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A Computational Hydrodynamics Method for Horizontal Axis Turbine
--- Panel Method Modeling Migration from Propeller to Turbine

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Abstract

A computational hydrodynamics method was formulated and implemented for horizontal axis tidal turbines. This paper
presents a comparative analysis between screw propellers and horizontal axis turbines, in terms of geometry and motion
parameters, inflow velocity analysis and the implementation methodologies. Comparison and analysis are given for a
marine propeller model and a horizontal axis turbine model that have experimental measurements available in literature.
Analysis and comparison are presented in terms of thrust coefficients, shaft torque/power coefficients, blade surface
pressure distributions, and downstream velocity profiles. The effect of number of blades from 2 to 5, of a tidal turbine on
hydrodynamic efficiency is also obtained and presented. The key implementation techniques and methodologies are
provided in detail for the propeller based panel method tool to migrate as a prediction tool for tidal turbine. While the
method has been proven to be accurate and robust for many propellers tested in the past, this numerical tool could be
validated further for turbines. To further refine and validate the panel method for various turbines, it requires substantial
additional experimental measurements. These measurements include downstream velocity profile by using LDV and/or
SPIV, which are essential for numerical wake vortices descritization.

Keywords: panel method; propeller; wind turbine, tidal turbine hydrodynamics;

Nomenclature

\[ \alpha \quad \text{--- angle of attack of blade section} \]
\[ \alpha' \quad \text{--- angle of attack of blade section with added induced velocity} \]
\[ \alpha_p \quad \text{--- geometric angle of attack of blade section} \]
\[ \alpha_o \quad \text{--- angle of zero lift of blade section} \]
\[ \alpha_v \quad \text{--- angle of inflow velocity (hydrodynamic angle of attack)} \]
\[ \alpha_v' \quad \text{--- angle of inflow velocity with added induced velocity} \]
\[ \alpha_e \quad \text{--- effective angle of attack of blade section} \]
\[ C_t \quad \text{--- thrust coefficient of turbine} \]
\[ C_{POW} \quad \text{--- turbine power coefficient} \]
\[ C_p \quad \text{--- pressure coefficient} \]
\[ D \quad \text{--- diameter of turbine or propeller} \]
\[ EAR \quad \text{--- expanded area ratio of blades of propeller disk, or solidity of turbine.} \]
\[ h_D \quad \text{--- hub diameter to blade diameter ratio} \]
\[ J \quad \text{--- shaft advance coefficient} \]
\[ K_t \quad \text{--- shaft thrust coefficient} \]
\[ K_q \quad \text{--- shaft torque coefficient} \]

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1. Introduction

Computational methods have been widely used for wind or tidal turbine research and development. A comprehensive review of these methods and their merits and limitations, for example, was given by Nicholls-Lee et al. [1]. Among these methods, the panel methods, in the most advanced and complicated method group, have both high computing efficiency and prediction accuracy as an engineering tool for turbine simulation and design optimization. While probably the blade element methods (BEM) are the most widely used as preliminary simulation tools for wind and tidal turbines, much fewer penal methods could be found for turbine in literature. To the author’s knowledge, these few panel methods for turbine simulation and prediction include a 2D panel method by Drela et al. mentioned in [1], a 3D time domain panel method for wind turbine by Hampsey [2], a rudder-propeller interaction panel code by Turnock [3] and a design based simulation work by Greco et al. [4]. Formulation and implementation of these panel methods for both wind turbines and tidal turbines are basically the same though different fluid properties such as viscosity/density and hence Reynolds number, require different scheme and numerical treatment. The primary difference for panel methods as tools for wind turbine and for tidal turbine is that tidal turbines are often exposed to cavitation that would cut off the negative pressure spike and reduces the energy extraction efficiency substantially even if there does not exist stall or separation. For wind turbine under high speed inflow conditions, compressibility might need to be a problem, at least in terms of turbine efficiency correction. A bare panel method developed from scratch also needs many other essential numerical components for different application cases. Establishing these numerical components for a newly developed bare panel method require substantial effort and implementation development, as described later in this section. In this paper, it shows an efficient turn-key solution that a well established propeller panel method can be quickly turned into a turbine simulation code.
Panel methods, are called the boundary element methods, or boundary integral methods (BEM in short as well). Lifting surface and panel methods have been widely used in research and development of aircraft wings, hydrofoils and both aerial and marine propellers. Zero thickness propeller blade simulated and computed by lifting surface theory in the computational fluid dynamics (CFD) field has a history of over 60 years. The use of the surface panel method for a simple body surface mesh can be traced back to the early 1960s. Hess and Vararezo [5] probably made the first panel method computation for propellers. To deal with complete aircraft geometry, panel method codes, PMARC (Panel Method Ames Research Center) developed by Katz [6] and VSAERO by Maskew [7] are the early examples of panel methods for aircraft wings and propellers. On the other hand, panel methods have also been used for marine propeller research development and early examples among those are publications by Kerwin [8] and Hoshino [9], just to name a few. A time domain unsteady panel method code OSFBEM (oscillating foil boundary element method) was developed by the author for oscillating propulsors of both chordwise and spanwise flexibility to simulate marine animals’ propulsion [10]. To respond the need in simulation of fluid-structure interactive hydrodynamics to predict ice blockage effects between ice sheet and ice-class propellers, a panel method code, PROPELLA [11] was developed in 1996, based OSFBEM. Since then, continued efforts were made to maintain and enhance the capability for the code. The capability for unsteady oblique flow and inflow wake were presented in early 1998 [12]. Automatic body surface generation for propeller of arbitrary number of blades, nozzle, rudder, ice blockage etc. was presented in 2001, along with velocity profile downstream prediction and wake vortices roll-up enhancement [13]. Cavitation predictive capability via an empirical formulation was established for PROPELLA and presented in 2001 [14]. At the meantime, a pre- and postprocessor was developed for the code by using OpenGL and Visual C++ of Microsoft Foundation Class, as a 3D unsteady data visualization tool [15] to view the geometry motion and colour blended results. A novel and robust numerical Kutta condition using Broyden’s iteration was developed and presented in 2002 [16]. Since 2003, this propeller panel method has been redeveloped with a multiple-body interaction formulation to deal with a propeller with pod and strut, called a podded propeller unit and recently for a podded propulsor interacting with an ice body at variable proximity (transient hydrodynamic response of a propeller moving towards an ice sheet) [17]. In the past decades, many panel methods have been developed and these panel methods along with their associated numerical schemes and techniques for propellers are well established. These existing panel methods with a minor or moderate revision, could be quickly used for tidal and wind turbine prediction. The aim of this work is to present the physics and numerical similarities and difference between propeller and turbine panel methods and then provide a detailed implementation techniques and treatments.
2. Method

The current panel method is a multiple-body interaction panel method. The fundamentals of panel method have been presented in detail in some textbooks, including the ones by Moran [18] for 2D foil sections and by Katz and Plotkin [19] for unsteady 3D body and wings. A detailed formulation and implementation for a low-order, time-domain panel method, were given by the author [10].

2.1. Flow Physics Similarity and Differences between Propeller and Tidal Turbine

Being rotary wings, flow around both propeller and turbine blade sections has both similarities and differences. Flow conditions around a wing section determine the hydrodynamic characteristics of the wing section. These characteristics are mainly determined by the effective angle of attack of the blade section. The similarity and differences of the flow around a blade section between propeller and turbine are shown in the velocity diagrams in Figures 1 and 2, respectively.

In Fig. 1, blade geometric pitch angle of attack is represented as $\alpha_p$, and the angle of zero life of the blade section is $\alpha_0$ due to the camber of the blade section. Therefore the effective geometric angle of pitch is $\alpha_p + \alpha_0$. The inflow velocity angle of attack, also called hydrodynamic angle of attack in the literature, is $\alpha_p = \tan^{-1}(\frac{V_a}{2\pi r n})$, where $V_a$ is propeller shaft forward velocity, also called inflow velocity in some literature, and $r$ is the blade section local radius and $n$ is the shaft rotation speed in revolutions per second. However, even in open water, as a propeller is rotating, the inflow velocity to the propeller plane is not unidirectional because of the induced tangential velocity $V_t$ and radial velocity $V_r$. In fact, these induced velocities are not in the same plane as shown in the figure. The total inflow velocity relative to the local blade section in 3D space is then $V_a + V_t + V_r$ with the resultant velocity angle of attack $\alpha_r$. Therefore, the effective angle of attack of the blade section, that determines the hydrodynamic characteristic of a blade section, is $\alpha_r = \alpha_p + \alpha_0 - \alpha_r$ for which induced velocities have been taken into account and is $\alpha = \alpha_p - \alpha_r$ without including the induced velocities and angle of zero life. It is obvious that for a propeller to produce positive thrust, i.e., to be in propulsion mode, the effective angle of attack must be positive. In the current version of the code PROPELLA, the angle of shed wake vortices at the blade trailing edge is taken as $\alpha$ at $r = 0.7000R$, where $R$ is the radius of the propeller, when the angle of zero life is
neglected. When shed wake vortices roll-up is taken into account, the angle of shed wake vortices is modified by the induced velocity during a wake vortex relaxation procedure. While the pitch angle of shed wake vortices has substantial effect on the prediction accuracy of thrust and torque prediction, it is even more important to the accuracy of the field velocity prediction downstream of a propeller. In addition, to avail a multiple-body computational capability, all bodies in the fluid domain move individually in an acquiescent fluid.

A similar blade section velocity diagram for turbine is shown in Fig. 2. With all the same variables as those for propeller, the offset coordinates of the surfaces of propeller blade section are now interchanged, i.e., the suction side and pressure side of the blade section are swapped for a turbine. The effective angle of attack now becomes \( \alpha_e = \alpha' - \alpha_p + \alpha_0 \) when the induced velocities are taken into account and \( \alpha = \alpha' - \alpha_p \) when the induced velocities and angle of zero lift are both neglected. It is also obvious that the effective angle of attack must be positive for the blade section to be in turbine mode. Now using the propeller code PROPELLA for turbine prediction is reduced to two key modifications when preparing for input, that is, the blade sectional coordinates interchange and the formulation and implementation of the angle of shed wake vortices. Using the same motion and geometric variables for propellers, the angle of shed wake vortices for a turbine is taken as \( \alpha \) at local radius of \( 0.7R \). In the following non-dimensional analysis, the wake pitch angle of turbine was obtained as follows:

\[
\alpha = \tan^{-1} \frac{V_a}{2\pi n} - \tan^{-1} \frac{np}{2\pi n} = \tan^{-1} \frac{JnD}{2\pi} - \tan^{-1} \frac{P_D}{D} = \tan^{-1} \frac{J}{\pi \frac{r}{R}} - \tan^{-1} \frac{P_D}{\pi \frac{r}{R}}.
\]  

(1)

where \( J \) is the advance coefficient, \( J = \frac{V_a}{nD} \), of turbine and \( P_D \) is the local non-dimensional pitch, \( P_D = \frac{P}{D} \), based on the diameter of turbine. The same as propeller simulation, turbine is moving in an acquiescent fluid; the pitch angle is taken for wake vortices formulation as \( P_{WD} = J - P_D \mid_{0.7R} \). The angle of shed wake vortices for a propeller by using the similar formulation gives good thrust and torque prediction and induced velocity estimation downstream. However, for turbine mode after extensive numerical test runs, the angle of shed wake vortices with an additional angle of inflow taken as \( P_{WD} = 2J - P_D \mid_{0.7R} \) gave better results. Hoshino [9] recommended a wake vortex discretization formulation based on LDV measurements downstream of a propeller, including wake pitch, wake contraction, ultimate wake region, and transition. This formulation is valid only for propulsion mode, not turbine mode. For turbine, if this wake vortex discretization is used, wake contraction becomes a substantial inflation (800 times). This occurred at a low tip speed ratio.
of about $TSR = 4.0000$ corresponding to $J = 0.7855$, with a ratio of $J/[p/D]$ at about 3, in the tidal turbine example (see the results and discussion section below).

Fig. 3 shows the described wake vortices behind a 3-blade P4119 propeller [21] at $J=0.8330$ and a 20° root pitch tidal turbine at $TSR = 7.0000$ ($J=0.4488$). In this paper, the tidal turbine refers to the tidal turbine model by Bahaj et al. [22]. For presentation purposes, wake vortices in Fig. 3 are shown only for one blade.

### 2.2. Parametric Interpolation between Propeller and Tidal Turbine

The geometric parameters between a propeller blade and a tidal turbine are the same. For example, the expanded area ratio EAR is the ratio of the total blade area to the area of the propeller disk. Therefore the solidity of a turbine rotor is equivalent to the EAR of a propeller. For a tidal turbine calculation using a propeller code such as PROPELLA, the only geometry manipulation needed to prepare for code input, is to interchange the blade upper side with the lower side, i.e. to swap the suction side and the pressure side. However, to extrapolate the results in propeller format corresponding to a turbine convention, three major variables need to be interpolated. They are advance coefficient $J$ versus tip speed ratio $TSR$, propeller thrust coefficient $K_t$ versus turbine thrust coefficient $C_t$, and propeller torque coefficient $K_q$ versus power coefficient $C_{pow}$. With the definitions of propeller advance coefficient, $J = \frac{V_a}{nD}$, propeller thrust coefficient $K_t = \frac{T_{thrust}}{\rho n^2 D^4}$, propeller shaft torque coefficient $K_q = \frac{Q_{torque}}{\rho n^2 D^3}$, turbine tip speed ratio, $TSR = \frac{\omega R}{V_a}$, turbine thrust coefficient, $C_t = \frac{T_{thrust}}{\frac{1}{2} \rho V_a^2 A}$, and turbine power coefficient $C_{pow} = \frac{Q_{torque} \omega R}{\frac{1}{2} \rho V_a^3 A}$, the three parameters for tidal turbine in terms of propeller, are then

\begin{align}
TSR &= \frac{\omega R}{V_a} = \frac{2 \pi n R}{V_a} = \frac{\pi n D}{V_a} = \frac{\pi}{J}, \quad (2) \\
C_t &= \frac{T_{thrust}}{\frac{1}{2} \rho V_a^2 A} = \frac{K_{t} \rho n^2 D^4}{\frac{1}{2} \rho V_a^2 \pi R^2} = \frac{8}{\pi J^2} K_{t}, \quad \text{and} \quad (3) \\
C_{pow} &= \frac{Q_{torque} \omega R}{\frac{1}{2} \rho V_a^3 A} = \frac{K_{q} \rho n^2 D^3}{\frac{1}{2} \rho V_a^3 \pi R^2} = \frac{16}{\frac{1}{2} \rho V_a^3 \pi R^2} \frac{16}{V_a^3} = \frac{16}{J^3} \frac{K_{q}}{J^3}. \quad (4)
\end{align}
Therefore, for a desired TSR a value of corresponding advance coefficient $J$ can be obtained for a propeller code. When the results $K_t$ and $K_q$ are obtained from a propeller code, they can be interpolated as $C_t$ and $C_{pow}$ respectively, for turbine as a function of non-dimensional speed $J$.

3. Results and discussion

3.1. Geometry and Motion Parameters

Predictions for turbine hydrodynamic characteristics were obtained for a tidal turbine base model plus two root pitch offsets. For these models, experimental measurements are available for comparison. Table 1 shows, as an example, a list of coordinates for a foil section with 18.0000% thickness and a section profile shape of the NACA 63-8xx [23]. The turbine blade’s sectional maximum thickness varies from 24.0000% at the root section to 15.6000% at the tip section.

A model propeller P4119 by David Taylor Model Basin (DTMB) along with its results is used for comparison [21, 24]. A surface panel mesh arrangement for the propeller and the tidal turbine model [12] are shown in figures 4.

To obtain a range of tip speed ratio TSR from 4.0000 to 10.0000, corresponding input values of $J$ and required shaft revolution speed $N$ in rpm for a 0.8000-m diameter tidal turbine model were obtained. These $J$ values were calculated based on a constant inflow speed of 1.5000 m/s to simulate the actual cavitation tunnel test conditions by Bahaj et al. [22] and are listed in Table 2.

Table 3 shows the sectional effective angle of attack (AOA) at shaft speed of $n = 4.0000$ rps corresponding to a TSR of 6.7021. The values of the effective angle of attack are estimated when the induced velocity and angle of zero lift are neglected. In the work by Bahaj et al. [22], a base turbine model has a root pitch angle of 15° with an offset degree of 5° and 10° to add another two more turbine models.

Examining the sectional effective AOA for the 15°, 20° and 25° root pitch angle turbines, it can be seen that the 15° root pitch turbine is the most heavily loaded. The effective AOA at the tip section is about 8.5000°, which will most likely result in a bad cavitation (cavitation analysis is a different topic so it is omitted here though PROPELLA has the capability to do so). The best sectional effective AOA among these three turbines is the 20° root pitch one, though a slight increase of effective AOA up to 5° at the tip might give a better hydrodynamics efficiency. The 25° root pitch turbine would give the
poorest hydrodynamic efficiency because the blade sections from $r/R = 0.8000$ to the tip are all in propulsion mode (the effective AoA is negative by turbine AoA definition).

### 3.2. Turbine $C_t$ and $C_{pow}$ versus Propeller $K_t$ and $K_q$

Figures 5, 6 and 7 show the thrust and power coefficient of the horizontal axis turbine with a root pitch angle of 15°, 20° and 25°.

As mentioned earlier in this section, the effective angles of attack of the blade section of the 15° root pitch turbine are abnormally large. These large values could most likely cause flow separation and stall. The separation and stall simulation is not implemented in the current method. It is noted too that the maximum power coefficient of the 15° root pitch turbine is just as high as about 0.4000 and this value diminishes at a TSR of about 6.0000. This small production of power coefficient are most likely attributed separation and stall where potential flow based methods like the panel methods could not simulate (the higher the TSR, the worse the stall and separation and hence the larger the discrepancy between the method and measurements). This indicates that the 15° root pitch angle turbine is operational only in a narrow range of tip speed ratio with pretty poor hydrodynamic efficiency.

In Fig. 7, a better hydrodynamic performance is shown for the 20° root pitch angle turbine. The current method produced a slightly lower thrust coefficient and higher power coefficient, compared with the measurements, than that of the 15° root pitch turbine, especially operating under at large tip speed ratio close to 10.0000. It noted that the maximum power coefficient occurred at the tip speed ratio $TSR$ of 6.0000 for measurements but occurred at about a $TSR$ of 7.5000 for the prediction.

A general agreement between the measurements and the prediction by the current method can also be seen for the 25° root pitch angle turbine. As mentioned earlier in this section, blade tip sections at more than 80%R radial locations have negative effective angle of attack and hence they do negative work. This caused a much small power coefficient compared with the 20°one (0.3900 versus 0.4500).

As mentioned earlier for wake vortices descritization, the wake pitch angle has a strong influence on the prediction accuracy of the method. Taking a proper value of wake pitch will ensure not only the accuracy of predicted thrust and
power coefficients but also for the velocity profile downstream. At the moment, lack of sufficient tidal turbine measurements on thrust and power coefficients, and experimental data by LDV or PIV for the velocity profile downstream of a tidal turbine, makes it difficult for numerical method validation. Fig. 8 shows a set of thrust and power coefficient for the DTMB propeller by the current method PROPELLA.

It can be seen that prediction by PROPELLA agreed very well with the measurements. As described above, the wake pitch was taken as the value of the angle of attack at $0.7000R$ and this wake pitch seems a proper one based on the close agreement between the prediction and the measurements. Further comparison will be given for the induced velocity at a location downstream of propeller and turbine.

### 3.3. Pressure Distribution of a Propeller versus the a Tidal Turbine

Fig. 9 shows the pressure distribution on the blade surface of the DTMB P4119 propeller at an advance coefficient of $J=0.8330$. It can be seen that the values of the pressure coefficient on the back side of the blade (viewing from downstream) are mainly negative and vice versa. Therefore the back side of a propeller blade is often referred to suction side and the face side is referred to the pressure side, when operating in the propulsion mode at the first quadrant (positive rotational speed and positive forward speed). After a robust numerical Kutta condition was applied, the pressure difference between the suction side and the pressure side at the trailing edge, as can be see in the figure, is close to zero.

Fig. 10 shows color blended pressured distribution on the suction side of the DTMB P4119 propeller blade (viewing from upstream). Both the propeller and turbine were modelled as right-handed revolution propeller/rotor viewing from downstream.

Fig. 11 shows the pressure distribution on a blade of the 20°root pitch tidal turbine at $TSR = 7.0000$. It is noted that, contrary to propeller, the values of the pressure coefficients on the back side of the tidal turbine (viewing from downstream) are mainly positive and these on face side are mainly negative. The back side now becomes pressure side and vice versa. It also can be seen that the values of the suction side pressure of the tidal turbine could possibly cause sever cavitation that would deteriorate the hydrodynamic performance substantially, if the shaft immersion depth of tidal turbine is sufficiently small.
Fig. 12 shows the color blended pressure coefficient distribution on the surface of the right-handed rotation tidal turbine blades, viewing from downstream. It can be seen that high pressure occurs at the trailing edge of the blade sections and the pressure coefficient decreases with the chord location towards the leading edge.

### 3.4. Downstream Velocity Profile: Propeller versus Tidal Turbine

A comparison on velocity profile downstream of the propeller and the tidal turbine is discussed below. Fig. 13 shows the velocity profile in a plane at 0.16405 diameter behind the propeller predicted by the current panel method PROPELLA compared with the measurement by using LDV [21]. It is noted that the tangential velocity is in the same direction with the revolution direction of the propeller shaft. The axial velocity at the plane pointing downstream but radial velocity is negative, parallel to centre-line of the shaft. The negative radial velocity means that the shed wake vortices at this plane are in contraction.

Fig. 14 presents the predicted velocity profile at a plane of 0.16405 diameter downstream of the tidal turbine at $TSR = 7.0000$. Contrary to propeller, the radial velocity at the plane is positive which means that the wake vortices tend to inflate. The tangential velocity is opposite to the rotation of the turbine with negative sign except the tip. The axial velocity is negative as well indicating that the flow past the blades of the turbine has a decrease in velocity.

The opposing direction of the downstream velocities of a propeller against that of a turbine also indicates that propeller in operation transfer energy to the fluid to accelerate the inflow, while turbine extracts energy by absorbing the momentum of the fluid and ends up a slowing down the inflow.

### 3.5. Number of Blade Effect on Hydrodynamic Efficiency of Tidal Turbines

A numerical investigation was also made to examine the effect of number of blades on the hydrodynamic efficiency (power coefficient) for a turbine of 2, 3, 4 and 5 blades with the same solidity, i.e., expanded area ratio $EAR=0.0669$. Fig. 15 shows the surface solid modeling of the turbine geometry for the 4 turbines of different number of blades.

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4. Conclusion

A time-domain, low order panel method was developed for tidal turbine performance evaluation, design and optimization, based on a well established and robust propeller code PROPELLA. Familiarities and differences in hydrodynamic characteristic between propeller and turbine were discussed in terms of blade section flow velocity diagram and effective angle of attack. Detailed methodology and techniques were presented for propeller panel method applied to migrate to simulation for turbines. Predictions for a propeller and a turbine model were obtained and compared. These predictions showed a general agreement with the measurements. Blade surface pressure distributions and the downstream velocity profile of the tidal turbine model obtained are contrary to a propeller. A numerical investigation was performed for 4 turbines with the same geometry shape and same expanded area ratio with a different number of blades ranging from 2 to 5. Results indicate that with the increase of the number of blades, the power coefficients decrease dramatically and proportionally. It was also noted that the optimum \( TSU \) at which the highest power coefficient occurs shifts left (reduces) with the increase of the number of blades, from \( TSU = 8.0000 \) to \( 6.5000 \). While the propeller panel method was developed with a huge amount of measurement data to validate it, there is a need for substantial experimental data on wake vortices and downstream velocity profile by LDV and/or SPIV.

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