessment of the errors in testing "thick" specimens.

The computer calculations for standard testing conditions do not agree with the measurements. It would appear, therefore, that the computer program does not adequately simulate the true pattern of heat flow in a GHP, particularly in the area of the gap.

DISCUSSION

Errors in measurement with GHP apparatus arise from uncertainty in basic electrical and thickness measurements and from deviations from one-dimensional heat flow. The errors in basic measurements, for example, repeatability precision (for a given level of heat flux), are a characteristic of the equipment used. The extraneous heat leakage, i.e., unbalance error and edge loss, may also depend on the nature of the material under test.

Table 8. Comparison between Errors Measured and Calculated with Three-Dimensional Programs for 600 mm GHP

Description	Error, %
Measured at 150 mm $\lambda_{app} = 0.0398$ W/mK	1.8
Measured at 25 mm $\lambda_{app} = 0.0391$ W/mK	
Calculated for standard test conditions from measurements, the lateral flow test $T_h = T_c = 35^\circ\text{C}$, $T_a = 9.5$	1.2
Calculated for normal flow, model 2 or model 1 with $\Delta T_c = 0$.	Insignificant
Calculated for lateral flow test, model 2	0.9

The materials used in this study were selected to minimize errors caused by material inhomogeneity and testing conditions. Polystyrene with a density of 15 to 16 kg/m³ was selected as a relatively stable material, unaffected by humidity changes during testing. It does, however, exhibit a significant transmission of long wave radiation making the apparent thermal conductivity dependent on thickness even when tested with the same mean temperature and temperature difference. To reduce changes in the fraction of long wave radiation in the total heat flux, i.e., to eliminate the so called "thickness effect" a Kraft paper septum was used for each 25 mm thick layer (Pelanne³). Thus, 25 mm thick layers were tested without paper, 50 mm had one paper septum, and 75 mm had two septa.

The thermal conductivity determined for the polystyrene specimens stacked from 25 to 150 mm with and without septa are shown in Fig. 2. A gradual increase in apparent conductivity with the specimen thickness is evident.

Part of the error commonly attributed to the edge losses could well be an effect of increased heat exchange between metering and guard ring heaters, labelled in this paper as the unbalance errors.

Fig. 2. Apparent thermal conductivity of polystyrene stack with and without paper septa.

Temperature distribution on the plate, particularly in the guard ring area, can produce a variation along the gap. Thus a heat flow across the gap might occur even though the average output from the thermopile is zero. The guard ring heater has to compensate for the heat loss to ambient from the edge of the hot plate and from the edge of the specimen. These losses are proportional to the perimeter length so that when only a single guard ring heater winding is used there is proportionally less heater area to compensate for the heat loss at the corners than at the center of each side. Heat must, therefore, flow laterally and a reduced temperature may be expected in the corners.

The guard ring temperature will be more uniform if two guard ring heater windings are used, a narrow perimeter winding and an inner winding as discussed by Maréchal⁴, Rousselle⁵ and Bode⁶. The perimeter winding can be used to supply the edge loss without requiring lateral heat transfer in the face plates (Troussart⁷). One suitable technique for operating such an instrument consists of adjusting the heat flux on the inner guard heater to a level equal to or slightly lower than that on the main heater and using the differential thermocouples across the gap to control the heat output on the outer guard ring. Theoretically, the same heat flux should be used on both sides of the gap, but a slightly lower flux on the inner guard ring heater permits more convenient control of the unbalance temperature.

Increasing the number of differential thermocouple junctions permits a better assessment of the mean temperature difference, but the additional wire crossing the gap also increases the heat transfer. The influence of construction details on the errors of the GHP method has been discussed by Doussain⁸. With a limited number of thermopile junctions, the choice of their location becomes important. This has been discussed by De Ponte and Di Filippo⁹, and by De Ponte, Marotti and Strada¹⁰. The percentage of unbalance errors depends on the level of heat flux. This fact indicates that there is no single value of accuracy for a particular GHP apparatus but that the accuracy and precision depend on the temperature difference applied as well as on the thickness and resistivity of the test specimen.

SUMMARY

The basic estimated instrument error shown in Table 1 agrees well with the repeatability precision determined on a 25 mm thick, "ideal" specimen. At the 95% probability level the repeatability precision is 0.2%. The lowest uncertainty of the GHP's examined is estimated at 1.1%. (Comparison with National Bureau of Standards 1000 mm line-heat-source GHP apparatus discussed in these proceedings showed agreement within 0.3% on low density glass fiber specimens of 25 mm thickness.)

Uncertainty of R-value determination depends on thickness and resistivity of the specimen, as well as the temperature difference applied to hot and cold plates. Using the limits of total uncertainty of 2 to 2.5% the specimen thickness is limited to less than 60 mm on 450 mm GHP and to less than 100 mm on 600 mm GHP. Thicker specimens, even with careful conditioning and using edge insulation in excess of $2 \text{ m}^2\text{K/W}$, may produce an uncertainty larger than 2.5%.

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