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DEVELOPMENT OF A RADIANT HEATING AND COOLING MODEL FOR BUILDING ENERGY SIMULATION SOFTWARE

Abdelaziz Laouadi

Indoor Environment Research Program, Institute for Research in Construction,

National Research Council Canada,

1200 Montreal Road Campus, Ottawa, Ontario, Canada, K1A 0R6

Tel.: (613) 990 6868

Fax: (613) 954 3733

Email: aziz.laouadi@nrc-cnrc.gc.ca

ABSTRACT

Efficient radiant heating and cooling systems are promising technologies in slashing energy bills and improving occupant thermal comfort in buildings with low energy demands such as houses and residential buildings. However, the thermal performance of radiant systems in buildings has not been fully understood and accounted for in currently available building energy simulation software. The challenging tasks to improve the applicability of radiant systems are the development of an accurate prediction model and its integration in the energy simulation software. This paper addresses the development of a semi-analytical model for radiant heating and cooling systems for integration in energy simulation software that use the one-dimensional numerical modeling to calculate the heat transfer within the building construction assemblies. The model combines the one-dimensional numerical model of the energy simulation software with a two-dimensional analytical model. The advantage of this model over the one-dimensional one is that it accurately predict the contact surface temperature of the circuit-tubing and the adjacent medium, required to compute the boiler/chiller power, and the minimum and maximum ceiling/floor temperatures, required for moisture condensation (ceiling cooling systems), thermal comfort (heating floor systems) and controls. The model predictions for slab-on-grade heating systems compared very well with the results from a full two-dimensional numerical model.

INTRODUCTION

The use of radiant heating/cooling systems in houses has been growing in recent years. In Europe, for example, the use of radiant heating/cooling technique represents more than 50% in newly constructed houses [1-2]. The intent of radiant systems is to lower thermostat set point temperature in winter and to increase it in summer, resulting in a substantial energy savings for heating and cooling as compared with conventional systems. The energy savings from radiant heating/cooling may reach more than 30%, as demonstrated in some theoretical/experimental case studies [3-5]. However, these energy savings may not be achieved without an accurate prediction model, an appropriate control strategy and a careful design. The prediction model integrated within a building energy simulation program will aid building and system designers to specify and size energy-efficient radiant systems and to improve indoor thermal comfort

PROBLEM

Radiant heating and cooling systems are very complex systems since they involve different heat transfer mechanisms –conduction within the slab, radiation between the radiant surface and the surrounding surfaces, convection between the radiant surface and the adjacent zone air, and conduction between the slab and the ground. Integration of a radiant heating/cooling model into a building energy simulation program introduces another complexity to the modeling process of radiant systems. The radiant system has to interact with the operation of the building control system to ensure indoor thermal comfort and avoid overheating in winter and overcooling in summer. The prediction model, therefore, has to take into account the transient nature of the radiant system and the dynamic controls. The prediction model should adequately predict heat transfer from the slab to the surrounding space whether the control is ON or OFF. Radiant systems are multi-dimensional by nature. Energy simulation software that use the one-dimensional numerical modeling to compute heat flows within building structures over-predict the thermal capacity of the radiant system, and therefore result in over-sizing the radiant system with the added cost. Furthermore, such software cannot predict the cold and hot spots of the floor or ceiling surface, which may cause thermal discomfort, ceiling moisture condensation, and the moisture-related damage to the radiant surface (and repair expenses).

PREVIOUS WORKS

Extensive studies have been devoted to laboratory testing of radiant systems with the main aim to quantify the thermal performance and response of radiant systems upon using conventional or newly developed control strategies [6-11]. However, from the modeling point of view, there are a few analytical studies for the prediction of thermal performance of radiant systems. Earliest models by Kollmar and Liese [12] assumed that the slab operates as a plate fin and loses heat from its upper surface. Leal and Miller [13] used an analytical-numerical method to solve the

temperature within pavement heating installations. Zhang and Pate [14-15] developed a two-dimensional finite-difference method for ceiling panels, which then was used to develop a simplified model for radiant slab heating. Athienitis [16] developed a one-dimensional finite-difference method for floor radiant heating. Kilkis et al. [17] developed a steady-state, composite-fin model to design radiant heating and cooling systems. Kilkis and Sapci [18] and Kilkis and Coley [19] subsequently used their composite-fin model to develop software for the design of floor and sub-floor heating and cooling systems.

Integration of radiant heating and cooling models into building energy and thermal analysis software has been very limited. The ESP-r program uses the one-dimensional transient numerical modeling and a node heat injection/extraction method to model radiant heating/cooling systems. Maloney et al. [20] developed a radiant heater model for the BLAST program. Strand and Pederson [21-22] used a conduction transfer function method to develop a radiant heating and cooling model for the EnergyPLUS program. Stetiu et al. [23] developed a detailed model for cooling radiant systems for the RADCOOL software that works under the SPARK environment. Some artifices have also been employed to model radiant heating systems. Chun et al. [24] used the SERI-RES program and modeled radiant slab heating as a fictitious zone that provides heat to the conditioned zone. Miriel et al. [25] used the TRNSYS program to model a ceiling radiant panel heating and cooling system.

It should be noted that the steady-state simplified models cannot be used for transient systems and are not valid when the control system is not operating. Furthermore, the one-dimensional transient models under-predict the circuit water temperature, and therefore over-predict the heating/cooling capacity of the boiler/chiller.

OBJECTIVES

The specific objectives of this paper are:

To develop a two-dimensional prediction model for radiant systems for integration in energy simulation software that use the one-dimensional numerical modeling; and

To validate the developed model using the numerical modeling approach.

MATHEMATICAL FORMULATION

Radiant heating/cooling systems consist of tubes or resistance heating elements embedded in a concrete slab or attached to a radiant surface. The tubes can be laid out within the medium with different patterns. Tubing patterns introduce additional complexity on the prediction of heat exchanges. Two of the most used tubing patterns are serpentine and spiral. In the serpentine pattern, heat transfer is mostly transversal (perpendicular to the tubing plane) while in the spiral

pattern the heat transfer is radial. A typical radiant system may also consist of several circuits serving different surface areas.

Figure 1 shows a typical slab-on-grade heating system. For simplicity purposes, the serpentine pattern is considered in this study although the results apply to both patterns. Figure 2 shows a typical circuit serpentine pattern. The average length of all circuits is noted L_c , and the tubes outside and inside diameters are noted as D_o and D_i , respectively. The tubes are uniformly spaced with a distance w . The transport fluid enters a given circuit with an inlet temperature T_{in} , and exists with an outlet temperature T_{out} . Owing to this temperature difference, heat transfer takes place between the transport fluid and the adjacent medium.

ASSUMPTIONS

The following assumptions are used in the mathematical modeling:

- The temperature gradients along tubing (z-axis) are neglected compared to those in the perpendicular plane. This assumption holds when the ratio of the tubing spacing to circuit length is very small ($w/L_c \ll 1$). In this regard, the outside surface temperature of tubing can be considered as uniform.
- The radiant system edges are adiabatic (well insulated).
- Temperature gradients within the tubing material are negligible. This assumption holds when the tubing thickness is very small compared to the interior diameter ($D_o/D_i \approx 1$).

HEAT TRANSFER WITHIN THE RADIANT SYSTEM

Since the radiant system configuration exhibits symmetry between tubing circuit passes, only half of the symmetry domain as shown in Figure 3 is considered for the mathematical modeling. The circuit tubing is considered as heat generation sources occurring in a finite volume of the medium. The temperature of the heat source is therefore equal to the temperature of the tubing-medium contact surface. Taking into account the previous assumptions, the heat transfer in the medium is governed by the following fundamental equation:

$$\rho \cdot c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + q \quad (1)$$

where:

- c : material specific thermal capacity (J/kg.°C);
- k : material thermal conductivity (W/m.°C);
- q : heat generation rate per unit material volume (W/m³);
- T : local temperature (°C) ; and

ρ : material density (kg/m^3).

The heat generation rate, which occurs in the volume occupied by the tubing is given by the following expression:

$$q(x,y,t) = \begin{cases} q_0(t) : x_q - \Delta x \leq x \leq x_q + \Delta x, \text{ and } 0 \leq y \leq \Delta y \\ 0 : \text{elsewhere} \end{cases} \quad (2)$$

where:

q_0 : the heat injection/extraction rate per unit volume (W/m^3).

x_q : x-coordinate of the tube's center point.

$\Delta x, \Delta y$: designates the dimensions of the heat generation volume.

Considering the heat generation occurs within the volume occupied by the pipe, one obtains the following relation:

$$\Delta x = \Delta y = \sqrt{\pi} \cdot D_o / 4 \quad (3)$$

The average heat transfer rate per circuit (Q_c) injected in or extracted from the medium is calculated as follows:

$$Q_c = 4 \cdot (\Delta x \cdot \Delta y \cdot L_c) \cdot q_0 \quad (4)$$

The total heat transfer rate (Q_{tot}) injected in the medium from all circuits is expressed as follows:

$$Q_{\text{tot}} = Q_c \cdot N_c \quad (5)$$

where N_c is the number of circuits.

The number of circuits is calculated based on the radiant surface area as follows:

$$N_c = A_{\text{rad}} / (w \cdot L_c) \quad (6)$$

where A_{rad} is the area of the radiant surface.

Equation (1) is subject to different boundary conditions. At the radiant surface ($x=0$), the surface exchanges heat with the indoor environment via radiation and convection. The surface temperature may therefore fluctuate from one point to another. The realistic way to describe this boundary condition is to assume that the heat transfer rate is uniform over the radiant surface, since the uniform temperature does not hold. The uniform heat transfer rate is consistent with the fact that the convection and radiation heat transfer coefficients are calculated/correlated based on the average surface temperature, and that the local values of the convection coefficients are not available in any form of relationship. The boundary condition at the insulated surface ($x = s$) may be adiabatic, or with uniform temperature if it is in contact with the ground. In practice, the adiabatic condition may not hold. For this reason, a uniform temperature may

represent the realistic condition since the ground temperature is uniform at a relatively deep depth. Finally, the symmetry condition holds across the tubes ($y = 0$) and at the mid-distance between them ($y = w/2$). These conditions are expressed as follows:

$$x = 0 : -k \frac{\partial T}{\partial x} = q_r + h_c (\bar{T}_s - T_{air}) \quad (7)$$

$$x = s : T = T_g(t) \quad (8)$$

$$y = 0, w/2 : \frac{\partial T}{\partial y} = 0 \quad (9)$$

where:

h_c : indoor convection heat transfer coefficient ($W/m^2\text{°C}$);

q_r : radiation heat transfer density from the radiant surface to the surrounding surfaces (W/m^2);

T_{air} : indoor air temperature ($^{\circ}\text{C}$);

T_g : ground temperature ($^{\circ}\text{C}$) at $x = s$; and

$\bar{T}_s$: surface average temperature ($^{\circ}\text{C}$) at $x = 0$.

The radiation heat transfer may be accurately calculated using the view factor method. A linearized form of the radiation heat transfer may also be used as follows:

$$q_r = h_r (\bar{T}_s - T_{mrt}) \quad (10)$$

$$h_r = \sigma \varepsilon_s (\bar{T}_s^2 + T_{mrt}^2) \cdot (\bar{T}_s + T_{mrt}) \quad (11)$$

where:

h_r : radiation heat transfer coefficient ($W/m^2\text{°C}$);

T_{mrt} : mean radiant temperature of the surrounding surfaces ($^{\circ}\text{C}$);

ε_s : radiant surface emissivity; and

σ : radiation constant.

Equation (1) is difficult to solve analytically. A two-dimensional numerical method should be employed to solve for the temperature within the medium. An alternative method is proposed. The alternative method uses a one-dimensional numerical method to solve for the y-averaged temperature within the medium and an analytical solution is appended to the average values. The method is described as follows:

Averaging equation (1) over the control volume shown in Figure 3, one obtains:

$$\rho c \frac{\partial \bar{T}}{\partial t} = \frac{\partial}{\partial x} \left(k \frac{\partial \bar{T}}{\partial x} \right) + \bar{q}(x, t) \quad (12)$$

Where the y-averaged temperature and heat generation rate are given as follows:

$$\bar{T}(x, t) = \frac{2}{w} \int_0^{w/2} T(x, y, t) \cdot dy \quad (13)$$

$$\bar{q}(x, t) = \frac{2}{w} \int_0^{w/2} q(x, y, t) \cdot dy \quad (14)$$

Taking into account equation (2), equation (14) reduces to the following:

$$\bar{q}(x, t) = \begin{cases} \frac{2\Delta y}{w} q_0(t) & : x_q - \Delta x \leq x \leq x_q + \Delta x \\ 0 & : \text{elsewhere} \end{cases} \quad (15)$$

Equation (12) is subject to the following boundary conditions:

$$x = 0 : -k \frac{\partial \bar{T}}{\partial x} = h_r (\bar{T}_s - T_{mrt}) + h_c (\bar{T}_s - T_{air}) \quad (16)$$

$$x = s : \bar{T} = T_g(t) \quad (17)$$

Consider that the local temperature may be expressed as the summation of a mean value $\bar{T}$ and a perturbation Φ as follows:

$$T(x, y, t) = \bar{T}(x, t) + \Phi(x, y, t) \quad (18)$$

where the function Φ is to be determined. Φ must satisfy the following condition:

$$\int_0^{w/2} \Phi \cdot dy = 0 \quad (19)$$

Substituting equation (18) in equation (1) and taking into account equation (12), one obtains the following equation:

$$\rho \cdot c \frac{\partial \Phi}{\partial t} = \frac{\partial}{\partial x} \left(k \frac{\partial \Phi}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial \Phi}{\partial y} \right) + q - \bar{q} \quad (20)$$

Equation (20) is subject to the following boundary conditions:

$$x = 0 : \frac{\partial \Phi}{\partial x} = 0 \quad (21)$$

$$x = s : \Phi = 0 \quad (22)$$

$$y = 0, w/2: \frac{\partial \Phi}{\partial y} = 0 \quad (23)$$

Introducing the Kirchhoff transformation variables as follows:

$$\frac{\partial \Psi}{\partial x} = k \frac{\partial \Phi}{\partial x}; \quad \frac{\partial \Psi}{\partial y} = k \frac{\partial \Phi}{\partial y} \quad (24)$$

one obtains the following equations:

$$\rho \cdot c \frac{\partial \Phi}{\partial t} = \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + q - \bar{q} \quad (25)$$

Equation (25) is subject to the following boundary conditions:

$$x = 0: \frac{\partial \Psi}{\partial x} = 0 \quad (26)$$

$$x = s: \Psi = 0 \quad (27)$$

$$y = 0, w/2: \frac{\partial \Psi}{\partial y} = 0 \quad (28)$$

Equation (25) may have an analytical solution if the function Ψ consists of a series of a product of separable functions. The function Ψ may be separable if the local thermal capacity ($\rho \cdot c$) and conductivity (k) are constant throughout the calculation domain, which is not the case for radiant systems. An attempt, however, is made to come up with an approximate solution by replacing the thermal capacity (ρc) by its average value over the medium.

Consider that the function Ψ may consist of separable functions as follows:

$$\Psi(x, y, t) = F(t) \cdot G(x) \cdot H(y) \quad (29)$$

where $F(t)$, $G(x)$ and $H(y)$ are functions to be determined. The space functions $G(x)$ and $H(y)$ should satisfy the heat balance equation at the transient and steady-state regimes. Substituting equation (29) in equation (25) at the steady-state regime, one obtains the following equation:

$$\frac{1}{G} \frac{d^2 G}{dx^2} + \frac{1}{H} \frac{d^2 H}{dy^2} = \frac{\bar{q} - q}{\Psi_0} \quad (30)$$

where Ψ_0 is the value of Ψ at the steady-state regime. The function Ψ_0 is separable if the following condition holds:

$$q - \bar{q} = C \cdot \Psi_0 \quad (31)$$

where C is a constant different than zero. The task is now to determine this constant.

Taking into account equation (31), equation (30) leads to two equations as follows:

$$\frac{d^2G}{dx^2} + A \cdot G = 0, \text{ with } A \neq 0 \quad (32)$$

$$\frac{d^2H}{dy^2} + B \cdot H = 0, \text{ with } B \neq 0 \quad (33)$$

where A and B are constants to be determined. The constant C is then given by the following equation:

$$C = A + B \quad (34)$$

The functions F, G and H satisfy the following initial and boundary conditions:

$$t = 0 : F = 0 \quad (35)$$

$$x = 0 : \frac{dG}{dx} = 0 \quad (36)$$

$$x = s : G = 0 \quad (37)$$

$$y = 0, w/2 : \frac{dH}{dy} = 0 \quad (38)$$

The possible solutions of equations (32) and (33) are expressed as follows:

$$G(x) = a \cdot \cos(\omega \cdot x) + b \cdot \sin(\omega \cdot x), \quad \text{with } \omega = \sqrt{A} \quad (39)$$

$$H(y) = a' \cos(\omega' \cdot y) + b' \sin(\omega' \cdot y), \quad \text{with } \omega' = \sqrt{B} \quad (40)$$

where a, b, a' and b' are constants to be determined. Applying the boundary conditions in equations (36) - (38), one obtains the following equations:

$$b = b' = 0 \quad (41)$$

$$\omega = (1 + 2i)\pi / (2s), \quad i = 0, 1, 2, \dots, n \quad (42)$$

$$\omega' = 2j\pi / w, \quad j = 1, 2, \dots, m \quad (43)$$

The general solution at the steady-state regime then consists of a superposition of the individual solutions, expressed as follows:

$$\Psi_0 = \sum_{i=0}^n \sum_{j=1}^m F_0 \cdot a_{ij} \cdot \cos(\omega_i \cdot x) \cdot \cos(\omega'_j \cdot y) \quad (44)$$

Where F_0 is the value of F at the steady-state regime.

Equation (44) is a Fourier series expansion of the function Ψ_0 . Taking into account equation (31), the series elements a_{ij} are calculated as follows:

$$a_{ij} = \frac{\int_0^s \int_0^{w/2} (q - \bar{q}) / F_0 \cdot \cos(\omega_i x) \cdot \cos(\omega'_j y) \cdot dy \cdot dx}{(\omega_i^2 + \omega_j'^2) \cdot s \cdot w / 8} \quad (45)$$

Since the coefficients a_{ij} are time-independent, one deduces the following relation:

$$F_0 = q_0 \quad (46)$$

Performing the integration in equation (45), one obtains the following relation:

$$a_{ij} = \frac{\sin(\omega'_j \cdot \Delta y) \cdot \sin(\omega_i \cdot \Delta x) \cdot \cos(\omega_i \cdot x_q)}{w \cdot s / 16 \cdot \omega_i \omega'_j \cdot (\omega_i^2 + \omega_j'^2)} \quad (47)$$

The original function Φ may also be expressed as a Fourier's series expansion as follows:

$$\Phi(x, y, t) = \sum_{i=0}^n \sum_{j=1}^m F \cdot b_{ij} \cdot \cos(\omega_i \cdot x) \cdot \cos(\omega'_j \cdot y) \quad (48)$$

Taking into account equation (44) and the feature of Fourier's series, one obtains the following relation between the coefficients a_{ij} and b_{ij}

$$a_{ij} = \sum_{k=0}^{\infty} b_{kj} \cdot (-\omega_k) \frac{2}{s} \cdot \int_0^s \left(\int_s^x k(\xi) \cdot \sin(\omega_k \cdot \xi) \cdot d\xi \right) \cdot \cos(\omega_i \cdot x) \cdot dx \quad (49)$$

Equation (49) may be reduced to:

$$a_{ij} = \sum_{k=0}^{\infty} b_{kj} \cdot \frac{2}{s} \int_0^s k(\xi_m) \cdot \cos n(\omega_k \cdot x) \cdot \cos(\omega_i \cdot x) \cdot dx \quad (50)$$

where the value ξ_m may vary in the interval $[s, x]$. Equation (50) can be further reduced to the following:

$$a_{ij} / b_{ij} = \bar{k} \approx \frac{1}{s} \int_0^s k(x) \cdot dx \quad (51)$$

From the transient equation (25), one obtains the following differential equation for the function $F(t)$:

$$\frac{dF}{dt} + \frac{\bar{k}}{\rho c} (\omega_i^2 + \omega_j'^2) \cdot (F - q_0) = 0 \quad (52)$$

Equation (52) cannot be solved analytically unless the function of the heat injection rate (q_0) is known *a priori*. The heat injection rate is usually determined by the thermal control system. The function F should, therefore, be solved using a numerical model. Using the fully-implicit scheme for differentiation, one obtains the following relation to predict the current values of the function F based on the values at the previous time:

$$F = \frac{F^{\text{old}} + \Delta t \cdot (\omega_1^2 + \omega_j'^2) \cdot \bar{k} / \rho c \cdot q_0}{1 + \Delta t \cdot (\omega_1^2 + \omega_j'^2) \cdot \bar{k} / \rho c} \quad (53)$$

where the exponent “old” designates the values at the previous time.

The temperature distribution within the medium is then given by:

$$T(x, y, t) = \bar{T}(x, t) + \sum_{i=0}^n \sum_{j=1}^m F_{ij}(t) / \bar{k} \cdot a_{ij} \cdot \cos(\omega_i \cdot x) \cdot \cos(\omega_j' \cdot y) \quad (54)$$

HEAT TRANSFER FROM TUBING CIRCUITS TO ADJACENT MEDIUM

Consider the general case where the adjacent medium to circuit tubing can be solid (e.g., concrete slab), or air space (e.g., sub-floor heating where tubing is emerged in an air space and attached to a surface). When the adjacent medium is solid, the heat transfer from the circuit tubing to the adjacent medium is by conduction. The perfect contact is assumed between the tubing outer surface and the medium (zero contact resistance). However, when the adjacent medium is an air space, heat transfer is more complex, where all the three modes of heat transfer -radiation, convection and conduction- occur at the same time. While the explicit modeling of this system is not attempted here, one would treat the air space as a solid medium with an equivalent thermal conductivity, pre-calculated based on some experimental correlations.

Consider the heat balance around a control volume as shown in Figure 4. Assuming that the tubing thickness is very small compared to the interior diameter, one obtains the following equations:

$$dQ_b = dQ_f + (\rho c A)_f \cdot dz \cdot \frac{dT_f}{dt} \quad (55)$$

$$dQ_f = dQ_c + (\rho c A)_t \cdot dz \cdot \frac{dT_{to}}{dt} \quad (56)$$

where:

A_f : fluid cross-sectional surface area (m^2).

A_t : tubing wall cross-sectional surface area (m^2).

c_f : specific heat capacity of fluid (J/kg °C).

c_t : specific heat capacity of tubing material (J/kg °C).

Q_b : heat transfer rate from boiler to fluid per circuit (W).

Q_c : heat transfer rate from circuit tubing to adjacent medium (W).

Q_f : heat transfer rate from fluid to circuit tubing (W).

T_{to} : temperature of outer tubing surface ($^{\circ}\text{C}$).

T_f : fluid bulk temperature ($^{\circ}\text{C}$).

ρ_f : fluid mass density (kg/m^3).

ρ_t : mass density of tubing material (kg/m^3).

The cross-sectional areas of fluid and tubing wall are expressed as follows:

$$A_f = \pi D_i^2 / 4; \quad A_t = \pi(D_o^2 - D_i^2) / 4 \quad (57)$$

The heat transfer rates, dQ_b , dQ_f , and dQ_c , are expressed as follows:

$$\frac{dQ_b}{dz} = -m \cdot c_f \frac{dT_f}{dz} \quad (58)$$

$$\frac{dQ_c}{dz} = (T_{ti} - T_{to}) / R_c \quad (59)$$

$$\frac{dQ_f}{dz} = (T_f - T_{ti}) / R_i \quad (60)$$

where:

m : circuit fluid mass flow rate (kg/s);

R_c : conductive thermal resistance per unit circuit length ($\text{m}^{\circ}\text{C} / \text{W}$);

R_i : convective thermal resistance per unit circuit length ($\text{m}^{\circ}\text{C} / \text{W}$); and

T_{ti} : temperature of inner tubing surface ($^{\circ}\text{C}$).

The conductive and convective thermal resistances are given by:

$$R_c = \frac{\text{Ln}(D_o / D_i)}{2\pi k_t} \quad (61)$$

$$R_i = \frac{1}{\pi D_i h_f} \quad (62)$$

where:

h_f : convective film coefficient at the inner tubing surface ($\text{W}/\text{m}^2 \text{ } ^{\circ}\text{C}$); and

k_t : tube thermal conductivity ($\text{W}/\text{m } ^{\circ}\text{C}$).

Taking into account equations (56), (59) and (60), one obtains the following equation:

$$\frac{dQ_c}{dz} = UP \cdot \left((T_f - T_{to}) - R_i \cdot (\rho c A)_t \cdot \frac{dT_{to}}{dt} \right) \quad (63)$$

where:

U : overall heat conductance (W/m² °C).

P : perimeter of the pipe cross-sectional surface (m).

The product UP is given by the following equation:

$$UP = \frac{1}{R_i + R_c} = \frac{\pi}{1/(D_i h_f) + \ln(D_o / D_i)/(2k_t)} \quad (64)$$

Summing up equations (55) and (56) and substituting equation (63) in the resulting equation, one obtains the following differential equation for the fluid temperature:

$$m \cdot c_f \frac{\partial T_f}{\partial z} = - \left\{ UP \cdot (T_f - T_{to}) + (\rho c A)_f \frac{\partial T_f}{\partial t} + UP \cdot R_c (\rho c A)_t \frac{dT_{to}}{dt} \right\} \quad (65)$$

Equation (65) is subject to the initial and boundary conditions as follows:

$$\begin{aligned} t = 0 : \quad T_f &= T_{to}, \quad 0 \leq z \leq L_c \\ z = 0 : \quad T_f &= T_{in}, \quad t > 0 \end{aligned} \quad (66)$$

Let's pose:

$$\Theta = T_f - T_{to} \quad (67)$$

Equation (65) reduces to:

$$\frac{\partial \Theta}{\partial z} = - \left\{ \frac{UP}{m \cdot c_f} \cdot \Theta + \frac{(\rho c A)_f}{m \cdot c_f} \cdot \frac{\partial \Theta}{\partial t} + \frac{(\rho c A)_m}{m \cdot c_f} \frac{dT_{to}}{dt} \right\} \quad (68)$$

with:

$$(\rho c A)_m = (\rho c A)_f + UP \cdot R_c (\rho c A)_t \quad (69)$$

Equation (68) may be rewritten as follows:

$$\frac{\partial \Theta}{\partial z} + C_1 \Theta + C_2 \frac{\partial \Theta}{\partial t} + C_3 \frac{dT_{to}}{dt} = 0 \quad (70)$$

with C₁, C₂ and C₃ are constants given by:

$$C_1 = \frac{UP}{mc_f}; \quad C_2 = \frac{(\rho c A)_f}{mc_f}; \quad C_3 = \frac{(\rho c A)_m}{mc_f} \quad (71)$$

Equation (70) is subject to the initial and boundary conditions as follows:

$$\begin{aligned} t = 0: \quad \Theta &= 0, \quad 0 \leq z \leq L_c \\ z = 0: \quad \Theta &= \Theta_{in} = T_{in} - T_{to}, \quad t > 0 \end{aligned} \quad (72)$$

The general solution of equation (70) is expressed as follows:

$$\Theta = e^{-C_1 z} \cdot \left\{ C_0 - \int e^{C_1 z} \left(C_2 \frac{\partial \Theta}{\partial t} + C_3 \frac{dT_{to}}{dt} \right) dz \right\} \quad (73)$$

where C_0 is a constant to be determined using the boundary condition at $z = 0$. Performing the integration in equation (73), one obtains:

$$\Theta = C_0 e^{-C_1 z} - C_3 / C_1 \frac{dT_{to}}{dt} - C_2 \cdot e^{-C_1 z} \frac{\partial}{\partial t} \int e^{C_1 z} \cdot \Theta \cdot dz \quad (74)$$

Equation (74) can further be reduced to:

$$\Theta = C_0 e^{-C_1 z} - C_3 / C_1 \frac{dT_{to}}{dt} - C_2 / C_1 \cdot \frac{\partial \bar{\Theta}}{\partial t} \quad (75)$$

where $\bar{\Theta}$ is the average value of Θ over the circuit length, expressed as follows:

$$\bar{\Theta} = \frac{1}{L_c} \int_0^{L_c} \Theta \cdot dz \quad (76)$$

Averaging equation (75) over the circuit length, one obtains the following differential equation for the average temperature:

$$C_1 \bar{\Theta} = C_0 / L_c \cdot (1 - e^{-C_1 L_c}) - C_3 \frac{dT_{to}}{dt} - C_2 \cdot \frac{\partial \bar{\Theta}}{\partial t} \quad (77)$$

Equation (77) is subject to the initial boundary condition as follows:

$$t = 0: \quad \bar{\Theta} = 0 \quad (78)$$

Taking into account equation (77), one obtains from equation (75) the following relation for the local temperature:

$$\Theta = \bar{\Theta} + (C_0 / C_1 L_c) \cdot (C_1 L_c \cdot e^{-C_1 z} + e^{-C_1 L_c} - 1) \quad (79)$$

Using the boundary condition at $z = 0$, equation (72), the coefficient C_0 is expressed as follows:

$$C_0 = \frac{C_1 L_c \cdot (\Theta_{in} - \bar{\Theta})}{C_1 L_c - 1 + e^{-C_1 L_c}} \quad (80)$$

Substituting equation (80) in equation (77) and rearranging, one obtains the following equations for the average and local fluid temperatures:

$$\frac{C_2}{C_1} \cdot \frac{\partial \bar{\Theta}}{\partial t} + \left(\frac{C_1 L_c}{C_1 L_c - 1 + e^{-C_1 L_c}} \right) \cdot \bar{\Theta} - \left(\frac{1 - e^{-C_1 L_c}}{C_1 L_c - 1 + e^{-C_1 L_c}} \right) \cdot \Theta_{in} + \frac{C_3}{C_1} \cdot \frac{dT_{to}}{dt} = 0 \quad (81)$$

$$\frac{\Theta - \bar{\Theta}}{\Theta_{in} - \bar{\Theta}} = \frac{C_1 L_c \cdot e^{-C_1 z} + e^{-C_1 L_c} - 1}{C_1 L_c + e^{-C_1 L_c} - 1} \quad (82)$$

The circuit and boiler heat transfer rates, and the outlet fluid temperature are given by the following equations:

$$Q_c = UPL_c \cdot \left\{ \bar{\Theta} - R_i \cdot (\rho c A)_t \cdot \frac{dT_{to}}{dt} \right\} \quad (83)$$

$$Q_b = m \cdot c_f (\Theta_{in} - \Theta_{out}) \quad (84)$$

$$\frac{\Theta_{out} - \bar{\Theta}}{\Theta_{in} - \bar{\Theta}} = \frac{(1 + C_1 L_c) \cdot e^{-C_1 L_c} - 1}{C_1 L_c + e^{-C_1 L_c} - 1} \quad (85)$$

Equation (81) cannot be solved without knowing the outside tube temperature (T_{to}). The later is equal to the tube-concrete contact temperature, given by equation (54) evaluated at the heat source point ($x = x_q, y = 0$). Equation (54) is coupled with equation (81) by the heat source injection rate given by equation (83). To solve for both the circuit fluid average temperature and the tube-concrete contact temperature, a numerical method combined with an iterative approach have to be employed. This is explained in the following procedure.

Using the fully implicit scheme, equation (81) is reduced to the following:

$$(A + B \cdot \Delta t) \bar{\Theta} - A \cdot \bar{\Theta}^{old} - \Delta t \cdot D = 0 \quad (86)$$

with the coefficients A, B and D given by the following equation:

$$A = \frac{(\rho c A)_f}{UP}; \quad B = \frac{C_1 L_c}{C_1 L_c - 1 + e^{-C_1 L_c}}; \quad D = \frac{(1 - e^{-C_1 L_c}) \Theta_{in}}{C_1 L_c - 1 + e^{-C_1 L_c}} - \frac{(\rho c A)_m}{UP} \cdot \frac{dT_{to}}{dt} \quad (87)$$

The Newton-Raphson method is then employed to solve for both $\bar{\Theta}$ and T_{to} . Starting with an initial guess of the solution, the following system of equation is solved at each iteration (k):

$$\begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} \delta \bar{\Theta} \\ \delta T_{to} \end{bmatrix} = - \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \quad (88)$$

where F_1 and F_2 are the functions to be zeroed and J_{ij} the Jacobian matrix elements. These are given by the following equations:

$$F_1 = (A + B \cdot \Delta t) \cdot \bar{\Theta} - A \cdot \bar{\Theta}^{old} - \Delta t \cdot D \quad (89)$$

$$F_2 = T_{to} - \bar{T}_{to} - \sum_{i=0}^n \sum_{j=1}^m F_{ij}(t) / \bar{k} \cdot a_{ij} \cdot \cos(\omega_i \cdot x_q) \quad (90)$$

$$J_{11} = \frac{\partial F_1}{\partial \Theta} = A + B \cdot \Delta t \quad (91)$$

$$J_{12} = \frac{\partial F_1}{\partial T_{to}} = \frac{(\rho c A)_m}{UP} + \frac{\Delta t \cdot (1 - e^{-C_1 L_c})}{C_1 L_c - 1 + e^{-C_1 L_c}} \quad (92)$$

$$J_{21} = \frac{\partial F_2}{\partial \Theta} = -\frac{UPL_c}{vol} \cdot \left(\sum_{i=0}^n \sum_{j=1}^m \left(\frac{\Delta t \cdot (\omega_i^2 + \omega_j'^2) \cdot \bar{k} / \rho c}{1 + \Delta t \cdot (\omega_i^2 + \omega_j'^2) \cdot \bar{k} / \rho c} \right) \cdot a_{ij} / \bar{k} \cdot \cos(\omega_i \cdot x_q) \right) \quad (93)$$

$$J_{22} = \frac{\partial F_2}{\partial T_{to}} = 1 - \left(\frac{R_i \cdot (\rho c A)_t}{\Delta t} \right) \cdot J_{21} \quad (94)$$

where Δt is the time step, vol is the volume occupied by the tubing, and the exponent (old) indicates the values at the previous time.

The new estimate of the solution is then calculated as follows:

$$\bar{\Theta}_{k+1} = \bar{\Theta}_k + \eta \cdot \delta \bar{\Theta}_k \quad (95)$$

$$T_{to,k+1} = T_{to,k} + \eta \cdot \delta T_{to,k} \quad (96)$$

where η is the relaxation coefficient ($0 < \eta \leq 1$). The iteration is repeated until convergence is reached. The convergence is declared when the following inequality is satisfied based on a given tolerance (Tol):

$$F_1^2 + F_2^2 < Tol \quad (97)$$

MODEL VALIDATION

The previously developed semi-analytical model is validated against a two-dimensional numerical solution. A slab-on-grade system as shown in Figure 1 is considered for the comparison. The material physical characteristics of the system are given in Table 1. Tubing is embedded in the middle of the slab and spaced apart at a distance $w = 0.30\text{m}$ (12"). A time-based control supplies the system with a constant heat injection rate equal to $Q_c/L_c = 20 \text{ Watt/m}$ ($q_0 = 186667 \text{ Watt/m}^3$) during a 12-hour heating cycle and ceases to operate the system during the rest of the day. The system exchanges heat with an indoor space maintained at an air temperature $T_{air} = 21^\circ\text{C}$ and surrounding surface mean radiant temperature $T_{mrt} = 18^\circ\text{C}$. The ground temperature is fixed at $T_g = 5^\circ\text{C}$. The convection heat transfer coefficient (h_c) is calculated based on experimental correlations taken from [26].

The employed two-dimensional numerical modeling is based on the control volume approach [27] coupled with the fully implicit scheme. After discretizing the partial differential equation (1), the resulting equations are solved using the line-by-line tri-diagonal matrix algorithm. The numerical mesh consists of 15 internal nodes in the y-direction, two nodes for the floor coverings, seven nodes for the concrete slab, five nodes for the insulation and 10 nodes for the gravel layer. The time step is fixed at $\Delta t = 15$ min. The semi-analytical solution is obtained with $n = m = 30$ elements (higher values of n and m would not significantly improve the solution).

Figure 5 shows the time profiles of the heat source temperature ($x = x_q, y = 0$), floor surface maximum temperature ($x = y = 0$) and floor surface minimum temperature ($x = 0, y = w/2$) predicted by the semi-analytical and numerical models. The predictions of the semi-analytical model compare very well with those of the numerical model. The maximum difference is less than 11%. This difference may be attributed mainly to the non-uniform physical characteristics of the system, and partially to the employed mesh size in the numerical model, and the number of the retained series elements in the semi-analytical model. In fact, the case of a hypothetical system (not shown here) having uniform physical characteristics (e.g., slab without floor coverings and insulation) showed that the semi-analytical model is in excellent agreement with the numerical model.

Figure 6 shows the temperature profiles of the left-side surface ($y = 0$), and the right-side surface ($y = w/2$) predicted by the semi-analytical and numerical models at the end of the heating cycle (time = 12h). Figure 7 shows the temperature profiles of the floor surface ($x = 0$), and a horizontal surface passing through the heat source point ($x = x_q$) predicted by the semi-analytical and numerical models at the end of the heating cycle (time = 12h). Again, the predictions of the semi-analytical model compare very well with those of the numerical model. The maximum difference is less than 9%. As in the previous results, this difference is attributed mainly to the non-uniform physical characteristics of the system.

The previous graphs clearly showed that a multi-dimensional model should be used to accurately predict the thermal performance of radiant systems. To demonstrate the usefulness of a two-dimensional over a one-dimensional model, the semi-analytical model is used to calculate the temperature corrections (Φ values) for the heat source point, and floor surface cold and hot spots. Figure 8 shows these temperature corrections. The temperature correction for the heat source can reach up to 25°C, and that for the floor surface may fluctuate between -5°C to +6°C. This means that when a one-dimensional model is used, the thermal capacity of the system will be significantly oversized. Furthermore, thermal comfort may be jeopardized at the cold and hot spots (for heating systems), and moisture condensation at the cold spot may occur (for cooling systems).

The previous results were for an ideal case where the heat injection rate is fixed. In practice, however, the heat injection rate varies with time. To illustrate this effect, figure 9 shows the time-response of a hydronic system in terms of the water average and outlet temperatures (T_{fm} and T_{out}) and the outside tube temperature (T_{to}) when coupled with the floor slab. The hydronic system has the following characteristics: $D_i = 12\text{mm}$, $D_{out} = 16\text{mm}$, $k_t = 0.1\text{ W/m}^\circ\text{C}$, $\rho_t = 1100\text{ kg/m}^3$, $c_t = 2000\text{ J/kg }^\circ\text{C}$. The system serves a surface area of 16m^2 (with one circuit of a total tubing length $L_c = 53.3\text{m}$). During a 12-hour heating period, the water enters the circuit tubing with an inlet temperature $T_{in} = 50^\circ\text{C}$ with a volumetric flow rate of 1 liter/min. The system is completely shut off during the rest of the day ($m = 0$). The system reaches the quasi-steady regime in about 2 hours. The water outlet temperature is about 8 degrees lower than the inlet temperature. The heat delivered from the boiler to the water (Q_b) is completely transferred to the concrete slab, except at the start of the heating period, suggestion minor thermal storage in the water and tubing masses. The heat transferred to the concrete slab (Q_c) decreases about 32% from the start to the end of the heating period.

CONCLUSION

A detailed model was developed for radiant heating and cooling systems for integration in energy simulation software. The model targets energy simulation software that use the one-dimensional numerical modeling to calculate the heat transfer within the building construction assemblies. The developed model is decomposed into two sets: one set uses the heat source method to calculate the heat transfer within the radiant medium, and the second set calculates the heat transfer within the circuit tubing. The heat transfer within the radiant medium is solved using the one-dimensional numerical modeling already built in the energy simulation software, and a two-dimensional analytical solution is appended to it. The heat transfer from the circuit tubing to the adjacent medium is solved using an analytical model. The two sets are coupled together via the temperature of the heat source node, i.e., the heat source node temperature equals to that of the circuit tubing outside surface. The advantage of this model over the one-dimensional model is that it accurately predict the circuit tubing-concrete contact surface temperature, which is required to compute the boiler/chiller thermal capacity, and the minimum and maximum ceiling/floor temperatures, which are required for moisture condensation (ceiling cooling systems), thermal comfort (heating floor systems) and controls. The model predictions compared very well with the results from a full, two-dimensional numerical model.

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Figure 1 Schematic description of a slab-on-grade floor radiant system

Figure 2 Typical serpentine tubing pattern

Figure 3 Calculation domain

Figure 4 Heat balance around a tubing element

Figure 5 Profiles of the minimum floor temperature ($x = 0, y = w/2$), maximum floor temperature ($x = y = 0$), and heat source temperature ($x = x_q, y = 0$) as a function of time - comparison between the numerical and semi-analytical models

Figure 6 Temperature profiles of the left side surface ($y = 0$) and right-side surface ($y = w/2$) predicted at the end of the heating cycle (time = 12h) - comparison between the numerical and semi-analytical models

Figure 7 Temperature profiles of the floor surface ($x = 0$) and heat source horizontal surface ($x = x_q$) predicted at the end of the heating cycle (time = 12h) - comparison between the numerical and semi-analytical models.

Figure 8 Temperature corrections (Φ values) calculated at the floor surface hot ($x = y = 0$) and cold ($x = 0, y = w/2$) spots and at the heat source point ($x = x_q, y = 0$)

Figure 9 Response of a typical hydronic system to the boiler heat injection

Table 1 Physical characteristics of a slab-on-grade radiant system

Layer	Thickness (m)	Conductivity (W/m·K)	Density (kg/m ³)	Specific heat (J/kg·K)
Gravel bed	0.100	0.520	2050	184
Insulation (Polystyrene)	0.050	0.027	55	1210
Concrete	0.10	0.38	1200	653
Floor covering (hardwood)	0.019	0.14	600	1210