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Original Paper

The application of dynamic clustering data to the sedimentation rates of concentrated suspensions

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Abstract—A modified Stokes equation is proposed for the settling of concentrated suspensions of spherical particles in Newtonian fluids. The proposed hindered settling model was derived semi-analytically incorporating data obtained on dynamic particle clustering in concentrated suspensions by Graham and Bird. The approach differs from previous models which either neglected the influence of one particle on another or treated these interactions in an empirical manner with minimal physical or mechanistic basis for their design. The phenomenon of dynamic clustering which has recently been observed experimentally and shown to occur in flowing suspensions and sedimentation provides the basis for this study. Cluster size distributions are estimated based on suspension properties and a summation of Stokes terms provides a settling velocity prediction of concentrated systems. The validity of the model is supported by comparing calculated sedimentation rates with measured values for a number of data sets obtained from a variety of different authors.

NOMENCLATURE

a	radius of sphere
C_i	multivariate regression coefficients
d	diameter of sphere
D	diameter of vessel in the Richardson–Zaki exponent
g	gravitational constant
$\bar{g}$	mean cluster size
g_i	probability of a cluster of size i occurring
i	cluster size integer index
K	single sphere correction factor from Eq. (8)
n	Richardson–Zaki model exponent
N	largest cluster size for a given suspension
Re	Reynold's number, equal to $dv\rho_s/\mu_f$
$\mathrm{Re}_{\mathrm{HFA}}$	Reynolds number in homogeneous flow apparatus, equal to $a^2\dot{\gamma}\rho/\mu$
S_i	dynamic cluster shape coefficients
$\bar{S}$	average suspension shape coefficient

Greek

$\dot{\gamma}$	shear rate
ε	suspension void fraction
μ_{f}	viscosity of fluid
μ_{r}	reduced viscosity coefficient
μ_0	viscosity of suspension

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v	velocity of sphere relative to fluid
v_s	overall suspension settling rate
ρ_f	density of fluid
ρ_s	density of sphere
ρ_ϕ	suspension density
ϕ	suspension concentration (volume fraction solids)
ϕ_{eff}	effective suspension concentration including bound liquid
Φ	Steinour model concentration functions
χ	Batchelor sedimentation model coefficient
χ_{ij}	Batchelor polydisperse sedimentation model coefficient

1. INTRODUCTION

The kinetics of sedimentation of solid particles in liquid media and the characteristics of the resulting sediments are of wide applicability in industry, relevant in such activities as ore processing, chemical and ceramics manufacturing, and waste water treatment. The design of settling units typically requires some means of predicting the sedimentation behavior of concentrated suspensions.

The objective of the present study was to investigate the influence of a phenomenon identified as dynamic clustering [1, 2] on the sedimentation behavior of concentrated suspensions of spherical particles in Newtonian fluids. The treatment extends from the sedimentation of single particles following Stokes law to cases of more concentrated suspensions where a simple model simulating dynamic clustering effects was implemented. A modified form of Stokes law is then developed incorporating data collected from observations of particle clustering. A further objective of the present work was to provide a model with a mechanistic basis for the sedimentation behavior of concentrated suspensions. Existing models (see discussion in Section 1.2) have not to this point, incorporated clustering phenomena as a mechanistic basis for the observed macroscopic behavior.

1.1. Theoretical framework

The terminal falling velocity v_0 of a particle in a fluid is obtained by equating the viscous drag to the effective gravitational force. The settling velocity of a single sphere in an infinite fluid is given by equation 1, which is well known as Stokes Law.

$$v_0 = \frac{d^2(\rho_s - \rho_f)g}{18\mu_f} \tag{1}$$

The above result is valid for $\mathrm{Re} \ll 1$, since convective acceleration terms were omitted from its derivation. In practice however, Stokes law is often accurate up to about $\mathrm{Re} = 10$. The Reynolds number is a dimensionless ratio of the inertial and viscous forces at work for a fluid under shear. For this case of a settling particle in a fluid it is given as,

$$\mathrm{Re} = \frac{dv\rho_s}{\mu_f} \tag{2}$$

1.2. Previous hindered settling models

Hindered settling functions describe the reduction in the particle settling velocities with increasing suspension concentration due to particle–particle interactions and/or arising from the fluid upflow created by the downward settling of particles. Numerous theoretical and empirical models of settling have been previously proposed in the literature. Stokes law (1) predicts the terminal settling velocity for one spherical particle in an infinite fluid medium. As suspension concentration increases (above 1% by volume), the settling velocity is hindered. A paper by Steinour [3], which studied the sedimentation of concentrated flocculated suspensions, considered that the settling velocity of a suspension could be represented as a function of particle concentration. The effect of concentration was to prescribe the suspension density and viscosity as well as the void fraction ε (tantamount to the liquid volume fraction) to account for the shape and size of flow spaces. Thus for concentrated suspensions of spherical particles the following modified form of Stokes law was determined for the motion of particles relative to the fluid.

$$v = \frac{d^2(\rho_\phi - \rho_f)g}{18\mu_f}\Phi\varepsilon \tag{3}$$

Above, ρ_ϕ is the overall density of the suspension. Experimentally determined explicit forms of the function Φ were given by Steinour for different Reynolds number regimes.

Richardson and Zaki [4] proposed a model based on the case of uniform particles falling with equal velocity. Each particle was assumed to displace the same volume of liquid, thereby making the effective buoyant force independent of the suspension density. They considered the sedimentation rate of the suspension relative to a fixed reference frame and re-expressed Eq. (3) as,

$$v = \frac{d^2(\rho_\phi - \rho_f)g}{18\mu_f}\varepsilon^2 \tag{4}$$

Dimensional analysis of the system yielded a relation for the sedimentation and fluidization of spherical particles upon which they proposed the equation,

$$v = v_0\varepsilon^n \tag{5}$$

where n is a constant. Plots of log v against log ε allow the calculation of n. For a Reynolds number smaller than 0.2 and having negligible inertial forces, the index n was found to be,

$$n = 4.65 + 19.5\frac{d}{D} \tag{6}$$

Batchelor [5] derived a model for the case of a dilute dispersion of identical particles. It was found that the average velocity relative to the containing vessel is given by,

$$v = v_0(1 + \chi\phi + O(\phi^2)) \tag{7}$$

and the sedimentation coefficient χ, has the value of -6.55 for rigid spherical particles which exert no direct force on each other except by touching. The term $O(\phi^2)$ represents neglected second order coefficients for ϕ. The reduction in sedimentation velocity was attributed to hydrodynamic interactions between pairs of particles and as such this model was accurate to the first order in particle concentration. The model devised for monodisperse systems was later expanded to consider polydisperse interacting systems [6] and provided the basis for other subsequent research.

Observations by Ham and Homsy [7] identified a cluster induced back flow effect giving local sedimentation rates which fluctuate about a mean. These fluctuations give rise to a dispersion process, 'self-induced hydrodynamic diffusion', which occurs during sedimentation. Their diffusivity term was found to vary with suspension concentration [8] for sedimenting non-colloidal particles.

Swanson [9] developed a model to predict free and hindered settling for any size or shape of particle in any fluid. Development of the model began with the assumption that the settling rate of a particle is influenced by the immobilized layer of fluid which accompanies the particle. It was determined that other factors also influenced hindered settling of the particles such as the volume fraction of solids in a suspension, apparent viscosity of the suspension and the effects of backflow (i.e. the displacement of fluids by the particles).

The Swanson model is more akin to a multi-condition algorithm which determines a number of coefficients and factors related to the system at hand. The general form, however, makes use of eight parameters: ρ_s, ρ_f, d, μ_f, ϕ, ϕ_{max} (the maximum possible solid volume fraction for a given system) and two coefficients associated with the boundary layer of fluid accompanying a particle. The Swanson model considers bound liquid accompanying settling particles which in effect is very similar to the theory of dynamic clustering where bound liquid becomes an important consequence of cluster behavior in concentrated suspensions. It should be noted, however, that for the case of hindered settling of spheres, Swanson's model relies heavily on several system-related parameters determined with substantial complexity.

1.3. Dynamic clustering phenomena

It has been suggested for some time [10] that the purely hydrodynamic interaction arising in a suspension undergoing shear will produce a transient situation where individual particles are formed into short-lived clusters. Indeed, theoretical models of the motion of spheres in a suspension suggest that transient or periodic cluster formation will occur [11]. Refinements of the Tory and Pickard model have been presented [12, 13], and these more recent results conclude that more compact cluster configurations are favored.

Experimentally, it has been shown that the sedimentation process evolves via the formation and disintegration of clusters [14]. Graham and Bird [1] conducted homogeneous shear flow experiments and observed that particles in both dilute and concentrated suspensions form clusters that are continually created and destroyed. These clusters translate and rotate as the suspension is sheared. Using a homogeneous flow apparatus, they measured directly the frequency distributions of sizes of clusters of spheres in dilute suspensions and obtained partial information on concentrated suspensions from observations of clusters of labeled spheres in transparent suspensions. Both volume concentration and Reynolds number were found to have an effect on the cluster-size distribution in sheared suspensions, thus:

$$g_i = f(\mathrm{Re}, \phi)$$

As concentration increased the average cluster size increased substantially, whereas it decreased with increasing Reynolds number.

Ham and Homsy [7] observed non-uniform sedimentation in concentrated systems and expressed this as a dispersion coefficient. They concluded that hydrodynamic interactions rather than Brownian diffusion were governing the behavior of these systems. Further, the lack of fit of their data with earlier models by Batchelor [5, 15], led them to deduce that multi-body interactions were an essential part of the mechanism leading to the random walk of a sedimenting particle.

Clustering has also been observed and modeled in suspensions under pressure-driven shear conditions [16, 17]. The phenomenon of particle cluster formation was found to be of great importance for many applications in liquid–solid and gas–solid suspension systems, such as sedimentation and fast fluidization. Chen *et al.* [16] conducted experiments in a two-dimensional, liquid–solid transport bed to study the mechanisms of particle cluster formation and disintegration. It was found that when a cluster forms, particles organize and gather in a vertical direction, while when clusters disintegrate, the clusters rotate to a horizontal position and gradually disperse into individual particles and/or small clusters.

Chen *et al.* [16] also state that the precise mechanisms of the continuous formation and disintegration of clusters in sedimentation and fluidization systems at low Reynolds number ranges (of the order of 0.01) are still uncertain. What is known is that a particle in a concentrated suspension is subject to two hydrodynamic forces which are different from the case of a single-particle system. These include the Stokes drag which is reduced due to the presence of other particles and the line-of-center force which acts in the downward direction along the line joining the centers of any two particles. Thus for a suspension of more than three particles, clusters may form in the Stokes regime merely due to these two hydrodynamic forces. It should also be noted that for very small particles, clusters may also be formed due to flocculation or aggregation, whereby electrostatic and Van der Waals forces come into play. However, these latter phenomena are beyond the scope of the present study.

At higher Reynolds numbers where the inertial term is not negligible, the 'wake' effect plays an additional role in cluster formation. When two identical particles fall in vertical alignment, the upper one falls faster and is accelerated to a greater extent, due to the wake effect of the lowe particle. At the intermediate range of Reynolds number (an order of 0.01 to 100) evidence of particle cluster formation exists where the wake effect is stronger than the other two effects, i.e. Stokes drag reduction and the line-of-center force. The wake effect enhances cluster formation by promoting a vertical alignment for a group of particles and draws them toward the leading particle. Too much of the wake effect, however, hinders cluster formation, usually for $Re \gg 100$, where colliding particles produce impacts strong enough to break down clusters as soon as they form.

The body of literature suggests that indeed, it would be appropriate to incorporate clustering phenomena into a model for the prediction of sedimentation rates.

2. PROPOSED HINDERED SETTLING MODEL

Most previous hindered settling models either neglect the influence of one particle on another or treat these interactions in a way which does not necessarily reflect any mechanistic basis. For example, Richardson and Zaki assumed uniform particles falling with equal velocity where each particle displaces the same volume of fluid. For concentrated systems, however, inter-particle hydrodynamic effects ensure that not all particles will fall at the same rate. The variation in fall velocities can be attributed to hydrodynamic interactions based on the local microstructure or arrangement of particles in the suspension. A close pair or cluster of particles will fall faster than the average, whereas a relatively isolated particle could be caught in the fluid backflow and fall more slowly than the average. The present model takes into account both particle concentration and the clustering phenomena (explained in more detail in Section 1.3) to produce a workable model for predicting mean settling velocity.

As observed by Graham and Bird [1], a suspension of spheres under shear conditions is composed of a cluster size distribution, g_i, consisting of single spheres ($i = 1$), clusters of two spheres ($i = 2$), clusters of three spheres and clusters of four or more spheres. Thus equation 4 can be re-expressed to give the individual component velocities of each cluster size i where,

$$v_i = K\frac{d_i^2(\rho_i - \rho_f)g}{18\mu_0}(\varepsilon_{\text{eff}})^2 \tag{8}$$

where: d_i is the effective diameter of cluster size i, ε_{eff} is the effective void fraction $(1 - \phi_{\text{eff}})$ of suspension and ρ_i is the effective density of cluster size i. If the fraction g_i of each cluster size i in the suspension can be determined then the mean settling velocity, v_s, of the suspension is taken as the weighted sum of the individual component velocities,

$$v_s = \sum_{i=1}^{i_{\max}} g_i v_i \tag{9}$$

The model proposed in Eq. (9) is constructed so as to reduce to Stokes law in the limit of $\phi \to 0$.

2.1. Probability distribution of cluster sizes

Graham and Steele [18] reconstructed estimates of the cluster size distribution in concentrated suspensions using partial information from measurements of cluster sizes in an experimental system of suspended spheres. The experiment featured a suspension of a colorless and transparent liquid as well as solid spheres of an identical index of refraction, making them thus invisible. Additionally, a certain number of opaque spheres were included and observations were made on this partial information. The clustering of the particles was observed in suspensions subjected to homogeneous shear flow under such conditions that only hydrodynamic forces exerted an appreciable effect. Probability distributions of the sizes of clusters of spheres were used to summarize the particle cluster data from a total of over 12 000 sphere observations. The statistical inference employed partial experimental information in combination with techniques derived from information theory [19, 20] and the Maximum Entropy Principle [21, 22] to reconstruct the least biased estimate of the cluster size distributions.

It should be noted that to test the effects of various assumed packings on the inferred results, a least-dense (string) packing was compared with a maximum density (hexagonal close-packed) packing. The close packed arrangement was found to be the most representative of the true physical situation for the modeling of suspension viscosities [2]. This assumption was confirmed by the analysis of Hesse [12].

Table 1 given below summarizes the results of Graham and Steele [18]. Further, for cases where $\phi = 0$, $\bar{g}$ can be set to 1.0.

Note that the Reynolds numbers originally given in the Graham and Steele paper [18], were defined for their homogeneous flow apparatus as:

$$\text{Re}_{\text{HFA}} = \frac{a^2\dot{\gamma}\rho}{\mu}$$

where a is the particle radius and $\dot{\gamma}$ is the shear rate in the flow apparatus. The term $\dot{\gamma}$ is generally taken in shear thinning falling ball rheometry as v/a. Thus, the conventional particle Reynolds number ($\text{Re} = Dv\rho_s/\mu_f$) is equal to 2Re_{HFA}. The Reynolds numbers given in Table 1 are the conventional particle Reynolds numbers, and they are used throughout this paper.

Table 1.
Summary of experimental results of Graham and Bird [1].

Parameter	Experiment no.					
	1	2	3	4	5	6
Volume percent spheres, ϕ, %	1.0	1.0	1.0	10.0	20.0	20.0
Sphere Reynolds number, Re	0.0220	0.0544	0.0842	0.0466	0.0506	0.1348
Average cluster size, $\bar{g}$	1.21	1.19	1.16	2.45	5.96	5.36
Probability of a cluster of one sphere, g_1	0.872	0.855	0.874	0.409	0.271	0.282
Probability of a cluster of two spheres, g_2	0.086	0.104	0.097	0.204	0.135	0.141
Probability of a cluster of three spheres, g_3	0.021	0.027	0.023	0.136	0.090	0.094
Probability of a cluster of four or more spheres, g_{4+}	0.021	0.014	0.006	0.251	0.504	0.483

The above probability distributions allow for the calculation of multivariate regression equations to predict a cluster size distribution (g_1, g_2, g_3, g_{4+}), as functions of two independent variables, Re and ϕ. The probability of a cluster size i occurring in a given system can be calculated using the function:

$$g_i = C_1 + C_2\,\mathrm{Re} + C_3\phi + C_4\phi\,\mathrm{Re} + C_5\,\mathrm{Re}^2 + C_6\phi^2 \tag{10}$$

Likewise, coefficients to determine $\bar{g}$ can be calculated. The calculated regression coefficients of C_1, C_2, C_3, C_4, C_5 and C_6 for each cluster size and the average cluster size are listed in Table 2. A response surface plot of the average cluster size is shown in Fig. 1. A four-term polynomial (where $C_5 = C_6 = 0$) produced the most plausible results for the probability distributions at both high and low concentrations compared with higher-order functions. Using a higher-order model, however, would have then limited this study to include only experimental sedimentation data from suspensions of concentrations no greater than 20 vol.% solids, which was the limit of the experimental data.

Now the fraction of each cluster size, f_i, for a particular suspension can be determined by:

$$f_i = \frac{g_i}{\sum_{j=1}^{4+} g_j} \tag{11}$$

Theoretically, $\sum_{j=1}^{4+} g_j = 1$, but since the four cluster size probabilities were determined independently (and numerically) the normalization procedure of Eq. (11) was adopted.

Table 2.
Multivariate regression coefficients for cluster size distributions.

Cluster size distribution	C_1	C_2	C_3	C_4	C_5	C_6
g_1	0.8990	0.5891	−3.8255	2.6525	0.0000	0.0000
g_2	0.0830	−0.4566	0.6187	−0.1901	0.0000	0.0000
g_3	0.0247	−0.1450	0.5891	−1.1248	0.0000	0.0000
g_{4+}	−0.0065	0.0250	2.6177	−2.6751	0.0000	0.0000
$\bar{g}$	1.0496	1.9835	4.5003	−28.7263	−17.6521	105.9617

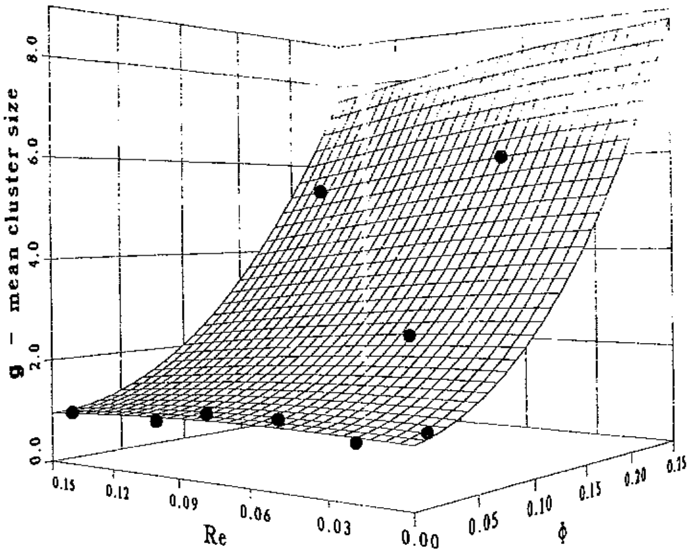

Figure 1. Mean cluster size $\bar{g}$ as a function of Re and ϕ.

2.2. *Effective suspension concentration*

For one spherical particle settling in an infinite fluid medium and in the absence of inertial effects, the Stokes equation for settling velocity is generally valid. However, in order to describe the sedimentation of particles in concentrated suspensions, many authors have modified Stokes theory to take into account mechanical interactions between particles. A paper by Vand [10] originated the concept of immobilized liquid in clusters. In concentrated suspensions of spherical particles, it is known that a quantity of immobilized fluid may be associated with particles undergoing shear [2], or specifically in this case, during sedimentation. This immobilized liquid would then contribute to an 'effective' volume fraction, ϕ_{eff} of spheres, which is greater than the actual solid volume fraction of the spheres, ϕ. This concept is illustrated schematically in Fig. 2.

When considering the effective volume fraction of sphere clusters, a shape factor is assigned to each class of cluster which is proportional to the volume of spheres. Assuming

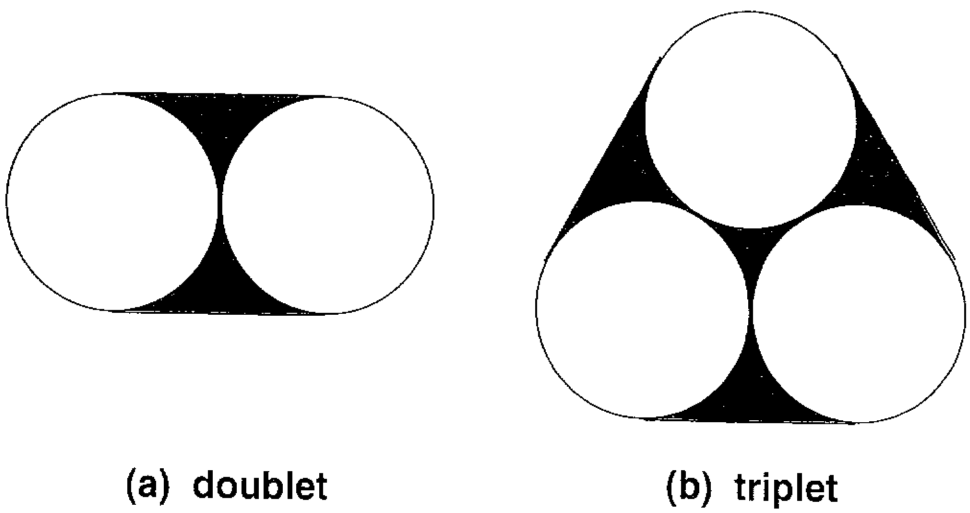

Figure 2. Effective volume in a doublet or triplet of spheres. The shaded area represents the immobilized liquid in the cluster of spheres which contributes to calculating the shape factors, S_i.

a single hexagonal-close packed arrangement of spheres the effective volume for each cluster size can be calculated by multiplying the initial volume by the appropriate shape factor, where:

$$S_1 = 1.00, \qquad S_2 = 1.250, \qquad S_3 = 1.333, \qquad S_{4+} = 1.350$$

Subscripts denote cluster size. For clusters of four or more particles (as denoted by the subscript 4+) the shape factor is essentially constant since with the assumed hexagonal packing, the absolute size of the cluster can vary without changing the proportion of bound liquid. Thus if the distribution of cluster sizes within a suspension can be calculated, the average shape factor $\bar{S}$ for the experiment is given by,

$$\bar{S} = \sum_{i=1}^{N} S_i f_i \tag{12}$$

where S_i is the shape factor for cluster size i in the suspension. Graham *et al.* [2] observed a small decrease in $\bar{S}$ as Re increased
and a rapid increase in $\bar{S}$ as ϕ increased from 0 vol.% to 20 vol.%.

The effective volume fraction of solids in the suspension can thus be calculated by,

$$\phi_{\text{eff}} = \bar{S}\phi \tag{13}$$

and the effective void fraction, ε_{eff}, is given by,

$$\varepsilon_{\text{eff}} = 1 - \phi_{\text{eff}}$$

3. IMPLEMENTATION OF THE THEORETICAL MODEL

In order to test our proposed hindered settling function (Eq. 9), we utilized previous hindered settling data obtained for monodisperse suspensions by a variety of investigators. All calculations were made using raw data available in literature. A total of 17 different data sets were used (see Section 3.2).

3.1. *Basic assumptions*

When presenting a hindered settling model, certain assumptions must be made about the type of suspension under consideration. Barnea and Mizrahi [23] provided a set of basic assumptions in their approach to particulate systems which can be applied to this study:

(a) The present discussion deals with systems of spherical particles, of common specific gravity, with a relatively narrow size distribution which can be reduced to an average characteristic size. Polydisperse suspensions are beyond the scope of this paper.
(b) There are no interactions of any kind between the particles, except hydrodynamic effects inducing dynamic clustering effects throughout the fluid. Surface chemical and colloidal body forces which induce flocculation and aggregation are beyond the scope of this paper.
(c) The relative positions of the particles in the suspension are completely random, without any segregation.
(d) The effect of the walls of the container can be neglected so that the concentration of particles is constant at any cross section in the vessel.
(e) Solid particles and the fluid in the suspension are essentially incompressible.

3.2. *Experimental data*

Listed below in Table 3 are the sources and properties of the raw data used to test the proposed hindered settling model.

Table 3.
Raw experimental data used for testing the hindered settling model.

Source	Material	Fluid	d (μm)	T (C)	μ_f (kg/ms)	ρ_s (kg/m^3)	ρ_f (kg/m^3)	K
Chhabra *et al.* [24]	glass beads	castor-oil	1300	27	0.603	2620.0	965.0	0.7823
Chhabra *et al.* [24]	glass beads	castor-oil	640	27	0.603	2620.0	965.0	1.1761
Yan and Masliyah [25]	glass beads	water	29	25	8.94×10^{-4}	2530.0	1000.0	0.4328
Yan and Masliyah [25]	glass beads	oil–water	29	25	2.00×10^{-3}	2530.0	1000.0	2.0606
Turian *et al.* [26]	coal	water	99.8	25	8.94×10^{-4}	1338.7	1000.0	0.3737
Oliver [27]	kalloduc spheres	water	64.5	25	8.94×10^{-4}	1911.0	1000.0	0.1677
Oliver [27]	kalloduc spheres	water	161	25	8.94×10^{-4}	1911.0	1000.0	0.1589
Whitmore [28]	methyl-methacrylate	$Pb(NO_3)_2$ (aq.)	95.4	20	1.046×10^{-3}	1188.1	1055.8	0.7248
Whitmore [28]	polystyrene spheres	$Pb(NO_3)_2$ (aq.)	98.2	20	1.024×10^{-3}	1055.3	1026.6	0.7541
DeVaney and Shelton [29]	magnetite	water	15.8	25	8.94×10^{-4}	5180.0	1000.0	1.1891
DeVaney and Shelton [29]	magnetite	water	25.7	25	8.94×10^{-4}	5180.0	1000.0	0.4924
DeVaney and Shelton [29]	magnetite	water	37.6	25	8.94×10^{-4}	5180.0	1000.0	0.3879
DeVaney and Shelton [29]	magnetite	water	51.7	25	8.94×10^{-4}	5180.0	1000.0	0.2225
DeVaney and Shelton [29]	galena	water	38.3	25	8.94×10^{-4}	7400.0	1000.0	0.1696
DeVaney and Shelton [29]	galena	water	56.0	25	8.94×10^{-4}	7400.0	1000.0	0.0815
Richardson and Zaki [4]	glass particles	water	154.94	25	8.94×10^{-4}	2932.0	1000.0	0.0146
Richardson and Zaki [4]	divinyl benzene	water	253	25	8.94×10^{-4}	1058.0	1000.0	0.8279

3.3 Calculation Procedure

The calculations were made by a numerically implemented procedure. Data consisted of experimental results of measured settling velocities at different concentrations from a variety of investigators. A value for the correction factor K (from Eq. 8), was determined either directly from the data or if unavailable, an extrapolation of the data was used to calculate a zero concentration sedimentation velocity, v_{max}. The deviation of v_{max} from the Stokes velocity (v_0) was then used as a correction factor in the proposed hindered settling function. A best estimate for K was calculated as,

$$K = \frac{v_{max}}{v_0} \tag{14}$$

The characteristic properties (i.e. μ_f, d, ρ_s, ρ_f) of the suspension for each experiment were obtained from the literature. For each data set a sequence of calculations was performed as outlined below:

ALGORITHM

(i) The settling rate, v_j, at an initial solid concentration, ϕ_j for the jth experimental concentration value was determined from the literature.

(ii) A mean Reynolds number was calculated iteratively by,
  (1) Beginning with Re = 0, determining $\bar{g}(\overline{\text{Re}} = 0, \phi)$ from Eq. (10).
  (2) Calculating $\bar{d}$ using an equivalent spherical volume from $\bar{g}$.
  (3) Substituting into the equation,

$$\overline{\text{Re}} = \frac{\bar{d} v_j \rho_s}{\mu_f}$$

  (4) Determining $g(\overline{\text{Re}}, \phi)$. When $(\bar{g}_{new} - \bar{g}_{old}) < 0.1$ the iteration was considered converged.
  (5) $\overline{\text{Re}}$ was finalized using the updated data.

(iii) The distribution fraction, f_i, of each cluster size and the effective void fraction, ε_{eff}, were determined through Eqs (11) and (13), respectively.

(iv) The absolute viscosity, μ_0, was determined as a function of concentration by multiplying the liquid viscosity by a reduced viscosity coefficient μ_r where,

$$\mu_0 = \mu_r \mu_f$$

A numerical relation for μ_r as a function of concentration was taken from the results of Mondy *et al.* [30].

(v) For each concentration, effective densities and effective diameters were determined for each cluster size i. The effective densities were calculated using the appropriate shape factor for each cluster size where,

$$\rho_i = \frac{\rho_s + (S_i - 1)\rho_f}{S_i}$$

Cluster diameters were estimated using an equivalent spherical diameter.

(vi) Finally, the component velocities for each cluster size were calculated using the hindered settling model and a comparison was made between the experimental settling velocities from the data and the net calculated suspension settling velocities from the model.

Table 4.
Experimental data and model calculations (data from Oliver [27]: kalloduc spheres in water, $\bar{d}$ = 161 μm).

ϕ	ϕ_{eff}	v_s (expt.)	v_s (calc.)	$\frac{v_s(\text{calc.})}{v_s(\text{expt.})}$	Re
0.05	0.0506	1.73×10^{-3}	1.71×10^{-3}	0.991	5.96×10^{-1}
0.10	0.1032	1.42×10^{-3}	1.33×10^{-3}	0.933	4.89×10^{-1}
0.15	0.1590	1.18×10^{-3}	1.05×10^{-3}	0.893	4.06×10^{-1}
0.20	0.2233	8.03×10^{-4}	8.73×10^{-4}	1.088	2.77×10^{-1}
0.25	0.2972	5.90×10^{-4}	6.17×10^{-4}	1.045	2.03×10^{-1}
0.30	0.3828	4.46×10^{-4}	4.01×10^{-4}	0.899	1.54×10^{-1}
0.35	0.4432	3.06×10^{-4}	2.57×10^{-4}	0.840	2.17×10^{-1}

Table 4 shows a sample listing of the results produced by the calculation procedure and compares them to the experimental data.

3.4. Results and discussion

Data sets from a total of seven different papers were used in this study. A variety of different particle sizes and densities as well as a variety of different Newtonian liquids were used to test the validity of the model. In most cases experimental data agreed with calculated results and plots comparing the calculated velocities against the measured sedimentation rates are given in Figs 3–10. Most predictions fell to within 15% of the experimental data.

Although the Stokes equation is valid for Re $\ll$ 1, the zero concentration Reynolds numbers in this study ranged from 1.28×10^{-2} to 14.36 and both extremes agreed with

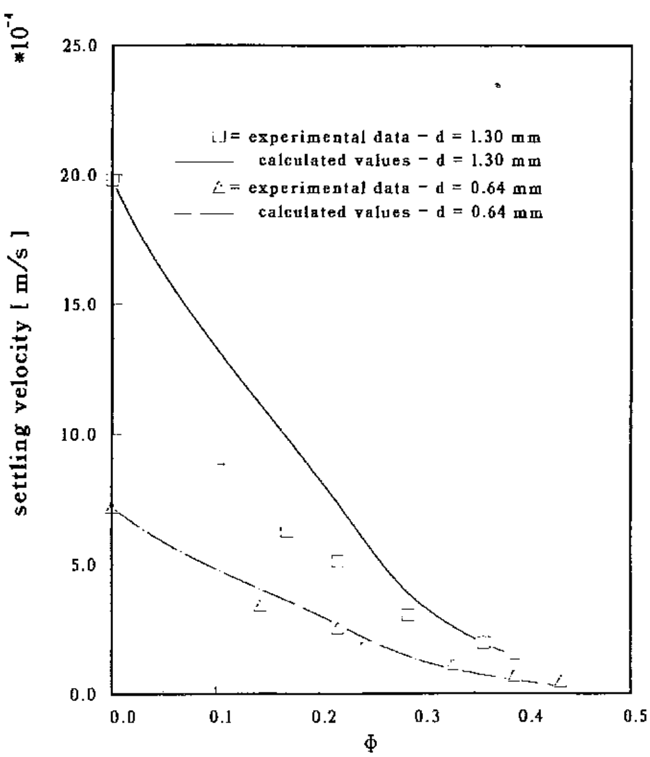

Figure 3. Sedimentation results of Chhabra *et al.* [24] for glass beads in castor oil.

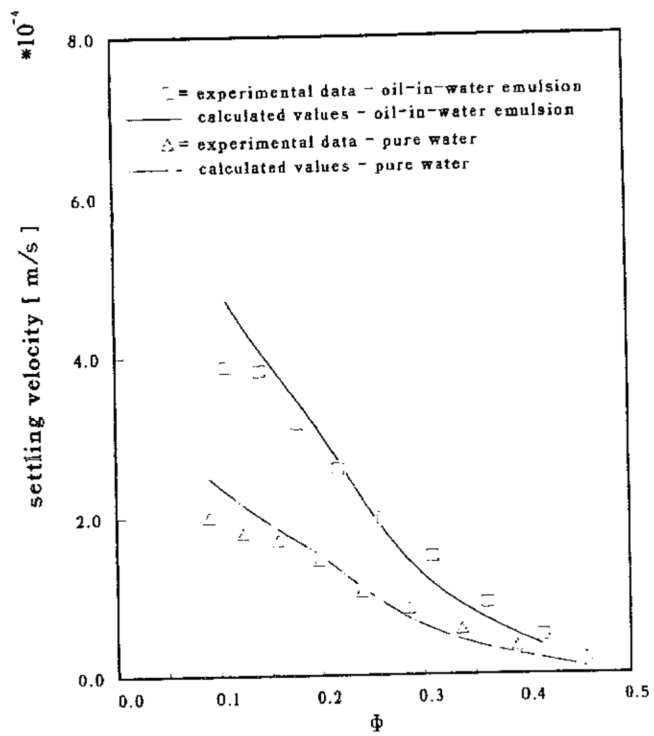

Figure 4. Sedimentation results of Yan and Masliyah [25]. Glass beads were tested in water and aqueous emulsion systems.

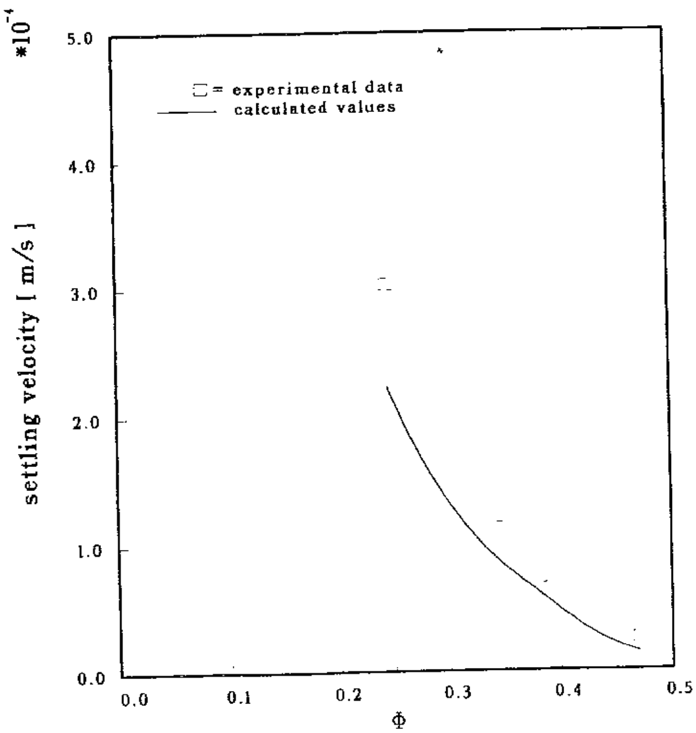

Figure 5. Sedimentation results of Turian *et al.* [26] for coal slurries.

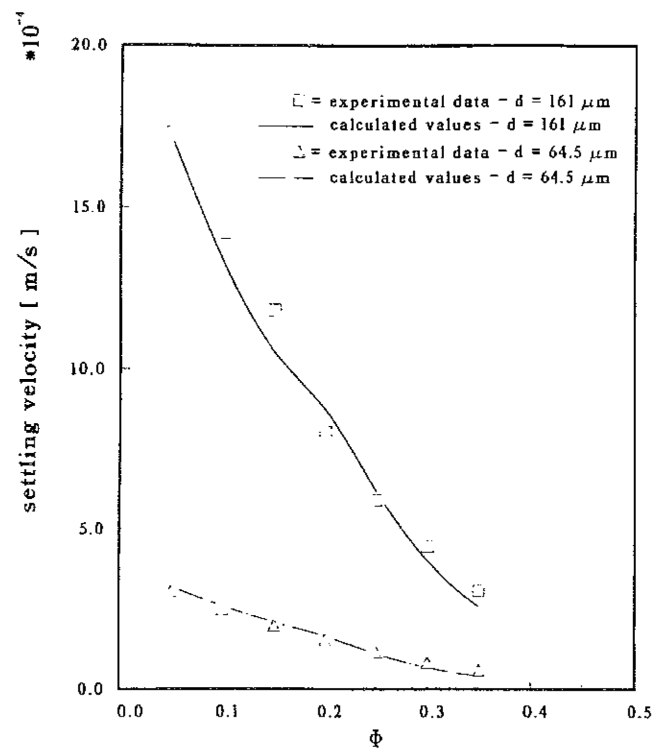

Figure 6. Sedimentation results of Oliver [27] for kalloduc spheres in water.

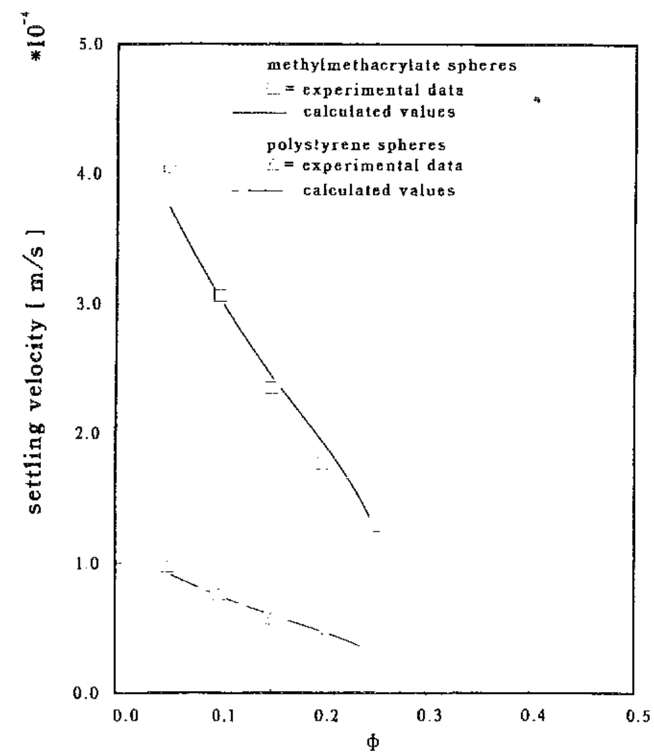

Figure 7. Sedimentation results of Whitmore [28]. Methylmethacrylate and polystyrene spheres were tested in aqueous solutions of lead nitrate.

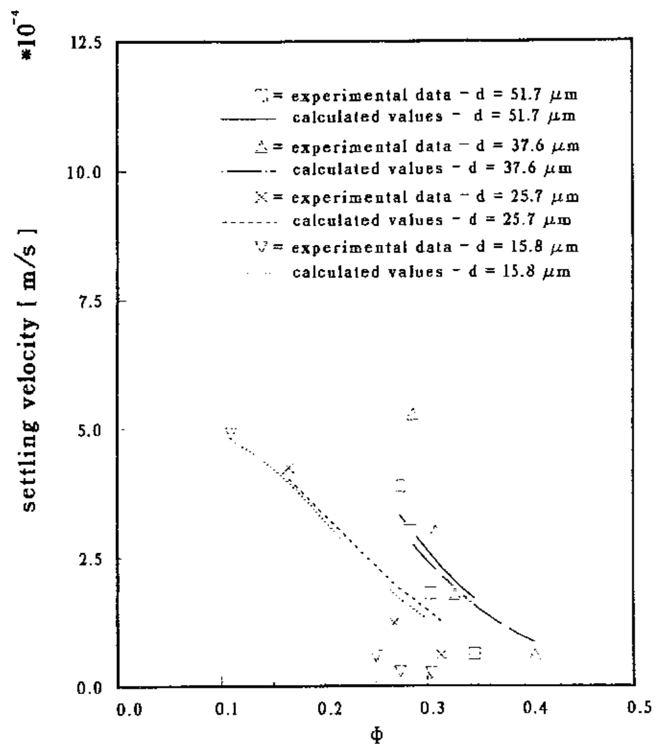

Figure 8. Sedimentation results of DeVaney and Shelton [29] for magnetite particles in water.

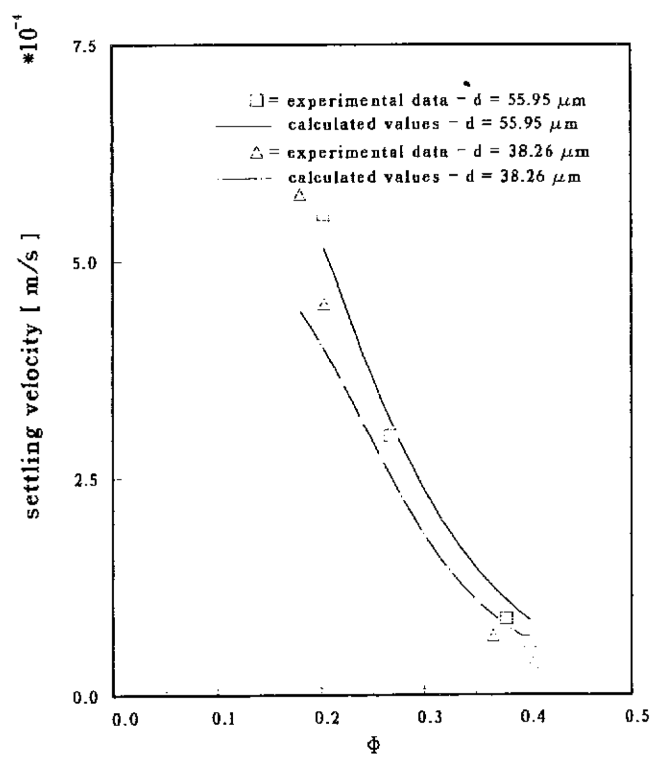

Figure 9. Sedimentation results of DeVaney and Shelton [29] for galena particles in water.

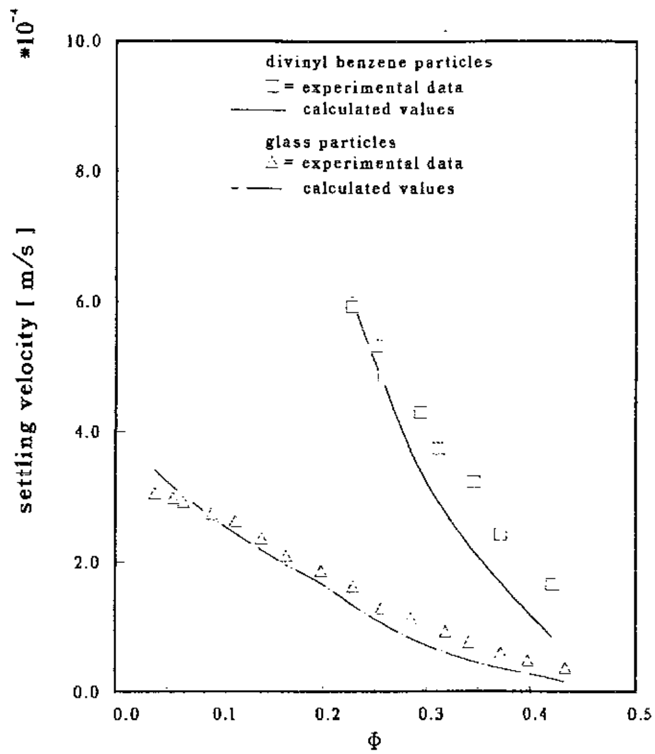

Figure 10. Sedimentation results of Richardson and Zaki [4]. Divinyl benzene particles and glass particles were tested in water.

the model. The calculated Reynolds numbers for the concentrated systems were in the range of 0.001–0.1 thus well within the Stokes regime.

The model did not make accurate predictions for systems which appeared to be flocculated or aggregated. In general, since the additional flocculation would be caused by surface or colloidal forces, the cluster sizes predicted on a purely hydrodynamic basis would be smaller than those occurring in reality. This extra complexity is not accounted for in the present model. Figure 11 shows some sedimentation data of small silica particles taken from Egolf and McCabe [31]. The present model (Eq. 9) underestimates the sedimentation velocities since the assumed cluster sizes are much smaller than those which arise in a flocculated system. Typically, a Richard–Zaki exponent (n in Eq. 5), of about 4.6 is found for sedimentation of colloidally disperse suspensions. When certain data sets produced higher values, this difference was shown to be attributable to flocculation and a corresponding inclusion of bound liquid by Fouda and Capes [32]. The data of Egolf and McCabe was indeed shown by Fouda and Capes to correspond to a flocculated system. In view of the present analysis, the Richard–Zaki exponent can be seen to be a measure of the degree of structure in the suspension. The smaller n becomes, the more continuous the clustering becomes. The present model considers only hydrodynamically induced clusters and it is expected that it should not properly model data for suspensions known to be flocculated. Mechanistically, for flocculated systems, the degrees of freedom for cluster motion are reduced and thus these flocs cannot align themselves according to the prevailing hydrodynamics in the same way that particles in a disperse system would.

Figures 3–10 showed consistent results where diameter ranges were from 29 μm to 1300 μm. Experimental data taken from DeVaney and Shelton [29] in Figs 8 and 9 showed satisfactory results for the settling of magnetite and galena particles where highly

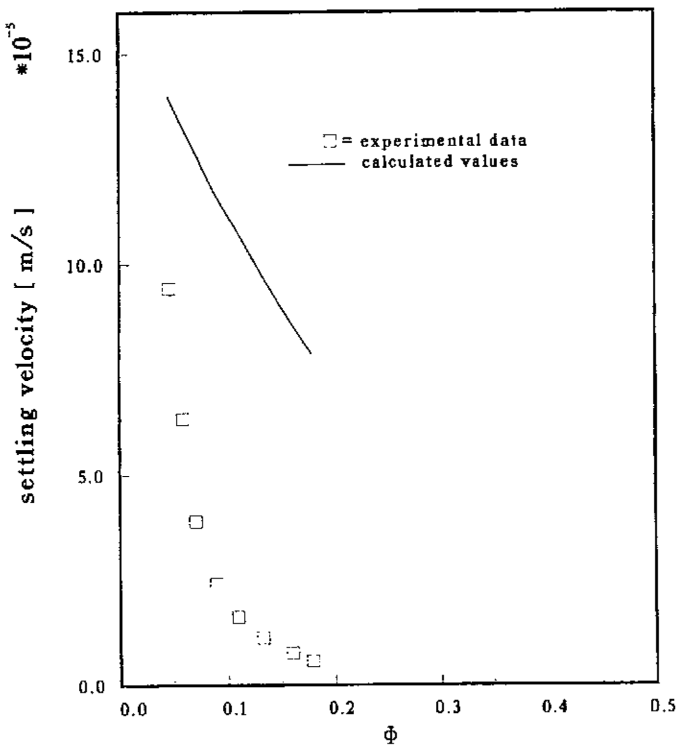

Figure 11. Sedimentation results of Egolf and McCabe [31]. Silica particles of 10.3 μm mean diameter were tested in water. This system was shown to have undergone colloidal flocculation, hence the poor fit of the model.

concentrated suspensions were used. Here diameters ranged from 15.8 μm to 56.0 μm. The smallest diameter of 15.8 μm produced the worst results from the data of DeVaney and Shelton. With particles this small, surface interactions would likely cause flocculation thereby making the data inappropriate for treatment with the model contained in Eq. (9). In view of all the results shown in the plots, it could be suggested that the mechanistic validity of the present model accounts for its following the curve patterns of the data for the various cases considered.

A point to note concerns the precision of the model. From the probability data of Graham and Steele [18], only six data points with suspension concentrations ranging from 0 to 20% were used as a basis for the regression curves. The lack of more detailed clustering data necessarily limits the accuracy of the model predictions, and this would be especially pronounced at concentrations above 20 vol.%, as the clustering behavior at higher concentrations was extrapolated. If a larger number of points with a greater concentration range existed the model would likely produce more accurate results especially at higher concentrations. Despite the limited cluster size distribution data, it has been clearly demonstrated that the model contained in Eq. (9) has been proven adaptable and accurate for all types of disperse sedimenting systems. The model is based on physical mechanisms, i.e. the application of the dynamic clustering phenomena to viscous settling, and required only a zero concentration (single-particle) sedimentation experiment to provide the parameter K, to be able to predict settling behaviour up to concentrations of over 40 vol.%. The present model does thus retain a some empirical content with the factor K and the regression estimates of the cluster size distribution.

The factor K, however, could be removed from the model if the sedimentation rates were expressed as v/v_0. This is often a method employed to test the universality of a model. In this case a global plot of v/v_0 versus ϕ could be considered only with Re as a parameter. The data available did not conform to this scheme, so the plots for individual samples had to be presented.

It is interesting to note that the clustering observations made by Graham and Bird [1] were in a very different physical system than those the present model is applied to. The success of the model using the clustering data applied to a different system indicates that dynamic clustering should be a univerisal phenomenon for concentrated suspensions under shear.

Likely the most common source of error may have occurred in the calculation of the correction factor K. The majority of data sets tested did not include a measured zero-concentration (one particle in a near infinite fluid volume) settling velocity which was used to correct for deviations from the Stokes velocity. When a zero-concentration settling velocity was not available it was still possible to extrapolate one from the data which in most cases provided acceptable results. However, even this proved faulty when all data were in a high concentration range or few data points were provided which made it difficult if not impossible to extrapolate a valid zero-concentration settling velocity. This point is underlined if one examines the data of Turian plotted against model predictions in Fig. 5. The ratio of $v_{s(\text{calc.})}/v_{s(\text{expt.})}$ was near constant with a value around 0.7. Since this ratio was near constant, a very precise match between model and experiment would actually be obtained with an improved K value. In this case, the lowest suspension concentration was 0.245 volume fraction, so it is not unreasonable that the zero-concentration settling velocity would be determined with considerable error. The fact that the experimental and theoretical curves remained parallel supports the soundness of the mechanistic basis of the hindered settling velocity model.

4. CONCLUSIONS

Recent theoretical predictions and experimental findings have determined that particles in sedimenting suspensions are subject to dynamic clustering. A model was conceived which integrated dynamic clustering data into a modified Stokes law. The proposed hindered settling function gave very sound results when compared with experimentally measured values. In total, 17 different data sets were tested. The results obtained corroborate the hypothesis of clustering phenomena as a mechanism to account for the hindered settling behavior observed with disperse concentrated suspensions. Consistently, the model could predict results parallel with non-linear experimental data of various forms.

Further study of dynamic clustering would be expected to consolidate the present findings on its influence on the sedimentation properties of concentrated suspensions. Experimental difficulties and expense have limited reported cluster size distributions to systems up to only 20 vol.% solids. Cluster size distributions for more concentrated suspensions were extrapolated in this study, but it is expected that expanded cluster size distribution data should provide more accurate sedimentation rate predictions by the present method. Results for systems known to be flocculated rather than disperse produced poor predictions, as the extra complexity of hierarchical clustering and the ensuing lack of freedom of particles in flocs to conform to the prevailing hydrodynamics in the same way as in a disperse suspension, would necessarily put such systems outside the scope of the model.

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