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Effective Theory of Electron-Hole Exchange in Semiconductor Quantum Dots

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Abstract. Semiconductor self-assembled quantum dots (QDs) are expected to become building blocks of novel optoelectronic devices capable of generating single photons and entangled photon pairs (EPPs) on demand. The major barrier to generation of EPPs is the so-called “bright exciton splitting” originating from the long-range electron-hole exchange interaction. A \( k \cdot p \) effective mass theory of electron-hole exchange in semiconductor quantum dots is presented (\textit{Phys. Rev. B} \textbf{81}, 045311). The matrix element responsible for the “bright” exciton splitting in the effective exciton Hamiltonian is identified and analyzed. An excitonic fine structure for a model quantum dot with quasi-two-dimensional anisotropic harmonic oscillator (2DLAHO) confining potential is considered as a function of the shape and external fields.

1. Introduction

One of the promising applications of semiconductor quantum dots (QDs) is the generation of entangled photon pairs (EPPs) on demand [1, 2, 3, 4]. Polarization entangled photon pairs can be efficiently generated using the biexciton cascade process (BCP) [5] in which a biexciton radiatively decays into the ground state via two different indistinguishable paths involving two intermediate dipole-active (“bright”) exciton states. The major barrier to EPPs generation in BCP is the splitting of the intermediate “bright” exciton levels which distinguishes the paths of radiative decay and, as a result, destroys the entanglement. The splitting of the two bright exciton states is controlled by the long range electron-hole exchange (LRE) interaction and depends on the dot’s shape, size, and chemical composition. External magnetic [6, 2, 4] and electric fields have been applied in the hope to manipulate the exciton fine structure [7, 8, 9].

Investigation of the electron-hole exchange interaction in QDs, particularly its LRE part, should help in the development of the EPP generation schemes. The LRE electron-hole interaction in bulk semiconductors was studied almost 40 years ago [10, 11, 12]. Recent advances in single QD spectroscopies motivated re-examination of the electron-hole exchange in semiconductor systems with spatial confinement of charge carriers within the framework of envelope function approximation [13, 14, 15, 16, 17, 18, 19] and, more recently, from the point of view of atomistic empirical pseudopotential and tight-binding approximation [20, 21, 22].

Within the envelope function approximation, Takagahara [13] have shown that if the electron-hole pair envelope contains only \( Y_{00}(\theta, \phi) \) angular momentum component, then the long-range part of the electron-hole exchange vanishes. In Ref. [17], Takagahara derived effective eight-band excitonic Hamiltonian which takes into account electron-hole exchange interaction. Within the
envelope function formalism of Refs. [13, 17], the LRE is a dipole-dipole interaction. It was demonstrated numerically in Ref. [17] that LRE vanishes for the ground state “bright” exciton doublet in QDs with rotationally symmetric 3D confining potential. A similar to that of Ref. [17] representation of the electron-hole exchange was discussed by Maialle et. al. [14] in connection with the exciton spin dynamics in quantum wells.

Within the empirical atomistic approach it has been demonstrated that the physical origin of the long-range exchange interaction in QDs might be different from that of bulk [20] due to the loss of local orthogonality on the unit cell scale between the electron and hole quasi-particle orbitals. It also has been shown that the “bright” exciton splitting can have either nonzero (dots of C2v point group) or zero (dots of D2h point group) magnitude in the case of “shape-symmetric” dots [22] and the importance of the dot’s true point group symmetry underlined by both the macroscopic shape and the zinc-blende crystal structure has been realized. Recently, Dalessi and Dupertuis extended [23] the effective \( k \cdot p \) approach to account for the true point group symmetry of the semiconductor nanostructures which might enable one to qualitatively predict excitonic fine structure based on the envelope theories.

In this work, we will re-examine electron-hole exchange within the framework of the envelope approximation. We derive an effective “four band” excitonic Hamiltonian which includes effects of the LRE interaction. The elements of the effective Hamiltonian are expressed explicitly in terms of the envelope functions. The “microscopic parts” of single-particle orbitals are integrated out and enter the effective Hamiltonian as numerical parameters. The number of bands is truncated to two conduction and two valence bands for the simplicity of the interpretation. We present explicit expression for the matrix element responsible for the “bright” exciton splitting and, therefore, establish a new sufficient condition for the LRE quenching.

Effective units of length and energy are used throughout unless otherwise specified. The lengths are measured in the effective Bohrs, \( \epsilon \) is dielectric constant and \( m^* \) is the conduction band effective mass. The energies are measured in effective Hartrees, \( 1 \ H = m^* \epsilon^4 / (e\epsilon)^2 = 2 \text{ Ry}^2 \). For example, using material parameters for GaAs, \( a_{eB} = 97.9 \text{ Å} \) and \( 1 \ H = 11.86 \text{ meV} \).

2. Theory of exciton fine structure in envelope function approximation

The excitonic states are obtained by diagonalizing the many-body Hamiltonian. Let \( c^\dagger (c) \) denote electron creation (annihilation) operator, respectively. The many-body Hamiltonian in a semiconductor is

\[
\hat{H} = \hat{H}_c + \hat{H}_h + \hat{H}_{\text{int}}, \quad \hat{H}_c = \sum_i \epsilon_i c_i^\dagger c_i, \quad \hat{H}_h = \sum_\alpha \epsilon_\alpha c_\alpha^\dagger c_\alpha,
\]

\[
\hat{H}_{\text{int}} = \frac{1}{2} \sum_{ijkl} c_i^\dagger c_j^\dagger c_k c_l \langle ij | V | kl \rangle + \frac{1}{2} \sum_{\alpha\beta\gamma\delta} c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta \langle \alpha\beta \gamma\delta | V | \alpha\beta \gamma\delta \rangle + \sum_{ij\gamma\delta} c_i^\dagger c_j^\dagger c_\gamma c_\delta \langle \langle i\beta \gamma\delta V \rangle - \langle i\beta V \rangle | \gamma\delta \rangle,
\]

\[
V_{ABCD} = \langle AB | V | CD \rangle = \iint \frac{\phi_A^\dagger(x_1)\phi_B^\dagger(x_2)\phi_C(x_2)\phi_D(x_1)}{\epsilon(r_1, r_2)|r_1 - r_2|} d\mathbf{x}_1 d\mathbf{x}_2
\]

(1)

The Hamiltonian (1) is obtained from the full many-body Hamiltonian by assuming that the quasi-particle orbitals belong to two distinct manifolds (conduction/valence band) and retaining the interactions within the conduction and valence bands (the first and the second terms in \( \hat{H}_{\text{int}} \)) as well as the interband direct \( \langle i\beta V | \gamma\delta \rangle \) and exchange \( \langle i\beta V | \gamma\delta \rangle \) terms. The indices \( \{i, j, k, l\} \) and \( \{\alpha, \beta, \gamma, \delta\} \) run over the valence and conduction band states, and \( \epsilon_\alpha (\epsilon_\alpha) \) denote the quasi-particle energies of conduction (valence) states. \( V_{ABCD} \) is a “screened” Coulomb-like matrix element involving “two-dimensional” integration over the combined space-spin variable \( \mathbf{x} = \{r, s\} \).

To obtain the excited states, the Hamiltonian (1) is diagonalized in the basis of all electron-hole pairs \( |X\rangle = c_{\sigma\tau}^\dagger c_{\sigma\tau} |g.s.\rangle \), \( |X\rangle = c_{\sigma\tau}^\dagger c_{\sigma\tau} |g.s.\rangle \), where \( |g.s.\rangle \) is a single Slater determinant.
with fully occupied valence and empty conduction bands \((c_1^v | g.s. = c_{ca|} (g.s.) = 0)\) and \(v\tau (\nu \tau')\) and \(\sigma\) (\(\sigma'\)) is a composite index describing valence and conduction band states, respectively. The matrix element of (1) is

\[
\langle X | H | X' \rangle = \langle g.s. | c_{v\tau} c_{ca\tau'} c_{v'\tau'} (g.s.) = \delta_{ca\sigma\sigma'} \delta_{v\tau \nu \tau'} (\varepsilon_{\sigma\tau} - \varepsilon_{\nu \tau} + E_{g.s.})
\]

\[
- \langle \sigma\nu\tau' | V | \nu\sigma'\tau' \rangle + \langle \sigma\nu\tau' | V | \sigma'\nu\tau' \rangle - \Sigma(\nu\tau', \nu\tau') \delta_{\sigma\nu\sigma'} + \Sigma(\sigma\nu, \sigma'\nu) \delta_{\sigma\nu\sigma'\nu'},
\]

(2)

where \(E_{g.s.} = \langle g.s. | H | g.s. \rangle\) is the Hartree-Fock energy of the ground state and \(\Sigma(A, B) = \alpha \langle \alpha | V | \alpha B \rangle - \langle \alpha | V | \alpha A \rangle\) is the so-called self-energy.

In this work, the quasi-particle states are computed in the effective mass approximation (EMA). Within the EMA, each quasi-particle state is uniquely specified by the band label of the periodic part \(u(r)\) of the Bloch eigenstate at \(k = 0 (v_jz, v_j', \sigma, \sigma')\) and by the index of the envelope function \(F(r) (q, r, s, p)\), for example, \(\phi_{v_jz}(r) = F^q(r) u_{v_j}(r)\). By “expanding” the composite indices as \(\nu\tau \rightarrow \{v_jz, q\}\), \(\nu\tau' \rightarrow \{v_j'z, r\}\), \(\sigma \rightarrow \{\sigma, s\}\), \(\sigma' \rightarrow \{\sigma', p\}\), neglecting the contribution from the self-energies, and “measuring” the excitations with respect to the ground state \((E_0 = 0)\), we arrive at the following expression for the matrix element (2)

\[
\langle g.s. | c_{v_jz} c_{ca\sigma} H c_{ca'\sigma'} c_{v_j'z} (g.s.) = \delta_{v_jz, v_j'z} \delta_{ca\sigma, ca'\sigma'} (\varepsilon_{ca\sigma} - \varepsilon_{v_jz, q}) - \langle ca\sigma, v_j'z | V | v_jz, q, \sigma' p \rangle + \langle ca\sigma, v_j'z | V | ca' \sigma' p, v_jz, q \rangle
\]

(3)

The effective Hamiltonian which involves only envelope functions is obtained by integrating out “microscopic" degrees of freedom (periodic parts of Bloch functions). The derivation follows that of Ref. [17]. First, we postulate the form of the “microscopic" functions. The conduction (valence) band “microscopic" functions are of \(s (p)\) symmetry with spin (hole angular momentum) projections \(\pm 1/2 (\pm 3/2)\)

\[
u_{1/2}(r) = |r| \chi_s(r) Y_0(\hat{r}) |\alpha \rangle, \quad \nu_{-1/2}(r) = |r| \chi_s(r) Y_0(\hat{r}) |\beta \rangle,
\]

\[
u_{3/2}(r) = |r| \chi_p(r) Y_1(\hat{r}) |\alpha \rangle, \quad \nu_{-3/2}(r) = |r| \chi_p(r) Y_1(\hat{r}) |\beta \rangle.
\]

(4)

The Coulomb attraction matrix element is approximated as

\[
\langle ca\sigma, v_j'z | V | ca' \sigma' p, v_jz, q \rangle = \frac{1}{\epsilon} \delta_{ca\sigma\sigma'} \delta_{v_jz, v_j'} \int d\mathbf{r}_1 d\mathbf{r}_2 \frac{F_{ca\sigma}^\dagger (\mathbf{r}_1) F_{ca'\sigma'}^\dagger (\mathbf{r}_1) F_{v_j'z}^\dagger (\mathbf{r}_2) F_{v_jz} (\mathbf{r}_2)}{|\mathbf{r}_1 - \mathbf{r}_2|},
\]

(5)

where \(\epsilon\) is a static dielectric function.

The exchange matrix element can be thought of as a Coulomb-like interaction of two “transition densities", where each “transition density" is a product of the electron and hole quasi-particle orbitals. First, we decompose the exchange integral into the short-range \(V_{ESR}^E\) (exchange within Wigner-Seitz cell) and long-range (exchange between the two Wigner-Seitz cells) \(V_{LR}^E\) contributions. The “effective" expression for the exchange matrix elements in terms of the envelope functions is obtained in a way which follows “standard” \(k \cdot p\) derivation, the only difference is the nature of the microscopic integrals to be parameterized. One postulates the “slowly varying nature" of the envelope functions and factors them out of the integral expression.

As a result of this procedure, we obtain the following “master" expression for the exchange matrix element

\[
\langle ca\sigma, v_j'z | V | ca' \sigma' p, v_jz, q \rangle = V_{ESR}^E(c_{a\sigma, v_j'z; ca' \sigma' p, v_jz, q}) + V_{LR}^E(c_{a\sigma, v_j'z; ca' \sigma' p, v_jz, q}),
\]

\[
V_{ESR}^E(c_{a\sigma, v_j'z; ca' \sigma' p, v_jz, q}) = E_{SR}(H_{SR}^{\text{int}}(c_{a\sigma, v_j'z; ca' \sigma' p, v_jz, q}) \int d\mathbf{r} \frac{F_{ca\sigma}^\dagger (\mathbf{r}) F_{ca'\sigma'}^\dagger (\mathbf{r}) F_{v_j'z}^\dagger (\mathbf{r}) F_{v_jz} (\mathbf{r})}{|\mathbf{r}_1 - \mathbf{r}_2|},
\]

\[
V_{LR}^E(c_{a\sigma, v_j'z; ca' \sigma' p, v_jz, q}) = -\frac{4\pi}{3} \mu^2 \left( \left[ \mathbf{d}_{ca\sigma v_jz}^0 \right] \cdot \mathbf{d}_{ca'\sigma' v_jz}^0 \right) \int d\mathbf{r} \frac{F_{ca\sigma}^\dagger (\mathbf{r}) F_{ca'\sigma'}^\dagger (\mathbf{r}) F_{v_j'z}^\dagger (\mathbf{r})}{|\mathbf{r}_1 - \mathbf{r}_2|} d\mathbf{r}_1 d\mathbf{r}_2
\]

(6)
where $E_{SR}$ and $\mu^2$ are two numerical constants parameterizing short- and long-range interaction, respectively, \(H_{\text{SR}}^{\text{int}(\sigma,\nu_j)}\) is an element of short-range exchange “microscopic” matrix

\[
H_{\text{SR}}^{\text{int}} = \begin{pmatrix}
|\sigma, \nu_j\rangle & |\frac{1}{2}, -\frac{3}{2}\rangle & |\sigma, \nu_j\rangle & |\frac{1}{2}, \frac{1}{2}\rangle \\
|\sigma, \nu_j\rangle & |\frac{1}{2}, \frac{3}{2}\rangle & |\sigma, \nu_j\rangle & |\frac{1}{2}, -\frac{1}{2}\rangle \\
|\sigma, \nu_j\rangle & |\frac{1}{2}, -\frac{3}{2}\rangle & 0 & 0 \\
|\sigma, \nu_j\rangle & |\frac{1}{2}, \frac{3}{2}\rangle & 0 & 0 \\
\end{pmatrix},
\]

(7)

and \(d_{\sigma\nu_j}^{0}\) is “microscopic” dipole

\[
\begin{pmatrix}
|\sigma, \nu_j\rangle \times (d_{\sigma\nu_j}^{0})_x \\
|\sigma, \nu_j\rangle \times (d_{\sigma\nu_j}^{0})_y \\
|\sigma, \nu_j\rangle \times (d_{\sigma\nu_j}^{0})_z \\
\end{pmatrix} =
\begin{pmatrix}
|\frac{1}{2}, -\frac{3}{2}\rangle \times -1 & -i & 0 \\
|\frac{1}{2}, \frac{3}{2}\rangle \times 0 & 0 & 0 \\
|\frac{1}{2}, -\frac{3}{2}\rangle \times 1 & -i & 0 \\
\end{pmatrix},
\]

(8)

The numerical parameters $E_{SR}$ and $\mu^2$ can be determined as to reproduce the excitonic fine structure in bulk semiconductors and, therefore, implicitly contain the screening effects.

Taking into account (7) and (8), a “block” Hamiltonian corresponding to the exchange interaction between two electron-hole pairs $F_p^pF_q^q$ and $F_{c}^pF_{v}^q$ can be presented in the form

\[
\begin{pmatrix}
|\sigma, \nu_j\rangle & |\frac{1}{2}, \frac{3}{2}\rangle & |\sigma, \nu_j\rangle & |\frac{1}{2}, -\frac{1}{2}\rangle \\
|\sigma, \nu_j\rangle & |\frac{1}{2}, \frac{3}{2}\rangle & 0 & 0 \\
|\sigma, \nu_j\rangle & |\frac{1}{2}, -\frac{3}{2}\rangle & 0 & 0 \\
|\sigma, \nu_j\rangle & |\frac{1}{2}, \frac{3}{2}\rangle & 0 & 0 \\
\end{pmatrix} =
\begin{pmatrix}
|\sigma, \nu_j\rangle & |\frac{1}{2}, \frac{3}{2}\rangle & |\sigma, \nu_j\rangle & |\frac{1}{2}, -\frac{1}{2}\rangle \\
|\sigma, \nu_j\rangle & |\frac{1}{2}, \frac{3}{2}\rangle & 0 & 0 \\
|\sigma, \nu_j\rangle & |\frac{1}{2}, -\frac{3}{2}\rangle & 0 & 0 \\
|\sigma, \nu_j\rangle & |\frac{1}{2}, \frac{3}{2}\rangle & 0 & 0 \\
\end{pmatrix},
\]

(9)

where we separated different contributions to the exchange based on their origin (SRE or LRE) and integral type (“local” and “nonlocal”). The contributions are

\[
\begin{align*}
\delta_{0}^{\text{SRE,L}} &= E_{SR}S, \\
\delta_{0}^{\text{LRE,L}} &= -\frac{8\pi\mu^2}{3}S, \\
\delta_{12}^{\text{LRE,N}} &= \mu^2(R_{xx} - 2iR_{xy} - R_{yy}), \\
\delta_{21}^{\text{LRE,N}} &= -\mu^2(R_{xx} + 2iR_{xy} - R_{yy}), \\
R_{\delta_7} &= \int \int \left\{ \left( \frac{\partial^2}{\partial r_1^2 \partial r_2^2} \right) F_{c}^p F_{v}^q \right\} \frac{F_p^{p\dagger}(r_2)F_q^{q\dagger}(r_2)}{|r_1 - r_2|} dr_1 dr_2.
\end{align*}
\]

(10)

With regard to the above exchange expressions we note the following. (a) The short-range exchange causes splitting between the “bright” \((|\frac{1}{2}, -\frac{3}{2}\rangle, |\frac{1}{2}, \frac{3}{2}\rangle\)) and “dark” doublets \((|\frac{1}{2}, \frac{3}{2}\rangle, |\frac{1}{2}, -\frac{3}{2}\rangle\)) by moving “bright” doublet up in energy by $\delta_{0}^{\text{SRE,L}}$. SRE does not split the “bright” doublet. The integral describing SRE is “local” in nature and involves overlap between two electron-hole pairs. (b) Long-range exchange contains expressions of two types: a “local” term $\delta_{0}^{\text{LRE,L}}$ which arises from $\mu^2 \left( (d_{\sigma\nu_j}^{\sigma\nu_j})^\dagger \cdot d_{\sigma\nu_j}^{\sigma\nu_j} \right)$ and “nonlocal” terms $\delta_{12}^{\text{LRE,N}}$, $\delta_{21}^{\text{LRE,N}}$, and $\delta_{21}^{\text{LRE,N}}$, which involve differentiation operators applied to the electron-hole pair envelopes. The “nonlocal” terms describe dipole-dipole interaction between the electron-hole transition densities. (c) The splitting between “bright” and “dark” doublets is affected by LRE through
Figure 1. Exciton fine structure as a function of anisotropy of the confining potential.

Within 2DLAHO model, the Hamiltonian for holes (electrons) is

\[
\hat{H}_{h(e)} = -\frac{1}{2M_{||,h(e)}} \left( \nabla_x^2 + \nabla_y^2 \right) + \frac{1}{2} M_{||,h(e)} \left( \omega_{x,h(e)}^2 x^2 + \omega_{y,h(e)}^2 y^2 \right) - \frac{1}{2} M_{\perp,h(e)} \nabla_z^2 + \frac{1}{2} M_{\perp,h(e)} \omega_{z,h(e)}^2 z^2,
\]

\[
\omega_{x,h(e)} = \omega_{h(e)}^0 (1 + t), \quad \omega_{y,h(e)} = \frac{\omega_{h(e)}^0}{1 + t}, \quad \omega_{z,h(e)} = \omega_{h(e)}^0 (1 + t) \quad \text{const.}, \quad \omega_{z,h(e)} \gg \omega_{h(e)}^0,
\]

where \(M_{||,h(e)}\) and \(M_{\perp,h(e)}\) are effective masses, \(\omega_\gamma (\gamma = x, y, z)\) denotes confinement frequency in \(x, y, \text{ and } z\) directions, respectively, \(\omega_{h(e)}^0\) determines the typical energy scale of the confining potential. The energies and lengths are expressed in the effective units.

3. Exciton fine structure of quasi-two-dimensional anisotropic harmonic oscillator (2DLAHO) quantum dot

In this section, we will investigate excitonic fine-structure for a model quantum dot in which the confining potential is of 2D-like anisotropic harmonic oscillator type. We note that HO oscillator spectrum has been observed in self-assembled quantum dots [24].
The excitonic fine structure is studied as a function of anisotropy $t$. For $t < 0$, $\omega_x < \omega_0$ and $\omega_y > \omega_0$ and the confinement along $x$ is weaker than along $y$. For $t > 0$, $\omega_x > \omega_0$ and $\omega_y < \omega_0$. In the case $t = 0$, the confining potential for holes and electrons is rotationally symmetric.

In Ref. [25], an explicit expression for matrix elements $\delta_{12}^{LRE,N}$ and $\delta_{21}^{LRE,N}$ was derived in the case of $s_e s_h$ envelop and it was demonstrated that $\delta_{12}^{LRE,N}$ and $\delta_{21}^{LRE,N}$ vanish provided that the “confinement” of electron-hole pair envelope is identical in the $x$ and $y$ directions ($\alpha_x = \alpha_y$). Figure 1a shows “bright” exciton doublet splitting as a function of lateral anisotropy $t$ as computed from the effective theory for four different combinations of the confinement frequencies. The splitting energy is defined as $E^y_X - E^x_X$, where $E^x_X$ and $E^y_X$ are ground state “bright” exciton energy levels polarized along the $x$ and $y$ directions, respectively. The “bright” exciton splitting decreases as the magnitude of the anisotropy $|t|$ decreases and vanishes at $|t| = 0$. The passing through $t = 0$ is accompanied by the change of polarization of the “bright” ground state. For $t < 0$ ($\omega_x < \omega_y$), $E^y_X > E^x_X$ and the “bright” ground state is dipole active along the $x$-axis, whereas for $t > 0$ ($\omega_x < \omega_y$) the “bright” ground state is dipole active along the $y$-axis. In our model, the “bright” ground state is always dipole-active along the axis of weaker confinement. Figure 1b shows the splitting between the “bright” ground state and the “dark” doublet.
The Figure 2 shows $R_{xx}$ (a) and $R_{yy}$ (b) “nonlocal” contributions to the LRE exchange as a function of anisotropy for several confinement frequencies. We can see that $R_{xx}$ increases and $R_{yy}$ decreases in magnitude as $t$ goes from -0.5 to 0.5. Due to this trend, the difference $R_{xx} - R_{yy}$ which determines the “bright” exciton splitting decreases as a function of $|t|$. At $t = 0$, $R_{yy} = R_{xx}$ and the exchange matrix element which couple two “bright” exciton levels vanishes. The “polarization” of the “bright” ground state is determined by the sign of $R_{xx} - R_{yy}$.

Figure 2c shows the overlap $S$ between $s_e s_h$ pairs which, within the envelop approximation, determines the SRE ($\delta_{0}^{SRE,L}$) as well as the “contact” contribution from the LRE ($\delta_{0}^{LRE,L}$). The electron-hole pair overlap $S$ is insensitive to the anisotropy $t$ due to the imposed “area conservation” condition $\omega_{x,h(e)}\omega_{y,h(e)} = (\omega_{h(e)}^{0})^{2} = \text{const}$. Figure 2d shows $R_{xx} + R_{yy}$ which determines the “nonlocal” LRE contribution to the “bright”-“dark” splitting. We find that $\delta_{0}^{SRE,L} > 0$ and its effect is to move the “bright” doublet up in energy. The “local” ($\delta_{0}^{LRE,L}$) and “nonlocal” LRE ($\delta_{0}^{LRE,N}$) act against each other.

4. Application of external fields

In the previous section, we have demonstrated that the magnitude of the “bright” exciton splitting can be controlled through the shape of the confining potential. It is, therefore, of great interest to examine the effects of external fields on the excitonic fine structure. For example, QDs can be placed between Schottky gates for the application of vertical and lateral electric fields. Recently, it has been demonstrated [9] that by applying an in-plane electric field it is possible to fine-tune photon cascades originating from the recombination of multie exciton complexes in QDs.

Within our model, the application of lateral electric field $\vec{F} = F_{e_x} + F_{e_y}$ of magnitude $|F|$ displaces the “origin” of electron and hole envelopes in $xy$-plane from $(0, 0)$ to $(x_{0}^{h}, y_{0}^{h})$ and $(x_{0}^{e}, y_{0}^{e})$, respectively. The single-particle energy levels are rigidly shifted by the Stark shift whereas the spacing between the single-particle energy levels of HO is not affected.

Evaluating the integrals which control “bright” exciton splitting $R_{xx}(F), R_{yy}(F), R_{xy}(F)$ as a function of field $F$ for $s_e s_h$ electron-hole envelope, we obtain

$$R_{xx}(F) = R_{xx}(0) \exp\left(\frac{-2\alpha_x\alpha_{xx}}{\alpha_x} \Delta x_0^2 - \frac{2\alpha_y\alpha_{yy}}{\alpha_y} \Delta y_0^2\right),$$
\[ R_{yy}(F) = R_{yy}(0) \exp \left( -\frac{2\alpha_{xc}\alpha_{xx}}{\alpha_x} \Delta x_0^2 - \frac{2\alpha_{gc}\alpha_{gy}}{\alpha_y} \Delta y_0^2 \right), \quad R_{xy}(F) = 0, \quad (12) \]

where \( R_{xx}(0) \) and \( R_{yy}(0) \) are matrix elements in the absence of the electric field \((F = 0)\). Field dependence of \( R_{xx}(F) - R_{yy}(F) \) which determines the “bright” exciton splitting is shown on Figure 3 for four different values of the lateral anisotropy.

At \( F = 0 \), the “bright” exciton splitting attains maximum value which is determined by the initial shape anisotropy of the confining potential. The larger the magnitude of the anisotropy, the larger is the initial \((F = 0)\) “bright” exciton splitting. As \( F \) increases in magnitude, the electron and hole envelopes are pulled out in the opposite direction and the magnitude of the splitting is reduced. Since the separation between the envelopes depends on \( F^2 \), the splitting is independent of the sign of the field \( F \).

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