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Practical Correlations for the Thermal Resistance of Vertical Enclosed Airspaces for Building Applications

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Abstract

Many parts of the building envelope contain enclosed airspaces. The thermal resistance (R-value) of an enclosed airspace depends on the emissivity of all surfaces that bound the airspace, the size and orientation of the airspace, the direction of heat transfer through the airspace, and the respective temperatures of all surfaces that define the airspace. The 2009 ASHRAE Handbook of Fundamentals (Chapter 26) provides a table that contains the R-values for an enclosed airspace. The ASHRAE table is extensively used by modellers, architects and building designers in the design of building enclosures. This table provides R-values for enclosed airspaces for different values of the thickness of the airspace, effective emittance, mean airspace temperature, and temperature differences across the airspace. The effect of the airspace aspect ratio (height/thickness of airspace) on the R-value is not included in the ASHRAE table. However, in a recent study on the R-value of reflective insulations using a numerical simulation model, it was shown that the aspect ratio of the airspace can affect the R-value of the enclosed airspace. The numerical simulation model used in this study had been benchmarked against experimental data obtained using two standard test methods: ASTM C-518 and ASTM C-1363.

In this paper, a numerical simulation study was conducted, that was based on previous work focused on enclosed airspaces, to investigate the effect of the aspect ratio on the R-value of vertical enclosed airspaces of different thicknesses and having a wide range of values for effective emittance, mean temperature, and temperature differences across the airspace. The R-values predicted from numerical simulation are compared with those provided in the ASHRAE table. Considerations were also given to investigating the potential increase in R-values of enclosed airspaces when a thin sheet is placed vertically in the middle of the airspace and whose surfaces have different values of emissivity. Finally, practical correlations are developed for determining the R-values of an enclosed airspace for future use by modellers, architects and building designers. The simplicity of these correlations suggests that these could be included in the ASHRAE Handbook of Fundamentals.

Keywords: Reflective insulation, low emissivity material, thermal modelling, R-value correlation, airflow in enclosed airspace, heat transfer by convection, conduction and radiation.

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INTRODUCTION

The design of building envelope roof and wall systems with the intent of achieving energy savings can necessarily help reduce building operating loads and thus the demand for energy over time [1, 2]. A straightforward means of reducing building operating costs is to limit heat transmission and thus energy loss through the building envelope. This evidently can be achieved by increasing the thermal resistance (R-value) of the building envelope. Reflective Insulation (RI) products are widely used in both space as well as terrestrial applications, for example, in building construction. For applications in building construction, and in accordance with installation guidelines of the Reflective Insulation Manufacturers Association International (RIMA-I) [3], RI products have at least one reflective surface facing an airspace. As well, RI products are typically being used in conjunction with mass insulation products, such as glass fibre, expanded polystyrene foam (EPS), and other similar insulation products. Reflective Insulation products can be installed in wall cavities, between ceiling and floor joists, to provide radiant energy barriers, and in metallic buildings that cannot readily accommodate loose-fill or batt-type insulations. Furthermore, RI products can be used as part of a roofing system either below the decking between rafters, within small air gaps between decking and roofing, and in air gaps created, for example, by paneling interior masonry walls [4].

An enclosed airspace contributes to the overall thermal resistance of a system whether or not a product having a reflective surface is installed in the system. However, it is known that the presence of a reflective surface augments the thermal resistance of that airspace [5–13]. The contribution of the reflective insulation on the thermal performance (R-values and energy savings) of above-grade, and above- and below-grade wall systems having a Furred-Airspace Assembly (FAA) have been investigated for which the results are available in a number of publications [7, 9–13]. In the wall systems described in these studies, a low emissivity foil material was installed within the furred-airspace assembly.

The thermal resistances (R-values) of enclosed airspaces were calculated by many investigators (e.g. see [14–17]) for various orientations of airspaces and reflective boundaries by using heat transfer coefficient data that was published by Robinson et al. [15, 16, 17]. The heat transfer coefficient data were obtained from measurements of panels of different thicknesses (0.625 in to 3.375 in (15.9 mm to 85.7 mm)) using the test method described in ASTM 236-53 [18]. In those studies, the steady-state heat transmission rates were corrected for the heat transfer occurring along parallel paths between the hot and cold boundaries. Thereafter, the convective heat transfer coefficients were obtained from the data by subtracting a calculated radiative heat transfer rate from the total corrected heat transfer rate; and the radiative heat transfer was calculated using an emissivity of 0.028 for the aluminum surfaces.

Generally, the value for the effective heat conductance, U-value (the reciprocal of the R-value), of an enclosed space (e.g. airspace) between two parallel planes depends on the physical properties of the gas, the temperature and emissivity of all the surfaces of the space, temperature differences across the space, the dimensions of the space, the direction of heat transfer through the space and the orientation of the space. The U-value accounts for the contribution of heat transfer in the enclosed space due to heat transfer by conduction, convection and radiation.

In the absence of heat transfer by radiation, the contribution of heat transfer due to convection in an enclosed space is normally given in terms of the Nusselt number, \( \text{Nu} = \frac{h \delta}{\lambda} \), where \( h \)
is convective heat transfer coefficient, \( \delta \) is the thickness of the space, and \( \lambda \) is the thermal conductivity of the fluid filling the space). According to many authors [19, 20], the convective heat transfer coefficient for an enclosed space can be given as:

\[
Nu = \frac{h \delta}{\lambda} = a (Gr \cdot Pr)^b \cdot A_R^c = a (Ra)^b A_R^c, \quad \text{and} \quad Gr = \frac{g \beta \cdot \Delta T}{\mu^2}.
\]

Where the coefficients \( a, b \) and \( c \) in Eq. (1) are dimensionless constants, derived from experiments, \( A_R \) is the aspect ratio of the enclosed space (\( A_R = \text{height (H)}/\text{thickness (\( \delta \))} \)), \( Gr \) is the Grashoff number, \( Ra \) is the Raleigh number (\( Ra = Gr \cdot Pr \)), and \( Pr \) is the Prandtl number. In order to derive the coefficients \( a, b \) and \( c \) (Eq. (1)), from which the heat transfer coefficient, \( h \), due to the convective component of heat transfer can be determined, the emissivity of all surfaces that bound the enclosed space must be zero (i.e. purely reflective surfaces). However, it is not possible in practice, to use materials having zero emissivity when conducting such experiments. Hence, to derive the coefficients \( a, b \) and \( c \) of Eq. (1) from experiments, as mentioned previously, the rate of radiative heat transfer across the enclosed space would be subtracted from the total rate of heat transfer across the space.

A number of correlations for the value of \( Nu \) in the form of the relationship given in Eq. (1) and for different ranges of values of \( Ra, A_R \) and \( Pr \) are provided in several studies as described in the IEA Annex XII report [19]. Some of these correlations showed the dependence of the \( Nu \) on the aspect ratio of the enclosed space (\( A_R \)). As such, it is anticipated that the effective thermal conductance or the effective thermal resistance of the enclosed space would be affected by the aspect ratio of the space, as will be shown later in this study.

Yarbrough [4] developed a one-dimensional, steady-state model, called “REFLECT” to provide R-values of reflective assemblies and the model was further used to help establish the sensitivity of R-values to the values of surface emissivity and as well, the location of foil surfaces inside an enclosed airspace. The model was used to calculate the R-values using the heat transfer coefficients derived by Robinson [15, 16, 17] combined with a term accounting for repeated reflection of energy between infinite parallel planes based on the Stefan-Boltzmann law. The R-values obtained using the REFLECT model are thus independent of the aspect ratio of the enclosed airspace [4].

The 2009 ASHRAE Handbook of Fundamentals (Chapter 26) [14] provides a table that contains the R-values for an enclosed airspace and these were determined on the basis of the heat transfer data reported by Robinson et al. [15, 16, 17]. These values were obtained by combining the convective and radiative components of heat transfer from which the total thermal resistance value for an enclosed airspace was provided for airspaces of different thickness (\( \delta = 13, 20, 40, \) and 90 mm), mean temperature (\( T_{avg} = 32.2, 10.0, -17.8 \) and -45.6°C), temperature difference across the airspace (\( \Delta T = 5.6, 11.1 \) and 16.7°C), effective emittance (\( \epsilon_{eff} = 0.03, 0.05, 0.2, 0.5 \) and 0.82), and direction of heat flow through the airspace. Note that the effective emittance (\( \epsilon_{eff} \)) of an enclosed airspace is given as [14]:

\[
1/\epsilon_{eff} = 1/\epsilon_1 + 1/\epsilon_2 - 1,
\]

where \( \epsilon_1 \) and \( \epsilon_2 \) are the emissivity of the hot and cold surfaces.

The ASHRAE table [14] is being used extensively by modellers, architects and building designers. The R-value of an enclosed airspace at other values of the parameters \( \delta, \epsilon_{eff}, T_{avg}, \) and \( \Delta T \) that are not listed in this table, is obtained by interpolation. It is worth mentioning that
the effect of the aspect ratio of the enclosed airspace on the R-value is likewise not included in this ASHRAE table [14].

MODEL DESCRIPTIONS AND BENCHMARKING

The NRC's hygrothermal model "hygIRC-C" was used in this study to predict the R-values of vertical enclosed airspaces having different $\varepsilon_{\text{eff}}$ and subjected to wide ranges of $T_{\text{avg}}$ and $\Delta T$. This model solves simultaneously the 2D and 3D moisture transport equation, the energy equation, the surface-to-surface radiation equation (e.g. the surface-to-surface radiation in the enclosed airspace, an example is provided in Figure 1) and the air transport equation in the various material layers. The air transport equation is the Navier-Stokes equation for the airspace layers (e.g. air cavities), and the Darcy equation (Darcy Number, $\text{DN} < 10^{-6}$) and Brinkman equation ($\text{DN} > 10^{-6}$) for the porous material layers. The present model was benchmarked [27] against the hygIRC-2D model previously developed at NRC [28, 29], and against experimental results from the evaluation of a number of different wall assemblies. A full description of the present model is available in previous publications (see references [1-2, 6-13, 24-27]).

In building applications that are similar to this study, the present model was benchmarked against thermal performance data for a wall assembly featuring a reflective insulation product. The data was obtained using a Guarded Hot Gox (GHB) (in accordance with ASTM C-1363 test method [21]) for a full-scale (8 ft x 8 ft) above-grade wall system. This wall featured 2 x 6 wood framing, stud cavities filled with friction-fit glass fiber batt insulation, and a layer of foil-lined fiberboard installed to the interior side of the framing, with the foil facing a furred-airspace. Results showed that the R-value predicted by the model for this wall system was in good agreement with the measured R-value (within 1.2%) [11, 22].

Recently, a number of tests were conducted at the Cold Climate Housing Research Center (CCHRC) [5] and the NRC [6, 8] that examined the thermal performance of different types of reflective insulations. The tests were conducted using a Heat Flow Meter, HFM (in accordance with ASTM C-518 test method [23]) capable of accommodating product samples with maximum dimensions of 12 in x 12 in x 4 in (length x width x thickness). In these tests, sample stacks including different types of reflective insulations were placed horizontally between the upper cold plate and lower hot plate of the heat flow meter (i.e. upward heat flow). Each sample stack consisted of three layers. The upper layer (12 in x 12 in x 1 or 2 in) was made of reflective insulation (foil adhered to the bottom surface). The bottom layer was made of gypsum (12 in x 12 in x ½ in) [5] or EPS (12 in x 12 in x 1 in) [6, 8]. To quantify the thermal resistance contribution of the reflective insulation component, an air cavity (8 in x 8 in x 1 in) was created in the center of an EPS layer (12 in x 12 in x 1 in), which was placed between the reflective layer and the bottom layer. As shown in references [6, 8], the heat fluxes predicted by the model were in good agreement with all measured heat fluxes from these tests (within ±1.0%). Thereafter, the model was used to investigate the contribution of reflective insulations to the R-value for specimens with different inclination angles, different directions of heat flow through the specimens, and a wide range of foil emissivity [6].

In the study [6], the prediction of the R-values using the model described above was compared with the R-values given in the ASHRAE table [14] for an enclosed airspace having a thickness of 20 mm. As previously indicated, the effect of the aspect ratio of the airspace is not accounted for on the basis of the R-values given in the ASHRAE table [14]. Given that in conducting a
numerical simulation, the length of the enclosed airspace is required to define the computational
domain. A sensitivity analysis was conducted in the study [6] to investigate its effect on the R-
value. Three airspace lengths were considered in this analysis: 4 in (101.6 mm), 8 in (203.2 mm) and 16 in (406.4 mm). For a horizontal airspace of different lengths and in the case of
upward heat flow when \( \varepsilon_{\text{eff}} = 0.05 \), \( T_{\text{avg}} = 32.2^\circ\text{C} \), \( \Delta T = 5.6^\circ\text{C} \), results showed that the R-value depended on the length of the airspace. For example, increasing the airspace length from 4 in (101.6 mm) to 8 in (203.2 mm) and from 4 in (101.6 mm) to 16 in (406.4 mm) resulted in an
increase in its R-value of 1.5% and 3.7%, respectively. Furthermore, results showed that the R-
value from the ASHRAE table [14] was 14.8%, 13.0% and 10.7% greater than the R-value
predicted by the model for respective airspace lengths of 4 in (101.6 mm), 8 in (203.2 mm) and 16 in (406.4 mm). For an airspace with a length of 8 in (203.2 mm), and for which the value of
effective emittance (\( \varepsilon_{\text{eff}} \)) ranged between 0.5 and 0.82 (high value range), the corresponding R-
values predicted by the model and taken from the ASHRAE table [14] were in good agreement
(within \( \pm 3.5\% \)). Whereas, for low values of effective emittance (\( \varepsilon_{\text{eff}} \)), the R-values obtained
from the ASHRAE table [14] were, respectively, 14.0%, 13.0% and 6.1% greater than that
predicted by the model for values of \( \varepsilon_{\text{eff}} = 0.03 \), 0.05 and 0.2. The results obtained for the low
emissivity values are in agreement with those of a recent study by Craven and Garber-Slaght
[5]. In this study [5] it was shown that the R-values taken from the ASHRAE table [14] were also
greater than R-values measured in the case of low emissivity materials.

The effects of the inclination angle and the direction of heat flow through an enclosed airspace
(20 mm thick, 8 in (203.2 mm) long) were also investigated and thereafter a comparison was
made of the R-values predicted by the model to those obtained from the ASHRAE table [14]. A
series of comparisons was undertaken for an airspace subjected to \( T_{\text{avg}} = 32.2^\circ\text{C} \), \( \Delta T = 5.6^\circ\text{C} \) for the following five cases [6]:

- Case I: horizontal airspace (\( \theta = 0^\circ \)) with upward heat flow;
- Case II: inclined airspace (\( \theta = 45^\circ \)) with upward heat flow;
- Case III: vertical airspace (\( \theta = 90^\circ \)) with horizontal heat flow;
- Case IV: inclined airspace (\( \theta = 45^\circ \)) with downward heat flow, and;
- Case V: horizontal airspace (\( \theta = 0^\circ \)) with downward heat flow.

The results of these comparison cases showed that for Case II and Case V, the R-values
predicted by the model and those given in the ASHRAE tables [14] were in good agreement for all values of \( \varepsilon_{\text{eff}} \) (within \( \pm 3.5\% \)). For Case III and Case IV, the R-values predicted by the model
and given in the ASHRAE tables [14] were in good agreement (within \( \pm 2.0\% \)) for high values
of \( \varepsilon_{\text{eff}} \) (0.5 and 0.82). Whereas in Case III at low values of \( \varepsilon_{\text{eff}} \), the R-values obtained from the
ASHRAE table [14] were 24.9%, 21.7% and 12.7% higher than those predicted by the model for \( \varepsilon_{\text{eff}} = 0.03 \), 0.05 and 0.2, respectively. Also, in Case IV for low values of \( \varepsilon_{\text{eff}} \), the R-values given
in the ASHRAE table [14] were 16.5%, 16.1% and 7.7% greater than those predicted by the
model for \( \varepsilon_{\text{eff}} = 0.03 \), 0.05 and 0.2, respectively (see [6] for more details).

In this paper, numerical simulations were conducted to predict the R-values of enclosed vertical
airspaces. The results of the numerical simulations described in this paper follow on the work of
a previous study [6] and in addition provide results for vertical enclosed airspaces (Case III)
subjected to different mean temperatures (\( T_{\text{avg}} \)) and temperature differences (\( \Delta T \)) across the
enclosure. As such, the objectives of this study are to:
• Investigate the effect of the aspect ratio (AR) on the R-value of enclosed airspaces of different thicknesses (δ = 13, 20, 40 and 90 mm) for a wide range of effective emittance values (εeff = 0 – 0.82).
• Compare the R-values predicted by the model and those given in ASHRAE [14] for airspaces having different values for δ, Tavg, ∆T, and εeff.
• Develop practical correlations for the R-values of enclosed airspaces covering a wide range of values for AR, Tavg, ∆T, and εeff for subsequent use in currently available energy simulation models (e.g. ESP-r, Energy Plus, DOE).
• Investigate the potential increase in the R-value of an enclosed airspace when a thin sheet having an emissivity ranging between 0.0 to 0.9 is placed vertically in the middle of the airspace.

RESULTS AND DISCUSSIONS

Numerical simulations were conducted in order to predict the effective thermal resistances (R-values) of vertical enclosed airspaces for the same parameters listed in ASHRAE table [14]: (a) thickness (δ = 13, 20, 40 and 90 mm), (b) mean temperature (Tavg = 32.2, 10.0, -17.8 and -45.6°C), (c) temperature difference across the airspace (∆T = 5.6, 11.1 and 16.7°C), and (d) effective emittance (εeff = 0 – 0.82). In order to quantify the effect of the aspect ratio (AR = H/δ) of the enclosed airspace on the R-values, for a given airspace thickness (δ), the numerical simulations were conducted at different heights (H = 8 – 96 in (203.2 – 2438.4 mm)). The purpose of considering a wide range of values for H was to permit the development of practical correlations for determining R-values that cover most of expected building applications having vertical enclosed airspaces. These applications include, for example: double glazed windows and curtain walls of different heights, and Furred-Airspace Assemblies (FAA) attached to thermal insulation (bonded by Low Emissivity Material, LEM) in wall systems having horizontal furring and vertical furring of different center-to-center spacing [6 – 13].

A schematic is shown in Figure 1a of a vertical enclosed airspace in which only one vertical surface of the airspace has a low value of emissivity (ε₁), and all other surfaces have a high value of emissivity (ε₂). This represents, for example, the case of low-e coating of a double glazed window or thermal insulation bonded by LEM (e.g. aluminum foil) that faces the enclosed airspace. Note that the value of the low emissivity surface can increase in the case where, for example, the surface accumulates dust (see the study by Cook et al. [30] for more details) or vapor condenses on this surface.

Most construction materials have an emissivity of 0.9 [14]. In this study, the numerical simulations were conducted for different values of ε₁ (ranging from 0 – 0.9) and ε₂ was taken equal to 0.9. According to Eq. (2), a wide range of the effective emittance, εeff (ranging from 0 – 0.82) was considered. Note that εeff = 0.82 represents the case when all surfaces of the enclosed airspace have an emissivity of 0.9 (i.e. no LEM installed on the surfaces of the enclosed airspace).

For an enclosed airspace with H = 12 in (304.8 mm), numerical simulations were also conducted in order to investigate the potential increase in R-values of the enclosed airspace when a thin sheet (0.1 mm thick) of sides having low emissivity (ε₁), is placed vertically in the middle of the airspace (see Figure 1b). The thin sheet divides the enclosed airspace into two cavities of equal thickness. It was assumed that no cross airflow occurs between the two cavities (i.e. each cavity is treated as a sealed airspace). The numerical simulations were
conducted for a wide range of values for $\delta$, $\varepsilon_1$, $\varepsilon_2$, $T_{\text{avg}}$, and $\Delta T$. In this paper, the simulation case for which there is no thin sheet material in the airspace is referred to as “1-Cavity” (Figure 1a), whereas “2-Cavities” refers to the case with a thin sheet in the airspace (Figure 1b).

**Comparisons between Simulation Cases “1-Cavity” and “2-Cavities”**

As indicted earlier, the modes of heat transfer in the enclosed airspace occur by conduction, convection, and radiation. Figure 2 through Figure 5 show the temperature and vertical velocity contours in the enclosed airspace ($H = 12$ in (304.8 mm)) with different values of $\delta$ for the simulation cases that relate to “1-Cavity” and “2-Cavities” when $T_{\text{avg}} = 10^{\circ}C$, $\Delta T = 16.7^{\circ}C$ ($T_H = 18.35^{\circ}C$ and $T_C = 1.65^{\circ}C$), $\varepsilon_1 = 0.05$, and $\varepsilon_2 = 0.9$. Note that different scales were used for the vertical velocity contours in Figure 2b through Figure 5b for simulation cases “1-Cavity” and “2-Cavities”.

Owing to the temperature differential across the enclosed airspace, a buoyancy-driven flow developed within the airspace. A mono-cellular airflow (one convection loop) is developed within each enclosed airspace for the cases of “1-Cavity” and “2-Cavities”. As shown in Figure 2b through Figure 5b, the vertical air velocities for the “1-Cavity” simulation case are significantly higher than that for the “2-Cavities” case, resulting in a stronger convection current with one convection loop for the “1-Cavity” as compared to the “2-Cavities” case for which there was one convection loop in each cavity. Consequently, the rate of heat transfer by convection for the “1-Cavity” case is greater than that of the “2-Cavities” case resulting in higher thermal conductance (i.e. lower thermal resistance) in the “1-Cavity” as compared to the “2-Cavities” case, as shown in Figure 6 through Figure 9.

The values of the maximum upward (+ve) and downward (-ve) vertical velocities ($u_y$) taken from Figure 2b through Figure 5b are summarized in Table 1 for “1-Cavity” and “2-Cavities” simulation cases at different airspace thicknesses ranging between 13 mm and 90 mm. These values indicate that the vertical upward or downward velocities in the airspace are greater for the “1-Cavity” as compared to the “2-Cavities” case, ranging by a factor of 7.4 at the smallest thickness (13 mm) to 1.6 at the largest airspace thickness (90 mm).

For enclosed airspace of $\delta = 13, 20, 40$ and 90 mm ($H = 12$ in (304.8 mm)) at different $\varepsilon_{\text{eff}}$, $T_{\text{avg}}$ and $\Delta T$, comparisons between the R-values for the cases of “1-Cavity” and “2-Cavities” are shown in Figure 6 through Figure 9. As shown in these figures, the R-values for the “2-Cavities” simulation case are higher than that of the “1-Cavity” case. As shown Table 2, the increase in the R-value due to the incorporation of a thin sheet in the middle of the enclosed airspace (see Figure 1b) and having an emissivity $= \varepsilon_1$ on both sides of the thin sheet, increases as $\delta$ increases from 13 mm to 20 mm and 40 mm. This percentage increase in the R-value was approximately the same by further increasing $\delta$ from 40 mm to 90 mm. For example, depending on the values of $T_{\text{avg}}$ and $\Delta T$, the percentage increase in the R-value for $\varepsilon_{\text{eff}} = 0.03$ changes: (a) from 10.6% to 34.4% for $\delta = 13$ mm, (b) from 29.8% to 100.0% for $\delta = 20$ mm, (c) from 120.3 to 136.9% for $\delta = 40$ mm, and (d) from 116.8 to 128.6 for $\delta = 90$ mm (Table 2). Similarly, for $\varepsilon_{\text{eff}} = 0.82$, the percentage increase in the R-value changes: (a) from 48.8% to 59.4% for $\delta = 13$ mm, (b) from 72% to 98.3% for $\delta = 20$ mm, (c) from 103.0 to 113.7% for $\delta = 40$ mm, and (d) from 97.8 to 102.7 for $\delta = 90$ mm (Table 2). In summary, the R-value could be doubled due to installing a thin sheet in the middle of the enclosed airspace. The results presented in this section could be important for future applications when a thin reflecting foil is placed in the middle of the airspace of double glazed windows and curtain wall systems so as to enhance the energy performance of these systems.
Comparison between R-values Predicted by model and Listed in ASHRAE Table [14]

For an enclosed airspace of $H = 12$ in (304.8 mm) and different airspace thicknesses ($\delta = 13, 20, 40$ and $90$ mm), Figure 6 through Figure 9 show the dependence of the R-value on the effective emittance, $\varepsilon_{\text{avg}}$, for the “1-Cavity” and “2-Cavities” simulation cases when the enclosed airspace was subjected to different values of $T_{\text{avg}}$ and $\Delta T$. For the purpose of comparison, the R-values from the 2009 ASHRAE table ($\varepsilon_{\text{avg}} = 0.03, 0.05, 0.2, 0.5, 0.82$) [14], which correspond to the predicted R-values for the “1-Cavity” simulation case are also shown in these figures.

For an enclosed airspace of $\delta = 13$ mm ($H = 12$ in (304.8 mm)) and for the “1-Cavity” case, Figure 6 shows that the predicted R-values are in good agreement with R-values of the ASHRAE table for all values of $\varepsilon_{\text{avg}}$ when $T_{\text{avg}} = 32.2^\circ\text{C}, \Delta T = 5.6^\circ\text{C}$ (Figure 6a), $T_{\text{avg}} = 10.0^\circ\text{C}, \Delta T = 5.6^\circ\text{C}$ (Figure 6c), and $T_{\text{avg}} = -17.8^\circ\text{C}, \Delta T = 5.6^\circ\text{C}$ (Figure 6e). For other values of $T_{\text{avg}}$ and $\Delta T$, the predicted R-values are in good agreement with R-values of the ASHRAE table at high $\varepsilon_{\text{avg}}$ (0.2, 0.5 and 0.82); however, at low values of $\varepsilon_{\text{avg}}$ (0.03, 0.05), the R-values in the ASHRAE table are greater than those predicted by the numerical model. For example, at $\varepsilon_{\text{avg}} = 0.03$, the R-values in the ASHRAE table are higher than the predicted R-values by 8.1% ($T_{\text{avg}} = 10.0^\circ\text{C}, \Delta T = 16.7^\circ\text{C}$, Figure 6b), 8.9% ($T_{\text{avg}} = -17.8^\circ\text{C}, \Delta T = 11.1^\circ\text{C}$, Figure 6d), 10.5% ($T_{\text{avg}} = -45.6^\circ\text{C}, \Delta T = 11.1^\circ\text{C}$, Figure 6f), and 6.0% ($T_{\text{avg}} = -45.6^\circ\text{C}, \Delta T = 5.6^\circ\text{C}$, Figure 6g).

For an enclosed airspace of $\delta = 20$ mm ($H = 12$ in (304.8 mm)), it is shown in Figure 7 that the predicted R-values are in good agreement with those given in the ASHRAE table at high values of $\varepsilon_{\text{avg}}$ (0.5 and 0.82) for all values of $T_{\text{avg}}$ and $\Delta T$. However, at low values of $\varepsilon_{\text{avg}}$ (0.03, 0.05 and 0.2) the R-values of the ASHRAE table are higher than the predicted R-values, whereas the difference between these values increases as $\varepsilon_{\text{avg}}$ decreases. At $\varepsilon_{\text{avg}} = 0.03$, the R-values given in ASHRAE are higher than the predicted R-values by 15.2% ($T_{\text{avg}} = 32.2^\circ\text{C}, \Delta T = 5.6^\circ\text{C}$, Figure 7a), 20.7% ($T_{\text{avg}} = 10.0^\circ\text{C}, \Delta T = 16.7^\circ\text{C}$, Figure 7b), 16.6% ($T_{\text{avg}} = 10.0^\circ\text{C}, \Delta T = 5.6^\circ\text{C}$, Figure 7c), 19.6% ($T_{\text{avg}} = -17.8^\circ\text{C}, \Delta T = 11.1^\circ\text{C}$, Figure 7d), 19.6% ($T_{\text{avg}} = -17.8^\circ\text{C}, \Delta T = 5.6^\circ\text{C}$, Figure 7e), 14.6% ($T_{\text{avg}} = -45.6^\circ\text{C}, \Delta T = 11.1^\circ\text{C}$, Figure 7f), and 20.1% ($T_{\text{avg}} = -45.6^\circ\text{C}, \Delta T = 5.6^\circ\text{C}$, Figure 7g).

For an enclosed airspace of $\delta = 40$ mm ($H = 12$ in (304.8 mm)), Figure 8 shows that the predicted R-values are in good agreement with ASHRAE R-values at high values of $\varepsilon_{\text{avg}}$ (0.2, 0.5 and 0.82) for all values of $T_{\text{avg}}$ and $\Delta T$ and at all values of $\varepsilon_{\text{avg}}$ for $T_{\text{avg}} = -45.6^\circ\text{C}, \Delta T = 11.1^\circ\text{C}$ (Figure 8f). At low $\varepsilon_{\text{avg}}$ (0.03, 0.05) the ASHRAE R-values are higher than the predicted R-values. At $\varepsilon_{\text{avg}} = 0.03$, the ASHRAE R-values are higher than the predicted R-values by 37.9% ($T_{\text{avg}} = 32.2^\circ\text{C}, \Delta T = 5.6^\circ\text{C}$, Figure 8a), 12.8% ($T_{\text{avg}} = 10.0^\circ\text{C}, \Delta T = 16.7^\circ\text{C}$, Figure 8b), 30.7% ($T_{\text{avg}} = 10.0^\circ\text{C}, \Delta T = 5.6^\circ\text{C}$, Figure 8c), 11.9% ($T_{\text{avg}} = -17.8^\circ\text{C}, \Delta T = 11.1^\circ\text{C}$, Figure 8d), 17.9% ($T_{\text{avg}} = -17.8^\circ\text{C}, \Delta T = 5.6^\circ\text{C}$, Figure 8e), 5.4% ($T_{\text{avg}} = -45.6^\circ\text{C}, \Delta T = 11.1^\circ\text{C}$, Figure 8f), and 13.2% ($T_{\text{avg}} = -45.6^\circ\text{C}, \Delta T = 5.6^\circ\text{C}$, Figure 8g).

Similarly, for an enclosed airspace of $\delta = 90$ mm ($H = 12$ in (304.8 mm)), Figure 9 shows that the predicted R-values are in good agreement with ASHRAE R-values at high $\varepsilon_{\text{avg}}$ (0.2, 0.5 and 0.82) for all values of $T_{\text{avg}}$ and $\Delta T$. At low values of $\varepsilon_{\text{avg}}$ (0.03, 0.05) the ASHRAE R-values are higher than the predicted R-values. At $\varepsilon_{\text{avg}} = 0.03$, the ASHRAE R-values are higher than the predicted R-values by 29.4% ($T_{\text{avg}} = 32.2^\circ\text{C}, \Delta T = 5.6^\circ\text{C}$, Figure 9a), 16.1% ($T_{\text{avg}} = 10.0^\circ\text{C}, \Delta T = 5.6^\circ\text{C}$, Figure 9b),
16.7°C, Figure 9b), 25.3% (T_{avg} = 10.0°C, ΔT = 5.6°C, Figure 9c), 14.9% (T_{avg} = -17.8°C, ΔT = 11.1°C, Figure 9d), 17.7% (T_{avg} = -17.8°C, ΔT = 5.6°C, Figure 9e), 12.6% (T_{avg} = -45.6°C, ΔT = 11.1°C, Figure 9f), and 15.2% (T_{avg} = -45.6°C, ΔT = 5.6°C, Figure 9g).

In summary, for enclosed airspaces of height of 12 in (304.8 mm) and different thicknesses, the R-values from ASHRAE are in good agreement with the predicted R-values derived from numerical simulation at high values of ε_{eff} (0.5 and 0.82). However, at low values of ε_{eff} (0.03, 0.05 and 0.2) the ASHRAE R-values are higher than the predicted R-values.

Dependence of the R-value on the Aspect Ratio

For enclosed airspaces of different thickness (δ = 13, 20, 40 and 90 mm), numerical simulations were conducted at T_{avg} = 32.2, 10.0, -17.8 and -45.6°C, ΔT = 5.6, 11.1 and 16.7°C, ε_{eff} = 0 – 0.82, and different aspect ratios, A_R (A_R = H/δ, H = 8 – 96 in (203.2 – 2,438.4 mm)). The R-values obtained from the results of the simulation are shown in Figure 10 through Figure 13. Because the predicted R-values for enclosed airspaces (δ = 13 mm, Figure 10) having heights of H = 56 in (1,422.4 mm) and 88 in (2,235.2 mm) were quite close, no numerical simulations were conducted at a height of H = 96 in (2,438.4 mm). For the purpose of comparisons, these figures also show the R-values taken from the ASHRAE table [14] for different values of airspace thickness, δ.

In recent studies [7, 9], the contribution of Furred-Airspace Assembly (FAA) to the R-values of wall systems with horizontal furring and vertical furring of the same center-to-center spacing were investigated. The FAA was attached to EPS layer (1 in (25.4 mm) thick), which was bonded by Low Emissivity Material (LEM) facing the airspace (1 in (25.4 mm) thick). The results from these studies [7, 9] showed that the contribution of FAA to the R-value of a wall with vertical furring was greater than that for a wall with horizontal furring. Note that the height of the enclosed airspace in the wall with vertical furring was greater than that in the wall with horizontal furring. In this study, it can be determined from a review of Figure 10 to Figure 13 that the R-value of an enclosed airspace of longer height is greater than that of an enclosed airspace of shorter height. For example, in Figure 10b for an enclosed airspace having a thickness of 13 mm (T_{avg} = 10.0°C, ΔT = 16.7°C), it is shown that increasing H from 8 in (203.2 mm) (A_R = 16) to 88 in (2,235.2 mm) (A_R = 172) resulted in an increase in the R-value by 20.3% at ε_{eff} = 0.03. This observation is in agreement with results from previous studies [7, 9].

In Figure 11b, it is shown that for an enclosed airspace having a thickness of δ = 20 mm ε_{eff} = 0.03 (T_{avg} = 10.0°C, ΔT = 16.7°C), an increase of H from 8 in (203.2 mm) (A_R = 10) to 96 in (2,438.4 mm) (A_R = 122) resulted in an increase of R-value by 48.4%. Likewise as given in Figure 12b, for enclosed airspace of δ = 40 mm at ε_{eff} = 0.03 (T_{avg} = 10.0°C, ΔT = 16.7°C), an increase of H from 8 in (203.2 mm) (A_R = 5) to 96 in (2,438.4 mm) (A_R = 61) resulted in an increase of R-value by 86.5%. Finally, as provided in Figure 13b, for enclosed airspace of δ = 90 mm at ε_{eff} = 0.03 (T_{avg} = 10.0°C, ΔT = 16.7°C), an increase of H from 8 (203.2 mm) in (A_R = 2) to 96 in (2,438.4 mm) (A_R = 27) resulted in an increase of R-value by 84.0%.

More observations were made from results obtained by numerical simulation to derive R-values and that are shown in Figure 10 through Figure 13 are the following:

- The results cover a wide range of aspect ratios (A_R) from 2 to 172 (16 to 172, 10 to 122, 5 to 61 and 2 to 27 for δ = 13, 20, 40 and 90 mm, respectively).
For a given value of $\delta$ and $\varepsilon_{\text{eff}}$, the increase in the R-value due to increasing the value of aspect ratio depends on the values of $T_{\text{avg}}$ and $\Delta T$.

The heights of airspace that result in the closest agreement between the predicted R-values and the ASHRAE R-values [14] are provided in Table 3.

Practical Correlations for the R-values

The resulting R-values that were presented in the previous section for an enclosed airspace of selected thicknesses ranging between $\delta = 13$ mm to $\delta = 90$ mm, and as provided in Figure 10 to Figure 13 inclusively were used to develop practical correlations for the R-values. For the wide application of these correlations, two types of correlations were considered:

I. For given set of values for $T_{\text{avg}}$ and $\Delta T$, as are also provided in the ASHRAE table [14], a correlation was developed for each value of thickness, $\delta$, to obtain the corresponding R-value in (ft$^2$hr°F/BTU) as a function of $A_R$ and $\varepsilon_{\text{eff}}$. This correlation is given in the following form:

$$ R - \text{value} = R_c(T_{\text{avg}}) + a_0 A_R^{a_1} + a A_R^{a_2} \varepsilon_{\text{eff}}^b + A_R^{a_3} \sum_{i=1}^{4} b_i \varepsilon_{\text{eff}}^i. \quad (3) $$

Where the coefficients $R_c(T_{\text{avg}})$, $a_0$, $a$, $b_1$, $b_2$, $b_3$, $b_4$, $a_1$, $a_2$, $a_3$, and $\beta$ in Eq. (3) are listed in Table 4 for different values of $\delta$. In Eq. (3), $R_c(T_{\text{avg}})$ is the R-value in (ft$^2$hr°F/BTU) of the enclosed airspace due to conduction only, which also can be given as:

$$ R_c(T_{\text{avg}}) = \delta / \lambda(T_{\text{avg}}), \quad (4) $$

where,

$$ \lambda(T_{\text{avg}}) = \sum_{i=0}^{4} f_i T_{\text{avg}}^i, \quad f_0 = -0.00227583562, \quad f_1 = 1.15480022E - 4, \quad f_2 = -7.90252856E - 8, \quad f_3 = 4.11702505E - 11, \quad f_4 = -7.43864331E - 15. \quad (5) $$

Note that $\lambda(T_{\text{avg}})$ in Eq. (5) is the average thermal conductivity of air in (W/mK), which is evaluated at the mean temperature of the airspace, $T_{\text{avg}}$ in (K). It is important to point out that the calculated value of $R_c(T_{\text{avg}})$ from Eq. (4) and (5) must be converted to be in (ft$^2$hr°F/BTU) in order to be used in Eq. (3).

II. A comprehensive correlation was developed for each value of $\delta$ to obtain the R-value in (ft$^2$hr°F/BTU) as a function of all parameters (i.e. $T_{\text{avg}}$, $\Delta T$, $A_R$ and $\varepsilon_{\text{eff}}$). This correlation is given in the following form:
\[ R - \text{value} = R_c(T_{\text{avg}}) + a_0A_R^aT_{\text{avg}}^{a_1}(\Delta T)^{a_2} + aA_R^a\epsilon_{\text{eff}}T_{\text{avg}}^{a_2}(\Delta T)^{a_3} + A_R^{a_3}T_{\text{avg}}^{a_4}\sum_{i=1}^{4}b_i\epsilon_{\text{eff}} \]  

(6)

In Eq. (6), \( R_c(T_{\text{avg}}) \) is given by Eq. (4) and (5) and its unit must be in \((\text{ft}^2\text{hr}^\circ\text{F}/\text{BTU})\), whereas units for \( T_{\text{avg}} \) and \( \Delta T \) must be in \((\text{K})\). The other coefficients in Eq. (6) \((a_0, a, b_1, b_2, b_3, b_4, \alpha_1, \alpha_2, \alpha_3, \beta, a_1, a_2, a_3, c_1, c_2, \text{and } c_3)\) are listed in Table 5. In order to minimize the error in developing this correlation, these coefficients were obtained for values of \( \epsilon_{\text{eff}} < 0.4 \) and \( \epsilon_{\text{eff}} \geq 0.4 \) (see Table 5).

Note that the first term in RHS of Eq. (3) and Eq. (6) represents the R-value due to heat transfer by conduction only. The second term on the RHS of these equations represents the reduction in R-value due to heat transfer by convection only. The final two terms on the RHS of these equations represent the reduction in R-value due to heat transfer by radiation only at different values of \( \epsilon_{\text{eff}} \).

For an enclosed airspace of a given thickness, the predicted R-values obtained using the numerical simulation model (see Figure 10 through Figure 13) are compared with those obtained using the correlations given in Eq. (3) and Eq. (6). These comparisons are shown in Figure 14 (\( \delta = 13 \text{ mm} \)), Figure 15 (\( \delta = 20 \text{ mm} \)), Figure 16 (\( \delta = 40 \text{ mm} \)), and Figure 17 (\( \delta = 90 \text{ mm} \)). As shown in these figures, most of the predicted R-values are in good agreement with those obtained using the correlation given by Eq. (3) (within \( \pm 3\% \), as given in Figure 14a to Figure 17a). Furthermore, most of the predicted R-values are in good agreement with those obtained using the correlation given by Eq. (6) (within \( \pm 5\% \), as shown in Figure 14b to Figure 17b).

SUMMARY AND CONCLUSIONS

The numerical simulation model used in this study, had been benchmarked against experimental data derived for reflective insulations, as reported in previous studies [6, 8, 11], on the basis of results derived from two standard test methods: (i) Guarded Hot Box (GHB) in accordance with ASTM C-1363, and (ii) Heat Flow Meter (HFM) in accordance of ASTM C-518. In the first portion of this study, the model was used to conduct numerical simulations to predict the thermal resistance (R-value) of vertical enclosed airspaces of different dimensions and effective emittance. The various airspaces were subjected to different mean temperatures and temperature differences. The R-values predicted by the model were compared to those provided in the ASHRAE table [14] for the different conditions. The dependence of the R-value on the aspect ratio of the enclosed airspace was investigated for different conditions. Depending on the thickness of the enclosed airspace and the boundary conditions, the results showed that the aspect ratio can have a significant effect on the R-value (e.g. up to \(-85\% \) for an effective emittance of 0.03). Considerations were also given to investigating the potential increase in the R-value of the enclosed airspace when a thin sheet having different values of emissivity on both sides was placed vertically in the middle of the airspace. Depending on the effective emittance, the results showed that the R-value could be doubled with the incorporation of this thin sheet along the middle of the enclosed airspace.

In the second portion of this study, practical correlations were developed to determine the R-values of enclosed airspaces of different thicknesses (\( \delta \)), and for a wide range of values for
various parameters, including: (a) aspect ratio \( (A_R) \), (b) temperature difference across the airspace \( (\Delta T) \), (c) mean temperature \( (T_{avg}) \), and, (d) effective emittance \( (\varepsilon_{eff}) \). Two types of correlations were developed:

I. For selected values of \( T_{avg} \) and \( \Delta T \) that are given in the ASHRAE table [14], a correlation was developed for each value of \( \delta \) to obtain the R-value as a function of \( A_R \) and \( \varepsilon_{eff} \). This correlation is given by Eq. (3). The predicted R-values from the numerical model and those obtained using these correlations were in good agreement (mainly within \( \pm 3\% \)).

II. A comprehensive correlation was developed for each value of \( \delta \) to obtain the R-value as a function of \( T_{avg}, \Delta T, A_R \) and \( \varepsilon_{eff} \). This correlation is given by Eq. (6). The predicted R-values from the model and those obtained using this correlation agreed within \( \pm 5\% \).

It is of practical importance in the design of building envelopes to determine the R-value of enclosed airspaces having different values of effective emittance under varying climatic conditions to avoid selecting oversized heating and cooling equipment. The practical correlations that were developed in this paper can be used by architects and building designers to determine the R-values of vertically enclosed airspaces having varying aspect ratios and values of effective emittance, and subjected to a wide range of mean temperatures and temperature differences across the airspace. These correlations can also be readily implemented in currently available energy simulation models (e.g. ESP-r, Energy Plus, DOE, etc.). Similar practical correlations for determining the R-values of flat and sloped roof applications having enclosed airspaces of different orientations and directions of heat flow will be reported in future publications.

Acknowledgments

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NOMENCLATURE

\( A_R \quad \text{Aspect ratio, the ratio between the height and the thickness} \)
\( g \quad \text{Gravitational acceleration,} \ 9.81 \text{ m/s}^2 \)
\( Gr \quad \text{Grashoff number} \)
\( h \quad \text{Convective heat transfer coefficient (W/(m}^2\text{.K})} \)
\( H \quad \text{Height of enclosed airspace (m)} \)
\( Nu \quad \text{Nusselt number} \)
\( Pr \quad \text{Prandtl number} \)
\( Ra \quad \text{Rayleigh number (Gr. Pr)} \)
\( T_{avg} \quad \text{Mean temperature (K)} \)
\( \Delta T \quad \text{Temperature difference across the enclosed airspace (K)} \)
\( \beta \quad \text{Thermal expansion coefficient (1/K)} \)
\( \rho \quad \text{Density of the gas (kg/m}^3\text{),} \)
\( \mu \quad \text{Dynamic viscosity (Pa.s),} \)
\( \delta \quad \text{Airspace thickness (m)} \)
\( \lambda \quad \text{Thermal conductivity (W/(m.K))} \)
\( \varepsilon \quad \text{Emissivity} \)
\( \varepsilon_{eff} \quad \text{Effective emittance} \)
REFERENCES


13. H.H. Saber, W. Maref, and M.C. Swinton, “Thermal Response of Basement Wall Systems with Low Emissivity Material and Furred Airspace”, Journal of Building Physics, Published online in 5 August 2011, DOI: 10.1177/1744259111411652, The online version of this article can be found at: http://jen.sagepub.com/content/early/2011/07/30/1744259111411652.


Table 1. Maximum upward and downward velocities in the cases of “1-Cavity” and “2=Cavities” (see Figure 2b through Figure 5b) when $T_{avg} = 10^{\circ}C$, $\Delta T = 16.7^{\circ}C$, $c_1 = 0.05$, $c_2 = 0.9$, $\delta = 13\text{ mm}$, $H = 12\text{ in (304.8 mm)}$

<table>
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<tr>
<th>Thickness of airspace, $\delta$ (mm)</th>
<th>Maximum Upward Velocity $(u_y)$ / Maximum Downward Velocity $(u_y)$ (mm/s)</th>
<th>$u_y$, 1-Cavity / $u_y$, 2-Cavities</th>
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<tbody>
<tr>
<td>13</td>
<td>54.6 / -54.5</td>
<td>7.4 / -7.7</td>
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<td>20</td>
<td>119.9 / -117.0</td>
<td>17.8 / -18.6</td>
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<td>40</td>
<td>114.5 / -113.9</td>
<td>62.0 / -64.2</td>
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<tr>
<td>90</td>
<td>112.0 / -112.3</td>
<td>69.2 / -70.6</td>
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Table 2. Percentage increase in R-value between “2-Cavities” and “1-Cavity” simulation cases for given value of $\varepsilon_{\text{eff}} = 0.03$ and 0.82 (H = 12 in (304.8 mm))

<table>
<thead>
<tr>
<th>Thickness of Airspace, $\delta$ (mm)</th>
<th>Increase (%) in R-value between “2-Cavities” and “1-Cavity” Simulation Cases for Given Value of $\varepsilon_{\text{eff}}$</th>
<th>Governing Simulation Conditions and Related Figures</th>
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<td>$\varepsilon_{\text{eff}} = 0.03$</td>
<td>$\varepsilon_{\text{eff}} = 0.82^*$</td>
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<td>$\delta = 13$ mm</td>
<td>10.6% 58.3%</td>
<td>$T_{\text{avg}} = 32.2\degree C, \Delta T = 5.6\degree C$ (Figure 6a)</td>
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<td>20.5% 59.4%</td>
<td>$T_{\text{avg}} = 10.0\degree C, \Delta T = 16.7\degree C$ (Figure 6b)</td>
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<td>11.2% 54.8%</td>
<td>$T_{\text{avg}} = 10.0\degree C, \Delta T = 5.6\degree C$ (Figure 6c)</td>
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<td>$T_{\text{avg}} = -17.8\degree C, \Delta T = 11.1\degree C$ (Figure 6d)</td>
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<td>13.0% 50.9%</td>
<td>$T_{\text{avg}} = -17.8\degree C, \Delta T = 5.6\degree C$ (Figure 6e)</td>
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<td>34.4% 59.2%</td>
<td>$T_{\text{avg}} = -45.6\degree C, \Delta T = 11.1\degree C$ (Figure 6f)</td>
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<td>17.8% 48.8%</td>
<td>$T_{\text{avg}} = -45.6\degree C, \Delta T = 5.6\degree C$ (Figure 6g)</td>
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<td>$\delta = 20$ mm</td>
<td>29.8% 72.1%</td>
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<td>50.7% 75.3%</td>
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<td>100.0% 98.3%</td>
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<td>70.7% 82.8%</td>
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<td>$\delta = 40$ mm</td>
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<td>132.9% 110.8%</td>
<td>$T_{\text{avg}} = 10.0\degree C, \Delta T = 16.7\degree C$ (Figure 8b)</td>
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<td>132.1% 107.0%</td>
<td>$T_{\text{avg}} = -17.8\degree C, \Delta T = 5.6\degree C$ (Figure 8e)</td>
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<td>$T_{\text{avg}} = -45.6\degree C, \Delta T = 11.1\degree C$ (Figure 8f)</td>
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<td>130.7% 113.7%</td>
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<td>$\delta = 90$ mm</td>
<td>128.6% 97.8%</td>
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<td>119.8% 100.9%</td>
<td>$T_{\text{avg}} = -17.8\degree C, \Delta T = 11.1\degree C$ (Figure 9d)</td>
</tr>
<tr>
<td></td>
<td>120.4% 99.6%</td>
<td>$T_{\text{avg}} = -17.8\degree C, \Delta T = 5.6\degree C$ (Figure 9e)</td>
</tr>
<tr>
<td></td>
<td>116.8% 102.7%</td>
<td>$T_{\text{avg}} = -45.6\degree C, \Delta T = 11.1\degree C$ (Figure 9f)</td>
</tr>
<tr>
<td></td>
<td>117.8% 101.4%</td>
<td>$T_{\text{avg}} = -45.6\degree C, \Delta T = 5.6\degree C$ (Figure 9g)</td>
</tr>
</tbody>
</table>

*0.82 means that all surfaces of the airspace have emissivity of 0.9
Table 3. Height of airspace, H, for which predicted R-value for an enclosed airspace, for all values of $T_{avg}$, $\Delta T$, and $\varepsilon_{eff}$ is in closest agreement with R-values given in the ASHRAE table [14].

<table>
<thead>
<tr>
<th>Thickness of Airspace, $\delta$ (mm)</th>
<th>Airspace Height, H (in) or (mm)</th>
<th>Accompanying Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>~ 24 in (609.6 mm)</td>
<td>Figure 10</td>
</tr>
<tr>
<td>20</td>
<td>~ 24– 36 in (609.6 – 914.4 mm)</td>
<td>Figure 11</td>
</tr>
<tr>
<td>40</td>
<td>~ 16 – 36 in (406.4 – 914.4 mm)</td>
<td>Figure 12</td>
</tr>
<tr>
<td>90</td>
<td>~ 16 – 36 in (406.4 – 914.4 mm)</td>
<td>Figure 13</td>
</tr>
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</table>
Table 4. List of correlation coefficients given by Eq. (3) for an enclosed airspace of different thicknesses (δ)

<table>
<thead>
<tr>
<th>T_{avg} (°C)</th>
<th>ΔT (K)</th>
<th>R_c(T_{avg})*</th>
<th>a₀</th>
<th>a</th>
<th>b₁</th>
<th>b₂</th>
<th>b₃</th>
<th>b₄</th>
<th>α₁</th>
<th>α₂</th>
<th>α₃</th>
<th>β</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>32.2</td>
<td>5.6</td>
<td>2.7805E+00</td>
<td>-5.6245E-01</td>
<td>2.2352E+02</td>
<td>-2.2839E+02</td>
<td>8.6103E+00</td>
<td>-9.0686E+00</td>
<td>3.7422E+00</td>
<td>-3.9510E-01</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td>16.7</td>
<td>2.9638E+00</td>
<td>-1.9285E-00</td>
<td>2.3712E+02</td>
<td>-2.4177E+02</td>
<td>8.7154E+00</td>
<td>-8.6060E+00</td>
<td>3.5098E+00</td>
<td>-4.9012E-01</td>
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<tr>
<td></td>
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<td></td>
<td>10</td>
<td>16.7</td>
<td>2.9638E+00</td>
<td>-7.3450E-01</td>
<td>2.2482E+02</td>
<td>-2.3014E+02</td>
<td>9.1270E+00</td>
<td>-8.8940E+00</td>
<td>3.6084E+00</td>
<td>-4.2199E-01</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-17.8</td>
<td>11.1</td>
<td>3.2363E+00</td>
<td>-2.6173E+02</td>
<td>2.3621E+02</td>
<td>-2.4123E+02</td>
<td>9.2490E+00</td>
<td>-8.7320E+00</td>
<td>3.5426E+00</td>
<td>-5.4937E-01</td>
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<tr>
<td></td>
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<td>-2.3347E+02</td>
<td>9.0639E+00</td>
<td>-8.6553E+00</td>
<td>3.5207E+00</td>
<td>-5.5177E-01</td>
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<tr>
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<td>-45.6</td>
<td>11.1</td>
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<td>-2.9292E+02</td>
<td>2.1573E+02</td>
<td>-2.1835E+02</td>
<td>4.0571E+00</td>
<td>-3.2542E+00</td>
<td>1.2326E+00</td>
<td>-4.0148E-01</td>
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<tr>
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<td></td>
<td>-45.6</td>
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<td>-1.7507E+02</td>
<td>3.1773E+00</td>
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<td>1.9647E+00</td>
<td>-1.3399E+00</td>
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<tr>
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<td></td>
<td>-17.8</td>
<td>11.1</td>
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<td>-7.3367E+01</td>
<td>5.1020E+01</td>
<td>-2.1613E+00</td>
<td>3.2659E+00</td>
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<td>1.3420E+00</td>
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<td>4.0571E+00</td>
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<td>1.2326E+00</td>
<td>-4.0148E-01</td>
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<td>-5.6</td>
<td>5.6</td>
<td>3.5750E+00</td>
<td>2.4750E+01</td>
<td>-6.4410E+00</td>
<td>4.2295E+00</td>
<td>-2.8103E+00</td>
<td>4.0681E+00</td>
<td>-3.7968E+00</td>
<td>1.5294E+00</td>
</tr>
</tbody>
</table>

* R_c(T_{avg}) is in (hr² hr °F/BTU)
Table 5. List of the correlation coefficients given by Eq. (6) for an enclosed airspace of different thickness (δ)

<table>
<thead>
<tr>
<th>δ (mm)</th>
<th>13</th>
<th>20</th>
<th>40</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>&lt; 0.4</td>
<td>&gt; 0.4</td>
<td>&lt; 0.4</td>
</tr>
<tr>
<td>ε&lt;sub&gt;eff&lt;/sub&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>-3.83570E+07</td>
<td>-5.14147E+10</td>
<td>-3.60829E+03</td>
<td>-7.76690E+03</td>
</tr>
<tr>
<td>b&lt;sub&gt;1&lt;/sub&gt;</td>
<td>2.59667E+09</td>
<td>4.04079E+06</td>
<td>4.82319E+00</td>
<td>4.03283E+03</td>
</tr>
<tr>
<td>b&lt;sub&gt;2&lt;/sub&gt;</td>
<td>-1.04989E-01</td>
<td>-3.57570E+00</td>
<td>-2.75887E-03</td>
<td>-1.29142E+00</td>
</tr>
<tr>
<td>b&lt;sub&gt;3&lt;/sub&gt;</td>
<td>2.16750E-01</td>
<td>4.08912E+00</td>
<td>5.58902E-03</td>
<td>1.40831E+00</td>
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<tr>
<td>b&lt;sub&gt;4&lt;/sub&gt;</td>
<td>-3.34605E-01</td>
<td>-1.87698E+00</td>
<td>-5.14488E-03</td>
<td>-6.12328E-01</td>
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<tr>
<td>α&lt;sub&gt;1&lt;/sub&gt;</td>
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<td>-1.90012E-01</td>
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<tr>
<td>α&lt;sub&gt;3&lt;/sub&gt;</td>
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<td>8.49586E-01</td>
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<tr>
<td>β</td>
<td>8.22318E-02</td>
<td>7.83258E-03</td>
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<td>-1.49296E+00</td>
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<td>-5.95124E+00</td>
<td>-4.16511E-01</td>
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<td>c&lt;sub&gt;3&lt;/sub&gt;</td>
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<td>-1.88410E-02</td>
<td>-3.13868E-01</td>
<td>-1.20773E-01</td>
</tr>
</tbody>
</table>
Figure 1. Schematics of enclosed airspace with and without thin sheet of low emissivity on both sides, placed in the middle of the airspace
Figure 2. Temperature and vertical velocity contours in enclosed airspace in the cases of “1-Cavity” and “2-Cavities” when $T_{avg} = 10^\circ\text{C}$, $\Delta T = 16.7^\circ\text{C}$, $\varepsilon_1 = 0.05$, $\varepsilon_2 = 0.9$, $\delta = 13$ mm, $H = 12$ in (304.8 mm)
(a) Temperature (°C)

(b) Vertical velocity (mm/s)

Figure 3. Temperature and vertical velocity contours in enclosed airspace in the cases of “1-Cavity” and “2-Cavities” when $T_{avg} = 10^\circ$C, $\Delta T = 16.7^\circ$C, $\varepsilon_1 = 0.05$, $\varepsilon_2 = 0.9$, $\delta = 20$ mm, $H = 12$ in (304.8 mm).
**Figure 4.** Temperature and vertical velocity contours in enclosed airspace in the cases of “1-Cavity” and “2-Cavities” when $T_{\text{avg}} = 10^\circ\text{C}$, $\Delta T = 16.7^\circ\text{C}$, $\varepsilon_1 = 0.05$, $\varepsilon_2 = 0.9$, $\delta = 40 \text{ mm}$, $H = 12 \text{ in (304.8 mm)}$
Figure 5. Temperature and vertical velocity contours in enclosed airspace in the cases of “1-Cavity” and “2-Cavities” when $T_{avg} = 10^\circ C$, $\Delta T = 16.7^\circ C$, $\varepsilon_1 = 0.05$, $\varepsilon_2 = 0.9$, $\delta = 90$ mm, $H = 12$ in (304.8 mm)
Figure 6. Dependence of R-value on $T_{avg}$, $\Delta T$ and $\varepsilon_{eff}$ ($H = 12$ in (304.8 mm) and $\delta = 13$ mm)
Figure 7. Dependence of R-value on $T_{avg}$, $\Delta T$ and $\varepsilon_{eff}$ ($H = 12$ in (304.8 mm) and $\delta = 20$ mm)
Figure 8. Dependence of R-value on $T_{avg}$, $\Delta T$ and $\varepsilon_{eff}$ (H = 12 in (304.8 mm) and $\delta = 40$ mm)
Figure 9. Dependence of R-value on $T_{\text{avg}}$, $\Delta T$ and $\epsilon_{\text{eff}}$ (H = 12 in (304.8 mm) and $\delta = 90$ mm)
Figure 10. Dependence of R-value on $T_{\text{avg}}$, $\Delta T$, $\varepsilon_{\text{eff}}$ and H for airspace of 13 mm thick.
Figure 11 Dependence of R-value on $T_{avg}$, $\Delta T$, $\varepsilon_{eff}$ and $H$ for airspace of 20 mm thick
Figure 12. Dependence of R-value on $T_{avg}$, $\Delta T$, $\varepsilon_{\text{eff}}$ and $H$ for airspace of 40 mm thick
Figure 13. Dependence of R-value on $T_{avg}$, $\Delta T$, $\epsilon_{eff}$ and $H$ for airspace of 90 mm thick.
Figure 14. Comparison of R-values obtained using the present model and correlations given by Eq. (3) (Figure 14a) and Eq. (6) (Figure 14b) for airspace of 13 mm thick
Figure 15. Comparison of R-values obtained using the present model and correlations given by Eq. (3) (Figure 15a) and Eq. (6) (Figure 15b) for airspace of 20 mm thick.
Figure 16. Comparison of R-values obtained using the present model and correlations given by Eq. (3) (Figure 16a) and Eq. (6) (Figure 16b) for airspace of 40 mm thick
Figure 17. Comparison of R-values obtained using the present model and correlations given by Eq. (3) (Figure 17a) and Eq. (6) (Figure 17b) for airspace of 90 mm thick.